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**A STUDY ABOUT THE DESIGN OF A SIMPLE  
KINEMATIC STIRLING HEAT-PUMP**

by

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## Objective

The aim of this project is to investigate possibilities of a simple kinematic configuration for a Stirling-cycle heat-pump with the prospect of a commercially competitive design. Stirling heat-pumps offer many advantages in contrast to traditional vapour compression systems. They can be operated by air, the COP of the ideal Stirling-cycle is the highest possible (Carnot-efficiency) – real Stirling heat-pump cycles are not less efficient than vapour compression cycles – and, not at last, Stirling-cycle machinery runs quietly and is nearly maintenance-free. Also, other than with Stirling-engines where expensive materials need to be used due to higher temperatures, Stirling heat-pumps can be made of low-cost materials such as plastics.

However, Stirling-cycle machinery, it seems, is characterised by inherent conflicts. Most of the parameters involved seem to mutually exclude each other. For instance, if one variable is to be optimised by a parameter, quite often this can only be done at the cost of another. In this project, the priority was to keep manufacturing costs at a minimum, even at the cost of a lower coefficient of performance (COP).

This report documents the investigation of various kinematic configurations. A number of ideas has been suggested which will be discussed in this report. Since all discussed designs are very distinctive, it is difficult to group them. However, two major distinctions can be made:

- Configuration: Rotary design; reciprocating mechanism
- Type of the cycle: Closed cycle; open cycle

In order to decide on the best solution for the set goal, one has to take all problematic areas of Stirling-cycle machinery into account:

- Regenerator: Good heat-transfer and heat-storage properties seem to exclude low flow losses and vice versa.
- Heat exchanger: The gas volume in the heat exchangers cannot be used for expansion or compression.
- Leakage/seal problems: If the crank case cannot be hermetically sealed, an atmospheric seal creates problems, especially with mono-atomic working gases.
- Lubrication: Wherever lubrication is necessary, the regenerator runs the risk of getting clogged.
- Slider crank mechanism: Side loads occur which accelerate the wear of piston rings. The mechanism is complicated, but well understood and experienced. Wherever cross-heads are used, lubrication is required and additional friction loss occurs.
- Balancing: The configuration should be able to be well balanced to provide smooth and quiet performance.

In this report, it was tried to look into each suggested configuration with consideration of above mentioned issues, keeping the main focus on a simple and low-cost design.

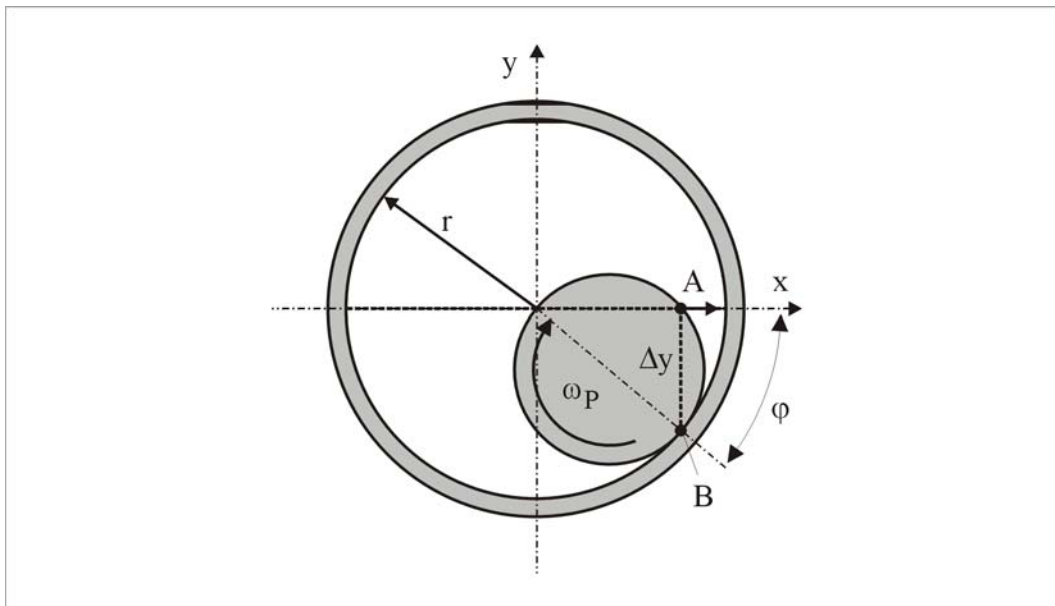
# 1. Epicyclic configuration

**Main characteristic:** No side loads

## Description

If a circular cylinder rolls without slip along the inner surface of a ring of double the diameter, a point on the circumference of the revolving cylinder carries out a linear motion along the diameter of the ring. In Fig. 1, point A travels only along the x-axis while the cylinder rolls in anti-clockwise direction on the inner surface of the ring.

This geometry can be applied to an epicyclic gear with the pitch-diameter of the planet being half the pitch-diameter of the annulus. A rod connected to a fixed point on the circumference would then perform a linear reciprocating motion with a stroke of the pitch-diameter of the annulus. This rod, in turn, could be connected to two pistons in two opposing cylinders (Fig. 2).



**Fig. 1: Linear motion of a point on a revolving cylinder**

The piston displacement is a sinusoidal function of the crank angle  $\phi$  as can be concluded from Fig. 1. If constant annular velocity  $\omega_P$  is assumed (the annular velocity of the planet is the same as of the spider in this case), then, the linear velocity of point A in x-direction becomes

