Design of Sparse Antenna Array with Compressive Sensing

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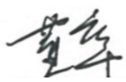
Statement of Originality

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person (except where explicitly defined in the acknowledgements), nor material which to a substantial extent has been submitted for the award of any other degree or diploma of a university or other institution of higher learning.



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Abstract

As compare to single-element antenna, antenna array can not only achieve better radiation performance, but also be able to control the radiation beam. Aiming to achieve the same radiation performance, sparse antenna arrays attempt to minimize the number of antenna elements and reduce the mutual coupling between antenna elements. For sparse antenna array, there are multiple design algorithms to optimize the antenna location, like nonlinear optimization methods including genetic algorithm (GA) and simulated annealing. Recently, a new method named compressive sensing (CS) was proposed. CS could accurately recover the original signal, even at a sampling rate lower than the Nyquist rate.

This thesis aims to design sparse antenna array that can achieve better performance compared with uniform antenna array, reducing the cost and optimizing antenna radiation patterns. Moreover, we aim to reduce the computational complexity. With CS, we start with a linear array and move on to a planar array, which is compared to the results obtained using the GA. Our results show that a sparse antenna array with 8 elements can be used to replace a uniform array with 100 elements. Next, both GA and CS can be used to optimize uniform planar antenna arrays. However, the computational time of CS is 350 times shorter than that of GA. In the future, we would like to apply CS in the design of three-dimensional (3D) sparse arrays.

Acronyms

CS	Compressive Sensing
DOA	Direction of Arrival
GA	Genetic Algorithms
AA	Annealing Algorithms
SA	Simulated Annealing
VSWR	Voltage Standing Wave Ratio
PCS	Personal Communication Systems
FM	Frequency Modulation
A/D	Analogue to Digital
MUSIC	Multi-Signal Classification
ESPRIT	Estimating Signal Parameters Vibrational Invariance Techniques
MVDR	Minimum Variance Undistorted Response
BP	Basic Pursuit
BPDN	Basic Pursuit Denoise
AAS	Adaptive Array Systems
SLL	Sidelobe Level
TDLs	Tapped Delay Lines
SDLs	Sensor Delay Lines
FIR	Finite Impulse Response
IIR	Infinite Impulse Response
RF	Radio Frequency
SOI	Signal of Interest
1-D	One-Dimensional Array
2-D	Two-Dimensional Array
3-D	Three-Dimensional Array
ULAs	Uniform Linear Arrays
SPGL1	Spectral Projected Gradient for l_1 Minimization
Aps	Wireless Access Points
SDMA	Space Division Multiple Access

Symbols

η	Antenna Efficiency
G	Peak Gain
D	Peak Directivity
R_i	Input Resistance
X_i	Input Reactance
M	Numbers of antenna elements
$x_0(t)$	Signal Received by First Sensor
$x_1(t)$	Signal Received by Second Sensor
$x_{\mathrm{m-1}}(t)$	Signal Received by M -1 Sensor
$y(t)$	Output Response
$w_0, \ldots, w_{\mathrm{M-1}}$	Weight Coefficients
d	Inter-element distance
λ	Wavelength
$S\ (\theta)$	Steering Vector
θ	Angle of arrival
φ	Azimuth angle
$p_0, \ldots, p_{\mathrm{M-1}}$	Array elements Vector
k	Wave Number
$B\ (\theta)$	Array Beam Pattern
x_m	x-coordinate of m
y_m	y-coordinate of m
ω	Angular frequency
$\tilde{T}$	Propagation Time
$e^{j\omega t}$	Impinging Complex Wave
$P\ (\omega,\ \theta)$	Response of the beamformer
τ	Propagation Delay
J	Unity Vector
$F\ (M)$	Aperture synthesis Cycle
T_e	Initial Temperature
Θ	Sparse Coefficient
Ψ	Sparse Matrix of $M \times N$
I	Vector Coefficient
ϕ_i	Column Vector
T_x	Transmitter
R_x	Receiver
L	Array Length
ρ	Spatial Resolution
O	Antenna Azimuth Dimension
$u_\mathrm{i}(t)$	Received Signal Amplitude
$Z(t)$	Phase of Received signal
g_{li}	Gain of First l Element to the i^{th} Signal
$n_\mathrm{i}(t)$	Noise of l_{th} element at time t
α	Difference of Desired response and Designed Response
β	Difference of Desired response and Designed Response

ϵ	Bound of Error Generated by Designed Response
ω	Angular Frequency
χ	Progress Phase Shift
E_r	Desired Response
$\|w\|_0$	l_o Norm Optimization
$\|w\|_1$	l_1 Norm Optimization
μ	Pareto Curve
s'	Error of Steering vector
c	Signal Propagation Speed
f	Three-Dimensional Spatial Frequency
V_G	Dummy Variable
σ	Difference of Desired and Designed Response

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Chapter 1

Introduction

1.1 Introduction

1.1.1 Background

Antennas transform the input electrical signals from a transmission line into electromagnetic waves [1]. They are widely used in wireless communications, such as cellular radios and television broadcast [2]. Due to reciprocity, an antenna can be used as a transmitting antenna or a receiving antenna [1]. In other words, an antenna is not only a device for radiating radio waves, but also an energy converter. It is an interface device between the circuit and the space. Antennas have many performance parameters, such as gain, radiation pattern and bandwidth [2]. The directionality of single-element antenna is limited [3], and it is hard to achieve beamforming. Therefore, antenna array was proposed to achieve better beamforming as compared to single-element antenna [4].

Antenna array consists of multiple antennas working at same frequency [5]. Antenna array can not only improve the direction of the radiation but also strengthen the intensity of the radiation [6]. The working principle of an antenna array can be regarded as the superposition of electromagnetic waves (electromagnetic fields). In general, antenna array can be divided into three categories by its dimensionality: (1) one-dimensional array, also known as linear array; (2) two-dimensional array can be defined as planar array [7] and (3) three-dimensional array. Due to the limited space, we focus on one-dimensional and two-dimensional arrays in this thesis.

As there are multiple elements associated with the arrays, the mutual coupling [4] between the elements degrades the overall performance of antenna arrays [8]. Besides, the more the number of antenna elements in the antenna array, the higher the cost to produce the array [9]. Sparse antenna array is a special type of phase-shifted antenna array, which can reduce the cost because it requires less number of antenna elements as compared with conventional antenna arrays. It can not only minimize the mutual coupling but also reduce the array size [10]. In particular, adjacent antenna elements in sparse antenna arrays that separate by a distance greater than half a wavelength can minimize peak side lobes [11]. As such, sparse antenna arrays have promising applications in wireless communications, radar and remote sensing.

Array signal processing is a significant area of the digital signal processing related to beamforming [5] [7] [12] [13]. Beamforming is known as a technology to achieve directional transmission and reception of wireless signals by using an antenna array. Beamforming algorithm is a key topic of researches on antenna arrays. Broadly speaking, there are two types of beamforming techniques according to the bandwidth, namely narrowband and wideband beamforming [7].

Recently, compressive sensing (CS) was proposed to optimize the locations of antenna elements while achieving the desired beamforming. This can be done by assigning different weights to different antenna elements in an antenna array. For example, the CS technology could recover the original data with adequate accuracy, even though they are sampled at a rate lower than the Nyquist rate. In other words, CS could recover certain signals and images (with sparsity) from fewer samples as compared to conventional methods based on Nyquist Sampling Theorem [11], [14]. An algorithm named SPGL1 is applied together with CS for the sparseness of antenna arrays. SPGL1 relies on matrix-vector operations and accepts both explicit matrices and functions that evaluate these products [15].

Antenna arrays should minimize the spacing between adjacent antennas, which is ideally less than a wavelength. [13]. Consequently, design of antenna arrays focuses on the optimization of antenna locations [16]. To achieve the desired radiation pattern while avoid mutual coupling, it is necessary to design the antenna array by using the minimum numbers of antenna elements, which is the key objective of sparse antenna arrays. Antenna locations can be optimized as a nonlinear problem [11] [17] [18], and then solved by using genetic algorithms [17] [18] and simulated annealing [11]. But these methods still have their respective drawbacks. For example, the required computational time is usually quite long [19], despite of their slow convergence speed that makes them difficult to obtain an optimal solution in practical applications. At the same time, CS implies a more efficient method to optimize sparse antenna array location compared with genetic algorithm (GA).

1.1.2 Motivation

Through the literature review, it is noticed that existing algorithms for antenna array design have their respective drawbacks and CS is a promising technology to complement these drawbacks for sparse array design. It thus motivates us to study the design of sparse antenna array using CS. The objective is to obtain a satisfied beamforming pattern while using the minimum number of antenna elements.

1.1.3 Objectives

The rationale behind CS is to assign different weights to antenna elements in a uniform array while minimizing the difference between the desired beamforming response and the designed beamforming response. To realize a sparse antenna array, the weights should have a maximum number of zeroes and a few non-zero values.

This study focuses on the design of one-dimensional linear sparse array and two-dimensional planar sparse array. For one-dimensional linear array, we use CS to compute the individual weight for each antenna element based on the desired beamforming pattern. If the corresponding weight of an antenna element is zero, it means that antenna element is no longer required in the sparse array, leading to an unequally spaced antenna array. For a two-dimensional uniform planar antenna array, there are $m \times n$ elements. If the weight of antenna array is zero, the positions of those antenna elements are no longer exist. Consequently, the resultant array is a sparse planar array.

1.1.4 Contribution

Sparse antenna array can avoid grating lobs, however, the tradeoff raised from using sparse array is their unpredictable behavior. As a result, it is necessary to optimize the antenna location. There are several methods to optimize the antenna location. As compared with other methods, CS is expected to reduce cost and computation time. Under CS, finding optimal antenna location can be normalized as a l_1 minimization problem, which is aimed to increase the sparseness. In this thesis, design of 2D sparse array has been studied with CS. Compared with GA, CS has its advantage of fast converging speed. The aim of this method is to bring the minimization of l_1 norm of weight coefficient closer to the l_0 norm. Previous work with CS normally focuses on traditional beamforming, where the steering vector is exactly known. In practice, there are various errors in antenna locations, mutual coupling etc. In this thesis, we have take into account these practical factors. The contribution of this research on sparse array design using CS include short computation time and minimized number of antenna elements to save the cost.

1.1.5 Thesis Organization

The thesis is organized as follows:

In Chapter 1, the background knowledge on antenna and antenna array was briefly revisited. Then, array signal processing related to beamforming was introduced. Drawbacks of existing beamforming algorithms were discussed, which motivates us to study how to apply compressive sensing in the design of sparse antenna array. The design objectives were briefly outlined, followed by the thesis organization.

Chapter 2 provides a thorough review of the literature related to sparse antenna array. It starts from the basic antenna theorem, then moves on to antenna arrays. The key connection between them is the theory on beamforming. Next, different types of beamforming techniques are discussed, followed by different algorithms to sparsify antenna arrays. Finally, CS is briefly introduced, followed by the model for radiation pattern calculation and the chapter summary.

Chapter 3 focuses on the design of one-dimensional sparse antenna array using CS. To achieve the optimization of antenna location [16], the problem can be solved as a weight assignment problem [14] [20]. CS forms a basis method to design sparse antenna arrays and obtain a satisfied beamforming pattern [6] [21]. Starting with a uniform antenna array [6], we can obtain the response of antenna array of conventional antenna array by calculating antenna weights w and steering vector d. Furthermore, for the sparse antenna array, we use SPGL1 algorithms to obtain the norm l_0 [15]. The purpose is to remove those antenna elements with a weight value equal to zero or approximately zero to achieve a sparse antenna array [6]. This chapter briefly discusses one-dimensional sparse array design by CS. The algorithm has described and numerical results are provided, followed by the chapter summary.

In Chapter 4, the design is expanded to a two-dimensional antenna array. We start with an m×n uniform planar array. For sparseness of a planar array, we use the SPGL1 algorithm to optimize it. Besides the CS method, GA algorithms are also discussed, which uses the model of simulating natural evolutionary processes to screen antenna weights. By removing those antenna elements with a weight value of zero or approximately zero, a sparse antenna array can be obtained. As compared with CS, GA has longer computational time. In this chapter, the first part briefly introduces the GA method, including its advantage and disadvantage. Numerical experiments have been described and performance comparison of CS and GA methods is provided, followed by the chapter summary.

Chapter 5 concludes this thesis and presents possible future work related to this topic.

Chapter 2

Literature Review

2.1 Fundamentals of Antenna

Antennas become popular with the flourish of wireless communications [22]. Antenna is a converter that transforms the electrical signals fed by a transmission line into electromagnetic waves, which are propagating in the unbounded medium [2]. Due to reciprocity, an antenna can be used as both a transmitting antenna and a receiving antenna with a transmit / receive switch (T/R switch) [2], thus the antenna is a key component in the radio frontends to transmit or receive electromagnetic waves [23]. In practice, antennas are widely used for cellular communications, television, radar, navigation, remote sensing, radio astronomy and other engineering systems [22].

2.1.1 Types of Antennas

There is not a unique taxonomy for antenna types. According to the nature of antenna characteristics, it can be divided into transmitting antennas and receiving antennas [22] [24]. By applications, it can be divided into cellular radio antennas, radio broadcasting antennas, television antennas, radar antennas and so on [22]. According to directivity, antennas can be categorized into directional antennas and omnidirectional antennas. According to operating wavelength (frequencies), it can be divided into ultra-long wave antenna, long wave antenna, medium wave antenna, short wave antenna, ultrashort wave antenna, microwave antenna and so on [22]. According to the structure and working principle, it can be divided into wire antennas and surface antennas.

According to dimensionality, it can be divided into two types: one-dimensional antenna and two-dimensional antenna [7]. For one-dimensional antennas, they usually consist of many wires. Unipolar and bipolar antennas are two basic one-dimensional antennas. For two-dimensional antennas, their shapes vary from flaky (a square metal), array (well-organized two-dimensional pattern), trumpet shaped to saucer shaped.

According to the different use scenarios, antennas can be divided into three categories: *handheld antennas*, *vehicular antennas* and *base station antennas*.

- Handheld antenna is an antenna used for personal handheld devices such as smartphones, or walkie-talkies [22]. For handheld antennas, there are two main types—rubber antenna and pull rod antenna.
- Vehicular antenna is used on vehicles, where a common type is sucker antennas. Vehicular antennas include quarter-wavelength antennas, central sense, 5/8 wavelengths, dual 1/2 wavelengths and other forms of antenna.
- Base station antennas play a very important role in the whole cellular communication system. The common base station antennas are fiberglass high gain antenna, four ring array antennas (eight ring array antenna) and directional antennas.

2.1.2 Antenna Characteristic Parameters

Antenna parameters describe basic antenna properties, as well as their performance metrics. Characteristic parameters of antennas are radiation pattern, gain, efficiency, directivity coefficient, input impedance, radiation efficiency, polarization and bandwidth.

2.1.2.1 Radiation Pattern

The antenna radiation pattern is also called the radiation pattern or the far-field pattern. The radiation pattern of antennas is a mathematical function or graphical representation used to represent the antenna directivity. Because the reciprocity principles, the radiation pattern is the same for antenna in the receiving or the transmitting modes [22].

The antenna pattern can be divided into horizontal plane pattern and plumb surface pattern. Antenna pattern is an important figure to measure the performance of the antenna, and the parameters of an antenna can be observed from the antenna pattern. In order to compare the directional pattern characteristics of various antennas, some characteristic parameters are required. It mainly includes main lobe width, sidelobe level, front to back ratio, directional coefficient, which are discusses as follows.

- Main lobe width is the physical quantity that measures the sharpness of the maximum radiation area of the antenna.
- Sidelobe level is the level of the first side lobe nearest the main lobe and the highest level, usually expressed in decibel.
- Front to back ratio is the ratio of the maximum radiation direction (forward) level to its opposite direction (backward) level, usually in decibel units.
- Directional coefficient is the ratio between radiation power flow density of antenna in the maximum radiation direction and ideal non-directional antenna radiation power flow density with the same radiation power at the same distance.

Because the antenna pattern is usually looks like a flower, it is also known as the lobe map. The beam within the first zero radiation direction is called the main lobe on both sides of the maximum radiation direction. The beam opposite to the main lobe is called the back flap. The beam between the other zero radiation directions is called the sidelobe.

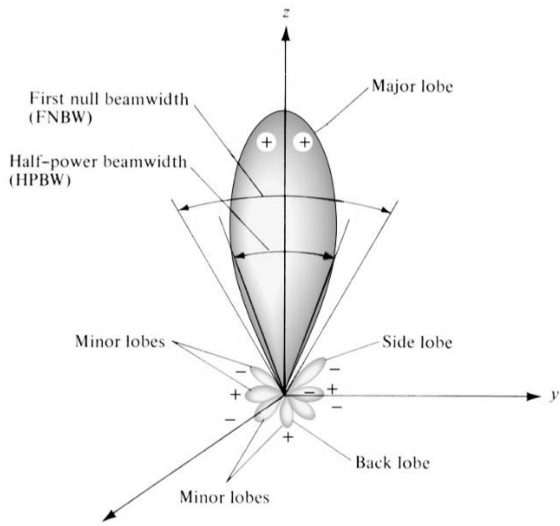

Figure 2.1 Radiation lobes and beam width of an antenna pattern [2]

Antenna gain can be defined as the ratio of the actual antenna power density to the ideal radiation unit generated at the same point in the space when the input power is equal. It quantitatively describes the degree of input power radiation. Antenna gain is used to measure the ability of antennas to transmit and receive signals in a specific direction. For example, it is one of the most important parameters for selecting base station antennas. Antenna gain is very important for the quality of mobile communication system because it determines the signal level of the cell edge. Increasing the gain can increase the coverage of the network in a definite direction or increase the gain margin within a certain range. Any cellular system is a two-way process, increasing the gain of the antenna can reduce the gain budget of the bidirectional system. The gain is obviously related to the antenna pattern. The narrower the main lobe of radiation pattern, the smaller the sidelobe and the higher the gain. Generally, gain enhancement mainly depends on reducing the width of the vertical radiating lobe, while maintaining the omnidirectional radiation performance on the horizontal plane. The antenna gain cannot be obtained directly from radiation pattern, but the directional coefficient is obtained from the radiation pattern. Antenna gain is the product of directional coefficient and antenna efficiency. The direction coefficient is usually greater than the gain. The directivity is calculated by measuring the radiation pattern related to the gain. This can be written by approximating the integral as a finite sum. The efficiency is the ratio of the peak gain to the peak directivity [22].

$$\eta = \frac{G}{D} \tag{2.1}$$

2.1.2.2 Directivity Coefficient

To determine directional coefficients of directional antennas, ideal omnidirectional antennas are usually used as the criteria for comparison. To quantitatively describe the intensity of antenna direction, the ratio of the power density at the maximum radiation direction in the far field and same power density of the omnidirectional antenna is defined as the antenna directivity coefficient. According to the above definition, because the radiation intensity of the directional antenna varies in all directions, the directional coefficient of the antenna varies with the position of the observation point, and the directional coefficient is the largest in the direction of the maximum radiation field. Generally speaking, the directional coefficient of the directional antenna should be taken as the directional coefficient of the maximum radiation direction. The directivity coefficient can be used to compare the strength of different antenna directivity.

2.1.2.3 Input Impedance

The connection between the antenna and the feeder is called the input or feed point of the antenna. For wire antennas, the ratio of the voltage to the current of the antenna input is called the input impedance of the antenna. For surface type antennas, the impedance matching property of the antenna is usually represented by voltage standing wave ratio (VSWR) on the feeder. Generally, the input impedance of the antenna is complex. The real part is called input resistance, expressed in R_i, and the imaginary part is called input reactance, expressed in X_i.

