UNIVERSITY LECTURERS' VIEWS ON THE TRANSITION FROM SECONDARY TO TERTIARY EDUCATION IN MATHEMATICS:

An International Survey

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This paper deals with a very practical issue. In many countries there is a gap between school and university mathematics. The transition period from school to university can be hard for many students. Even students with good marks in school mathematics experience difficulties at university and sometimes fail the first year university mathematics courses. Different parties – school teachers, university lecturers, first year university students, administrators, researchers – might have different views on the reasons for the gap and the ways to narrow or fill it. The purpose of this paper is to present and analyse responses of university lecturers worldwide to a short survey concerning the transition period between the school and university mathematics.

Keywords: transition; school; university; lecturers

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INTRODUCTION.

This paper addresses the issue of the transition from school to tertiary study in mathematics, one which is of great concern and the subject of considerable attention around the world. It deals with perspectives of university lecturers from 24 countries. Many university lecturers worldwide feel that there is a gap between the school and university mathematics and there is a need to investigate the ways of reducing the gap. A serious concern was expressed in the report *Tackling the Mathematics Problem* commissioned by the London Mathematical Society (LMS, 1995): "There is unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates….The serious problems perceived by those in higher education are:

1. a serious lack of essential technical facility – the ability to undertake numerical and algebraic calculation with fluency and accuracy;
2. a marked decline in analytical powers when faced with simple problems requiring more than one step;
3. a changed perception of what mathematics is – in particular of the essential place within it of precision and proof.

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This is no way restricted to those 'new undergraduates' who ten years ago would not have proceeded to higher education. The problem is more serious; it is not just the case that some students are less well-prepared, but many 'high-attaining' students are seriously lacking in fundamental notions of the subject."

Many researchers writing on the transition period from school to university education in mathematics also indicate mathematical under-preparedness of students entering university (Luk, 2005; Kajander & Lovric, 2005; Guzman et al., 1998; Leviatan, 2004; Hourigan & O'Donoghue, 2007; Barnard, 2003; Selden, 2005). They provide a number of reasons for that under-preparedness (a recent trend of moving from elite to mass university education, lowering the mathematics standards at school and university, inadequate funding, etc.). Research has shown that mathematics students from UK (Hoyles, Newman & Noss, 2001), Hong Kong (Luk, 2005) and Ireland (Hourigan & O'Donoghue, 2007) tend to adopt a surface learning approach in schools but are expected to apply deep learning in tertiary mathematics. The Irish situation, according to a Chief Examiner's report of Leaving Certificate examinations (Hourigan & O'Donoghue, 2007) highlights the problem of poor relational understanding. Based on case studies of two schools, Hourigan and O'Donoghue (2007) found that the examination-oriented nature of the educational system tends to promote a faster pace of teaching, routine mastery of algebraic procedure and 'learned helplessness'. Consequently, surface learning is seen as a quick fix in the schools and creates a culture of learning that fails to prepare students

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for tertiary level Mathematics (Hourigan & O'Donoghue, 2007). Brandell, Hemmi and Thunberg (2008) provide a good overview of the situation in Sweden. The present Swedish national curriculum came into effect in 1994; thus the stability of the curriculum allows researchers to focus on issues such as the comparison between the "goals and ambitions of mathematics education in Swedish upper secondary school with the expectations of the new students held by the tertiary level" (p. 39). In one Swedish study a questionnaire to secondary teachers asked them to grade how well prepared they thought their students would be to tackle a variety of typical problems from a tertiary transition programmes preparatory course. Specific questions were found to be outside the curriculum and for others the teachers believed that even the better students lacked the necessary concepts and skills to "make sense of the exercise" (p. 41). A study at a New Zealand university by James, Montelle and Williams (2008) analysed students' performance over the years when moving from school to university. Their analysis though led to many more questions. "Have changes in assessment in fact affected the ways in which New Zealand teachers deliver their material, or has it only required some minor changes and adjustments in their curriculum? Is this a phenomenon just applicable to mathematics? After all, mathematics is a cumulative discipline that may be entirely suitable for modularization, unlike some other disciplines. Furthermore, is this initial study too early for full effects to be recognizable? And for those favourably disposed towards modularization, is the 'status quo' result disappointing to the supporters and