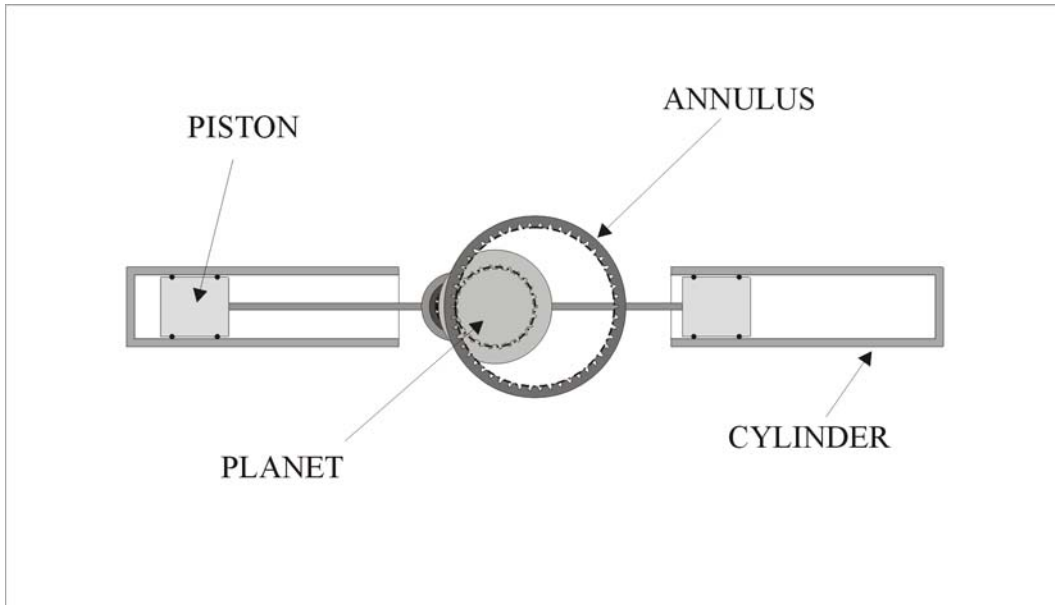
$$\dot{x} = \Delta y \omega_P$$

with  $\Delta y$  being the perpendicular distance to the point of instant rotation (point B). Therefore, with

$$\Delta y = r \sin \phi ,$$

the linear velocity of point A is

$$\dot{x} = r \omega_P \sin \phi .$$



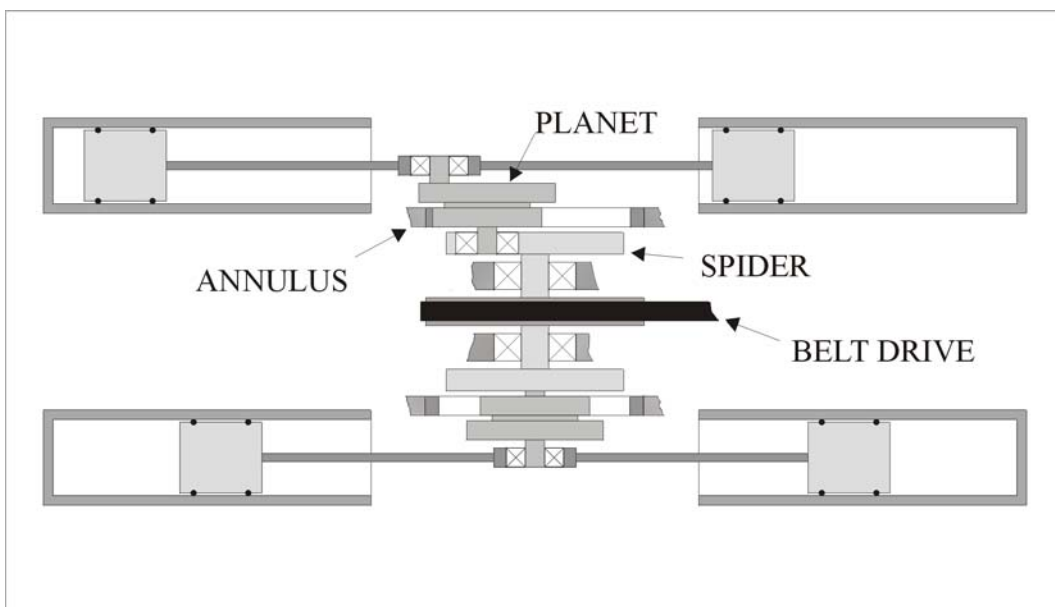
**Fig. 2: Epicyclic configuration for two opposing cylinders**

Since the distance  $\Delta y$  becomes smaller towards top and bottom dead centre, the piston force  $F$  reaches a maximum (even infinity for  $\varphi = 0^\circ$  and  $180^\circ$ ) if a constant torque  $T_S$  on the crankshaft is assumed:

$$F = \frac{T_S}{r \sin \varphi}.$$

This is an advantageous feature of this arrangement as the maximum piston force is usually required when the gas is in maximum compression.

Fig. 3 shows a possible arrangement of two cylinder pairs at a phase difference of  $90^\circ$ . Two Stirling-cycles in an  $\alpha$ -configuration would sit back to back.



**Fig. 3: Stirling-cycle heat-pump in epicyclic configuration**

## Discussion

The main characteristic of this arrangement is that no side-loads occur at all due to the strict linear motion of the pistons. Unfortunately, however, it is impossible to use one annulus only for two of these arrangements to provide a phase-angle of  $90^\circ$  in order to simulate the Stirling-cycle. Therefore, two annuli are required with two spiders sitting on each end of a shaft driven by a belt (Fig. 3). With no side loads occurring in this configuration, the wear of the seal rings is reduced as well as the friction loss. Also, the design can be held relatively compact. However, with two epicyclic gears, the manufacturing effort is relatively high and therefore expensive. Additionally, gears need to be lubricated which requires a housing and seals on the shaft. Another issue might be the noise of the gears. As a conclusion, the proposed epicyclic configuration might be more suitable for an engine (as a substitution for a swash plate or wobble yoke mechanism) than for a low-cost heat-pump.