The input impedance of antenna is related to the geometry and size of the antenna, the location of the feed point, the working wavelength and the surrounding environment. When the diameter of the antenna is relatively large, the input impedance changes gently with the frequency, and the impedance bandwidth of the antenna is wide. When antenna array is considered, the antenna elements in the array interact in a complex way. This phenomenon is called *mutual coupling*. The result makes the current on one antenna element not only related to its own excitation, but also related to the current on the adjacent antenna.

The main purpose of studying antenna impedance is to achieve the impedance matching between antennas and feeders. To match the transmitting antenna with the feeder, the input impedance of the antenna should be equal to the characteristic impedance of the feeder. Next, to match the receiving antenna with the receiver, the input impedance of the antenna should be equal to the conjugate complex of the load impedance. When the impedance of the antenna is complex, it is necessary to use the matching network to remove the reactance part of the antenna and to make the resistance part of the antenna equal.

2.1.2.4 Radiation Efficiency

Antenna efficiency refers to the ratio of the power radiated by the antenna (i.e., the power that is effectively converted to electromagnetic waves) to the active power input to the antenna. The ratio should be a constant less than unity.

The efficiency of antenna is generally defined as the ratio of the radiation power to the input power of the antenna, which naturally connects the directivity coefficient and the gain. Antenna efficiency measures the quality of the antenna and the loss of the echo at a predetermined frequency. It is obtained through the specific heat capacity of the antenna according to the material of the antenna. The antenna is divided into multiple equal parts according to the area. When the antenna reaches the heat balance, the temperature values of each equal working temperature and the ambient temperature around the antenna are measured, and the dissipative heat of the antenna is calculated according to the specific heat capacity, the mass, the working temperature value and the ambient temperature value, the input power of the antenna is calculated according to the predetermined power and the echo loss. Antenna efficiency is calculated by input power and dissipative heat.

2.1.2.5 Polarization and Bandwidth

Antenna polarization is the parameter to describe the vector space direction of antenna radiation electromagnetic wave. Because of the constant relationship between electric field and magnetic field, the direction of polarization of electromagnetic radiation is usually taken as the direction of the space vector of the electric field.

The polarization of antenna is divided into linear polarization, circular polarization and elliptical polarization. Linear polarization is subdivided into horizontal polarization and vertical polarization. Similarly, circular polarization is subdivided into left circular polarization and right circular polarization.

Linear polarization can be defined by a confinement of the electric field to a given plane along the direction of propagation. Circular polarization has the following properties: (1) the angle between the polarization surface of the radio wave and the line surface of the earth changes from 0~360 degree periodically; (2) the size of the electric field is constant; (3) the direction changes with time; (4) the trajectory of the end of the field vector is projected as a circle in the plane perpendicular to the direction of propagation.

The antenna bandwidth is generally related to the working frequency, which ensures the range of the allowable frequency for the designed electrical parameters. The working bandwidth of the general omnidirectional antenna can reach 3-5% of the working frequency range, and the working bandwidth of the directional antenna can reach 5-10% of the working frequency.

2.2 Antenna Array

In a communication system, especially in a point to point communication system, the antenna is required to be directional. In other words, the antenna can radiate most of the energy at a certain direction. However, a single symmetrical antenna increases with the electrical length of the symmetrical antenna arm. The main lobe of its direction is narrowed, and the directionality will be improved. However, when the power length is greater than 0.5, the reverse current will appear on the antenna. This will cause the main lobe to become smaller and the sidelobe larger, degrading the directivity of an antenna.

It is not feasible to increase the direction of the antenna by increasing the length of the antenna. Therefore, antenna array is proposed.

Antenna array is multiple antennas working at same frequency instead of one single antenna. An array has two or more antenna elements which spatially produce a directional radiation pattern [25]. Antenna array not only improves the direction of the radiation but also strengthens the intensity of the radiation. The working principle of the antenna array can be regarded as the superposition of electromagnetic waves (electromagnetic fields). For a series of electromagnetic waves, when they are transmitted to the same area, according to the superposition principle, the electromagnetic wave will generate vector superposition. The result of superposition is related not only to the amplitude of electromagnetic waves, but also to the phase difference between them. There are many different types of antenna arrays including straight row array, vertical array, end-fire array, round rod antenna etc.

- The straight row array is a type of the vertical array, and each antenna element is placed along the same straight axis line. They are omnidirectional in horizontal direction, but the radiation angle in the vertical plane is small. Therefore, they are suitable for making very good base station antennas for mobile radio systems. Many cellular wireless systems and base stations of PCS systems use straight row arrays.
- The vertical array is the array that antenna units are arranged in the same direction as multiple columns. The spindle of the array is perpendicular to the spindle of the unit, and can form a vertical array, so that the spindle is placed vertically. Although the antenna element in the array is not straight row, it is still in the same

phase. The polarizing method of array is different from straight row array.

- The dipole antenna is an example of end-fire array. The cross connection of the feed lines of each adjacent antenna element in the above array is changed to parallel connection, which makes the phase difference between the two antennas adjacent to each antenna element is 180°. The radiation from an antenna element is counteracted with adjacent radiations in the vertical direction.
- Round rod antenna also uses the dipole antenna element. For example, the performance of the rod antenna in the horizontal plane is obtained, and the polarization mode is horizontal. The difference between the feed phase of the dipole antenna is 90°. The rod antenna is often used for FM broadcast reception. In this application, the rod antenna does not need the rotor to show proper performance in all directions.

For smart antenna signal processing, it can be used in two types of array systems—phased array system and adaptive array systems (AAS). Phase array systems have a fixed patterns number [26], it can be switched to the required pattern according to the desired direction. Adaptive array systems use to adaptive beamforming, it has an infinite patterns number that can adjust the requirement in real time [26].

The working principle of the antenna array can be regarded as the superposition of electromagnetic waves (electromagnetic fields). For a series of electromagnetic waves, when they are transmitted to the same area, according to the superposition principle, the electromagnetic wave will generate vector superposition. The result of superposition is related not only to the amplitude of electromagnetic waves, but also to the phase difference between them.

The phase composition of electromagnetic waves consists of three parts: time phase, spatial phase and initial phase. As far as the initial phase is concerned, the initial phase is determined when the transmitting antenna and the working frequency are determined, and the time phase is determined at the time of the encounter of several columns of electromagnetic waves. Only the space phase may change, because the electromagnetic waves from each unit of the antenna array are different, and the electromagnetic waves from each other are transmitted to the same connection. The spatial path of the area is different, which results in different spatial phase values. It is due to the difference in the spatial phase caused by the electromagnetic wave transmitted by the transmitting antenna in different positions to the same receiving area, which will inevitably cause several electromagnetic waves to form the same phase superposition in the meeting area, the total field intensity enhancement, the reverse phase superposition and the total field intensity weaken. If the enhancement and weakening of the total field intensity is relatively fixed in space, the structure of the radiation field of a single antenna is changed by the antenna array, which is how the antenna array changes the size and direction of the radiation field.

2.2.1 Electromagnetic Waves Superposition

Array antennas can achieve interesting radiation characteristics that are different from conventional single-element antennas. It can form stronger radiation pattern by pointing its main lobe to a direction in the space because the radiated electromagnetic waves from multiple coherent radiation units are superimposed. As shown in Figure 2.2, the resultant radiation electromagnetic waves be strengthened in some regions and weakened in other regions.

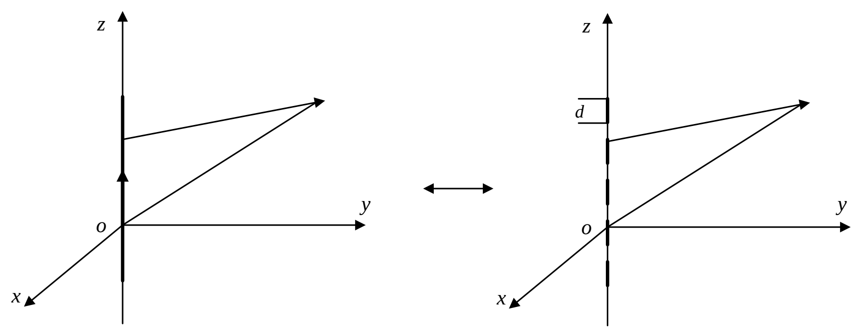

Figure 2.2 The superposition of electromagnetic waves

The continuous current distribution appears discrete as several small parts. The radiation field formed by small current can fully replace the original distribution of the continuous current. This is the theoretical basis for the existence of array antennas.

Furthermore, it is possible to use several small parts of current in antenna arrays instead of the continuous current distribution of single antenna to achieve more benefits. The current distribution on the conductor or medium as an antenna depends on its boundary conditions. Once those factors, including material type, shape, structure, installation position, and excitation mode of the antenna are determined, the distribution of current on the antenna is determined. It is difficult or even impossible to adjust the distribution mode of the current and form certain required radiation pattern. Because the radiation characteristics and radiation field are also determined, it is difficult for one to perform the adjustment and control.

However, when a single antenna is included as a radiating element in an array antenna (e.g., composed of multiple radiant units), the objective current distribution can be controlled by adjusting the amplitude and phase of feeding signal to each small radiating unit. This method can be used to obtain designed antenna radiation characteristics without changing the antenna material, shape and structure. This gives us the primary advantage of array antenna.

2.2.2 Principles of Pattern Multiplication

Consider an array antenna adopting similar or identical radiation elements, without considering the mutual coupling between adjacent elements, it can be deduced that the normalized direction of each element is the same.

Since the element factor only represents the radiation characteristics of each unit of the array antenna, it depends on not only the form, but also the direction of the unit. Therefore, the unit factor is the normalized pattern function of single antenna element at the origin of coordinates. The array factor is only dependent on the shape of the array, the interval of the element, the amplitude and phase of the element excitation current, which is independent of the form and orientation of the element. The unit factor and the array factor are independent and separable, which determine the radiation characteristics of the array antenna separately.

Define the element factor as $f_1(\theta,\varphi)$ and the array factor as $f_2(\theta,\varphi)$. The array radiation pattern can be written as

$$f(\theta,\varphi) = f_1(\theta,\varphi) \times f_2(\theta,\varphi) \tag{2.2}$$

This is known as pattern radiation multiplication principle. Based on this theorem, the radiation characteristics of the array antenna are only determined by the organization mode of the array. The radiation characteristics of the array antenna can be obtained by multiplying the array factor and element factor. Figure 2.3 illustrates this principle more clearly.

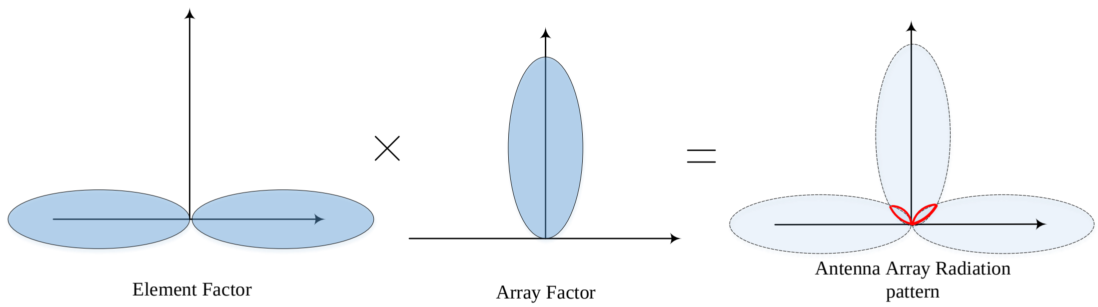

Figure 2.3 Radiation multiplication principle

Suppose element factor $f_1(\theta,\varphi)$ is symmetric ellipse and the array factor $f_2(\theta,\varphi)$ is vertical ellipse. The antenna radiation pattern is the common area of element factor and array factor.

2.2.3 Uniform Linear Array

Uniform linear array is considered as an array with M antenna elements which are located with equal spacing. The first antenna receives the signal $X_0(t)$ and the second signal $X_1(t)$ received by the second antenna. Finally, $(M\text{-}1)^{\text{th}}$ antenna receives the signal as $X_{M\text{-}1}(t)$. w_0, w_1, … $w_{M\text{-}1}$ are the weights of the array.

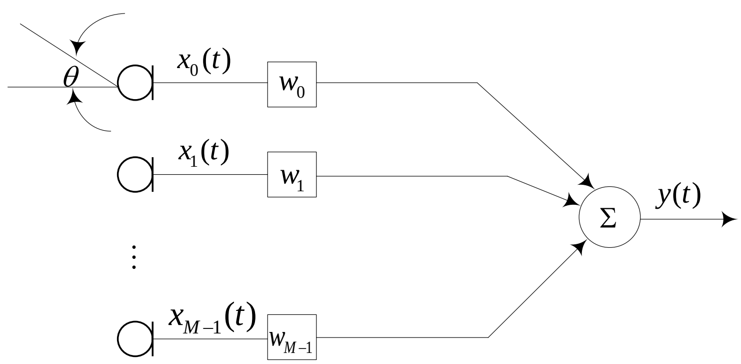

Figure 2.4 Uniform linear array with 0~N antenna elements.

Figure 2.4 shows an array with M elements that are uniformly located. Steering vector is the array response vector, which is also known as directional vector. d is the distance between two antenna elements. λ is the wavelength.

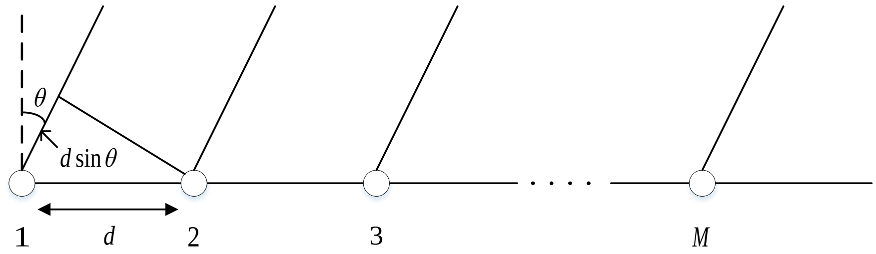

Figure 2.5 An Equal spaced linear array with M elements.

$\frac{2\pi}{\lambda}d\sin\theta$ is the phase difference between two adjacent antenna elements. The steering vector is

$$S(\theta)=\left[1,e^{j\frac{2\pi}{\lambda}d\sin\theta},\ldots,e^{j(M-1)\frac{2\pi}{\lambda}d\sin\theta}\right] \tag{2.3}$$

The array beam pattern can be defined as

$$B(\theta)=w^H S(\theta) \tag{2.4}$$

w is the weight coefficient of array, *w* can be expressed as $w=[w_0,w_1,\cdots,w_{M-1}]$.

When *w* is uniformed, the beam pattern can be defined as

$$B(\theta)=\frac{1}{M}\frac{\sin(\frac{\pi Md}{\lambda}\sin\theta)}{\sin(\frac{\pi d}{\lambda}\sin\theta)} \tag{2.5}$$

When *n*=0, the maximum value of $B(\theta)$ is the main beam. When *n* equals other values, the maximum value of $B(\theta)$ is a side beam.

When $\sin\theta=n\frac{\lambda}{Md},\sin\theta\neq n\frac{\lambda}{d},n=1,2,\cdots$, $B(\theta)$ is 0, $B(\theta)$ is at the zero point. The first zero point is at $\pm\sin\theta=n\frac{\lambda}{Md}$, the width of the first pair of zero point is $2\frac{\lambda}{Md}$. Half frequency main beam width is $\frac{\lambda}{Md}$ If the maximum response direction adjusted to θ_0 direction, the beam pattern can be written as

$$B(\theta) = \frac{1}{M} \frac{\sin[\frac{\pi M d}{\lambda}(\sin\theta - \sin\theta_0)]}{\sin[\frac{\pi d}{\lambda}\sin\theta - \sin\theta_0]}, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \tag{2.6}$$

To ensure there is no side lobe between $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, the following condition is required:

$$\frac{d}{\lambda} \le \frac{1}{1 + |\sin\theta_0|} \tag{2.7}$$

If $-\frac{\pi}{2} \le \theta_0 \le \frac{\pi}{2}$, Thus, $\frac{d}{\lambda} \le \frac{1}{2}$. If $u = \sin\theta$, $B(u)$ can be written as

$$B(u) = \frac{1}{M} \frac{\sin(\frac{\pi M d}{\lambda} u)}{\sin(\frac{\pi d}{\lambda} u)} \tag{2.8}$$

2.2.4 Non-uniform Linear Array

Minimum redundant linear array is a typical non-uniform linear array. It is widely applied in radio telescopes and microwave radiometer, which receive signals from the observed object passively. The received signal is uncorrelated. It uses interferometry techniques to obtain different outputs of base line. The longest base line decides the spatial resolution. The integrity of other base lines decides the coverage of spatial frequency. Minimum redundant linear array utilizes the minimum number of antenna units to obtain the longest base line while maintaining the coverage of other base lines.

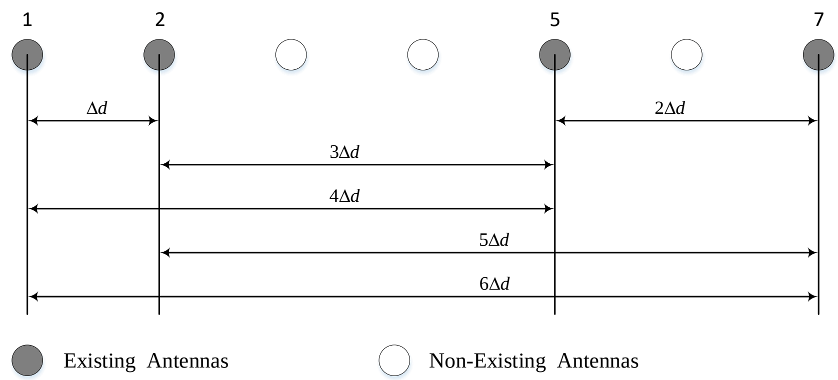

Figure 2.6 A diagram of minimum redundant linear array

From Figure 2.6, there are four units located at 1, 2, 5, 7. The combination of location difference is from 0 to 6 and at least one combination is existing. The optimal minimum redundant linear array can be defined as there are no same number from the location difference combination except 0. If the number of units exceed 4, the minimum redundant linear array is non-optimal. The sparseness of an array is a selection problem related to the number of possible antenna locations.

2.2.5 Planar Array

Planar arrays are versatile, widely applied in tracking radars, remote sensing and wireless communications. A planar array can be defined as an array with all the array elements located at a same planar. θ is the elevation angle while φ is the azimuth angle. As shown in Figure 2.7, let us consider a model of a linear antenna array system consisting of four antennas with one desired signal and one interference signal. P_0, P_1, $\cdots$, P_{M-1} are the array elements located at a planar array. The steering vector is

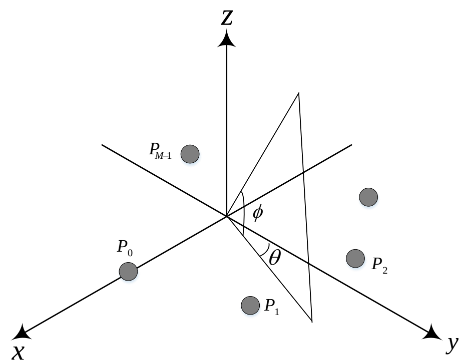

Figure 2.7 A plane wave propagating in the direction of z-axis

$$S = \begin{bmatrix} e^{jk^T P_0} \\ e^{jk^T P_1} \\ \vdots \\ e^{jk^T P_{M-1}} \end{bmatrix} \tag{2.9}$$

where k is the wavenumber.

$$k = \frac{2\pi}{\lambda}\begin{bmatrix} \cos\varphi\sin\theta \\ \cos\varphi\cos\theta \\ \sin\varphi \end{bmatrix} \tag{2.10}$$

The array beam pattern can be defined as

$$B(u_x, u_y) = \sum_{m=0}^{M-1} \omega_{M-1} \exp\left[jk_0 (x_m u_x + y_m + u_y) \right] \tag{2.11}$$

$u_x = \cos\varphi\sin\theta, u_y = \cos\varphi\cos\theta$, $k_0 = |k| = \frac{2\pi}{\lambda}$, x_{m} is x-coordinate of m. y_{m} is y-coordinate of m.