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instigators of the new qualifications? Would they have preferred to see rather a marked increase in the abilities and capabilities of the prospective tertiary level student, rather than a maintaining of standards?" (p. 1048). While much can be learned by teachers in the secondary school and tertiary sectors from studies that recognise the existence of curriculum and other gaps in transition, this is only the first step. The next step must surely be to analyse its causes and then try to do something about it. A widening gap appears to be a worldwide phenomenon and in many countries there is concern that differences in emphasis between school and tertiary mathematics may be increasing and several authors emphasise the importance of the issue of transition from school to university mathematics for students' success in university mathematics (Crawford et al., 1994; Gusman et al., 1998; Anthony, 2000).

RESEARCH FRAMEWORKS

In this study, practice was selected as the basis for the research framework and, it was decided "to follow conventional wisdom as understood by the people who are stakeholders in the practice" (Zevenbergen & Begg, 1999). The idea of this study has arisen from and is based on teaching practice. This study is primarily a practice-based research study with the aim of identifying and promoting pedagogical strategies that may make the transition period smoother and more beneficial in terms of learning. It is the teaching/research nexus.

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The theoretical framework of the study is based on Piaget (1985) concept of cognitive conflict and David Tall's (1991, 1997, 2004a, b) works on advanced mathematical thinking. One possible reason for transition problems is that there may be both process and conceptual differences in the approaches used at the secondary and tertiary level. A developing theory by Tall (2004a, b) suggests that mathematical thinking exists in three *worlds*, the embodied, symbolic and formal. The embodied is where we make use of physical attributes of concepts, combined with our sensual experiences to build mental conceptions. The symbolic world is where the symbolic representations of concepts are acted upon, or manipulated, where it is possible to switch from processes to *do* mathematics, to concepts to *think* about mathematics. The formal world is where properties of objects are formalised as axioms, and learning comprises the building and proving of theorems by logical deduction from these axioms.

It is hypothesised that the main reason for the gap between the school and university mathematics would be the difference in thinking. Many students are exposed to a formal deductive approach in mathematics for the first time only upon entry to university and may therefore experience a significant amount of cognitive conflict in their first year. "At school the accent is on computations and manipulation of symbols to 'get an answer', using graphs to provide imagery to suggest properties. At university there is a bifurcation between technical mathematics that follows this style (with increasingly sophisticated

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techniques) and formal mathematics, which seeks to place the theory on a systematic, axiomatic basis" (Tall, 1997). "During the difficult transition from pre-formal mathematics to a more formal understanding of mathematical processes there is a genuine need to help students gain insight into the concepts" (Tall, 1991). It is not an easy task and requires transition from one stage to another in the Piagetian stage theory where the previous knowledge conflicts with new ideas (Piaget, 1985). "The formal presentation of material to students in university mathematics courses – including mathematics majors, but even more for those who take mathematics as a service subject – involves conceptual obstacles that make the pathway very difficult for them to travel successfully. And the changes in technology, that render routine tasks less needful of labour, suggest that the time for turning out students whose major achievement is in reproducing algorithms in appropriate circumstances is fast passing and such an approach needs to move to one which attempts to develop much more productive thinking" (Tall, 1991). A number of research papers related to the transition period support this claim (Luk, 2005; Barnard, 2003; Hourigan & O'Donoghue, 2007; Selden, 2005; Kajander & Lovric, 2005; Clark & Lovric, 2008, 2009; Hong et al, 2009).

