Role of Mathematical Modelling and Applications in University Mathematics Service Courses: An Across Countries Study

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Abstract

The aim of this study was to find out what university students, studying mathematics as a service course, think about the role of the mathematical modelling process and application problems in their studies. For this purpose a questionnaire was given to more than 500 students from 14 universities in 9 countries. The research was not a comparison of countries or universities: an across countries study approach was chosen to reduce the effect of differences in education systems, curricula, cultures. The results of the questionnaire were analysed and presented in the paper. In particular, an attempt was made to identify which step of the mathematical modelling process the students found most difficult.

1. INTRODUCTION

> I suppose that model building might be designated the third of the built-in drives of the human organism - eating and reproduction being perhaps slightly more pressing.
>
> George Stibitz / *Mathematics in Medicine and the Life Sciences.*

> Model building is an activity which students often find difficult and sometimes rather puzzling. The process of model building requires skills other than simply knowing the appropriate mathematics.
>
> Donald A.R.George / *Mathematical Modelling for Economists.*

Mathematical modelling skills are important for students not majoring in mathematics. Many academics and practitioners consider these skills the most useful ones that should be developed while studying mathematical courses (Leijonhufvud, 1981; George, 1988). Most of these students will not develop advanced mathematical techniques and algorithms to solve practical problems in their work places after graduation. Thus, the skills in modelling of real world problems - as model builders and model users - are important for them. They may also apply those skills (which include, among other things, selecting essential information, differentiating the important conditions from the not important, making appropriate assumptions and simplifications, and so on) to many real life situations that have nothing to do with mathematics. However not many universities have special courses in mathematical *modelling* for students not majoring in mathematics.

Students often study modelling as a natural part, but usually not a large part, of general mathematics courses and do some application problems.
Most examination questions from first and second year mathematical courses consist of finding limits, derivatives, and integrals of *given* functions; and solving *given* equations – that is the examinations check the students' mathematical techniques. This leads to the emphasis on manipulation. Students cannot see the wood for the trees. Often these students are not happy because 'for users, the motivation comes from an interest in their particular subject matter. Even compulsory courses in statistics will be viewed with more enthusiasm if the student feels it is relevant to his or her career' (Weldon, 1996). We would assume that this applies to mathematics in the same way. A recent International Survey of Undergraduate Mathematics Courses (Engelbrecht, 1998) shows that 'the majority of our students do not care about abstract mathematical theories, about rigorous mathematical proofs for every statement, about mathematics that cannot be applied directly in the field that they really came to study at university. They want relevant, applicable subject content which they can apply now, not some day, not perhaps.' We believe that the earlier students start to develop mathematical modelling skills the better. It is hard to not agree with the statement that 'the development of sophisticated particular techniques goes hand in hand with the development of mathematical thinking in general, and modelling in particular' (Mason & Davis, 1991). Even further - 'mathematical modelling is a way of life' (Houston, 1997).

2. ASSUMPTIONS AND FRAMEWORKS

Our personal assumptions are that mathematics courses and *assessment* for non-mathematics major students should be context based because this encourages creativity, forces interest, helps motivate the students. For these students we believe that mathematics should not be seen as a stand-alone discipline but as connected to their work and their ways of thinking about it. We agree that 'whilst a theoretical framework may provide structure and a coherence to a research project, it may also restrict the project in a number of ways. A theoretical standpoint may engender the legitimising of the academic discourse over the practical discourses. A well-versed criticism of educational research is that the agendas and issues have been hijacked by academics who, in their pursuit of scholarly excellence, are at risk of reporting and disseminating findings within very strong and theoretical discourses. The upshot of such actions is that there is potentially little impact on the very participants that the project was aimed at informing' (Zevenbergen & Begg, 1999). In this study we select practice as the basis for the research framework and decide 'to follow conventional wisdom as understood by the people who are stakeholders in the practice' (Zevenbergen & Begg, 1999).