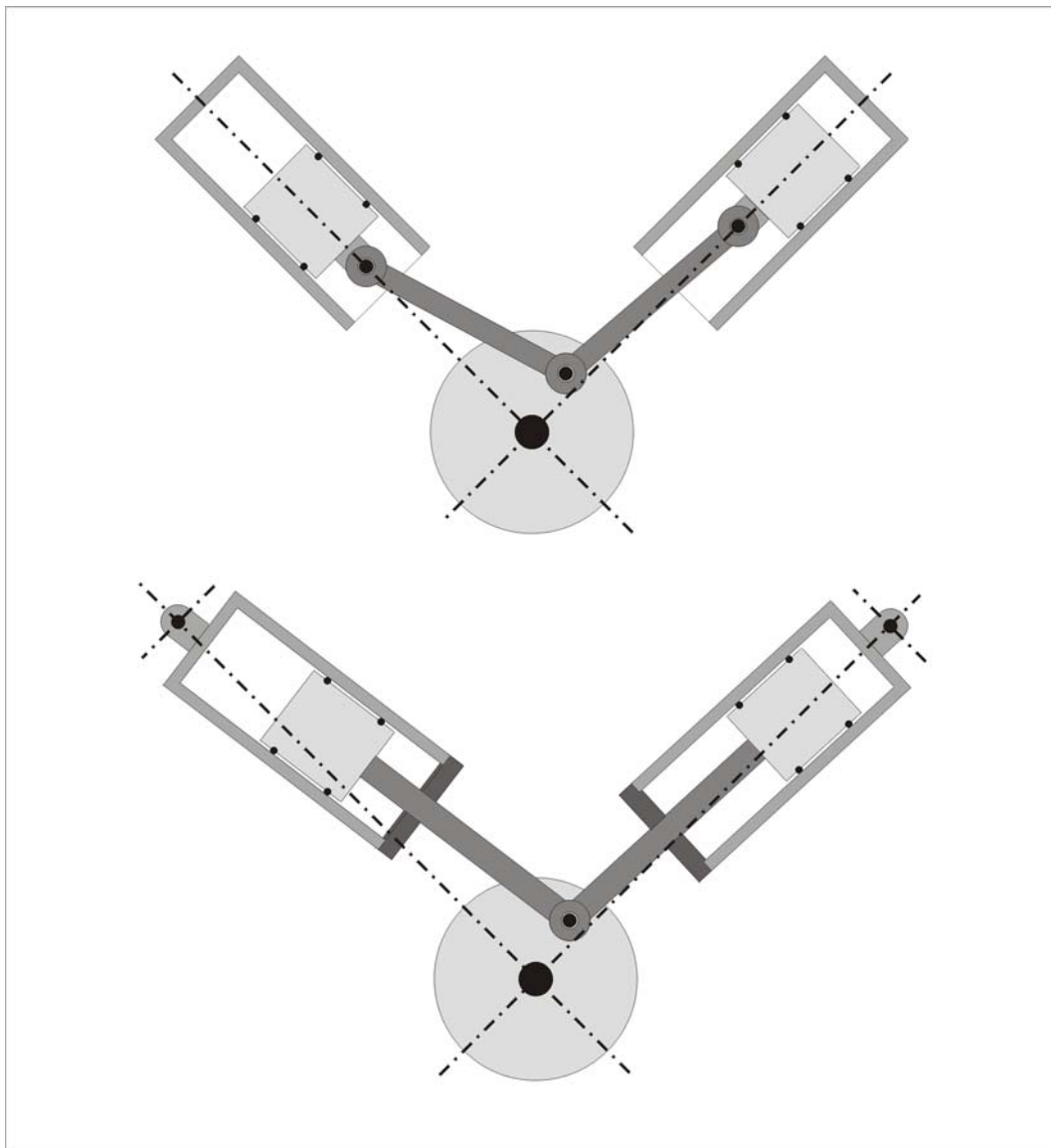
<b>PROS</b>	<b>CONS</b>
No side loads	Costly manufacture
Compact design	Lubrication required
Convenient piston force/crank angle relation	Noise of gears
Balancing?	

## 2. Pivoting cylinders

**Main characteristic:** No side loads

### Description

The side loads on pistons can be eliminated in shifting the pivot from the gudgeon pin to the foot of the cylinder, a system which was used for early steam engines. Fig. 4 shows a traditional V-twin arrangement with two fixed cylinders and with two pivoting cylinders. As it can be seen, the number of required bearings remains unchanged.



**Fig. 4: Fixed cylinders versus pivoting cylinders**

### Discussion

With hardly any additional effort, the side loads on the pistons can be taken away in making the cylinders pivotable. This reduces the wear of the seals as well as the friction loss. Although it would be possible to use this arrangement for an  $\alpha$ -configured Stirling-cycle with flexible

ducts connecting the cylinders, a design with moving cylinders including their incorporated heat-exchangers is problematic. Additionally, although depending on the design, the moving mass of the cylinders will increase the moment of inertia of the system and therefore the amount of required work-input.

<b>PROS</b>	<b>CONS</b>
No side loads	Moving components (cylinders, heat-exchangers) complicates the design
No additional manufacturing costs	Increased moment of inertia



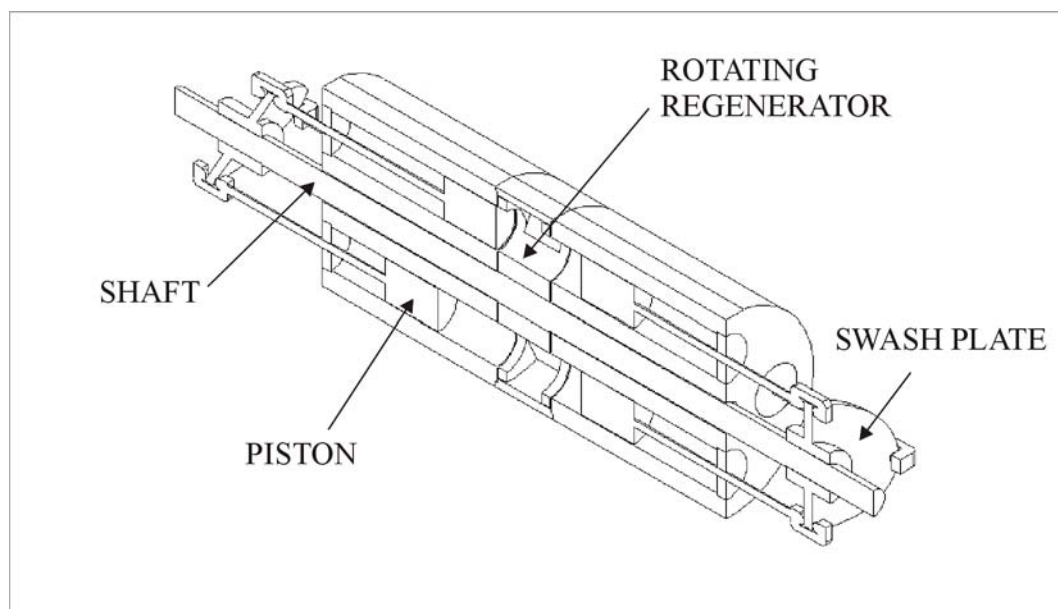
### 3. Open Stirling- cycle heat-pump

**Main characteristic:** Open cycle, no heat-exchangers

#### Description

The elimination of heat exchangers in Stirling-cycle heat-pumps offers many advantages. To start with, the manufacturing effort is smaller as these components are usually complicated to make. The second advantage is that the temperature difference between working gas and surroundings can be much smaller and heat losses are therefore lower.