METHODOLOGY

Our aim was to present and systematise the responses of university lecturers from different countries to a short questionnaire about the transition from school to university education in mathematics. A cross-country approach was chosen to reduce the

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differences in cultures, curricula and education systems. The lecturers were surveyed using a combination of two non-probability sampling methods - judgement and convenience. The results of the survey can be treated as a pilot study. The questionnaire was sent to selected participants of international conferences on mathematics education who either teach university mathematics or write papers on mathematics education at university level or both. The response rate was 36% (63 lecturers from 24 countries). The questionnaire comprised of three open ended questions: about the reasons for the gap, the successful remedies that work at their universities and about ideas on what else can be done to narrow the gap.

We summarized the responses by categorizing the participants' answers to the questionnaire and calculating percentages. We also presented the most common answers, strategies and ideas expressed by the participants of the study and added our own comments. We believe that such an exchange of good strategies that work at some universities and suggested ideas that are worth trying can make readers implement them at their own institutions.

THE STUDY

The Questionnaire

The questionnaire given to the university lecturers consisted of the following 3 questions:

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Question 1. What do you think are the reasons for the gap between the school and university mathematics?

Question 2. What is your Department doing to reduce the gap?

Question 3. In your opinion what else can be done to make the transition period smoother?

The Participants' Responses

Below we report on the most common responses to the questionnaire. The percentage of participants that identified a similar response (broadly speaking) is indicated. Each participant was randomly assigned a number between 1 and 63, and this number is used to identify a participant with his or her response.

Question 1. What do you think are the reasons for the gap between the school and university mathematics?

Table 1. Reasons for the gap.

Reasons for the gap	Percentages
Higher level of thinking at university mathematics	72
Emphasis on passing the exam at school	37
School syllabus is too broad	34
Too optimistic assumptions and expectations of university lecturers	33

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Different ways of teaching/learning	30
Lack of mathematics background of mathematics teachers in schools and lack of teaching skills of university lecturers	26
Lack of communication between school and university	17
Changes in environment	15

Reason 1. Higher level of thinking at university mathematics (72%)

Different emphasis: on calculations, techniques, algorithms, manipulations at school versus on theory, proof, conceptual understanding at university. This difference is reflected in textbooks and assessment.

"The teaching style in schools encourages rote learning of disjointed 'facts' and algorithms which are not underpinned by understanding of the meaning of them or of the fundamental relationship between them." (20)

> "They learn maths almost without theoretical explanation and only calculation in high school days." (18)

"The school mathematics is aimed to coach for a formal solution of as many exercises as possible, with only superficial understanding the theory, under the everyday instructor supervision. On the contrary, university mathematics is aimed to give in-depth theoretical understanding through the formal delivering lectures with the minimal instructor supervision." (15)

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"High school math is very mechanical and situational. University math is more theoretical and eventually becomes proof-oriented. The material is qualitatively different and we expect more out of the students." (19)

"Most of our students have not seen formal proof before entering university." (22)

"There is a major jump in thinking level into the abstract world of proofs." (39)

"…'recipes' for doing standard problems. The result of this is that many students don't *have* to understand the ideas behind the problems, just *do* them, and others don't even realise that there is more understanding to have. Being able to perform the right steps is what maths is about." (51)

To illustrate the above concerns we give two real examples from final school year mathematics exams (university entrance).

Example 1.

Show that the equation $x^2 - \sqrt{x} - 1 = 0$ has a solution between $x = 1$ and $x = 2$.

The model solution given to the markers of the exam reads: "If $f(x)$ = $x^2 - \sqrt{x} - 1$ then $f(1) = -1 < 0$ and $f(2) = 1.58 > 0$. So graph of f crosses x-axis between 1 and 2."

The suggested solution is based on the special case of the Intermediate Value Theorem which has 2 conditions: the continuity of $f(x)$ on $[a, b]$ and the condition $f(a)$ x $f(b) < 0$. But only the second condition is checked and the first is ignored as if it was not essential. It was a written exam and all working was required to be shown. The fact that the condition of continuity of the function $f(x)$ was not required by the examiners to get full

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marks for this question was very disappointing. The message is clear – the manipulations are important but the properties of functions are not. No wonder that students don't pay attention to all conditions of the theorem and properties of the functions – it is simply not required.