2.3 Beamforming

Beamforming is a technology to achieve focused wireless transmission by using a digital signal processor [20] [27]. Beamforming techniques can be used to create a desired required antenna directive pattern to achieve required performance under certain conditions [26]. It is usually used with an antenna array system [26].

Beamforming algorithm is the core content of smart antenna research. As a powerful signal processing method, beamforming is generally applied in radar and wireless communication systems. This method has its advantage of avoiding the noise and interference by thoroughly processing the received signal.

As a traditional filter, beamforming can be referred to as the space extension [19]. This method is used to obtain the designed signal while reduce the noise and interference [28]. In addition, it is noticed that DOA estimation is front-end processing of the beamforming [29].

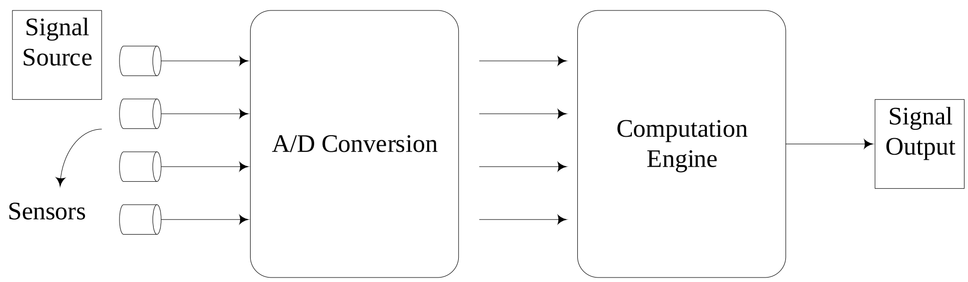

Figure 2.8 A digital beamforming system

The sensors of array are responsible to receive the signals. The signals from the signal source are transmitted to the computation engine. During the transmission, signals will be sent through the analogue to digital (A/D) conversion [30]. The desired signal is obtained after these processes. The obtained signal will be the output for further processing [31].

The beamforming algorithm can be divided into an adaptive algorithm based on direction estimation [32]. A method based on a training signal or a reference signal. The beamforming method based on the signal structure; it is divided into a non-blind algorithm and a blind algorithm based on whether a reference signal is needed.

The adaptive algorithm based on direction estimation can divided into two cases. In the first case, referring to the direction of the user signal, adaptive weights can be calculated according to different criteria, such as linear constrained minimum variance criterion, maximum likelihood criterion and maximum signal to noise ratio criterion. In the second case, the direction of the reference user signal is unknown. At this time, DOA can be estimated according to multi signal classification (MUSIC) [29], rotation invariant technology, estimating signal parameters vibrational invariance techniques (ESPRIT) [33] and so on. Although this method is more convenient in analysis, it has some problems such as high computational complexity and high sensitivity to errors.

2.3.1 Narrowband Beamforming

Narrowband means that the bandwidth of the impinging signal should be narrow enough to make sure that the signals received by the opposite ends of the array are still correlated with each other.

Signal of interest (SOI) including noise and interference can be obtained from specific directions. The sensors from different positions collected the spatial samples and aimed to eliminate the interference signals [31]. Figure 2.9 shows a simple structure base on a linear array. The output $y(t)$ at time t is given by an instantaneous linear combination of these spatial samples $x_{M-1}(t)$.

$$y(t) = \sum_{m=0}^{M-1} x_{M-1}(t)\omega_{M-1}^{*} \tag{2.12}$$

where * denotes the complex conjugate.

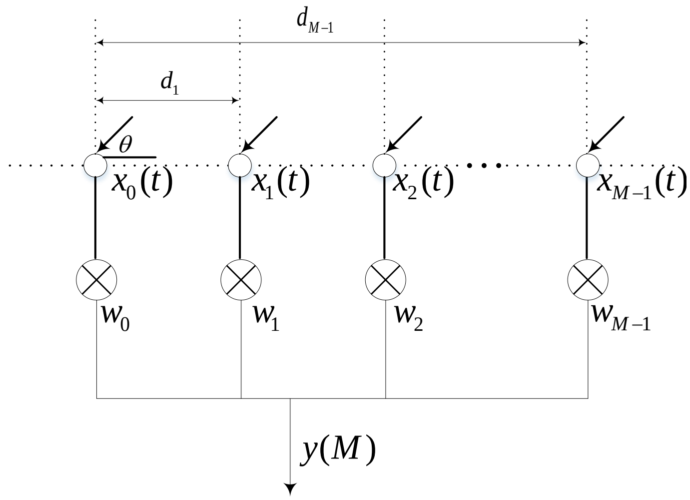

Figure 2.9 General narrowband beamforming structure

Imagine there is an impinging complex plane wave $e^{j\omega t}$, ω is the regular frequency. θ is DOA angle. As shown in Figure 2.9, let us assume the phase is 0 at first antenna. Then the signal received by the first antenna is $x_0(t) = e^{j\omega t}$, and by the M-1th antenna is $x_{M-1}(t) = e^{j\omega(t-\tau_{M-1})}$, $m = 1, 2, \cdots, M-1$. τ_{M-1} is the propagation delay for the signal from antenna 0 to antenna *m*. Then, the beamformer output is:

$$y(t) = e^{j\omega t} \sum_{m=0}^{M-1} e^{-\omega\tau_{M-1}} \omega_{M-1}^{*} \tag{2.13}$$

If $\tau_0 = 0$, The response of beam former can be written as

$$P(\omega, \theta) = \sum_{m=0}^{M-1} e^{-j\omega\tau_{M-1}} \omega_{M-1}^{*} = w^H d(\omega, \theta) \tag{2.14}$$

where the weight coefficient *w* has the *M complex conjugate* coefficients of the sensor. *w* is given by,

$$w = [w_0, w_1, \cdots w_{M-1}] \tag{2.15}$$

$d(\omega, \theta)$ which refers to steering vector can be given by

$$S = d(\omega,\theta) = [1e^{-j\omega\tau_1} \cdots e^{-j\omega\tau_{M-1}}]^T \tag{2.16}$$

Steering vector is also referred to as direction vector. Because a steering vector means several sets of phase delays a plane wave experiences, evaluated at a set of array antenna elements [22]. The phases are specified with an arbitrary origin[22] [2]. The operators $\{.\}^T$ and $\{.\}^H$ are transpose and Hermitian transpose, respectively [8].

The signals have same angular frequency ω and wavelength λ. DOAs are different with θ_1 and θ_2. $d(\theta_1,\omega) = d(\theta_2,\omega)$

$$e^{-j\omega\tau_{M-1}(\theta_1)} = e^{-j\omega\tau_{M-1}(\theta_2)} \tag{2.17}$$

The response of the uniformly spaced narrowband beamformer is

$$P(\omega,\theta) = \sum_{m=0}^{M-1} e^{j\omega\pi\sin\theta}\omega_{M-1}^{*} \tag{2.18}$$

From (2.18), the amplitude response result $|P(\omega,\theta)|$ with θ defined as direction of arrival angular can be obtained. $|P(\omega,\theta)|$ is the beam pattern. It describes the sensitivity of the beamformer.

$$BP = 20\log_{10}\frac{|P(\theta,\omega)|}{\max|P(\theta,\omega)|} \tag{2.19}$$

2.3.2 Wideband Beamforming

Narrowband beamforming structure has a degraded performance when the signal bandwidth increases. In wideband beamforming, (TDLs) or the FIR/IIR filter can be performed a temporal filter to form the dependent frequency response of each received signals [7]. Sensor delay lines (SDLs) can be used to achieve the frequency dependent weights [7]. Ideally, the response of SOI cannot be changed and interfering signal response should be equal or close to zero. Figure 2.10 shows a general wideband beamforming structure.

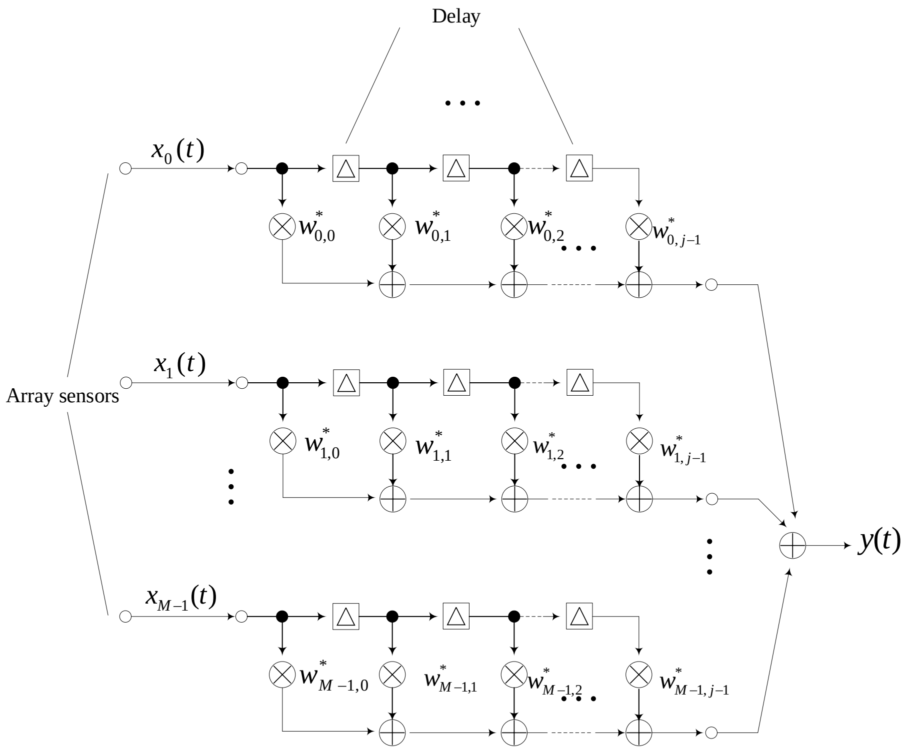

Figure 2.10 A general structure for wideband beamforming

The weight coefficients can be expressed as:

$$w = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_{j-1} \end{bmatrix} \tag{2.20}$$

where each vector w_i, $i = 0, 1, \cdot\cdot\cdot, j-1$, contains the *M complex conjugate* coefficients found at the i^{th} tap position of the *M* TDLs, and is expressed as:

$$w_i = [w_{0,i} w_{1,i} \cdots w_{M-1,i}]^T \tag{2.21}$$

The weight coefficients should be constant in narrowband beamforming. Therefore, narrowband beamforming structure will not be useful in the wideband beamforming structure.

The wideband beamforming response is

$$y(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{J-1} x_{M-1}(t - i\tilde{T}_{M-1}) w_{M-1} \tag{2.22}$$

$j-1$ is the amount of delay elements.

2.3.3 Adaptive Beamforming

As indicated by the name, adaptive beamforming can configure the antenna radiation response by using adaptive beamforming algorithms iteratively to approximate the optimum weights. As one of the core of adaptive array signal processing, the adaptive beamforming algorithm is developing very fast. Adaptive beamforming began with the pioneer work of Howells [34] and Applebaum [25]. Since then, many beamforming algorithms were developed. The traditional adaptive beamforming algorithm is derived under ideal conditions. However, in practical applications, it has difficulty in handling various model errors, such as array position error, channel error, mutual coupling and interference coherence [35]. This leads to performance degradation of the traditional adaptive beamforming. Therefore, it is necessary to study the robustness of adaptive beamforming algorithm [20] [36] [28].

The early representative adaptive beamforming algorithm was a multi-point constrained LCMV robust beamforming algorithm [37]. In order to deal with low output signal to noise ratio (SNR) caused by the errors, covariance matrix method was proposed to estimate the adaptive beamforming [5]. Nowadays, robust beamforming algorithm is based on sample covariance or feature subspace [38].

Adaptive beamformer can be regarded as a frequency selective filter [30]. The purpose of this method is to mitigate the effect of noise and interference signals. Adaptive beamforming combines the signal properties received from the arrays to form a desired radiation pattern.

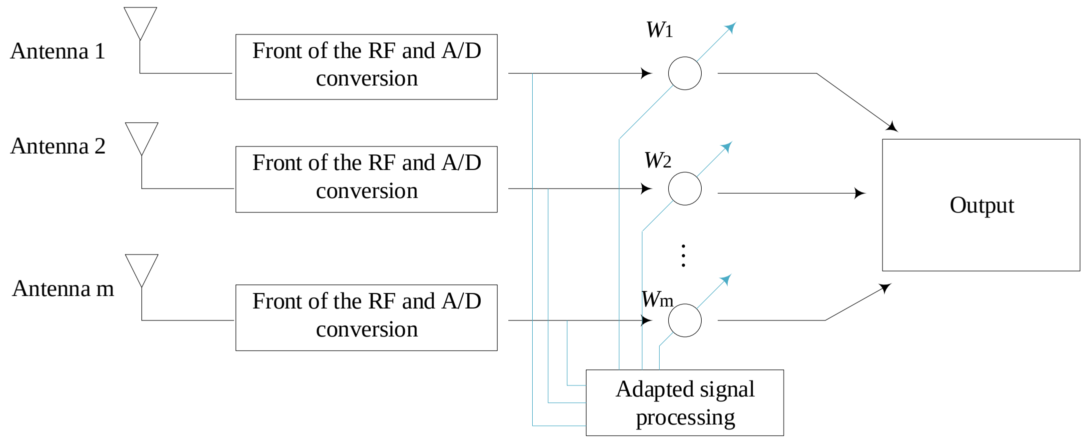

Figure 2.11 Adapted antenna array system

Figure **2.11** shows a traditional antenna adaptive array structure. It consists of three parts—antenna array, radio frequency frontend (including the A/D conversion) and adaptive signal processing section. It is designed such that the performance is optimized, and the noise output is suppressed.

The robust adaptive beamformer offers better capability of resolution and interference suppression than traditional adaptive beamformer. That is why researchers are interested in improving optimization algorithms of robust adaptive beamformer in the past decade.

2.3.4 Conventional beamforming

Conventional beamformer adopts fixed beamforming [39], or the delay-and-sum beamformer [40]. The magnitude weights of the array are equal. Therefore, it is also known as the simplest beamformer. The phase of excitation current fed to each antenna element in the array provides us a method to steer the array antenna in a particular direction [41], which is defined as the look direction [40]. This weighted array has desired response in the look direction.

The weights of array can be calculated as:

$$w = \frac{1}{L} S \tag{2.23}$$

The conventional beamformer can be considered as shown in Figure 2.12.

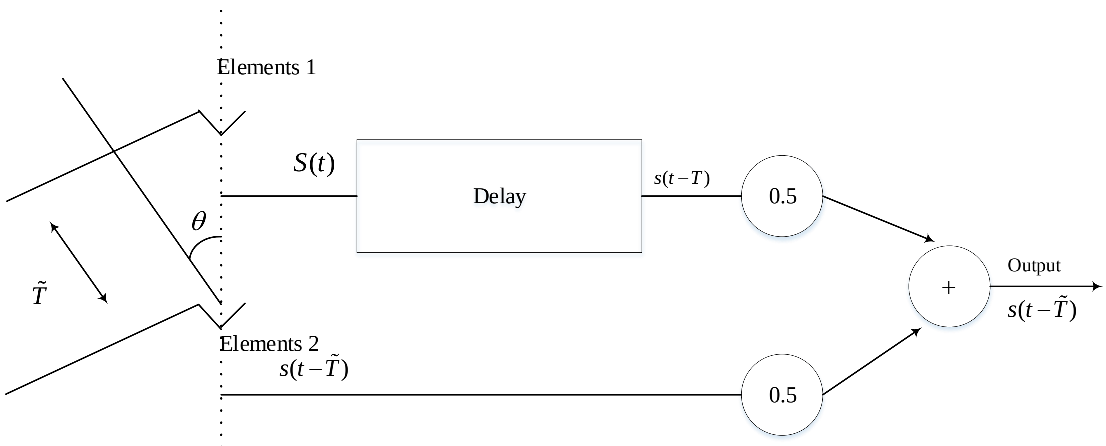

Figure 2.12 The structure of two- elements conventional beamformer [40]

Figure 2.12 shows a two-element array. Set the spacing between these two elements is *d*. $\tilde{T}$ is the time delay.

$$\tilde{T} = \frac{d}{c} \cos\theta \tag{2.24}$$

2.4 Sparse Antenna Array

The sparse antenna array is a special type of phase antenna array. The antenna arrays become increasingly important because it can not only provide a steerable beam but also a diversity gain in multipath signal reception[17]. In addition, it can reduce the cost of normal antenna array. The sparseness means fewer elements in antenna array, but it can achieve better performance. It is widely applied in telecommunications, radar and remote sensing.

As there are multiple elements associated with the arrays, the mutual coupling between the elements reduced the performance of antenna arrays[18]. Moreover, it increases the cost and weight. The sparse antenna arrays can not only avoid the mutual coupling but also minimize the array size. Furthermore, the adjacent sensor in sparse arrays are separate by a distance greater than half a wavelength to avoid side lobes [11].

Sparse antenna array of particularly use in many applications where the weight and size of antennas are extremely limited [19] [42], such as phased array radar, satellite communications [43], [44], [3]. Up to present many analytical models were proposed for the antenna problem, such as Dolph-Chebyshev [45] and Taylor methods [46]. This kinds of methods generally based on an assumption that the antenna element is equally spaced with a uniform distribution, thus requiring a large number of antenna element [3].

Antenna array is used to replace single antenna because the radiation of antenna can be improved and higher gain can be achieved. Besides, antenna array can do better beamforming due to phenomenon of interference, multiple radiation waves to enhance the power radiated in some specific direction.

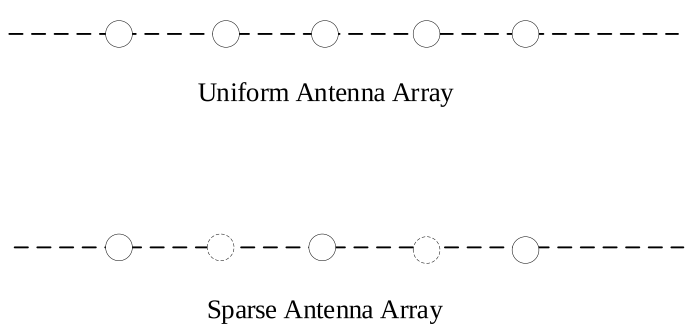

Figure 2.13 Comparison of uniform antenna array and sparse antenna array.

Sparse antenna array can achieve a desired radiation pattern with the minimum number of antenna elements.

2.4.1 Existing Design Methods for Sparse Arrays

2.4.1.1 Design with Annealing algorithm

It is a multivariable optimization problem to determine the optimal location of sparse array elements. Simulated annealing (SA) algorithm is a heuristic search algorithm, it can avoid the local optimal solution. It was first used to study combinatorial optimization problems. It is based on the similarity between the metal annealing process and the general combinatorial optimization problem. When solid is cooled from high temperature to low temperature, the particles inside the solid are gradually ordered from disorder, reach the equilibrium state at each temperature, Finally, the ground state is reached at normal temperature, internal energy is minimized. The probability is larger for acceptance of new state with higher energy than the current state at high temperature. Conversely, the acceptance probability is small of new state with higher energy than the current state at low temperature [18]. When the temperature is zero, there is no chance to accept the energy of new state is higher than the current state.