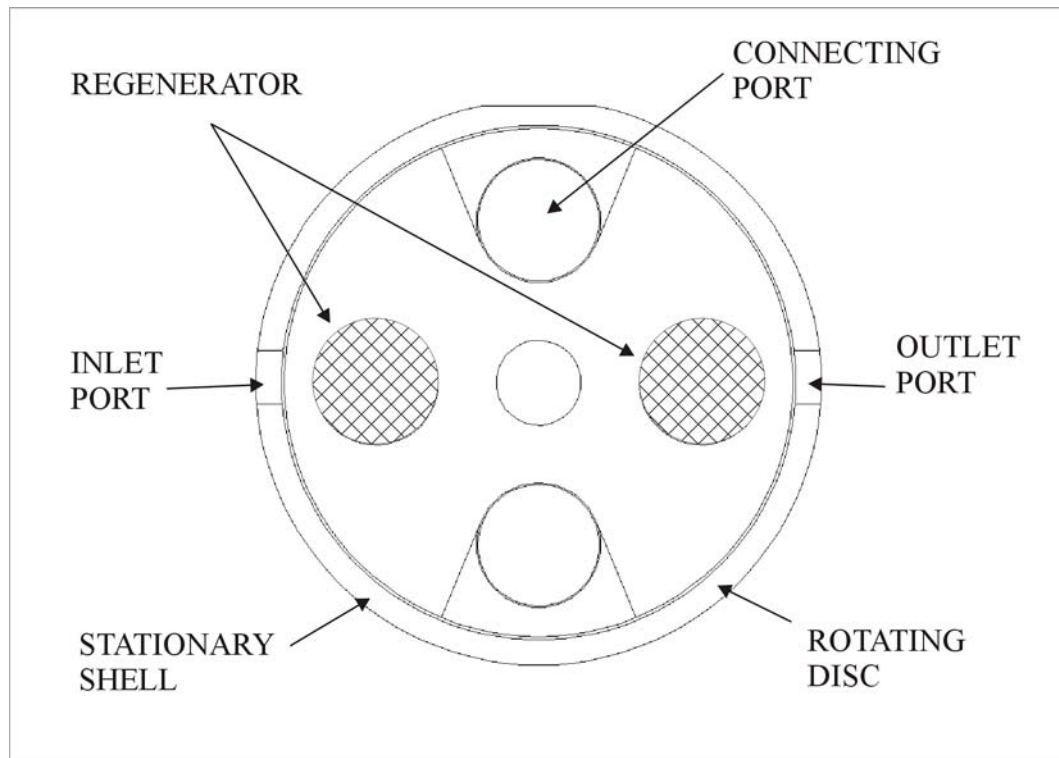
Fig. 5 depicts the design of four pairs of double-acting pistons driven by two swash plates. The two swash plates sit on the same shaft and are out of phase by  $90^\circ$  in order to simulate the Stirling-cycle. Also sitting on the shaft is a rotating disc which contains sections of regenerator material and ducts (Fig. 6).



**Fig. 5: Sectional view of an open Stirling-cycle heat-pump**

The space between each opposing cylinder pair is alternately filled either by regenerator material or a connecting duct, depending on the crank angle. Two cylinder pairs represent a closed cycle with two isochoric and two adiabatic processes, whereas the other two cylinder pairs perform an open cycle consisting of two isobaric and two isochoric processes. Each cycle is displayed in Fig. 7.

**Open cycle:** The connecting ports in the rotating disc also dispose of a radial duct. As soon as this duct is adjacent to the inlet port in the stationary shell, the two piston pairs move in opposite direction and take air in at constant pressure (1 - 2). During the isochoric process, the trapped gas flushes the hot regenerator and takes up heat (2 - 3). With further rotation of the shaft, the radial duct in the connecting port lines up with the outlet port, while the two pistons move towards each other and push out the hot gas (3 - 4). The cycle is finished with flushing the cold regenerator isochorically (4 - 1).



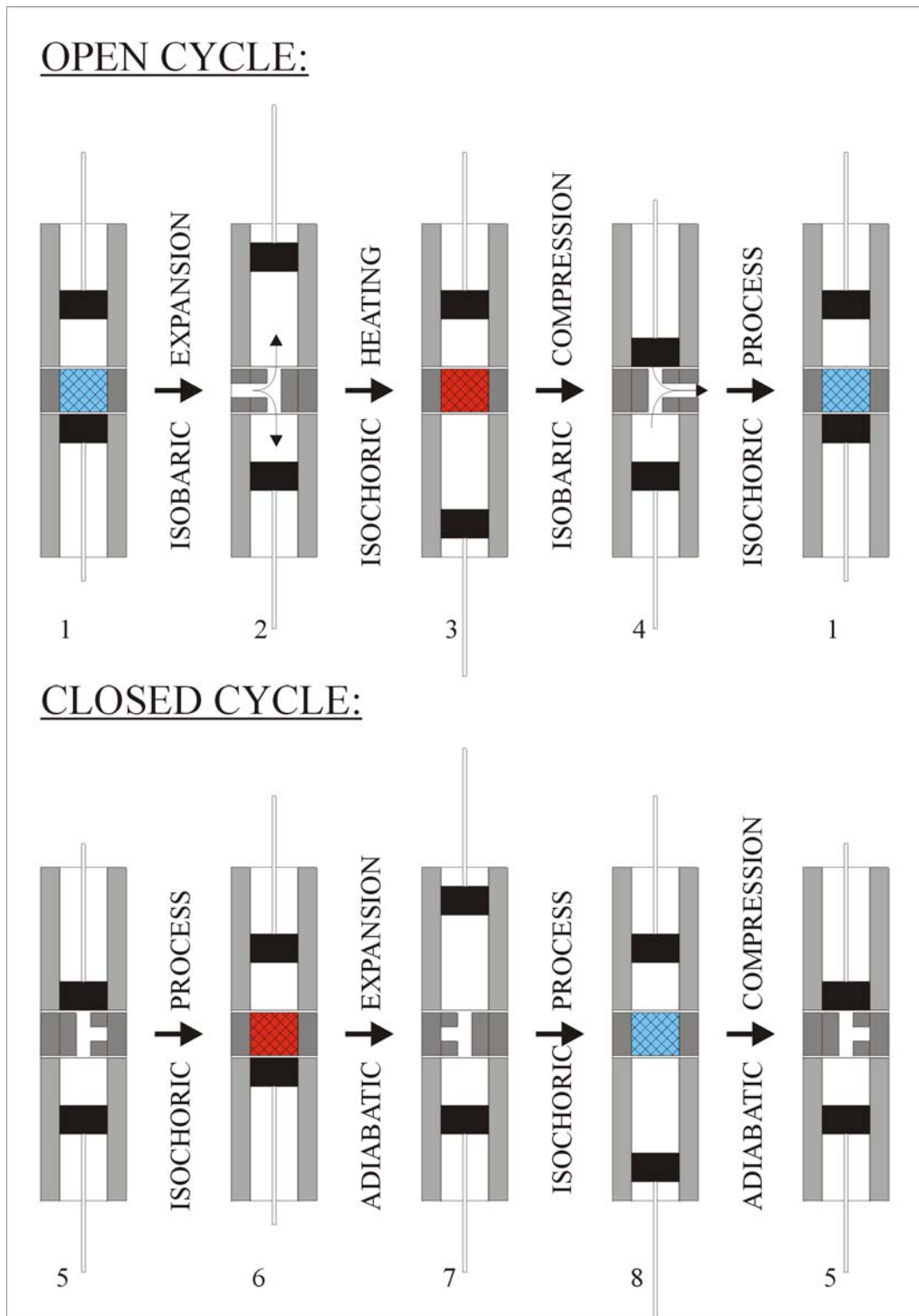
**Fig. 6: The rotating disc with regenerators and ports**