Example 2.

Solve the equation $\log_2 (9x\text{-}1) - \log_2(x+2) = 3$.

Shown below is the model solution given to examiners:

$$\log_2 \frac{9x-1}{x+2} = 3$$
$$\frac{9x-1}{x+2} = 2^3$$
$$9x-1 = 8(x+2)$$
$$x = 17.$$

According to this solution, a check of the validity of the answer seems not to be essential. But ignoring the domain of the logarithm function may lead to the wrong answer as further illustrated by the following example:

$$\log_2(9x-10) - \log_2(2x-3) = 0$$
$$\log_2 \frac{9x-10}{2x-3} = 0$$
$$\frac{9x-10}{2x-3} = 1$$
$$9x-10 = 2x-3$$
$$x = 1.$$

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It was a written exam and all work was required to be shown. Again the message to students was clear – you can get full marks for a question if you only know how to perform calculations.

The above examples illustrate another concern of our respondents (and discussed later) – lack of communication between school and university, particularly in setting school mathematics exams. It is hard to imagine a university lecturer who would accept the model solutions for Examples 1 and 2 above as complete solutions for which the student would receive full mark.

Reason 2. Emphasis on passing the exam at school (37%)

> "Assessment culture at school means that many students do not learn to understand, they learn to pass exams." (21)

"School mathematics is not truly 'learned' and stored in long term memory but is quickly lost after the final examination is safely passed." (20)

Some participants expressed concerns that the mathematics education at the school level is more like training or drilling for certain skills and procedures. The following interesting fact illustrates the point. In New Zealand every three years there is a notable drop followed by a two year increase in the students' performance at the final school year mathematics exam. It was reported by a chief school mathematics examiner that the reason for the regular drops was the change of a chief school examiner every three years. A new chief examiner used their own language style of setting up exam questions which

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was different from the wording used by their predecessor. The students are sensitive to the wording of the examination questions and it is reflected in their performance.

Reason 3. School syllabus is too broad (34%)

"The high school curriculum is too thick for students to understand whole, so students pick up some maths classes so they loose the maths understanding." (31)

"The amount of math taught at secondary level is too big to enable students to really understand. The result is that they are trying to remember only instead of understand." (27)

Reason 4. Too optimistic assumptions and expectations of university lecturers (33%)

"… we often expect that (a) all students learn in the same way we did – and that's the best way (b) what students learned at school was the same as we did, and with the same depth." (51)

Reason 5. Different ways of teaching/learning (30%)

"Students are not prepared to assume responsibility for their learning – rather, they expect continuation of spoon-feeding from high school." (14)

"Many students have problems adjusting to studying at university. In particular they are used to a teacher planning their study for them and at university they have to do this themselves." (55)

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Reason 6. Lack of mathematics background of mathematics teachers in schools and lack of teaching skills of university lecturers (26%)

"The lack of well qualified teachers of maths in schools means that students do not all get a good background in maths in schools." (21)

"University staff are appointed because of their knowledge and research record in many cases, not because of their teaching skills, and this puts an extra onus on students to find their own way to understand content." (38)

Reason 7. Lack of communication between school and university (17%)

"We at Uni level … aren't keeping up with what is happening at schools." (48)

"We have almost no communication with the schools. The university and school sectors have almost no overlap." (51)

"Our … lack of knowledge and understanding of what is currently taught at school." (52)

Reason 8. Changes in environment (15%)

"For students, coming to university everything is new: new people, environment, social contexts, norms, expectations. Drastic decline in the amount of personal attention students get from their teachers, compared to high school. Large classes create intimidating situations." (14)

"Some students cannot cope with the freedom they have being away from home for the first time." (26)

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Question 2. What is your Department doing to reduce the gap?

Table 2. Existing remedies for reducing the gap.