Assume there are antenna elements (subarray) unequally located from 1 to *l*, *M* is the number of elements. *P* is the optimal location vector of *M*. $P = [1, 2, \cdots, l]_{1\times M}$, *J* is the unity vector, $J = [1,1,\cdots 1]_{1\times M}$, the objective function is min(*M*), because the equivalent phase center is located between the middle of receiver and transmitter subarray. The constraint conditions are 1, 1.5, 2, …, *l*-0.5, $l \in F(M)$.

$$F(M) = (P_{1\times M}{}^{T} J_{1\times M} + J_{1\times m}{}^{t} p_{1\times M}) / 2 \tag{2.25}$$

$F(M)$ is an aperture synthesis cycle which obtain all set of the phase centers. Because the sum of the locations of any two elements in a half times array should include 1, 1.5, 2, …, l-0.5, the location of 1, 2, l-1 and l has existence of physical subarray. These four locations can be fixed constant and no need to optimize. P is the location vector of subarrays after optimization. Because the linear array has symmetric characteristics. P should satisfy

$$\begin{cases} P(m) = a \\ P(M - m + 1) = l - a + 1 \end{cases} \tag{2.26}$$

$3 \le a \le l - 2$, $3 \le m \le M - 2$, The number of phase center is $2l - 1$, the distance between phase centers is the half of minimize subarray distance.

The number of phased center obtained from sparse antenna array is for multiple input and multiple output and the objective array can be equivalent to annealing internal energy.

The procedure of simulated annealing algorithm is as follows:

- Initialization: Set the initial temperature T_e, the number of iterations at each temperature is Q, annealing rate is r, according to (2.25), the initial sparse array location vector P.
- Step 1: for n =1, 2, …, Q carry out steps 3 to 6;
- Step 2: according to the formula (2.25), randomly change the location of the two elements of the array to get a new array;
- Step 3: calculate the sum of any two locations of half current array and half new array, the missing numbers in the result at 1, 1.5, 2, …, l- 0.5, l are k_0 and k.
- Step 4: if $k < k_0$, a new array layout can be obtained, if $k \ge k_0$, and $e^{(k_0 - k)/T_e} > x$ the

new array layout can also be accepted, x is a random number that follows a uniform distribution between 0 and 1;

- Step 5: if the termination condition k =0 is satisfied, then output the current array configuration and terminate the program;
- Step 6: T_e gradually decreases with speed r, $T_e = r \times T_e$, and $T_e > 0.1$, and then return to step 2.

Higher temperature corresponds to a higher probability of a poor array formation. At the beginning of optimization, the temperature variables can be set high enough because high temperature can provide a perturbation that jumps out of the local optimization to achieve global optimization. With the optimization, the temperature is gradually decreased. and the probability of choosing the worse array formation is decreased.

When the temperature is reduced to <0.1, the probability that the new array achieves performance improvement will become very small. Then, the optimization process can be terminated.

2.4.1.2 Genetic Algorithms

Genetic algorithm (GA) is used as an optimization algorithm in sparse antenna array design. The GA is a computational model of the evolutionary process of the natural selection and genetic mechanism of Darwin's biological evolution theory. It is a method to search the optimal solution by simulating the natural evolution process.

By using the theory of biological evolution, the genetic algorithm simulates the problem to be solved as a process of biological evolution. By copying, crossing, mutation and other operations, the next generation of solutions can be generated. The solution of low fitness function is rejected, and the solution of high fitness function is retained. In this way, it is possible to evolve individuals with high fitness function after the evolution of N generation.

GA is used as a global optimization algorithm [47], which is widely applied in antenna array in recent years. For the sparse array optimization, the variable parameters are $\{a_i, \phi_i, c_i\}$, because the amplitude and phase of the actual array element are dispersed by the power divider and the digital phase shifter, it is reasonable by using binary coding. According to the minimum adjustable step of amplitude, phase and its dynamic range, the coding coefficients of amplitude and phase can be recognized, respectively. Imagine the normalization range of the antenna is in the range of 0 to 1, use eight step quantization, then a 3-bit code is needed. The phase shifter uses switch to transfer phase shifter, 4-bit code can be expressed as the step phase shift in 22.5°, 0° ~360°. The excitation state of the antenna occupied 1 bit, it can be found that the code of p generation of q chromosome expression is

$$\text{chro}_{p,q} = \left\{ \underbrace{\overbrace{I_{a1}, I_{a2}, I_{a3}}^{a_1}, \cdots}_{3M} \underbrace{\overbrace{I_{\phi 1}, I_{\phi 2}, I_{\phi 3}, I_{\phi 4}}^{\phi_1}, \cdots}_{4M} \underbrace{\overbrace{I_{c1}}^{c}, \cdots}_{M} \right\} \tag{2.27}$$

2.4.2 Compressive Sensing

Compressive sensing (CS) is one of the most prominent achievements in the field of signal processing since twenty-first century and has been applied in the fields of magnetic resonance imaging (MRI) and image processing. CS is a technology for signal sampling, which compresses data during the sampling process.

A signal that is not sparse in a domain can be sparse in another transformation domain. Sparse signals can be represented by a few non-zero coefficients. Sparse antenna arrays can decrease the grating lobes because its random antenna elements positions [48]. The antenna weights are related to the antenna elements locations. Therefore, design of sparse antenna array is equivalent to the optimization of antenna weights [49]. If the signal is sparse, there is no need to collect those spatial coefficients with zero values. Instead, a small number of non-zero coefficients are obtained [10], which leads to the model of sparse antenna array.

Sampling theorem is the law of sampling band-limited signals. In 1948, it was described and proved by C.E. Shannon. Conventionally, we need to first sample the data at least using the Nyquist rate. Then we can compress the sampled data, transmit them and reconstruct them at the receiver end. It is obvious to see that the conventional approach can cause a waste of resources.

To ensure the transmission security, the technology of encryption is also set to encode the signal. This will bring problems to the secure transmission of data [16] [50]. Recently, D. Donoho and other scientists raised a new method to acquire the information, which leads to the CS technology [19]. For CS to work properly, the signal to be sampled should be sparse or compressible. CS can recover the original signal by using a sampling rate lower than the Nyquist rate. In other words, CS could recover sparse signals with fewer samples than what is required by the Nyquist sampling method [51].

Figure 2.14 shows the comparison between Nyquist sampling and CS method.

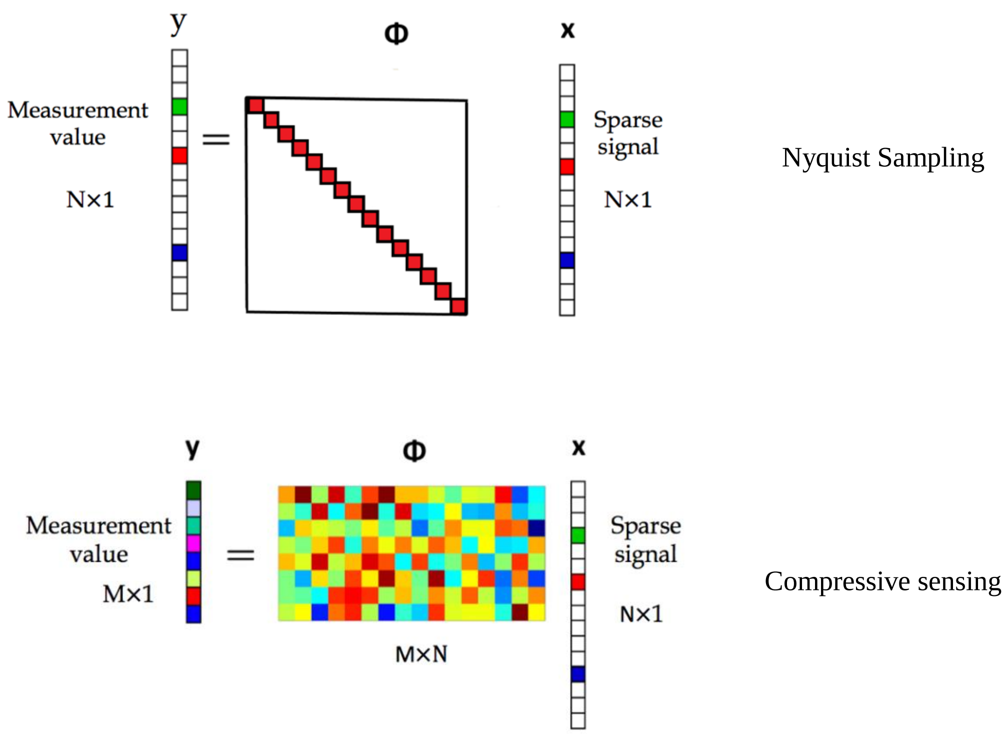

Figure 2.14 Comparison of Nyquist sampling and compressive sensing

There are three main aspects of CS:

- (1) sparse expression of natural signals;
- (2) designing the measurement matrix can reduce the signal dimension while ensuring the minimum loss of signal [52].
- (3) signal recovery algorithm, restoring original signal from M measurements without distortion or tolerable level of distortion) [53].

Generally, the natural signal x is not sparse, but it is sparse in some transform domain. Therefore, it can be expressed by the expression $x = \Psi\Theta$, Θ is sparse coefficient, Ψ is the sparse matrix of $M \times N$. This signal is dense in time domain but sparse in frequency domain. Sparse means that few elements are non-zero (or far more than 0), and most elements are 0 (or very small).

Above all, CS can receive super-resolved signals from few non-adaptive antennas. It requires no effort to understand the signal. The acquisition process is easier since it uses simply numerical process. It can reduce the amount of data required and huge image factors while restoring the original signals exactly [54].

2.4.2.1 CS method to reconstruct signal

At present, the reconstruction algorithms of compressed sensing are mainly divided into two categories [40]:

- Greedy algorithm, which is selected gradually by increasing the suitable solutions. To achieve the approximation of signal vectors, such algorithms mainly include matching tracking algorithm and orthogonal matching algorithm, matching pursuit algorithm, complementary space matching pursuit algorithm and so on [55].
- Convex optimization algorithm, which relaxes the norm l_0 to norm l_1 through linear programming. This kind of algorithms include gradient projection method, base pursuit method and minimum angle regression method [4] [56].

Convex optimization algorithm is more accurate than greedy algorithm, but it needs higher computation [57]. Other algorithms include matching pursuit algorithm, complementary space matching pursuit algorithm, and minimum norm l_2 reconstruction.

Norm l_2 measures the energy of the signal. Define norm l_P, finding the l_2 smallest vector in the converted zero space. Imagine I is vector coefficients of $M \times 1$

$$(\|I\|_p)^p = \sum_{i=1}^{M-1} |I_i|^p \tag{2.28}$$

$$\hat{I} = \arg\min \|I'\|_2 \qquad \text{subject to } \xi I' = y \tag{2.29}$$

where $\xi = \Delta\phi$, is a matrix of $M \times N$, ϕ_i is the column vector of the matrix $M \times N$,

ϕ_i can be written as $\phi = \phi_1, \phi_2, \cdots, \phi_N$. When a signal x defined as $x = \sum_{i=1}^{M-1} R_i \phi_i$, y can be expressed as $y = \Delta x = \Delta \phi I$

Minimum norm l_0 reconstruction works as follows. Norm l_0 calculate the sparseness of signal. It calculates the non-zero coefficient in I.

$$\hat{I} = \arg\min \|I'\|_0 \qquad \text{subject to} \quad MI' = y \tag{2.30}$$

The heavy computation is the unstable problem.

Minimum norm l_1 reconstruction works as follows.

$$\hat{I} = \arg\min \|I'\|_1 \qquad \text{subject to} \quad MI' = y \tag{2.31}$$

This can reconstruct the sparse signal accurately.

2.4.3 Advantage and Disadvantage of Different Methods

Table 2-1 Comparison of GA, SA and CS

method	Advantages	Disadvantages
GA	This method does not have many requirements. Even the researchers do not have any idea about the domain, GA can also be used to solve the optimization problem.	1. Potentially long computation time. 2. The convergence possibility is a non-optimal solution.
CS	1. CS can recovery the signals by using fewer measurements when certain conditions are met. 2. The computation time can be obviously decrease than other methods.	This method has its limitation, for example, the signal need to be sparse in some domain.
SA	1. This method can guarantee of finding an optimal solution. 2. For complex problems, it is easy to code. It even can deal with cost functions.	Long computation time will increase the cost especially the cost function is very expensive.

2.4.4 Compressive Sensing via Sparse Antenna Array

Different types of algorithms were applied to optimize the location of sparse antenna array [11] [17] [18], such as nonlinear optimization methods like genetic algorithm [17] [18] and simulated annealing [11]. CS defines as an alternative and effective method to design sparse antenna array. Antenna location optimization is the design problem to achieve a desired beamforming pattern by using fewer antenna elements [49] [58].

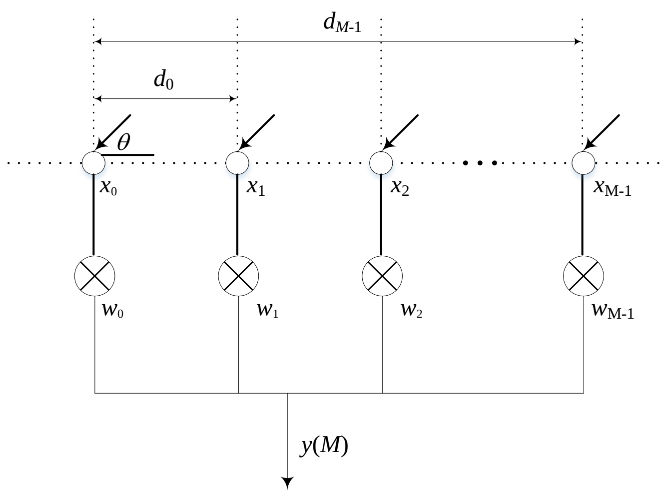

Figure 2.15 A general structure of equal spaced linear array

Figure 2.15 shows a general structure of equal spaced linear array. The output is

$$y(M) = x_0 * w_0 + x_1 * w_1 + \cdots + w_{M-1} * w_{M-1} \tag{2.32}$$

$$y(n) = x * w \tag{2.33}$$

This problem can be considered as weights optimization.

If weight is uniform weighting,

$$w = [w_0, w_1, \cdots, w_{M-1}] \tag{2.34}$$

$$w_i = 1/M, i = 0,1,\cdots, M-1 \tag{2.35}$$

2.4.4.1 Equivalent phase center

As shown in

Figure 2.16, when the receiver and transmitter of antenna is separated, the receiving signal is compensated for a phase related to the direction of the beam, which can be equivalent to a spontaneous and self-received antenna located in the middle of the transceiver antenna, the position of the equivalent antenna is equivalent phase center.

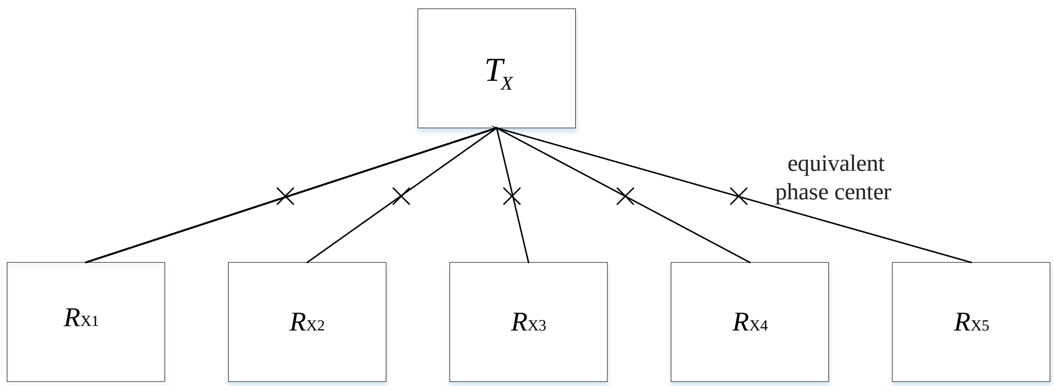

Figure 2.16 A structure of equivalent phase center

T_x is the transmitter and R_x is the receiver. Consider that the transmitter array is located at (x_1, y_1), and the receiver array is located at (x_2, y_2)

Then, the equivalent phase center is located at

$$\begin{cases} x_e = \dfrac{x_1 + x_2}{2} \\ y_e = \dfrac{y_1 + y_2}{2} \end{cases} \tag{2.36}$$

2.4.4.2 Aperture Synthesis

The aperture synthesis concept comes from radio astronomy, and it refers to the resolution obtained by a plurality of smaller antenna structures corresponding to the large aperture single antenna. It means multiple equivalent phase centers of the array are combined into equivalent arrays.

2.4.4.3 Directional resolution

For the length *L* of the array, the distance between two elements is *d*, far field signal received from the direction of θ , the wave range difference of two adjacent array elements is $\frac{d}{\lambda}\sin\theta$. When a narrow beam is used for both transceivers, the wave range difference of two adjacent array elements is $\frac{2d}{\lambda}\sin\theta$. It is equivalent to expand the distance to two times.

Define the resolution of array under received status is $\frac{\lambda}{L}$, so when a narrow beam is used for both transceivers, the spatial resolution of the direction of the normal direction is

$$\rho_a = \frac{\lambda}{2L} \tag{2.37}$$

2.4.4.4 Distance between adjacent elements

When the maximum response direction of the array is in the normal direction, the first grating lobe appears when the system works in the receiving and transmitting mode is located at $\pm\arcsin(\frac{\lambda}{2d})$, The beam width of the antenna is $\frac{\lambda}{D}$, *D* is the azimuth dimension of the antenna, to avoid beam coverage. The ambiguity in the range needs to be

$$\arcsin\left(\frac{\lambda}{2d}\right) \geq \frac{\lambda}{D} \tag{2.38}$$

It uses subarray structure, and the general subarray spacing is

$$\frac{\lambda}{2d} \geq \frac{\lambda}{D} \tag{2.39}$$

To avoid blurring, the subarray spacing should be

$$d \leq \frac{D}{2} \tag{2.40}$$

But when two subarrays are receiving at the same time, according to the principle of receiving the equivalent phase center, the phase center interval is 1/2 of the physical space of the subarray, so the spacing of the subarray can be written as

$$d = D \tag{2.41}$$

2.4.4.5 Array Pattern

In the subarray, each radiation unit is an omnidirectional unit with a distance of 1/2 wavelengths. The array pattern is the product of the subarray pattern and the array factor.

$$F(\theta) = F_a(\theta)F_e(\theta) \tag{2.42}$$

The expression of array factors is the same as the expression of the beampattern as $y(\theta) = w^H S$.

2.5 Summary

This chapter starts with a brief introduction to antenna basic theorem including antenna types and antenna measurement characteristics. Then it is extended to antenna array. A comparison of single antenna and antenna arrays is provided to illustrate why antenna array is an important research topic. There are many types of antenna array, we focus on three different classes of antenna divided by dimensions. These antenna arrays are linear antenna array (1-dimensional), planar antenna array (2-dimensional), and 3-dimensional array. Linear array and planar array are introduced in detail. After the introduction of antenna and antenna array, a detailed discussion of beamforming is provided. There are different types of beamforming. Three main types of beamforming are introduced. It included how to obtain the desired beamforming pattern and calculate the beam pattern. Sparse antenna array is introduced after beamforming. This part discusses not only the basic theory of sparse antenna array, but also discusses two analyses for existing sparse antenna array design methods, GA and SA. Both methods are discussed with detailed mathematical models. Finally, compressive sensing via sparse antenna array is discussed. A method related to CS algorithms is briefly presented.

Chapter 3 Sparse Antenna Array Design using Compressive Sensing

3.1 Introduction

3.1.1 Array Signal Processing

The array signal processing is an important area of the signal processing [7] [48]. Array signal processing has been widely used for microphone, radar, and telecommunications [5], [13], [12]. Multiple sensors from different directions such as microphones and antennas are applied to process the signal [12] [7]. DOA [29] is presented by the parameters of an elevation angle θ.