Closed cycle: The closed cycle is performed in the two remaining cylinder pairs. After the adiabatic compression (8 – 5) of the gas through the connecting duct, the hot gas flushes the regenerator isochorically and rejects heat to it (5 – 6). Adiabatic expansion follows through the connecting port between the two cylinders (6 – 7). The regenerator is flushed by the now cold gas in the isochoric process 7 – 8. The cycle is completed in compressing the working gas adiabatically (8 – 5).

### **Discussion**

The fact that the medium that is to be cooled or heated is in direct contact with the regenerator material offers a big advantage in terms of heat transfer. Without the need of heat-exchangers and therefore additional thermal resistance, temperature differences can be smaller and heat losses can be reduced. The core of the design (the two cylinder housings, pistons, rods and the rotating disc) can be easily manufactured and assembled. Also, the rotary aspect of the driving system offers advantages in terms of size and simplicity. Two swash-plates, however, may outweigh these design advantages as they invoke issues like lubrication, friction and side-loads.

Another problematic aspect is the sealing of the rotating disc. Two face seals are required to stop the gas from getting from one cylinder to another. An answer might be to “clearance-seal” the rotating disc in keeping the gaps to the cylinder housing on each side as small as possible. This can be achieved quite easily in using soft material for the disc and embed it by grinding (?). It remains to be shown if a compromise can be achieved in keeping the time for possible leakage as short as possible. The fact that the compression ratio does not need to be so high in an open cycle might assist this compromise. Another possibility would be the use of a low-wear pad that can easily slide on the surface of either the cylinder housing or the regenerator disc.



**Fig. 7: The working principle of the two cycles**

An additional source of loss can be found in “puffs” of working gas that remains in the regenerator matrix and gets from the closed cycle to the open cycle and vice versa.

The suggested cycle incorporates many uncertainties and its performance is very difficult to predict. Its efficiency strongly depends on how small leakage can be kept and on how good the

interaction of the rotating regenerator disc with its ducts and the thermodynamic processes of the working gas can be achieved.

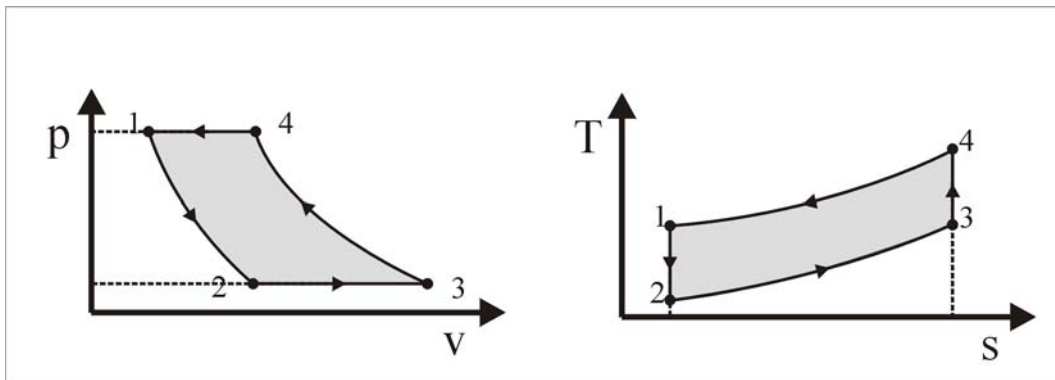
<b>PROS</b>	<b>CONS</b>
Direct heat transfer	Swash plates are complicated
Rotary system	Lubrication necessary
Relatively easy to manufacture	Side loads
	Rotating disc is difficult to seal
	Enthalpie loss through “air-puffs”

## 4. Open Joule- cycle heat-pump

**Main characteristic:** Open cycle, Joule-cycle, no heat-exchangers

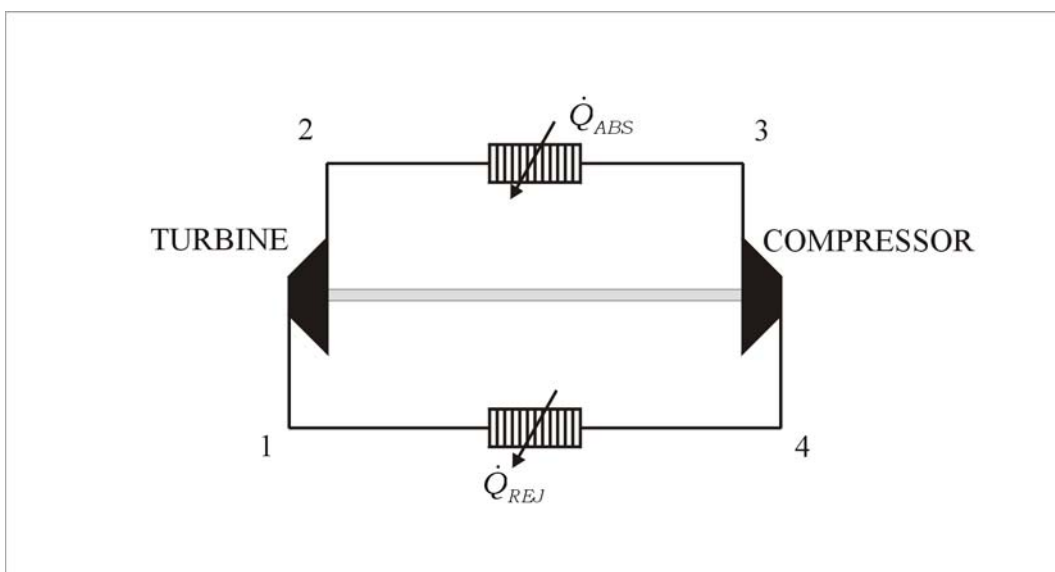
### Description

Although this non-Stirling-cycle does not, strictly speaking, belong to this series of suggested cycles, it should be listed here as it meets some of the critical requirements: Its set-up is simple and it can be operated as an open cycle in a rotary configuration. As opposed to the Stirling-cycle, the Joule-cycle consists of two isobaric heat-transfer processes and two adiabatic pressure changes. Fig. 8 shows both the  $p,v$ - and the  $T,s$ -diagram of the Joule cycle.