Existing remedies for reducing the gap	Percentages
Personal approach	55
Bridging courses	52
Developing different pedagogical strategies	32
Improve communication between school and university	16
Change in assessment: weekly tests, oral exams, detailed feedback	16
Lower the standard	12

Remedy 1. Personal approach (55%)

- Learning support centres
- Small classes
- Individual consultations
- Streaming after diagnostic tests
- Extra tutorials

Remedy 2. Bridging courses (52%)

- Different levels
- Different length

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- Different emphasis – e.g. "courses concentrating on mathematical thinking (proof) rather than just pushing content." (21)

Some participants indicated that the bridging courses often don't fill the gap. The two main reasons mentioned where:

a) The courses are too short in duration. Many bridging courses are just a few weeks or, at most, months long. During that time some students are not able to master the material usually covered at school during several years.
b) The mathematical background of the students often is so poor that the emphasis in the bridging courses is on the basics of mathematics: rules, techniques, manipulations, and algorithms. There is no time to teach students higher level of thinking (proofs, reasoning, etc). So the gap in thinking is not filled.

Some universities however offer bridging courses that are much longer, sometimes over 1-2 years that aimed at filling the difference in thinking. In the transition programme described in (Leviatan, 2008) there are 4 units, one semester each:

- Introduction to Advanced Mathematics
- Reading, Writing and Reasoning in Mathematics
- Number Systems
- Definitions and Proofs in Mathematics (the post-calculus stage)

Some of the goals of that programme are:

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- An introduction to the mathematical "*culture*": its language, its logic rules, etc.
- Exposure to typical mathematical activities: generalization, deductions, definitions, proofs, etc.
- Introducing basic mathematical concepts, such as number systems, sets, functions, sequences, convergence, etc., all these concepts are defined rather vaguely at school.

Remedy 3. Developing different pedagogical strategies (32%)

- "We give one lecture on study skills for mathematics and problem solving techniques at the beginning of the year. Each member of our department acts as a mentor to our first year students." (55)
- "Setting weekly homework which is peer assessed in class following lecturer's working on board, thus trying to encourage an early engagement with new material and a revision of school material." (17)
- "A daily one-hour help session taught by current Masters students but this is basically a patch-up rather than developmental assistance." (20)
- "We aim to take things fairly slowly, with hand-out notes that have detailed explanations. We try to communicate to students just what is expected of them and how to go about achieving their goals in maths." (51)

Remedy 4. Improve communication between school and university (16%)

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- "In the summer we offer workshops for teachers, and each year we invite one high school teacher to join our department as a "visiting master teacher" to teach lower level courses and engage with the college faculty in discussions about mathematics and pedagogy. As a consequence, many members of our department explore new methods of teaching; several have received national grants to support this work." (16)
- "In-service day for secondary school teachers. Summer school for year 10, 11 students." (54)
- "Visit high schools and talk about university math courses, and the expectations that those courses place on students … do sessions for high school students (problem solving sessions and presentations on various math topics)." (14)
- "Many of our Department professors go to teach maths at school because they are able to tune proficiently themselves up to the children perception. Professor can imbue the minds of schoolchildren with interest to intriguing, challenging tasks, can give them a taste for non-standard solutions and half-open the curtain to what they will do in the university." (15)

Remedy 5. Change in assessment: weekly tests, oral exams, detailed feedback (16%)

- "We try to give them plenty of feedback on how they're progressing, with assignments and quizzes. We try to listen and respond to as much feedback

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from students as possible/reasonable in terms of pace, timing of assessment or other administrative areas." (51)

Remedy 6. Lower the standard (12%)

- "Taking out or reducing the mathematics requirements in courses so students can pass!" (38)

Question 3. In your opinion what else can be done to make the transition period smoother?

Table 3. Other ideas to smooth the transition.