First of all, as compared with the traditional single-element antenna, antenna array has the advantages such as high signal gain and strong ability in suppressing interference. The array signal processing extracted useful feature information (including the receiving signal) to reconstruct the original signal in different domains, including time domain, frequency domain and spatial domain. Time domain signals have spectrum or power spectrum. Space spectrum has spatial spectrum. System response can be obtained by time domain processing. The radiation pattern can be obtained by space processing. Time domain filtering is to enhance or suppress signals with different frequencies. Spatial filtering is to enhance or suppress signals with different directions. The essence of array signal processing is a spatial filtering method. The effect is to adjust the radiation patterns of antenna array according the direction of arrival.

The array signal processing is mainly to calculate the correlation between multiple signals aimed to locate the signal source or to estimate the desired signal. Array signal processing is mainly applied in several fields:

- Beamforming techniques, it is used to make the main beam pattern of antenna array propagating at desired directions.
- Zero-point formation technology, zeroes of antennas are aligned to the direction of interference.
- Spatial spectrum estimation: the super-resolution estimation of DOA for spatial signals is mainly because the spatial spectrum estimation technology has the ability to distinguish the high spatial signal and can break through and further improve the resolution of the different space from the signal in the width of a beam.
- Signal source estimation, determine the elevation and azimuth of the array to the source, including the frequency, delay and distance.
- Source separation, determine the signal waveform generated by the source, the array arrivals from each source is from different directions using to separate the signals. Even the signals are superimposed both in the time domain and frequency domain, it still can be separated.

The common application of array signal processing includes the DOA estimate and beamforming. The DOA estimation is to determine the direction of the signal from the received data, whether the signal is a useful signal or the interference signal. Beamforming is the space expansion of traditional filtering. Its fundamental purpose is to effectively extract useful signals and to suppress noise and interference. In the direction diagram, a useful signal direction is considered to form a peak and the interference direction forms a zero. DOA can be regarded as an estimation of front processing of beamforming, after the desired signal and the direction of interference are determined, the array forms a beam to the desired signal direction and forms zero in the direction of the interference.

3.1.2 DOA Estimation

First, Capon proposed a minimum variance undistorted response (MVDR) algorithm based on beamforming in 1969 [59], which is different from the traditional beamforming. The MVDR method can break through the intrinsic limit of the array aperture, i.e., the Rayleigh limit. The research on high resolution technology of signal processing has played an important role, but this algorithm is also restricted by the array structure and array aperture and does not make full use of the structure of the array data correlation matrix and cannot achieve the true sense of high resolution.

A fundamental problem for array signal processing is the DOA estimation of spatial signals and is also an important task in many fields such as radar and sonar. The basic problem of DOA estimation is to determine the spatial position of multiple signals that are interested in a certain area of space (i.e., the direction angle of each signal arriving at the array reference element). The basic theory of spatial spectrum estimation cannot be separated from the basic principle of array signal processing. It uses the phase difference to determine one or several parameters to be estimated, such as the azimuth angle, the pitch angle and the number of signal sources.

The resolution depends on the array length, and its resolution is determined after the array length is determined. It is called the Rayleigh-limit. The super Rayleigh limit method is called super resolution. The earliest super resolution methods are mainly MUSIC and ESPRIT algorithms. As a classic DOA estimation method, the MUSIC algorithm was proposed by Dr. Schmidt in 1979 [60], and its full name is "*multiple signal classification*”. MUSIC is an estimation algorithm of signal parameters, which can finally give information about the number of incident signals, the direction of arrival of each source signal, the intensity of the signal, the correlation between the incident signal and the noise. ESPRIT algorithm is short for *estimating signal parameters vibrational invariance techniques*. These two algorithms belong to the subspace method of the eigen structure [61]. This method is based on such a basic observation that if the number of antennas is more than the number of sources, the signal component of the array data must be in a low rank subspace. Under certain conditions, the subspace will uniquely determine the DOA of the signal and can use the singular value of the numerical stability to decompose the exact determination of the DOA.

Spatial spectrum estimation uses the spatial array to estimate the parameters of spatial signals. The entire spatial spectrum estimation system consists of three parts: spatial signal incidence, space array reception and parameter estimation. Correspondingly, it can be divided into three spaces: target space, observation space and estimation space.

Target space is the space formed by the parameters of the signal source and the complex environment parameters. For the spatial spectral system, the unknown parameters of the signal are estimated from the complex target space by some specific methods.

For the estimation space, it uses spatial spectrum estimation technique to extract feature parameters from complex observation data. Consider the N remote narrowband signal incident on a space array, the array antenna is composed of *M* elements. If the number of elements is equal to the number of channels, under the condition of signal source is narrowband, the signal can be expressed as follows

$$s_i(t) = u_i(t) * e^{j(\omega_0 t + Z(t))} \tag{3.1}$$

$$s_i(t-\tau) = u_i(t-\tau) * e^{j(\omega_0 (t-\tau) + Z(t-\tau))} \tag{3.2}$$

$u_i(t)$ is the amplitude of the received signal. $Z(t)$ is the phase of the received signal, and ω_0 is the frequency of the received signal. Under the assumption of a narrowband far-field signal source, the following formula is established.

$$u_i(t-\tau) \approx u_i(t) \tag{3.3}$$

$$Z(t-\tau) \approx Z(t) \tag{3.4}$$

$$s_i(t-\tau) \approx s_i(t) * e^{-j\omega_0 \tau_1}, \quad i = 0,1,\cdots N \tag{3.5}$$

Then the receiving signal of *l* element can be obtained

$$x_i(t) = \sum_{i-1}^{N} g_{li} s_i(t-\tau_{li}) + n_l(t) \tag{3.6}$$

g_{li} is the gain of the first *l* element to the i$^{\text{th}}$ signal. and $n_l(t)$ represents the noise of the l^{th} element at time t. τ_{li} indicates the delay of the i^{th} signal relative to the reference element when it reaches the l^{th} element. It is written in a vector form

$$X(t) = AS(t) + N(t) \tag{3.7}$$

$X(t)$ is a $M \times 1$ dimensional snapshot data vector. $N(t)$ is a $M \times 1$ dimensional noise data vector. $S(t)$ is an $N \times 1$ dimensional vector of space signal. A is an $M \times N$ dimension flow pattern matrix (Guide matrix) for space array. The above model is very useful in array signal processing.

3.1.3 Beamforming

Another topic in signal processing is beamforming. The early beamforming technology is mainly applicable to narrowband signals, and wideband signals become common now. In 1972, Frost proposed the structure of broadband digital beamforming. After that, the technology of broadband adaptive beamforming was developed rapidly. Some scholars proposed several broadband beamforming algorithms. In this section, we will go through the narrowband beamforming. Although the pattern of the array antenna is omnidirectional, the output of the array can be adjusted to the direction of the array receiving after the weighted sum, and the gain is gathered in one direction, which is equivalent to forming a "beam", leading to the physical meaning of the beamforming. Beamformer is the foundation of digital ultrasound imaging and the guarantee of high performance color ultrasound.

Beamforming is the theory that data received by the antenna is weighted to form a certain beam shape. The function of beamforming serves for two purposes—allowing the direction of interested signal pass through to form a gain and suppressing the signal which is not interested in direction. In general, beamforming is used to enhance signal of interest and suppress interference by forming a certain beam.

Suppose there are M antenna elements in the array. Then the antenna factor can be referred as:

$$F(\theta)=\sum_{i}^{M} R_i e^{jkd_i\cos\theta} \tag{3.8}$$

R_i is defined as the antenna excitation coefficient where i located at $x = R_i$. The wave number can be defined as $k=\frac{2\pi}{\lambda}$. Figure 3.1 shows a non-uniform linear symmetric array [6].

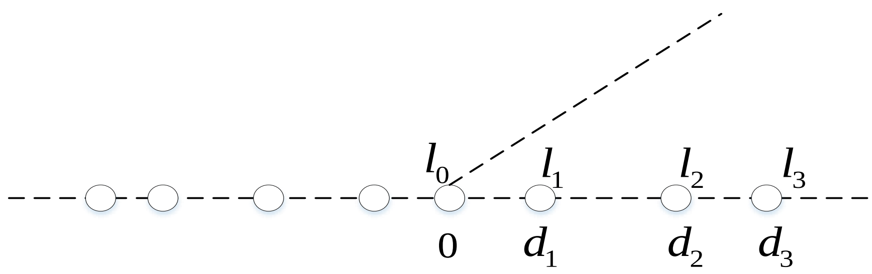

Figure 3.1. A non-uniform linear symmetric array

The response of array is:

$$F(\theta) = 2\sum_{i}^{\lceil M/2 \rceil} R_i \cos(kd_i \cos\theta) \tag{3.9}$$

$\lceil M/2 \rceil$ is the minimum integer no less than $\frac{M}{2}$. Consider there are M antenna elements with uniform spacing from $-X_m$ to X_m.

(3.9) can be re-arranged into a matrix format:

$$[f]_{m\times 1} = [A]_{m\times n} r_{n\times 1} \tag{3.10}$$

Suppose m is the number of data which sampled the antenna radiation pattern, n is the minimum integer no less than $\frac{2x_m}{\Delta d}$. The step size Δ is from $-X_m$ to X_m [6].

This can be regarded as the matrix optimization.

$$f = F(\theta_1), F(\theta_2), \cdots, F(\theta_n)^T \tag{3.11}$$

$$r = [R_1, R_2, \cdots, R_n]^T \tag{3.12}$$

$$A_{ij} = 2R_I \cos(kd_i \cos\theta_j) \tag{3.13}$$

After the optimization of w, those non-zero weight coefficients remain, which lead to a sparse antenna array.

In the actual applications, it is desired to minimize the difference between the desired response and the designed response.

The design should be further studied on robust beamformer to improve the performance. Various methods were applied to design robust beamformers, such as diagonal loading, worst case optimization and robust Capon beamformers [29] [30] [31].

3.2 Sparse Array Design

3.2.1 Array Modelling

In sparse array design, the antenna position randomness leads to the representation of grating nodes [20]. Therefore, we can say that the position optimization of the antenna array is the most important part of the design.

When CS is applied to optimize the locations of antenna elements in a sparse array, it attempts to find the minimum number of non-zero weight coefficients in antenna arrays. Consider a ULA with M antenna elements, we need to find M weights w_{M} in the sparse antenna array design. However, some of these weights may have a value of zero. This means some antenna elements can be removed without affecting the performance of the array, leading to a sparse array.

For a narrowband linear array consists of M elements. $w=[w_0, w_1, \cdots w_{M-1}]$, w multiplies the steering vector S with an input angular θ is the array response. From Figure 3.2, the output is obviously equals the sum of the product of $x_{\mathrm{i}}(t)$ and weights w_m.

The uniform array output according to the pattern multiplication principle is

$$y(m) = x_0(t)w_0 + x_1(t)w_1 + \cdots x_{M-1}(t)w_{M-1} \tag{3.14}$$

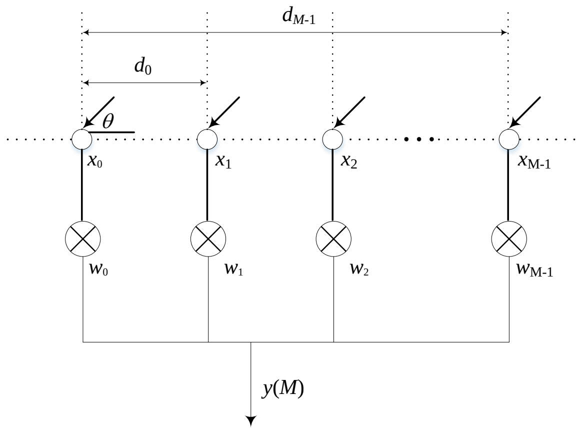

Figure 3.2 The output structure of *M* elements narrowband linear array

S= $x_i(t)$ is the steering vector of antenna array. w_M is the weight of antenna array.

The steering vector *S* is also called array response vector, steering vector is used to avoid confusion with the response vector. It can be written in vector form as

$$S(\omega,\theta)=\begin{bmatrix}1 & e^{-j\omega\tau_1} & \cdots & e^{-j\omega\tau_{M-1}}\end{bmatrix}^T \tag{3.15}$$

where τ is the propagation delay. The delay item can be written as

$$\tau_m = m\frac{2\pi d\sin\theta}{\lambda} \tag{3.16}$$

ω is the angular frequency. λ is the corresponding wavelength. θ is the DOA. $\theta\in\begin{bmatrix}-\frac{\pi}{2} & \frac{\pi}{2}\end{bmatrix}$. Set $d=\frac{\lambda}{2}$. w is the weights coefficient with M sensors. It can represent the location of antenna elements.

$$w=\begin{bmatrix}w_0 & w_1 & \cdots & w_{M-1}\end{bmatrix}^T \tag{3.17}$$

The response of the uniform narrowband beamformer in vector form can be rewritten as

$$P(\omega,\theta)=\sum_{m=0}^{M-1}e^{-j\omega\tau}w_m^* \tag{3.18}$$

The response of the signal can also be written as

$$P(\omega,\theta) = w^H S(\omega,\theta) \tag{3.19}$$

3.2.2 Sparseness Optimization

Sparseness for an antenna array means some elements can be removed without affecting the performance of the array with respect to a traditional uniform array. s. The purpose for this is to minimum the size of antenna, thus reducing the cost.

For the narrowband beamforming in equal spaced antenna array, the way to obtain a satisfied set of weights is to use norm l_0 to remove the zero values in the weights. As we know, the optimization of beamforming is equivalent to the optimization of weights transforming the equal spaced array into an unequally spaced antenna array with a new set of d_i

The weight optimization problem can be written as

$$\min\|w\|_0 \text{ subject to } \|E_r - w^H S\|_2 \leq \alpha \tag{3.20}$$

α is the upper limit for the tolerable difference between the two responses [48]. Ideally, the value of α should be zero. However, it is impossible to achieve that in practice. The array output will be different from the designed value if there is any error in the models. Nevertheless, it is still possible to let α be a tolerable small value. w is the weight of array. E_r refers to the desired response which can be written as

$$E_r = w_r^H S_r \tag{3.21}$$

$\|w\|_0$ denotes norm l_0, which is the number of non-zero weight coefficients for the antenna array. In CS design, it uses norm optimization method to minimize the number of weights. The result will be simplified and achieve sparseness to optimize a minimum size array. S is the steering vectors of all the angles [48].

$$S = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \exp(-j\omega\tau_1(\theta_0)) & \exp(-j\omega\tau_1(\theta_1)) & \cdots & \exp(j\omega\tau_1(\theta_{M-1})) \\ \vdots & \vdots & \ddots & \vdots \\ -j\omega\tau_{M-1}(\theta_0) & \exp(j\omega\tau_{M-1}(\theta_1)) & \cdots & \exp(j\omega\tau_{M-1}(\theta_{M-1})) \end{bmatrix} \tag{3.22}$$

$$w_r = [w_{r,0} w_{r,1} \cdots w_{r,M-1}] \tag{3.23}$$

(3.22) can be rearranged as a l_1 norm minimization [48] as

$$\min \|w\|_1 \text{ subject to } \|E_r - w^H S\|_2 \leq \beta \tag{3.24}$$

l_1 norm is used because l_0 norm requires longer computation time and generates more redundant. β is the upper limit for the difference in the l_1 norm. This problem can be simplified as

$$A - Sw \leq \beta \tag{3.25}$$

A is the desired response while S is steering vector. w_m is the weight coefficient. β should be a number that is small enough to ensure there is little difference between desired and designed response.

3.2.3 Algorithms Explanations

SPGL1 is a MATLAB solver for optimizing large scale complex system using one-norm regularized least-squares [15]

By creating a generated randomly data, it needs to create a matrix and sparse solution vector. SPGL1 feature can be indicated as basis pursuit (BP), basis pursuit denoise (BPDN) and least absolute shrinkage and selection operator (LASSO).

l_1 minimization is used to optimize the locations of antenna elements. The sparse signal recovery design has been achieved by the BPDN method for decades [62] [63]. As an algorithm, WSPGL1 can revise the spectral projected gradient by using l_1 norm.

The problem of SPGL1 algorithm named BP is given by:

$$\min \|w\|_1 \text{ subject to } Sw = E_r \tag{3.26}$$

$$\|w\|_1 = |w_1| + |w|_2 + \cdots + |w|_n \tag{3.27}$$

For l_0 minimization, the problem usually defined as

$$\min \|w\|_0 \text{ subject to } Sw = E_r \tag{3.28}$$

l_0 norm minimization can be considered as a combinatorial problem and its dimension increased rapidly [63]. l_0 norm represent the number of non-zero values. l_0 norm optimization usually requires longer computation time because of its own calculation characteristics. Therefore, l_1 norm minimization can be used to replace the l_0 norm minimization problem.

$$\min \|w\|_1 \text{ subject to } \|Sw - E_r\|_2 \leq \in \tag{3.29}$$

This forms the basis of BPDN [64], where $\in$ represents the upper limit of the noise level in the data.

There are several algorithms that aimed to fill the gap between l_0 and l_1 norm. For instance, weighted l_1 minimization method (w-BPDN) can be used to instead single l_0 norm with $0 < p < 1$. Because this method incorporates such information into the reconstruction stage.

$$\min \|u\|_1 \text{ subject to } \|Su - E_r\|_2 \leq \in \tag{3.30}$$

Where $w \in (0,1)^N$ and $\|u\|_{1,w} = \sum_i w_i |u_i|$ is the l_1 norm [65], [66], [67].

LASSO was used to solve the sequence problem [62] by using SPGL1 solver [68]. Another algorithm for SPGL1 is the LASSO regression. The sequence of LASSO problems is replaced by a sequence of weighted LASSO with constant weights [63] [69]. It can be written as

$$\min\left\|Sw - E_r\right\|_2 \ \text{ subject to } \ \left\|w\right\|_1 \le \mu \tag{3.31}$$

3.2.4 SPGL1 Algorithm with BPDN Problem

The SPGL1 algorithm solves the BPDN problem as to efficiently solve a sequence of LASSO subproblems [70]. The key idea of this algorithm is to use SPGL1 algorithm to obtain the solution of BPDN by solving a sequence of LASSO problems. It uses the spectral projected gradient algorithm for the solution [71]. Then, let the result be close the result of BPDN.

$$\min\left\|Sw - E_r\right\|_2 \text{subject to} \left\|u\right\|_1 \le \mu(LS_\mu) \tag{3.32}$$

The single parameter μ determines a Pareto curve $\phi(\mu) = \left\|r^\mu\right\|_2$, $r^\mu = E_r - Sw^T$, w^T is the solution of (LS_μ). The SPGL1 algorithm is initialized at point w^0, $\mu_0 = \left\|w^{(0)}\right\|_1$

The parameter μ is then updated according to the following rule

$$\mu_{t+1} = \mu_t + \frac{\left\|r^{\mu t}\right\|_2 - \varepsilon}{\dfrac{\left\|A^H r^{\mu t}\right\|_\infty}{\left\|r^{\mu t}\right\|_2}} \tag{3.33}$$

*^{H} indicates Hermitian transpose. $\varepsilon = \left\|e\right\|_2 = \left\|E_r - Sw\right\|_2$.

Figure 3.3 shows the flowchart of this algorithm.