**Fig. 8:** The  $p,v$ - and the  $T,s$ -diagram of the Joule-cycle

The Joule-cycle can be operated in two ways: In a closed cycle (Fig. 9) where the working gas is conserved in the system and where the heat is being transferred by heat-exchangers. Or, as an open system in which air is drawn in by a turbine and adiabatically expanded at the same time (1 – 2). With the air now being colder, heat can be transferred from the environment by an isobaric heat exchanger (2 – 3). Once the air has reached ambient temperature, however at lower pressure, compression can take place and bring back the air to ambient pressure.



**Fig. 9:** The flow chart of the closed Joule-cycle

In this adiabatic process (3 – 4), the end temperature of the air is above the initial temperature at point 1 and the cycle can be used as a heating device. Conversely, if the air is first adiabatically compressed (2 – 1), then isobarically cooled down to ambient temperature (1 – 4) and finally expanded back to ambient pressure (4 – 3), now at lower temperature, the cycle works as a refrigerator. In both cases (open and closed cycle), compressor and turbine can be connected by a shaft (as symbolized in Fig. 9) in order to conserve the expansion work.

It seems that the Joule-cycle is more suitable for a turbo-machine than for a reciprocating piston configuration, as the isobaric process is difficult to be carried out in the latter. Depending on the efficiency of turbine and compressor, the optimum compression ratio seems to be around 1.2 in order to achieve the maximum COP, which is approximately 2.

Unfortunately, the static pressure of axial fans and blowers is much too low in order to achieve a reasonable compression ratio. However, the performance data of vacuum cleaners suggest that their turbines are in the range of the required specifications as the examples in the following table show:

MODEL	VACUUM	FLOW RATE	POWER	SOURCE
KV-150	215 mbar	?	1.2 kW	<a href="http://www.koreaeng.co.kr/vacuum.htm">www.koreaeng.co.kr/vacuum.htm</a>
AP2551	137 mbar	80 l/s	1.5 kW	<a href="http://www.mca.es">www.mca.es</a>
E100	310 mbar	?	1.5 kW	<a href="http://www.cyclovac.com">www.cyclovac.com</a>
S548	250 mbar	66 l/s	1.1 kW	<a href="http://www.mielevacuums.com">www.mielevacuums.com</a>
Activa CH30	197 mbar	43 l/s	0.85 kW	<a href="http://www.lindhaususa.com">www.lindhaususa.com</a>

An interesting application for an open Joule-cycle would be to incorporate a counter-flow heat-exchanger and use this device for a ventilation system. Thus, the temperature of used, warm air could be increased by the adiabatic compression stage, followed by the isobaric heat-exchange with fresh, cold air which has been cooled down even further by adiabatic expansion. After the heat-exchange, adiabatic compression of the fresh air and adiabatic expansion of the used air complete the cycle.

Fig. 10 shows an example of an open Joule-cycle with a counterflow heat exchanger. As it turns out, it is more efficient to have only one flow adiabatically compressed and expanded, preferably the hot air flow as its density is higher due to the compression, which supports the heat transfer. The maximum COP in the given example occurs at a compression ratio of about 1.3. Depending on how efficient the heat exchanger works (in this example, a minimum temperature difference between the flows at both ends of 5 K was assumed), the COP seems to converge towards 2.0 with a decreasing compression ratio. However, a compromise has to be found as the output heat also gets smaller with a decreasing compression ratio.

## Discussion

The most appealing features of an open Joule-cycle are the rotary aspect and the fact that the airflow can be directly used for heating or cooling. The set-up of this heat-pump is very simple and mostly standard components like vacuum cleaner turbines could be used.

Red: INPUT DATA		Black: CALCULATED VALUES																		
Ambient pressure	1	bar																		
Outside temperature	-20	°C			OUTSIDE	adiabatic		isobaric		adiabatic										INSIDE
Inside temperature	20	°C				expansion		heat exchange		compression										
Minimum $\Delta T$ for HX	5	K			OUTWARD FLOW	←														
Efficiency electric motor	90	%			Pressure / bar	1.00														
Efficiency turbine	85	%			Temperature / °C	-33.6														
Efficiency compressor	85	%			Compression ratio	←	1.3	←												
Volumetric flow rate (@ 20°C)	2.00	m <sup>3</sup> /min			Adiabatic work / kW	←	-0.631	←												
Gas constant	287	J/kgK					TURBINE		COUNTERFLOW HEAT EXCHANGER			COMPRESSOR								
Specific heat ratio	1.4	-			INWARD FLOW	→														
Specific heat capacity	1004.5	J/kgK			Pressure / bar	1.00														
Input power	0.486	kW			Temperature / °C	-20.0														
Heat (massflow * $\Delta T$ inside)	0.709	kW			Compression ratio	→	1.0	→												
COP	1.5	-			Adiabatic work / kW	→	0.000	→												
ASSUMPTIONS:	* Inward and outward flow rates are equal																			
	* No heat recovery in machinery																			
	* No complete temperature equilibrium in counterflow HX. $\Delta T$ remaining on both sides																			

Fig. 10: Open Joule-cycle with counterflow heat exchanger

Apart from this, the suggested open cycle is ideal for ventilation applications.

However, despite its simplicity, a few aspects of this configuration are disadvantageous and might even rule out this idea. One of the problems could be the noise created by the turbines. Even filtering of the air might not reduce the noise to an acceptable level, as experience with vacuum cleaners shows. Also, it is well known that the efficiency of turbo-machinery decreases with smaller sizes, as flow losses through the gaps between the tips of the blades and the stationary walls become more and more significant. Finally, the achievable COP is not very high, however, given the simple set-up of the configuration plus the open-cycle aspect, the open Joule-cycle at this small scale might even find its application.

<b>PROS</b>	<b>CONS</b>
Simple set-up; standard components can be used	Lower efficiency with decreasing size
Open cycle: Direct cooling or heating	Noise
Ideal for ventilation applications	Low COP



## 5. Linear motor / Solenoid

**Main characteristic:** Linear motor drive

### Description

The fact that a rotating electric motor is used as a drive for reciprocating piston movement in traditional systems virtually suggests the shortcut in using a linear motor. Instead of converting circular motion into a linear motion by a more or less complicated mechanism involving the need for space, lubrication and, despite lubrication, creating friction, linear motors seem to be perfect for this application.