3. THE ORIGIN OF THE RESEARCH QUESTIONS

Some years ago one of the authors set up *all* tests and exams questions for his first year economics students in the relevant context. The purpose was to emphasise the relationship between mathematics and their major subjects, to enhance students' motivation, 'to illustrate the creativity involved in model construction and to develop in students the confidence to formulate their own models rather than have to rely on existing models in text-books' (Neal & Shone, 1976). Instead of pure mathematical questions which checked the mathematical techniques the students were given the application problems. Those problems were partly mathematised. Here are two examples.

1. Instead of the question 'Calculate the definite integral $\int_0^4 (\sqrt{49-6x}-5)dx$' there was the question:

'Find the equilibrium price and the consumer's surplus if the supply function is p(x) = x+1, and the demand function is p(x) = $\sqrt{49-6x}$ '.The students could draw the diagram and revise some other mathematical and economics concepts:

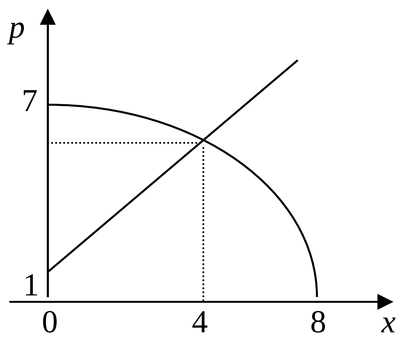

Figure 1.

2. Instead of the question

'Find the particular solution of the differential equation y'=0.001y(1000-y) with the initial condition y(0)=200 at the point t=2' the students received the following question:

'To promote its product a company has made an advertisement on radio and TV. The estimated number of all potential buyers of the product is 1000. From previous statistical studies it is known that after the advertisement the rate at which the number of people knowing about the product increases is proportional with the coefficient 0.001 to the number of people already knowing about the product, and the number of people who don't know about the product among all potential buyers. Find the number of people who will know about the product 2 months after the advertisement has been seen or heard, if initially 200 people knew about the advertisement '.

Another reason for emphasising mathematical modelling and application problems in tests and exams is a growing usage of computers and powerful graphics and symbolic calculators such as TI-89 or TI-92. ‘We can now concentrate on the beautiful ideas and concepts in mathematics without hesitation because the bulk of the manipulation can be left to the computer’ (Engelbrecht J, 1998).

4. THE STUDY

The main objectives of this study were to find what students think about the role of mathematical modelling in their studies, and identify the step they find most difficult in the mathematical modelling process. For this purpose a questionnaire was given to them. The questions from the questionnaire did not require solving any problems. Knowledge of senior school mathematics was enough to understand the only application problem from the questionnaire. We would not have expected broad and full understanding of the term

‘mathematical modelling’ from the first-year students. At this stage we are using the term ‘mathematical modelling’ to mean solving application problems which is a part of modelling in its broader sense.
A sample of 502 first and second-year students from 14 universities in 9 countries was selected. These students were all studying mathematics as a supplementary (non-major) subject. The countries were: United Kingdom, New Zealand, Australia, Russia, Ukraine, Finland, France, South Africa, Spain.

The Questionnaire

The questionnaire comprised the following six questions:
Question 1. Which of the following steps of the mathematical modelling process of solving an application problem (real world problem, practical problem, word problem, real life problem) is the hardest for you:

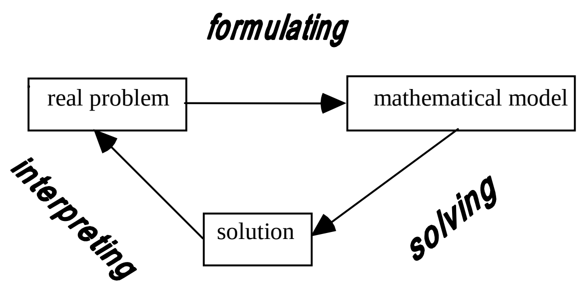

Figure 2.

a) formulating mathematical model
b) solving mathematical model
c) interpreting solution of mathematical model
Why?

Questions 2-6 are about the following two problems. You don't need to solve them. Just read.

Problem 1. A manufacturer can sell x tonnes of his product at a price of $(600 - 5x) per tonne. The fixed cost of production which doesn't depend on output is $10500, and the cost of production of one tonne of the product is $100. Determine the maximum profit and corresponding level of output for the manufacturer.