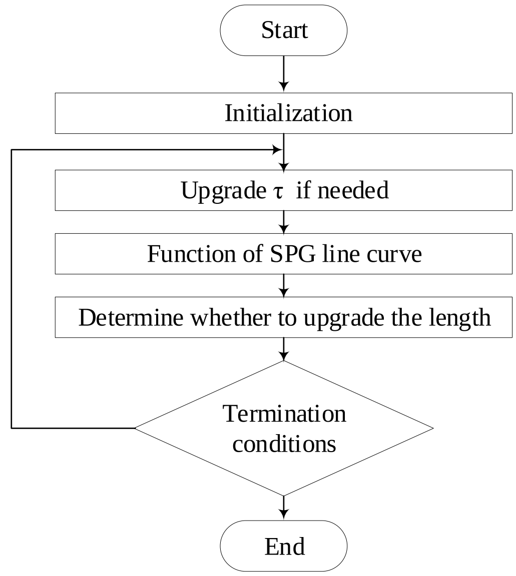

Figure 3.3 SPGL1 Algorithm uses in this project

3.2.5 Simulation Setup and Result

Imagine there is a uniform linear array with 100 antenna elements. In case I, II, and III**,** α decreases to ensure the allowed difference between desired and designed response is small enough. β is set to 0.1 to observe the result effected by α. Results on the effect of β are shown in Appendix A. A MacBook Pro 2017 with a 2.9GHz Intel core i7 is used to run MATLAB 2015a (Build 8.5.0.197613 64-bit Maci64) for the simulation. In this design, the designed response should be approximated enough to the desired response.

Case I — α **= 0.1,** β **= 0.1,** θ_k **= 90**$^{\circ}$, ε **=1**.

The uniform antenna array consists of 100 possible antenna elements.

For BPDN, $\|w\|_1$ subject to $\|S*w-b\|_2 \leq \alpha$, S is steering vector matrix and w is the weight coefficient. $\|.\|_2$ denotes l_2 norm. α is the limit of allowed difference between desired and designed response. Sparseness is achieved by selecting the weight coefficients to give fewer active antennas while matching the designed response to the desired one [48]. The actual steering vector should be $S = s + s'$, s is the designed steering vector while s' is the error caused by model perturbations [48]. $\|s'\|_2 \le \varepsilon$, ε is the upper bound. The difference between designed and desired response is given by

$$\left|w^H s - w^H \tilde{s}\right| = \left|w^H s'\right| \le \varepsilon \|w\|_2 \tag{3.34}$$

$\varepsilon \|w\|_2 \le \beta$, β is the acceptable level of difference in array response.

Consider $\theta = 90^\circ$ at main beam and sidelobe region at $\theta_{SL} = [0^\circ, 85^\circ] \cup [95^\circ, 180^\circ]$, Table 3-1 shows the results about d_i and w_i.

Table 3-1 Weights and antenna locations with α = 0.1

n	d_n/λ	w_n	n	d_n/λ	w_n
1	0.81	0.752	8	5.71	0.768
2	1.51	0.749	9	6.41	0.669
3	2.19	0.834	10	7.1	0.701
4	2.89	0.867	11	7.7	0.768
5	3.6	0.768	12	8.47	0.822
6	4.33	0.768	13	9.17	0.768
7	5.03	0.782			

Figure 3.4 shows the desired response and designed response as solid line and dotted line, respectively. The designed response is close to the desired response. The main beam mainly overlapped and the sidelobe is lower than the designed response. Therefore, when α = 0.1 the result of the design is satisfied. The difference between the designed antenna elements location is about 0.7λ. In this result, 13 antenna elements are used in the sparse array to achieve the performance of 100 antenna elements in the conventional array. The computational time is 0.077484s.

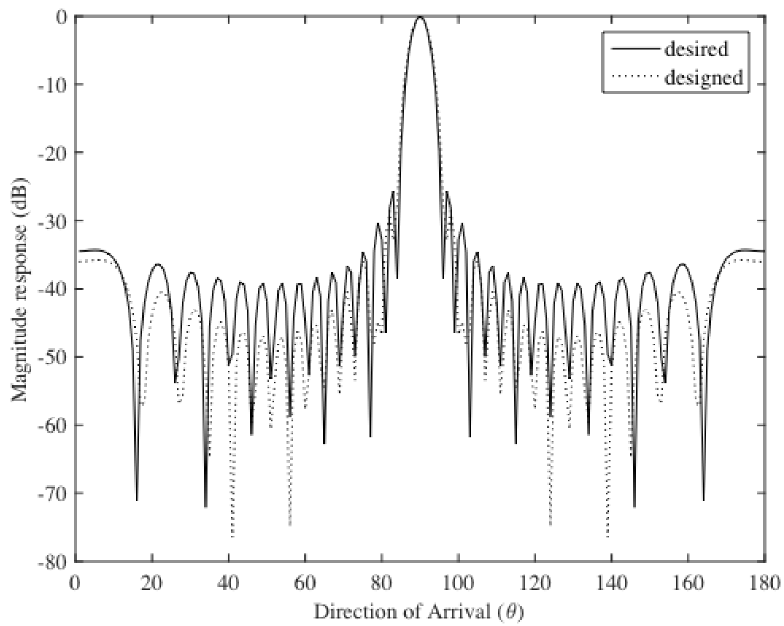

Figure 3.4 Design results with CS, α = 0.1

Case II — α = 0.05, β = 0.1, θ_K= 90°, ε =1.

Figure 3.5 shows that results for Case II. The result of designed response approached to the desired response mostly. Design results of Case II lead to a sparse antenna array with eight elements as compared to 100 antenna elements in the conventional array. The computational time is 0.087330s.

Table 3-2 Weights and antenna locations with α = 0.05

n	d_n/λ	w_n
1	0.64	0.7568
2	1.51	0.7400
3	2.46	0.6892
4	3.35	0.5912
5	4.25	0.4565
6	5.17	0.3164
7	6.06	0.2001
8	6.93	0.1137

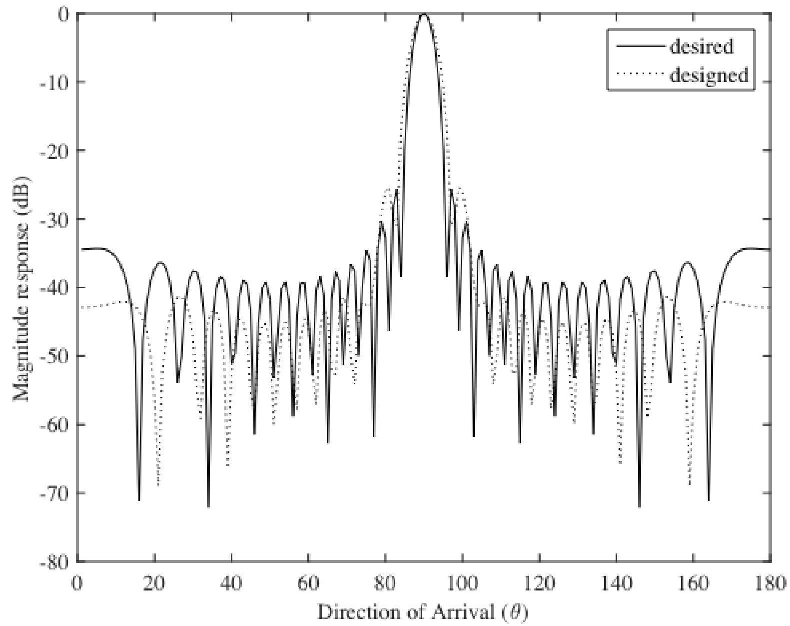

Figure 3.5 Design results with CS, α = 0.05

Case III — α = 0.095, β = 0.1, θ_K= 90°, ε =1.

As different parameters should work out a better performance. Other parameters are the same with Case I and Case II. As shown in Figure 3.6, the result is better than the other because the difference between designed and desired response is smaller. Case III leads to a sparse antenna array with eight antenna elements to achieve the performance of 100 antenna elements. The computational time is 0.079138s.

Table 3-3 Weights and antenna locations with α = 0.095

n	d_n/λ	w_n
1	0.31	0.8478
2	1.11	0.8243
3	1.99	0.7675
4	2.79	0.6714
5	3.59	0.5455
6	4.39	0.4117
7	5.19	0.2901
8	5.99	0.1863

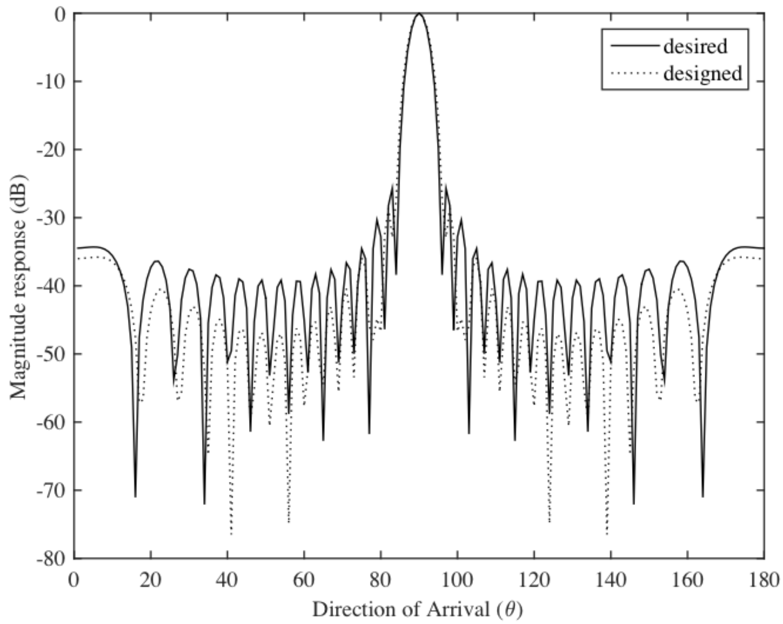

Figure 3.6 Design results with CS, α = 0.095

3.3 Summary

In this chapter, we first revisited the background knowledge about array signal processing. Two methods for the array signal processing were briefly discussed. DOA method is a pre-processing method in array signal processing. Different types of beamforming can be divided by the bandwidth. Narrowband beamforming was considered in this chapter. Next, array modelling and detailed method on sparse antenna array design using CS was explained. Finally, computer simulation results were discussed to demonstrate the effectiveness of CS method in designing sparse antenna array.

In particular, we focused on design a sparse antenna array based on the compressive sensing method in narrowband beamforming. This design aims to obtain satisfied radiation pattern with the minimum number of antenna elements. This will not only reduce the cost but also suppress the mutual coupling between two adjacent antenna elements. We start with an equal spaced antenna array with 100 antenna elements as a benchmark model to obtain a desired response. According to the product of steering vector and weights, by change the weights, different set of weights with non-zero coefficient can be obtained. They were obtained by using SPGL1 to perform a norm calculation by setting a tolerable level of discrepancy between desired response and designed response. This difference (denoted as α) is usually set less than 0.01. If the designed response matches the desired response well, the design is considered as a successfully sparse antenna array. After obtaining a set of non-zero weight coefficients, the locations of the array elements can be re-calculated.

For future work related to this chapter, the sparse array should be modeled with antenna locations errors, which can lead to an optimized result to reduce the overall error. Other error factors can also be included in the model.

Next, CS was demonstrated in a successful design of a sparse antenna array in narrowband beamforming. We can possibly extend the design to wideband beamforming. One possible way is to change the channel to a wideband channel. CS appears to be the most effective method in the design of sparse antenna arrays.

Chapter 4 Sparse Planar Antenna Array Design

4.1 Introduction

4.1.1 Antenna Array

With the development of science and technology, wireless communications become more and more popular. In modern life, it is common to see mobile phones, TV, radio, wireless access points (APs), radio navigation, radar and so on, all of which reflect the convenience brought by wireless communications [72]. Antennas play an important role in the proper operation of these wireless devices. In a wireless system, it is desired to focus the antenna radiation beams between the transmitting and the receiving antennas, which can help us to improve the energy efficiency. To achieve that, antenna array is required.

The antenna array can be regularly or randomly arranged by more than two antenna units, and properly excited to obtain predetermined radiation characteristics [73]. The antenna array is not just a simple collection of multiple antennas but requires them to be fixed at particular locations. Antenna arrays can be formed according to the different parameters of the antenna feed current, spacing, and electrical length to obtain the best radiation beam. Adaptive array antennas, also known as smart antennas, can adjust the radiation beam according to the needs. Thus, smart antennas become increasingly popular in modern wireless communications.

Antenna arrays can be divided into linear arrays, planar arrays and 3-dimensional arrays according to the element arrangement. The most common linear arrays have elements arranged equally in a straight line. The spacing between adjacent elements can be changed so that non-uniform linear arrays can be obtained. Furthermore, if multiple linear arrays are arranged on a plane at a certain interval, they form a planar array.

Advantages of antenna arrays include adaptive radiation beamforming and large antenna gain (directivity). For example, adaptive array radar can accurately track and recognize the target in the observation range and can track the dynamic of multiple targets at the same time, with a supercomputer to analyze the feedback information. It can obtain the target information in the set space. According to the target, it can quickly and flexibly change the antenna beam and the direction of the direction. Besides, the shape of the radiation pattern can be determined and controlled from the antenna array by controlling the relative phase and amplitude of the input signal to each antenna elements. Antenna arrays have better capability in controlling the radiation beam as compared to single-element antenna at the expense of a higher system cost.

As abovementioned, antenna arrays are based on the technology of beamforming. Understanding antenna arrays requires good knowledge of beamforming technology. Take cellular communications as an example, the traditional communication mode is the transmission from single antenna to single antenna between a base station and a cell phone. With beamforming technology, the base station has several antennas, which can automatically adjust the phase of the transmitting signals of each antenna to form the superposition of the electromagnetic wave at the receiving point, thus achieving the purpose of improving the intensity of the receiving signal. From the base station, the superposition effect produced by the digital signal processing is like the construction of the virtual antenna pattern of the base station. That is why it is called *beamforming*. Through this technology, the radiation can be focused in the direction of the user, rather than other directions. Consequently, the base station can track the DOA of the user's signal in real time, and ensure the signal received by the mobile device at any time is in the superposition state. In other words, traditional cellular communication link is like a light bulb to illuminate the whole room, while modern cellular beamforming is like a flashlight, enabling smart focus of brightness towards the direction of a mobile device.

In practical applications, a base station with adaptive array antenna can serve multiple users at the same time, construct different beams towards multiple target customers and effectively reduce the interference between each beam. This multi-user beamforming in space effectively separates the electromagnetic wave between different users. This is the basis of antenna arrays. Antenna arrays support multi-user beamforming. Imagine that if several hundreds of antennas are arranged at the base station, multiple mobile devices can be served by their respective beams. In addition, space-division multiple access (SDMA) can be achieved by transmitting multiple signals simultaneously on the same frequency resource. SDMA can enable us to effectively utilize the scarce frequency resources and increase network capacity several times. Antenna arrays do not simply increase the number of antennas but provide qualitative change with the quantitative change.

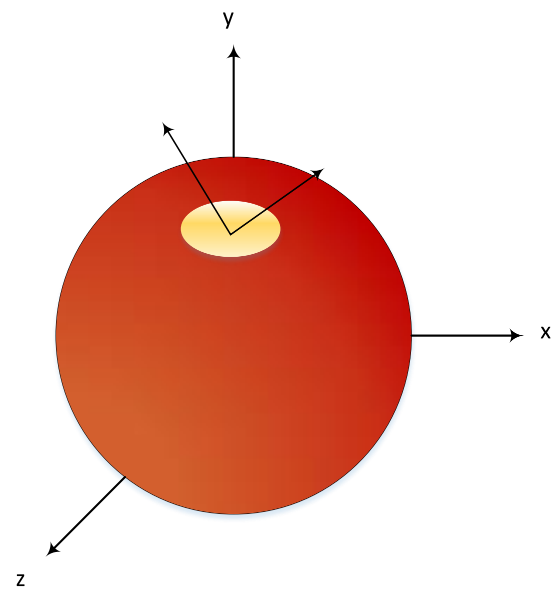

Figure 4.1 The radiation direction of omnidirectional antenna

Figure 4.1 shows the direction of ordinary omnidirectional antenna. The radiation coverages all directions

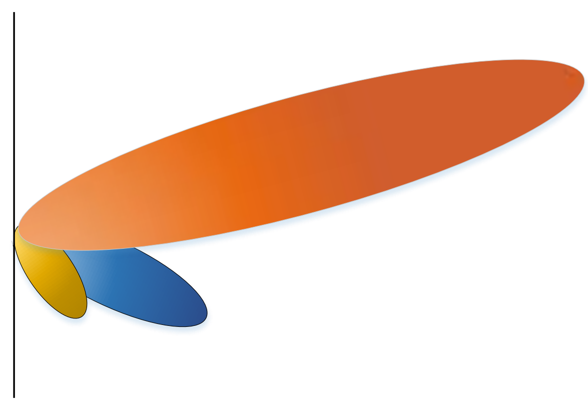

Figure 4.2 The antenna radiation direction after beamforming

Figure 4.2 shows the antenna radiation pattern after beamforming, with most of the electromagnetic energy gathering to one direction. In the single antenna to single antenna transmission system, due to the complexity of the environment, the phase of the electromagnetic wave may be opposite to each other at the receiving point after multipath propagation. If the channel is experiencing severe fading, the quality of the signal received by the user will be deteriorated. When the number of base station antennas is increased, the number of channels between the base station and a mobile device is also increased. They are independent of each other, and thus the probability of falling into severe fading is greatly reduced. This will improve the overall performance of the system.

The benefits of antenna arrays include improved network capacity. In addition, due to the simultaneous constructive effect of antennas, the superposition gain of beamforming will make each antenna only need to transmit a small amount of power, thus avoiding the use of expensive power amplifiers with a large dynamic range, reducing the cost of hardware. Furthermore, the flat fading channel created by the law of large numbers makes it possible for the base station and the mobile device to communicate with low latency. To combat deep fading channels, traditional communication systems needs to use channel coding and interleaving to disperse the continuous burst errors caused by deep fading to different time periods. The receiver has to accept all data at a time completely including information and delay. With antenna array, deep fading is weakened by the large number theorem. The channel becomes good, the process of the depth of confrontation can be greatly simplified, so the delay can also be greatly reduced.

4.1.2 Two-dimensional Antenna Array

Over the years, the synthesis problem of low sidelobe antenna pattern has always been an interesting research topic. Dolph [45] presented the best result of the array weights in an isometric array, which could minimize the beam width when the maximum sidelobe level (SLL) is given. Taylor [46] obtained an aperture distribution, which can generate equal amplitude sidelobe near the main lobe area while gradually decreasing the sidelobe level in the far field. Due to the dual relation between directional response of the isometric linear array and the frequency response of the finite impulse response (FIR) filter, the design method of low sidelobe for FIR filter can be applied to array design to form low sidelobe beam pattern. There are many other array synthesis methods, such as Woodward method [74], Perturbation method [75], statistical method and etc. The limitation of these traditional array synthesis methods is that it requires arrays to be uniform and array elements should have certain regularity. Especially when the array is integrated, the directional characteristics of the array elements cannot be considered. In recent years, the array synthesis algorithm development with the adaptive array signal processing theory can overcome the shortcomings of the above methods. It effectively solves the problem of non-omnidirectional array element, the different pattern between the array elements, unequal spacing of elements or even the irregular array synthesis problem with random array elements.

The simplest method of pattern synthesis for wide rectangular two-dimensional antenna arrays is the push line array method. If the aperture distribution is two-dimensional separable, the radiation pattern of the planar array is equal to the product of the two orthogonal arrays. The resulting pattern can achieve the required sidelobe level in the corresponding main plane, while the sidelobe level is lower than the design target in other planes, leading to the widening of the main lobe.

In one dimensional arrays, the weights are used to compensate for the phase propagation of a wave in a desired direction [22]. The extension to 2-D arrays is straight forward [22].

Planar arrays have higher directivity, lower side lobes and more symmetrical pattern [76], thus planar arrays have multifunction compared to linear arrays. Planar arrays have direction control in two dimensions and can be used to scan the main beam toward any point in space, making them useful in many applications [76]. The most common planar arrays are rectangular arrays (See Figure 4.3), hexagonal arrays and circular planar arrays (See Figure 4.4).