There is a variety of linear motors which can be subdivided into direct-current linear motors and alternating-current linear motors. The latter are basically the same as a rotary induction motor, however, cut along a radial plane and unrolled to a flat configuration. One application of these devices were oscillating machines as used in the textile industry for weaving looms [52]. Two stators with fields in opposite directions were placed end to end, while the “rotor” could travel along the common axis. It is reported that an oscillating mass of 30 g oscillated at a frequency of 500 traverses per minute over a length of 36 cm, whereas the input power was about 100 W.

A reciprocating generator was developed by Laithwaite and Mamak [51] in 1962, which consists of a single loop of conducting material travelling between a d.c. pole structure. This configuration can be used both to convert mechanical power directly from a piston or as a motor to drive compressors.

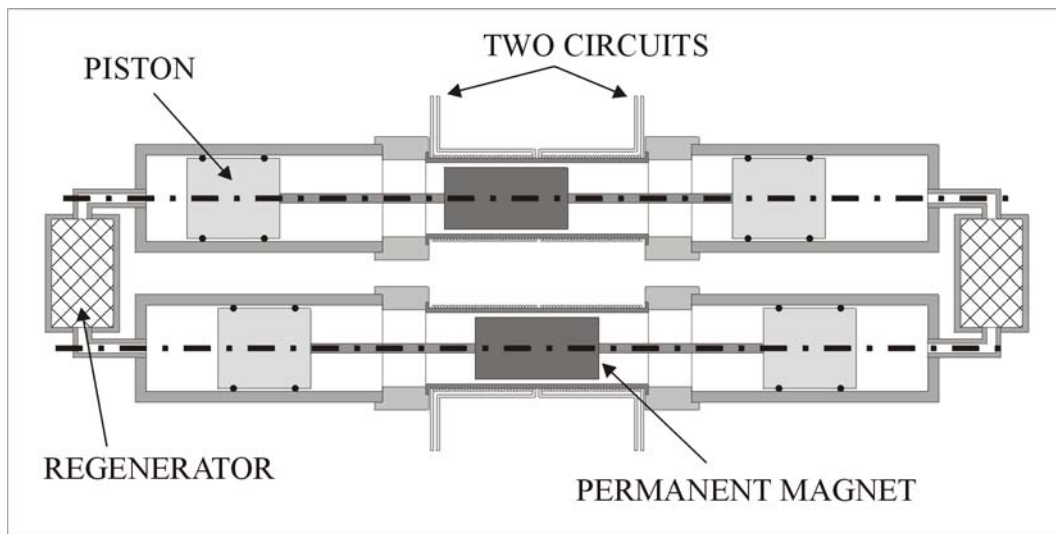
This is only to give a few examples of the variety of possible linear motors. It seems, however, that the most appropriate devices for this application are either voice coils (non commutated dc linear actuators) or an ac linear conduction motor as they have the following specifications:

- Voice coils: - small size  
- high force to weight ratio (peak force: 1.3 kN, continuous force: 0.4 kN, stroke: 50 mm [3])  
- no moving leads as they use a permanent magnet  
- no maintenance
- AC linear conduction motors:  
- non-contact  
- high speed (up to 45 m/s)  
- easy to install and control  
- Peak force of 0.9 kN and a continuous force of 0.2 kN [3]

Unfortunately, the efficiency of linear motors is much lower than that of their rotary counterparts. This is because the air gap between the armature and the magnet poles is usually wider than in rotary systems, which has a critical impact on the efficiency. Values of only between 30 % and 40 % can be expected for the maximum efficiency of linear devices.

A possible arrangement of a linear motor system in a Stirling-cycle heat-pump is shown in Fig. 11. Two pistons are attached to one rod, which is driven by a permanent magnet sitting in the centre of two circuits. Each piston seals the working gas towards one end, while the other end is

connected to a second system via a duct being 90° out of phase in order to simulate the Stirling-cycle. Thus, two alpha-configurations sit back to back.



**Fig. 10: Double alpha-configuration with linear motors**

### Discussion

This set-up offers many advantages. The constant volume in between the two pistons can be pressurized up to the mean pressure of the working gas in order to minimize the pressure difference across the piston rings, and thus reduce the driving force for leakage. Also, the complete system can be hermetically closed and therefore no seal problem occurs at all. Another advantage is that without the need of a crank mechanism, the design can be held very compact. The fact that the pistons are driven by electromagnetic forces additionally offers the opportunity to influence the piston motion via an electronic control system. Thus, the Stirling-cycle is not confined to a sinusoidal piston movement, but can be much better approached by ideal processes. Finally, mechanical problems such as side-loads can almost be neglected.

As convincing as these advantages are, there are still some drawbacks on the other hand. Balancing might become difficult unless more units are used counter-balancing each other. An additional disadvantage seems to be low efficiency of linear motors. However, since the air gap between the moving circuit and the stationary armature is of critical importance for the efficiency, an appropriate design could improve the situation significantly.

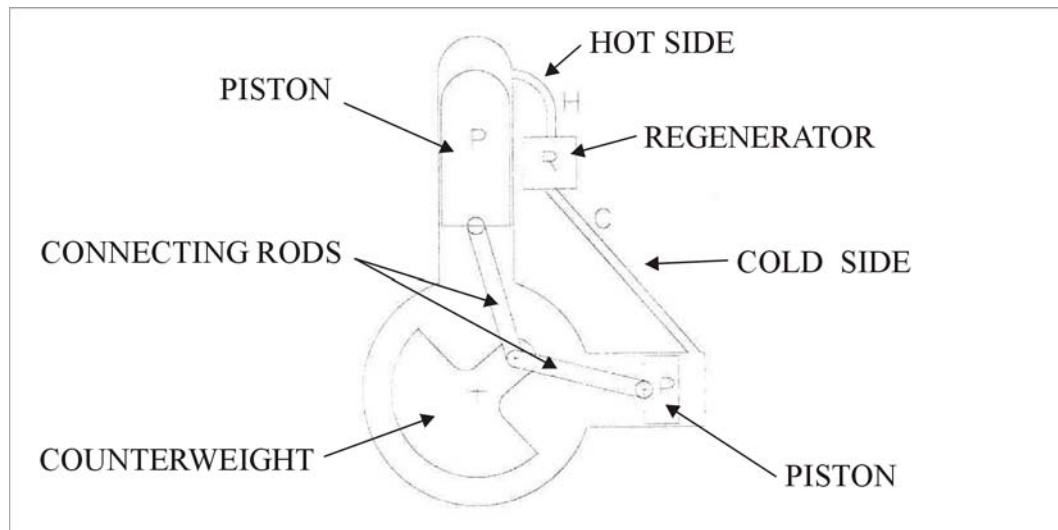
PROS	CONS
Compact design	Low efficiency
Can be hermetically sealed	Balancing might be difficult
“Crank-case” can be pressurized	
Electronic control of the piston movement	
No side-loads	

# 1. V-Twin configuration

**Main characteristic:** Classical design with slider-crank mechanism

## Description

The traditional design of a piston machine uses a slider crank mechanism in order to convert the circular motion of the work input source (usually an electric motor) into a linear and reciprocating piston movement. In order to circumvent the drawbacks of this design such as side loads and the need for lubrication, a number of approaches have been suggested. In a project report about the design of a 50 Watt Stirling engine battery charger carried out at the Department of Mechanical Engineering at the University of Canterbury, various configurations of rhombic drives and V-twin designs have been compared [1]. With the objective to keep the manufacturing costs at a minimum, the outcome of this investigation was in favour of the traditional V-twin configuration (Fig. 12).