Problem 2. Find the maximum point of the function $y = -5x^2 + 500x - 10500$ where x is from R^+.

Question 2. Which of the above two problems is more interesting for you to deal with?
a) problem 1 b) problem 2
Why?
Question 3. Which of them looks more difficult?
a) problem 1 b) problem 2
Why?
Question 4. Which of them develops more useful abilities for you in terms of your future studies or work?
a) problem 1 b) problem 2

What are these abilities?

Question 5. How much time approximately do you need to show that the problem 2 is the mathematical model of the problem 1?

a) less than 1 minute b) between 1 and 5 minutes c) more than 5 minutes

Question 6. Should **all** of the tests and exam questions be formulated in the form of the above problem 1 rather than problem 2?

a) yes b) no

Why?

5. RESULTS AND DISCUSSION

The main findings from the questionnaire are presented in the following table:

Number of students	Q1-steps of modelling			Q2-interest problem		Q3-difficult problem		Q4-useful problem		Q5-time in min			Q6-exam problem	
	a)	b)	c)	1/	2/	1/	2/	1/	2/	<1	1-5	>5	1/	2/
502	376	37	89	373	129	437	65	475	27	140	257	105	222	280
100%	75	7	18	74	26	87	13	95	5	28	51	21	44	56

Table 1. Summary of findings from the questionnaire

- The hardest step in solving application problems for the majority of the students in all countries is the first step - formulation of the mathematical model (75%). The main reason for that, according to the students' answers, is lack of practice in such an activity.
- The application version of the problem for most students is perceived to be more difficult (87%) than the pure maths version. Many students reported that the application version had more words and the additional step of moving from the word problem to the mathematics language (equations, constraints, graphs and so on) required more thinking and understanding. Below are typical comments:
  - 'So many words and the hardest part for me is putting the words into mathematical terms'.
  - 'More words make me confused'.
  - 'Because I have problems deciding what equation should be used'.
  - 'Have to put information together to form an equation which can be very hard'.
  - 'Would take a lot more thinking'.
  - 'There is more information which has to be analysed and used'.

The above two points perfectly match with the survey of non-academic mathematicians that has been done by the Society for Industrial and Applied Mathematics (SIAM). Here are some extracts from its summary: 'The hardest task for a mathematician is developing the real problem requirements... Problem never come in formulated as mathematics problem...The most important traits in non-academic mathematicians include skills in formulating, modelling...' (The Society for Industrial and Applied Mathematics (SIAM). Report on Mathematics in Industry, 1995).

- The application version of the problem for most students is perceived to be more useful in terms of their future studies and work (95%). This is expected and understandable, taking into consideration the fact that the students were majoring in subjects other than mathematics, with the vast majority of them in business or economics.
- More than half of the students (56%) opposed having tests exclusively made of applied questions in contrast to the trends pointing to the interest (74%) and usefulness (95%) of applications. The main reason for this, according to the students' comments, was – 'it is harder to pass'. Those students were more concerned about their test results rather than acquiring useful skills. There was a wide variation among universities in answers to question 6. In some universities about 90% of students reported that they would prefer the application version of the problem in a test in spite of the fact that it became more difficult.

We are very aware of the limitations of the study. It was not an international comparison. It was intended more as a pilot study to check our assumptions and share the findings with university lecturers and the mathematics education community.

6. CONCLUSION AND RECOMMENDATIONS

For those courses we would assume that there is a need to:

- teach students how to formulate mathematical models of application problems explicitly;
- introduce powerful symbol and graphics calculators (at least for demonstration purposes) as a means to reduce emphasis on mathematical manipulations and techniques and consequently to give more time for learning mathematical concepts and modelling;
- eliminate mathematical techniques questions, that can be done by symbol and graphics calculators, from tests and exams;
- set up problem oriented curricula and assessment. That is, to implement the fundamental statement of the Mathematical Modelling Topic Group from the 7th International Congress on Mathematics Education on 'integration of mathematical modelling as a continuing expectation in the everyday mathematics curriculum of students' (Gaulin C, Hodgson BR, Wheeler DH, Egsgard JC, 1994).

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