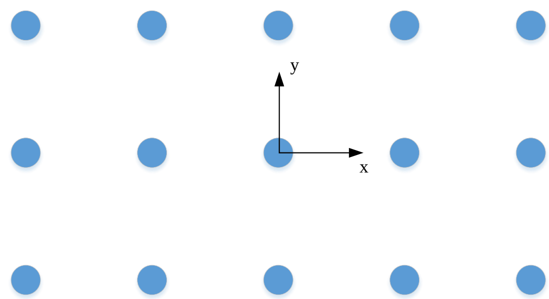

Figure 4.3 A uniform rectangular array structure

Figure 4.3 illustrates a 5×3 rectangular array structure with 0.5λ spacing.

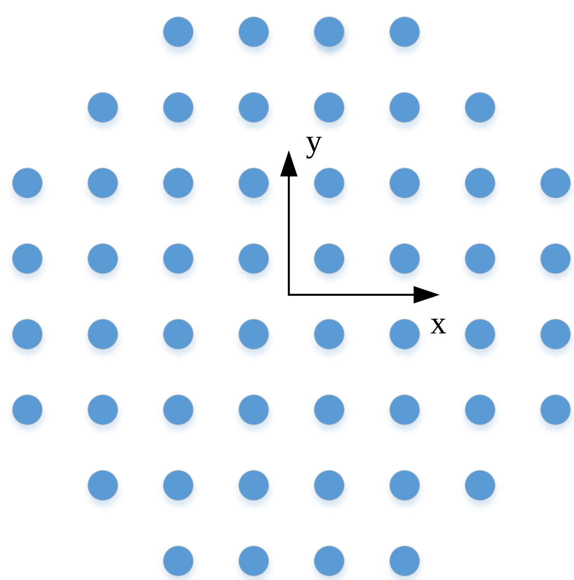

Figure 4.4 A circular planar array structure with 52 elements.

Figure 4.4 shows a general circular planar array with 0.5λ spacing, radius of 2λ.

The array factor of rectangular array is

$$AF = \left[\frac{1}{M} * \frac{\sin(M * \frac{\psi_x}{2})}{\sin(\frac{\psi_x}{2})} \right] * \left[\frac{1}{N} * \frac{\sin(N * \frac{\psi_y}{2})}{\sin(\frac{\psi_y}{2})} \right] \tag{4.1}$$

$$\begin{aligned} \psi_x &= kd_x \sin\theta cos\phi + \chi_x \\ \psi_y &= kd_y \sin\theta cos\phi + \chi_y \end{aligned} \tag{4.2}$$

Where d is the distance between elements in y-plane or x-plane. θ is the angle of arrival of x-axis, ϕ is the azimuth angle. χ is the progressive phase shift between elements in y-plane or x-plane.

The linear antenna arrays have advantages of compendious structure and flexible measurement of antenna parameters. In a linear antenna array, if the number of antenna elements is larger, the diversity of the array is higher. It provides electronic steering and hence cumbersome mechanical steering using servo motors can be avoided, hence beam can be moved in the desired direction in less than milliseconds [77].

But it has its limitations. Firstly, the number of elements in the linear array is fixed. These elements are static structures. Static means the memory is allocated at compilation time their memory used by them cannot be reduced or extended [78]. So, it is not easy to define a specific objective function of linear arrays. Another disadvantage is the limitation coverage to 120 degrees in azimuth and elevation [77].

In future multi-antenna systems, the use of two-dimensional antenna array structure is an inevitable trend. With the increase of the antenna element, a narrower beam can be formed by using different weighting vectors, so that the directional transmission of a single UE can be realized [79]. Traditional antenna arrays are usually fixed beams, beam pointing and beam width are relatively fixed [80]. With the emergence of the active antenna array, the beam direction can be controlled in real time by the digital way.

However, once the array elements are fixed, the beam width is fixed, only the beam direction can be adjusted.

4.1.3 2D Array Design Algorithms

Tseng and Cheng proposed a method of Chebyshev rectangle plane array design [81]. It can make the radiation pattern generated by the planar array best Chebyshev- patterns in each section. However, the method requires the number of row elements to be equal to the number of column elements. Besides, it cannot consider the characteristic of the radiation pattern of the array. Stutzman and Coffey proposed an iterative sampling algorithm for planar array synthesis [82]. Firstly, an initial map is obtained from other algorithms or experimental data, and the sample is corrected by using the properties of the orthogonal function $\sin(x)/X$ to match the pattern at the sampling point and the target pattern. The sampling iteration process is repeated until a satisfactory direction is obtained. This method is relatively simple, but the integrated pattern will fluctuate between sampling points, and it cannot consider the directional characteristics of the element.

In 2D array design, beamforming is a method for weights optimization. Figure 4.5 shows a plane wave propagating in Z-axis. The plane wave propagates with frequency f and can be defined by z = constant. Where θ is the elevation angle of the signal (DOA angle) and φ is the azimuth angle.

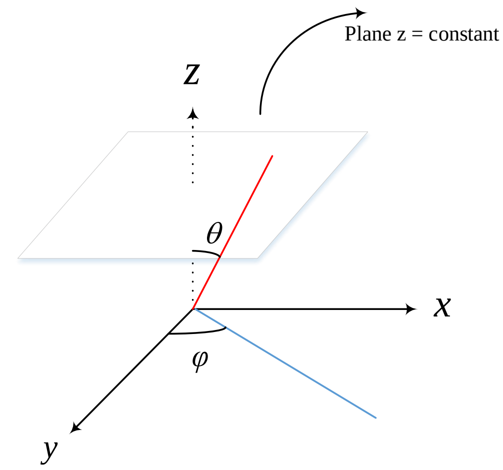

Figure 4.5 A plane wave propagating in Z-axis

The plane wave propagates in 2-D arrays. The phase can be defined as:

$$\varphi(t,z) = 2\pi ft - kz \tag{4.3}$$

Where t is the time, and k is the wave number expressed as:

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \tag{4.4}$$

ω is the temporal angular frequency. c refers the signal propagation speed in the specific medium. λ is the wavelength. In a temporal interval t, the phase of the signal can be written as ωt . k is a three-dimension spatial frequency. Its direction is opposite to the propagating directions of the signal [7]. In a Cartesian coordinate system, it can be denoted by a three-element vector[7]

$$k = \left[k_x, \quad k_y, \quad k_z\right]^T \tag{4.5}$$

With a length of

$$k = \sqrt{k_x^2, \quad k_y^2, \quad k_z^2} \tag{4.6}$$

which is referred to as wavenumber vector. In Figure 4.5, when $k_x = k_y$ =0, $k_z = -k$.

$k = \frac{2\pi f}{c}$.

If the signal impinges upon the array from an elevation angle θ and an azimuth angle φ, the wavenumber k can be referred to as

$$k = \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix} = k \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix} \tag{4.7}$$

The phased weights for 2-D planar array is towards (θ_d, φ_d)

$$w_n = e^{jk_d r_n} = e^{j(k_{xd}x_n + k_{yd}y_n + k_{zd}z_n)} = e^{j\frac{2\pi}{\lambda}\sin\theta_d(\cos\varphi_d x_n + \sin\varphi_d y_n)} \tag{4.8}$$

k_d is wave vector, as a vector that describes the phase variation of a plane wave [22]. The magnitude of wave vector is the wave number.

The 2D antenna array has many algorithms to optimize the weights coefficient. For example, Dolph Chebyshev optimization method for weights [45]and Taylor synthesis method [46]. GA method is also applied for antenna array beamforming. GA was first proposed by Professor J.Holland of Michigan University in 1975 [75], and it is according to the Mendel's theory of heredity [75] and Darwin's theory of evolution [75]. The algorithm has been applied to many scientific fields, and it has also been applied to the optimization design of antennas. GA is also known as an evolutionary algorithm. Nowadays, the interesting topics of genetic algorithm in antenna research include optimization of antenna array structure design, such as C. M. Coleman self-configuring antenna [83]; other methods combines with GA to optimize the broadband antenna; It also includes antenna feed mode optimization by GA. In the optimization of antennas, GA is a difficult problem for multi-objective optimization of antenna arrays. Most scholars focused on research on one-dimensional linear antenna arrays, but there are few studies on

2D and 3D, especially for 2D and 3D array elements distributions.

An introduction to the biological background of GA is necessary.

- *Population*: the evolution of organisms takes place in the form of groups. Such a group is called a population.
- *Individual*: a single organism which can forms a population.
- *Gene*: a genetic factor.
- *Chromosome*: contains a set of genes.
- *Survival competition*, or *survival of the fitness*: individuals with high adaptability to environment will have more chances to participate in reproduction, and more posterity will be born. Individuals with low fitness will have fewer opportunities to participate in reproduction and have fewer posterity.
- *Inheritance and mutation*: a new gene that inherits a part of both parents and has a certain probability of genetic mutation.

In simple terms, after the breeding process, Crossover, gene mutation, and the fitness of individuals will be phased out. Then after the natural selection of the *N* generation, the individuals preserved are with high fitness, as it is highly likely to include the individual with the highest fitness in history.

Figure 4.6 shows the process of GA, and the main procedure is related to the selection, cross, variation progress.

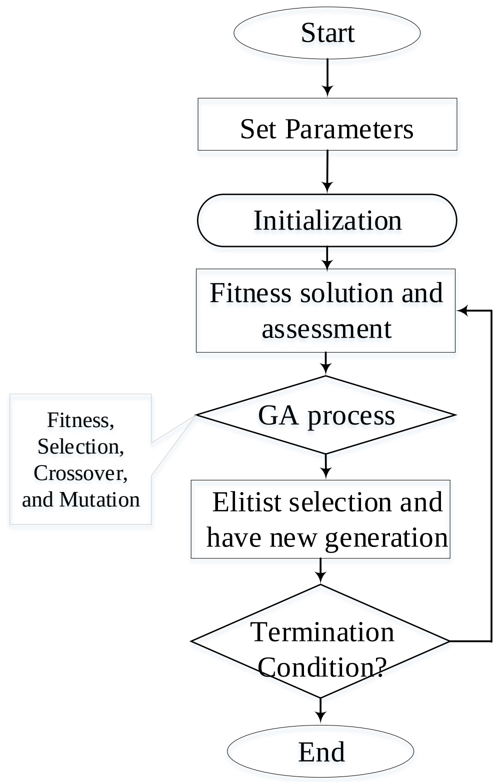

Figure 4.6 The diagram of genetic algorithm

In GA, the solution of the first generation is a random or according to a certain rule result. The result is unsatisfied. For further optimization, every generation should go through the selection, crossover, mutation process to obtain a better solution.

Selection is the operator to act on the whole group. The purpose of selection is to directly transfer the optimized individuals to the next generation, or to generate new individuals through pairing crossover and then to inherit the next generation. The selection operation is based on the fitness evaluation of individuals in a group.

Crossover operator is applied to the group. Crossover operators play a central role in genetic algorithms. The mutation operator is applied to the group because the gene values of some individuals are changed.

The population is the basis of GA, this algorithm imitating the biological population. The number of iterations is the number of evolutionary generations of the population.

Fitness function in GA is the key to solve the problem. The initial population is randomly generated, and the fitness should be chosen reasonable. Generally, the selection mechanism uses betting wheel method. This method has large probability to select the better population solution. The solution after chosen through crossover and mutation process will be in progress of another selection. The good result will replace the poor result. The crossover rate is a number between 0-1, the bigger the value is, the process will make the genes cross as much as possible. However, the mutation rate should as small as possible to prevent the better result from dropping off after mutation.

The advantage of GA algorithm is that it can solve most optimization problems. Whatever the question is, it can give a relatively better result in certain time.

4.2 Two-Dimensional Sparse Antenna Array design

4.2.1 Array Modelling

For a planar rectangular array, imaging the antenna elements are uniform placed, as shown in Figure 4.7.

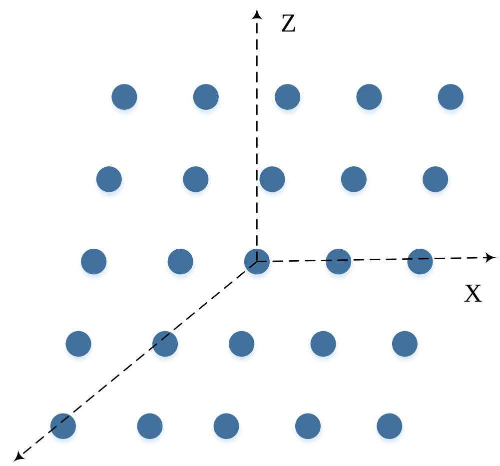

Figure 4.7 A uniform 5×5 Rectangular array in planar array

In a 5×5 equal spaced rectangular array, the distance between each element for λ is:

$$d = (a\frac{\lambda}{2}, b\frac{\lambda}{2}, 0), \text{ for a, b=0, 1, 2} \tag{4.9}$$

The array factor is

$$AF = \sum_{b=0}^{2}\sum_{a=0}^{2} e^{-j\pi\sin\theta(a\cos\phi + b\sin\phi)} \tag{4.10}$$

When the antenna array is steered towards direction θ_d

The weights are given by

$$w_n = e^{jknd\cos\theta_d} \tag{4.11}$$

n is an antenna element located at $r_n = (x_n, y_n, z_n) = (0, 0, nd)$.

The inter element spacing is a constant and equal to *d*. The array factor is

$$AF = \sum_{n=0}^{N-1} w_n e^{-jkr_n} = \sum_{n=0}^{N-1} e^{-jknd\cos\theta_d} e^{-jkr_n} \tag{4.12}$$

The above is related to the wave vector, as (4.7),

$$k * r_n = knd\cos\theta \tag{4.13}$$

The array vector can be written as:

$$AF = \sum_{n=0}^{N=1} e^{jknd(\cos\theta_d - \cos\theta)} = \sum_{n=0}^{N=1} G^n \tag{4.14}$$

V_G is the dummy variable, and can be given by

$$V_G = e^{jkd(\cos\theta_d - \cos\theta)} \tag{4.15}$$

Make the *V* simple as:

$$\sum_{n=0}^{N=1} V^N = \frac{1-V^N}{1-V} \tag{4.16}$$

The array factor can be rewritten as:

$$AF = \frac{1-e^{jkNd(\cos\theta_d - \cos\theta)}}{1-e^{jkd(\cos\theta_d - \cos\theta)}} \tag{4.17}$$

If the magnitude is only cared of the array factor, the results of array factor in magnitude can be referred to as:

$$AF = \frac{-e^{jkNd(\cos\theta_d - \cos\theta)/2}}{-e^{jkd(\cos\theta_d - \cos\theta)/2}} \left[\frac{e^{jkNd(\cos\theta_d - \cos\theta)/2} - e^{-jkNd(\cos\theta_d - \cos\theta)/2}}{e^{jkd(\cos\theta_d - \cos\theta)/2} - e^{-jkd(\cos\theta_d - \cos\theta)/2}} \right] \tag{4.18}$$

If use a general function of sin*x*(*)

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \tag{4.19}$$

The magnitude of array factor reduces to

$$AF = \frac{\sin\left(kNd(\cos\theta_d - \cos\theta)/2\right)}{\sin\left(kd(\cos\theta_d - \cos\theta)/2\right)} \tag{4.20}$$

Therefore, the array factor can be simplified as

$$AF = \left(\frac{e^{-j3\pi\sin\theta\cos\phi/2}}{e^{-j\pi\sin\theta\cos\phi/2}}\right)\left(\frac{e^{-j3\pi\sin\theta\sin\phi/2}}{e^{-j\pi\sin\theta\sin\phi/2}}\right) * \left(\frac{\sin(3\pi\sin\theta\cos\phi/2)}{\sin(\pi\sin\theta\cos\phi/2}\right)\left(\frac{\sin(3\pi\sin\theta\sin\phi/2}{\sin(\pi\sin\theta\sin\phi/2}\right)$$

(4.21)

It can be written as:

$$\left|AF\right| = \left(\frac{\sin(3\pi\sin\theta\cos\phi/2)}{\sin(\pi\sin\theta\cos\phi/2}\right)\left(\frac{\sin(3\pi\sin\theta\sin\phi/2}{\sin(\pi\sin\theta\sin\phi/2}\right) \tag{4.22}$$

To plot the functions, the variables *u* and *v* are as follows:

$$u = \frac{k_x\lambda}{2\pi} = \sin\theta\cos\varphi \tag{4.23}$$

$$v = \frac{k_y\lambda}{2\pi} = \sin\theta\sin\varphi \tag{4.24}$$

4.2.2 2D Sparse Array Design using CS

Spatial spectrum estimation is based on beamforming, while the basic theory of beamforming is related to weight optimization. A MacBook Pro 2017 with a 2.9GHz Intel core i7 is used to run MATLAB 2015b (Build 8.6.0.267246 64-bit Win64) for the simulation.

Firstly, consider a uniform10×10 rectangular array, the weights according to the antenna elements should have 100 values. The desired response is a uniform rectangular array with 0.5λ spaced of each element. From Figure 4.8, θ is from [-90°, 60°], ϕ has the same direction of θ. As

$$\left|Ax - b\right| \le \sigma \tag{4.25}$$

If σ=0, the desired response equals the designed response, there would be no interference between the array elements. Therefore, σ should be as small as possible. In this design, σ is set 0.1, For sparseness of this array, a norm l_0 should be used to remove the zero-valued weight coefficients. Because l_0 norm requires large scaled computing, l_1 norm is used to replace the optimization of l_0 norm.

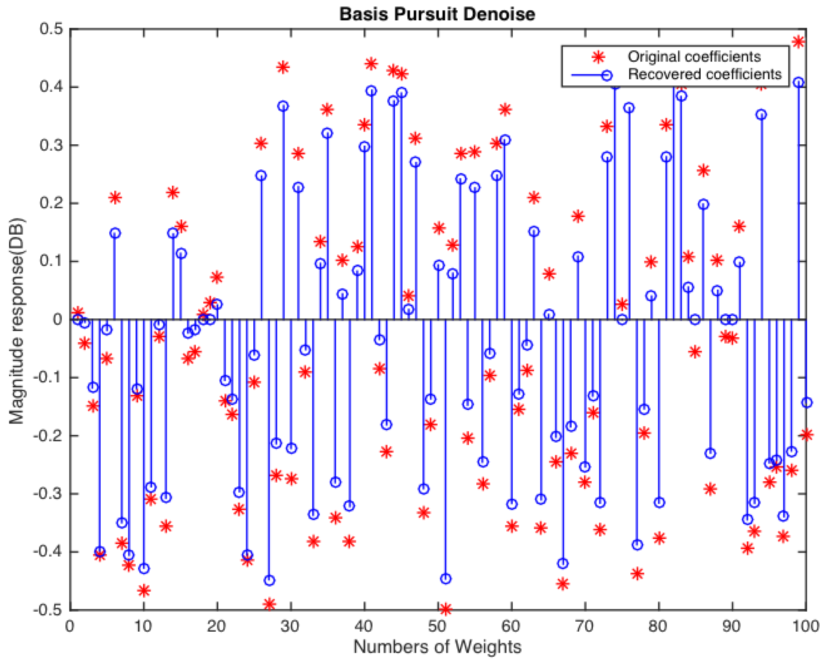

Figure 4.8 The recovery weights difference of rectangular antenna array

Table 4-1 Weights after sparseness optimization

n	w_n	n	w_n	n	w_n	n	w_n
1	0.209	10	0.126	19	0.290	28	0.445
2	0.218	11	0.355	20	0.302	29	0.405
3	0.162	12	0.440	21	0.362	30	0.257
4	0.304	13	0.428	22	0.210	31	0.160
5	0.434	14	0.421	23	0.178	32	0.405
6	0.286	15	0.313	24	0.333	33	0.477
7	0.134	16	0.158	25	0.440		
8	0.362	17	0.129	26	0.422		
9	0.101	18	0.285	27	0.333		

Table 4-1 shows the result of weights after the optimization by SPGL1 with l_1 norm. The number of weight coefficients is reduced from 100 to 33. From one dimensional array, we know that weight related to the antenna array locations. This can also be deduced in two-dimensional array. The result with 33 antenna elements can be inferred.