Other ideas to smooth the transition	Percentages
Establish a system to monitor quality at schools and universities	60
Extras: tutorials, courses, learning support, pastoral care, streaming, time (slower pace; 'adjustment' semester; summer school)	56
Improve communication between school and university	38
More attention to mathematics education at universities	18

Idea 1. Establish a system to monitor quality at schools and universities (60%)

- Better preparation of school teachers
- Improving school curriculum (less content, more proof, depth and rigour)

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- Improving teaching skills of university lecturers - have tertiary teaching qualification
- "More deductive (although not necessarily formal) reasoning in high school would help.""A bit more depth and rigour can be included in school mathematics so that the transition can be smoothened." (3)
- "Making it compulsory for all tertiary teachers of mathematics to have a tertiary teaching qualification as well as their mathematics qualification, hence making them aware of ideas about teaching and strategies for teaching." (21)

Idea 2. Extras: tutorials, courses, learning support, pastoral care, streaming, time (slower pace; 'adjustment' semester; summer school) (56%)

- "Involve the university learning centre as much as possible." (46)
- "Talk to first-year university students about how to study *efficiently*. Teach them how to read a maths textbook, i.e. how to do maths on their own." (14)
- "Differentiating among the newcomers, not the first day but after a month or so. Based on the student's own conception of her/his ability, motivation and background and on results from school and the first period at university the student should get an offer to choose among different strands, maybe leading to the same goal but with options for teaching/learning methods, time spent on the material and so on." (57)

Idea 3. Improve communication between school and university (38%)

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- "Better contacts with secondary school teachers, in the hope that there could be changes from both sides and especially more information about what is expected from students and how to choose the best preparation for future studies." (26)
- "We must improve the network connecting university mathematics and school mathematics at all levels." (57)
- "Bring final year students into the university to see what it is like here." (36)

Idea 4. More attention to mathematics education at universities (18%)

- More research in didactics
- Establishing mathematics education units in mathematics departments
- "Set up a Centre with focus on maths/stats education." (54)
- "Including in mathematics departments a "Mathematics Education Group". Such a group might: legitimize pedagogical studies as a legitimate research area for tertiary teachers of mathematics hold regular educational seminars within the department that others could attend provide support for young faculty members who lack educational expertise mean that some maths ed journals are subscribed to, that would also perhaps influence the culture of the department." (21)

CONCLUSIONS AND LIMITATIONS

According to participants' responses the major reason for the gap between the school and university education in mathematics is due to the higher level of thinking in university

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mathematics (72%). The difference is a direct result of where the emphasis is placed by school teachers (calculations and manipulations) and university lecturers (conceptual understanding and rigor). This finding is supported by the theoretical research on the transition issue by Tall (1991, 1997, 2004a, b). As Tall (1991) writes "advanced mathematics, by its very nature, includes concepts which are subtly at variance with naïve experience. Such ideas require an immense personal reconstruction to build the cognitive apparatus to handle them effectively. It involves a struggle with inevitable conflicts which require resolution and reconstruction".

It is clear from the participants' responses that there is a lack of knowledge and awareness by university lecturers of what is happening at school. This shows a need for closer communication between teachers and university lecturers and their institutions, to include understanding of the unique nature of teaching and learning in each sector. To facilitate this will require a mechanism for greater sharing of ideas and practice between the two groups.

As we mentioned earlier the idea of this study has arisen from and is based on teaching practice. It was primarily a practice-based research study with the aim of identifying and promoting pedagogical strategies that may make the transition period smoother and more beneficial for student learning. In their responses, participants presented possible reasons

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for the gap, and a variety of remedies for bridging the gap that are employed within their own institutions. They have also expressed other ideas on the issue that are worth exploring in the future.

We are well aware about the limitations of the study. We did not do a systematic analysis of the existence of the gap between the school and university mathematics. The study presupposes the existence of the gap based on numerous publications and conference and personal communications that indicated that the gap exists. The sample from the population was taken mainly from lecturers who attend international conferences on mathematics education and therefore have strong interest in and very particular opinion on mathematics education at the university level. For this reason they might not be good representative of the population of university mathematics lecturers. Nevertheless it was interesting to notice that the lecturers from 24 countries with clearly different school environments still express the same concerns. It is hoped that the participants' responses to our questionnaire will cause the readers to reflect on their own teaching practice, and may wish to implement some of the suggested ideas at their own institutions.

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