**Fig. 11: Traditional 90°-V-twin configuration (from [1])**

## Discussion

Since all parts used in this configuration are standard components, a great advantage is that the design is well understood and the components are easily available or manufactured. The design itself is simple and utilises identical pistons and connecting rods, which is important in hindsight of serial production. With only five moving parts, the number of moving components is relatively low. Due to the 90°-angle between the two pistons, a phase-difference is provided in order to simulate the Stirling-cycle. Also, this arrangement can be well balanced.

This design, however, is not free from side loads unless cross-heads are used. Cross-heads, on the other hand, require lubrication and more room. Another disadvantage of the V-twin arrangement is that the transfer passage between the two cylinders becomes relatively long. This not only leads to heat losses to the environment but also increases the flow friction in the ports.

<b>PROS</b>	<b>CONS</b>
Use of standard components	Side loads
Simple design	Long transfer passage
Easy to balance	

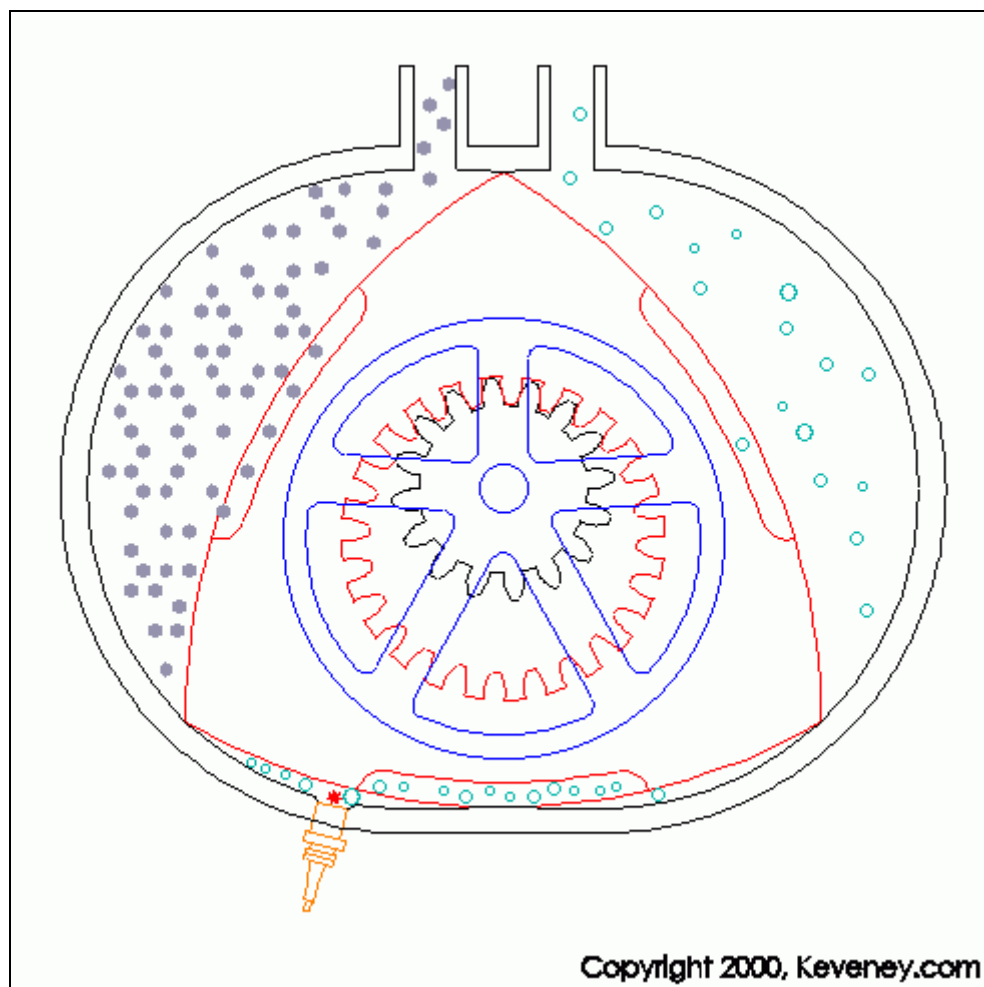
## 7. Rotary designs

**Main characteristic:** Rotary systems

### Description

The appeal of rotary machines lies in its compact design, light weight and the lack of reciprocating masses as is the case with pistons. Usually, an eccentrically supported rotor of an appropriate geometrical shape (here: triangular) creates chambers of varying volumes in rotating about a second point (Fig. 11). This configuration was developed by Felix Wankel in the 1950s. A classification of the almost infinite number of possible configurations was done by Wankel and can be found in [1].

As experience with Wankel-engines shows, this type of configuration is easy to balance and provides smooth running. There are no connecting rods and fewer parts as opposed to reciprocating piston machines. One of the disadvantages of this principle, however, when used in a combustion engine, is the high ratio of surface area to volume. In Stirling-cycle heat-pumps, on the other hand, this characteristic is not necessarily a disadvantage.



**Fig. 12: Principle of a Wankel-engine (taken from [www.keveney.com](http://www.keveney.com))**

As interesting as this type of mechanism seems, there are also a number of disadvantages attached to it. To begin with, the manufacture of the rather complicated shape of the rotor and the housing is expensive. Secondly, sealing of the sliding points of contact is a serious problem not only in combustion engines. Additionally, one tends to forget in looking at cross-sectional views only that a comparatively large face-area needs to be sealed on both sides.

### Discussion

It is tempting to use a rotary design for a Stirling-cycle heat-pump, since its design is compact and the machine can be directly driven by an electric motor without a crank mechanism. However, even if the relatively high manufacturing cost could be overcome, there is still the seal-problem that wants to be solved on a long-term basis.

<b>PROS</b>	<b>CONS</b>
Compact design	Large area to be sealed
No crank mechanism	High manufacturing cost
Smooth running	

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