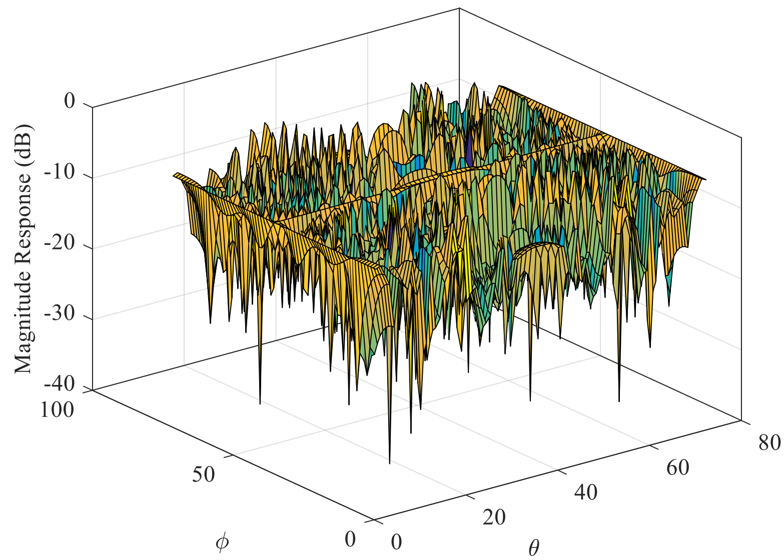

Figure 4.9 The desired response of a rectangular planar array

Figure 4.9 shows the radiation pattern of a uniform planar array. Since the rectangular is 10×10, there are 100 weights of values in this antenna array. Figure 4.10 shows the difference between the designed sparse planar array and desired uniform planar array. From the result, these radiation patterns do not have significant differences. Figure 4.11 is the difference value of desired and designed array. The difference between two sets of response values is quite small. Therefore, it is safe to say that we can use 33 antenna elements can achieve the performance that is originally achieved by using 100 antenna elements.

In addition, for the CS method, the iteration time is 0.157018s.

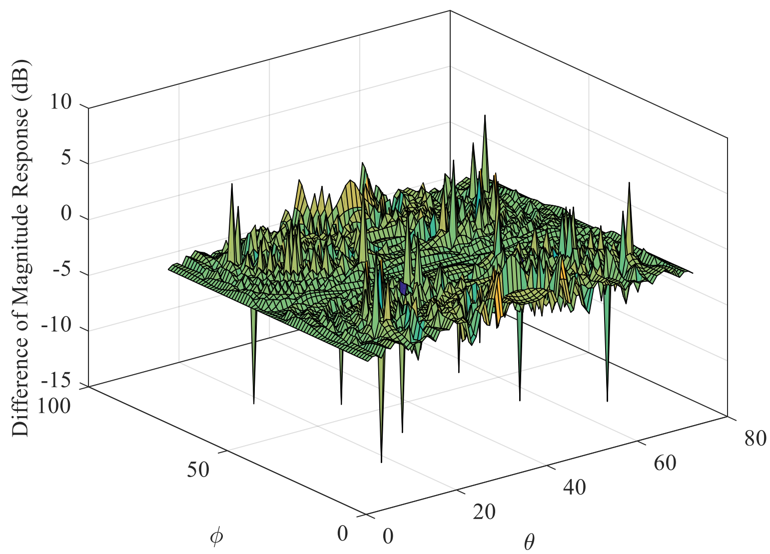

Figure 4.10 The difference of desired array response and designed array response.

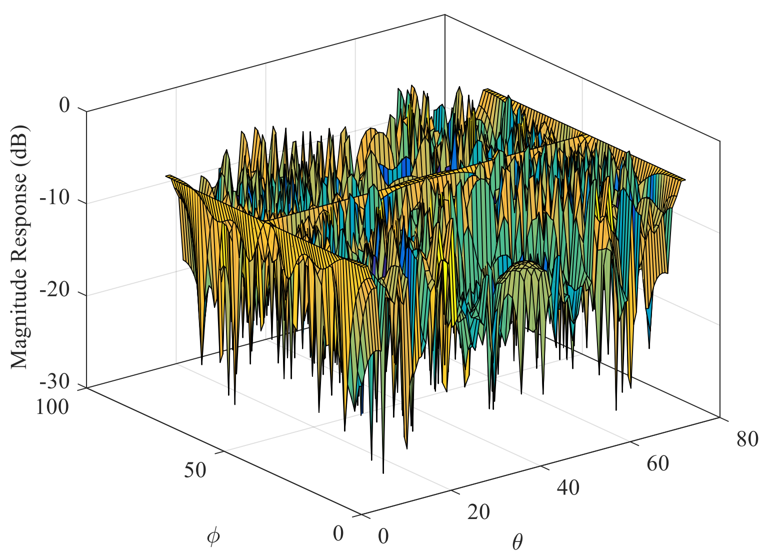

Figure 4.11 The designed response of planar array.

4.2.3 Result of GA Method

As aforementioned, the idea of GA is based on the theory of biological evolution, and it simulates the problem as a process of biological evolution. Its major operations include crossover and mutation to produce the next generation of solutions, and selection to gradually eliminate the solution to adapt the low fitness function value and increases the degree of adaptation to higher fitness solution. This evolution of N generations is likely to evolve the fitness function of the individual values towards a higher value.

For the design of sparse planar array using GA, the key idea is to find better solution of weights. Through the selection, crossover and mutation progress, GA adapts the non-satisfied weight (the value of weights is zero or relative), and finally selects the qualified weight coefficients. After GA, a non-uniform sparse planar array can be obtained. A MacBook Pro 2017 with a 2.9GHz Intel core i7 is used to run MATLAB 2015b (Build 8.6.0.267246 64-bit Win64) for the simulation as for the CS method.

The population=40, crossover=90% and generation set to 500.

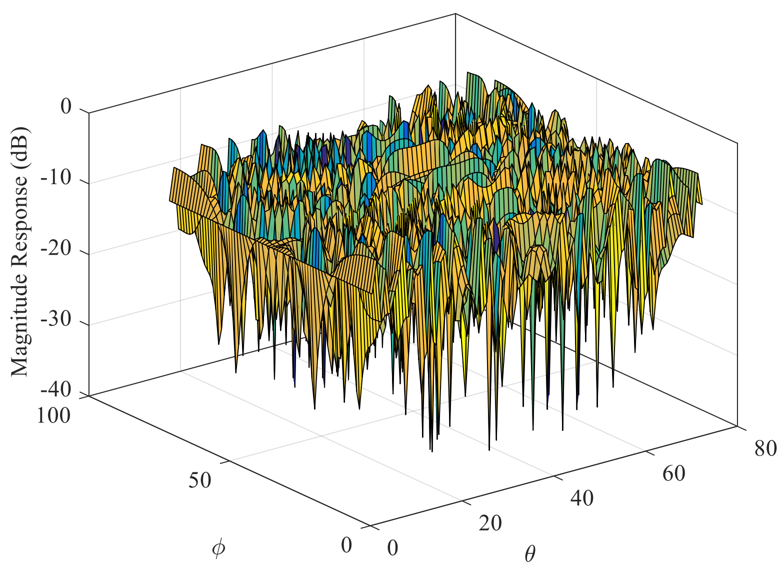

Figure 4.12 The Response of designed antenna array in GA method.

Figure 4.12 is the sparse planar array response with weight optimization by GA. The distribution of weight coefficients is shown in Figure 4.13

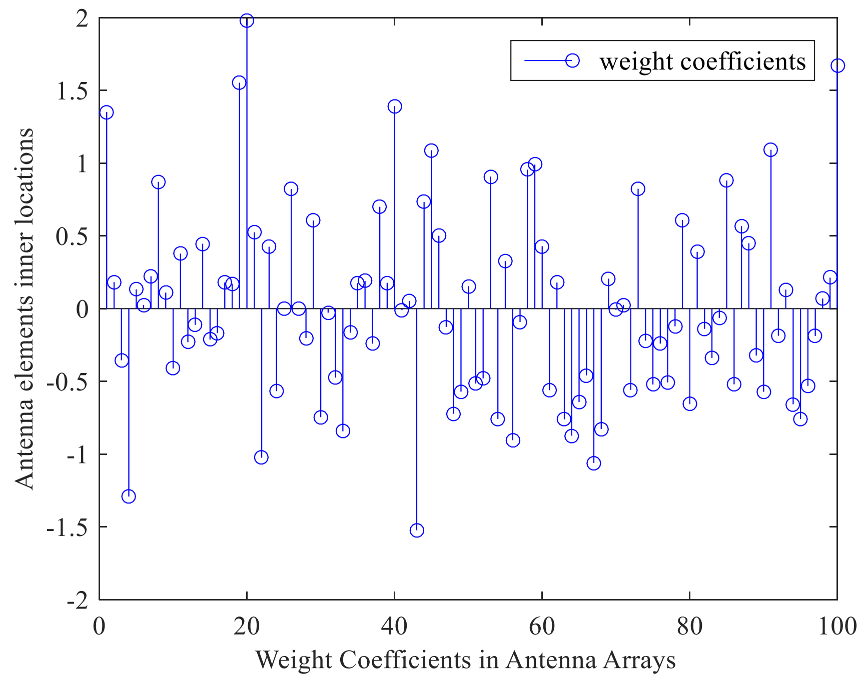

Figure 4.13 Weight coefficients distribution in antenna array.

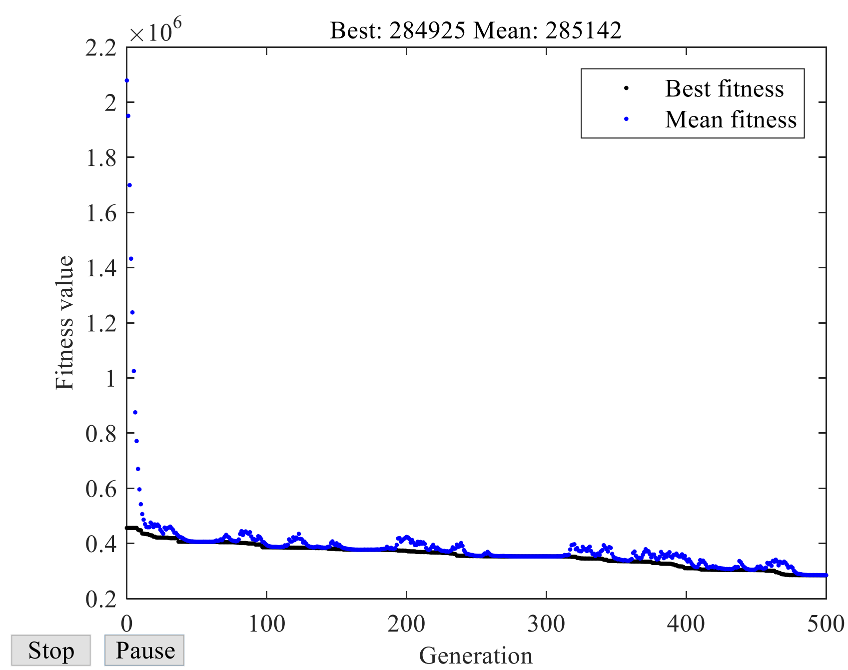

Figure 4.14 The weight coefficients fitness optimization.

As shown in Figure 4.13,there are 100 weights coefficients for the conventional uniform planar array. Figure 4.14 shows the evolution of fitness value of best weight obtained by GA optimization.

Table 4-2 The weight after optimization by GA selection.

n	w_n	n	w_n	n	w_n	n	w_n
1	0.297	12	0.542	23	0.729	34	0.487
2	1.493	13	1.900	24	0.326	35	1.763
3	1.235	14	0.291	25	0.341	36	0.376
4	1.215	15	0.434	26	1.432	37	0.910
5	2.734	16	1.267	27	0.167	38	0.139
6	1.078	17	0.757	28	0.232	39	1.239
7	0.555	18	0.303	29	0.469	40	1.929
8	1.020	19	0.867	30	1.779	41	2.432
9	0.269	20	0.644	31	1.260		
10	1.067	21	0.949	32	0.317		
11	0.400	22	0.934	33	0.803		

Table 4-2 lists the values of weights, from which we can see that the GA method obtained a sparse planar array with 41 elements to achieve the same performance as a conventional uniform planar array with 100 elements.

By the CS method, 33 elements are used to design the sparse planar array. While for the GA method, it requires 41 elements. The computation time for GA and CS is 76.476652s 0.253679s, respectively. Thus, CS can shorten the computation time by 350 times as compared to GA. In addition, GA requires larger system memory.

In summary, the GA method does not have advantage to design sparse planar array as compared to the CS method because it requires longer computation time and larger system memory. Nevertheless, the GA method does not require the signal to be sparse or incoherent, which is the limiting factor for the CS method. In general, CS has the great advantage to design multi-dimensional array because of its characteristics of fast computation and precise result.

4.3 Summary

In this chapter, we firstly studied the design of a sparse planar antenna array based on the CS method in narrowband beamforming, followed by the design of sparse 2D antenna array by GA. This design objective is to obtain a satisfied radiation pattern with the minimum number of antenna elements. 2D array can better control the beam patter than linear array. However, the cost and iteration computation time are unavoidably increased. For sparse optimize of the 2D array, the number of antenna elements would be reduced thus the overall cost will be lower. Therefore, the sparser the antenna array, the lower the cost.

For the CS method, we started with a 10×10 rectangular planar array, where antenna elements are equally spaced. This is the conventional uniform array model. The response is the product of steering vector and weights. To obtain the weights, it uses SPGL1 to perform a norm calculation by setting a tolerable difference between desired response and designed response according to one-dimensional linear array. This difference usually set less than 0.01. If it is set to zero, the desired response equals the designed response, indicating there is no error and interference in the array. This is not possible in practice. After obtaining the weights and removing the values of zero or relative to zero weight coefficients, the remaining weights will lead to a sparse antenna array. The location of the array can be calculated.

The second considered method GA is inspired by Darwin's theory of evolution and inspired by the evolutionary process of biology. GA usually uses selection, crossover, mutation process to evolve towards a better solution. The solution of the first generation is a random result, which is not satisfied. Every generation should go through the selection, crossover, mutation process to have better solution for further optimization. Fitness function in GA is the key to solve the problem. The initial population is randomly generated, and the fitness should be chosen reasonable. We used betting wheel method as the selection mechanism in this study. GA has advantage that no matter how complex of the array is, it can give an acceptable solution. The disadvantage is related to the long computation time and larger memory requirement.

In this chapter, we first reviewed the background knowledge about the advantage of antenna array. Then, it briefly discussed about 2D antenna array including its advantages and shapes, followed by the two methods to design the rectangular planar array. It briefly introduced the design in CS method and GA method. Next, the simulation results are presented. From the results with the CS method, the design is successful by using 33 antenna elements to achieve the performance of a uniform antenna array with 100 antenna elements. In addition, for the GA method, it uses 41 antenna elements to achieve the same performance. However, the iteration time is tremendous as compared to that of the CS method. This leads to our conclusion that the CS method is more suitable for the design of sparse antenna array as compared to the GA method, provided that the considered signal is sparse.

For future work, the sidelobe should be lower than the main lobe in order to further improve the performance. If the sidelobe is not low enough, it will generate the interference and mutual coupling between antenna elements. In addition, the number of weight coefficients after optimization could be further reduced. The decrease of number of antenna elements will not only lower the cost, but also help to avoid the mutual coupling between the elements.

In summary, as compared to GA method, CS is no doubt the most efficient way to design antenna array. The CS method should be used to improve the robustness of the design to avoid interference and model errors.

Chapter 5 Conclusions and Future Work

5.1 Conclusions

Beamforming emerged as a new technique to improve the system capacity and improve the energy efficiency in the future fifth generation (5G) mobile cellular networks. To achieve adaptive beamforming, antenna array is the necessary and currently it is usually implemented in the uniform form. In this thesis, we study how to design sparse antenna array (with less antenna elements), which adopts less antenna elements to adequately achieve the same performance of uniform antenna arrays. The less the number of antenna elements, the lower the overall cost and the lower the mutual coupling interference between antenna elements. We studied two methods, namely the compressive sensing (CS) method and the genetic algorithm (GA) method. We found that CS method can provide better results in a short computation time as compared to the GA method, provided that the considered signal is sparse. Nevertheless, GA method is more general, and it can be applied to any type of signals.

In chapter 1, antenna array background was briefly revisited. The design motivation and objectives were introduced, followed by the thesis organization to give a clearly frame of this thesis.

Chapter 2 is related to the basic theory of antenna and antenna arrays. It starts from single antenna and then moves on to antenna arrays. Besides, beamforming as a method for array signal processing has been introduced. Different types of beamforming have been listed in detail. Furthermore, it is expanded to sparse antenna array and give two common method GA and SA for sparse antenna array design.

In this thesis, one-dimensional array design is studied in Chapter 3 and two-dimensional array design is studied in Chapter 4.

In chapter 3, CS is used to design the linear antenna array which is benchmark of one-dimensional array. The desired response is that of an equal spaced linear antenna array. For sparseness, we used a norm calculation to remove the zero values of weights. The remaining weight coefficients forms a sparse linear antenna array that is the designed response. The whole design is to match the designed response near the desired response. The method to match these two curves is to set a parameter of the difference of these two responses. If the parameter is zero, it means there is no error in the array. Most of time, this parameter should be as small as possible.

In Chapter 4, there are two methods to design a 2-dimensional array. In this design, a rectangular planar array is used for the model of 2-dimensional array. For the CS method, it is close to the linear antenna array design. Notice that θ and ϕ are used to present the signal of arrival and azimuth angle. The result still matches the designed response to the desired one. 2-D array has the advantage of easier beam pattern control as compared with 1-D array. However, the computation and iteration time is increased. GA method is usually used to design 1-D antenna array, few researchers paid attention on 2-D or 3-D array. This is because GA requires longer computation time and better computing equipment as compared to the CS method.

5.2 Future Work

For the future work linear array designing, firstly the sparse array should be considered in errors in antenna locations. Then, the design can be expanded to a robust antenna array after the consideration of the model errors in antenna locations and other factors. The design can be extended to wideband beamforming. In wideband beamforming, there are two methods to design the array. One is to transform the channel to a narrowband one, then process according to narrowband signal. Second method is to focus on specific frequencies and then process the signal.

For the future work of 2-D rectangular array design by CS, sparseness should be considered further. The sparser the antenna is, the lower cost and the shorter computation time. This 2-D structure can be expanded to 3D antenna array. For sparseness in 3D array, CS has its advantage of fast computation time. SA can be used to compare with CS. Furthermore, other method or combined method can be studied for sparseness in array. This study can be expanded to wideband beamforming because of its wide application. However, narrowband study is necessary because the wideband signal can be decomposed into a bundle of narrowband signals.

Though GA has its limitation of long computation time, the condition of GA algorithms is quite relaxed. It can be used to solve some specified problems. In future design works, GA can be combined with CS to solve those problems that cannot be solved by solely using CS. The aim is to decrease the computation time of GA, and this requires further investigation.

Appendix A

Appendix A provides more simulation results in both 1-D and 2-D array designs in Chapter 3 and Chapter 4.

Section A.1 Sparse linear antenna array design using compressive sensing:

When $\alpha = 0.09$, $\beta=0.15$, $\theta_K = 90$;

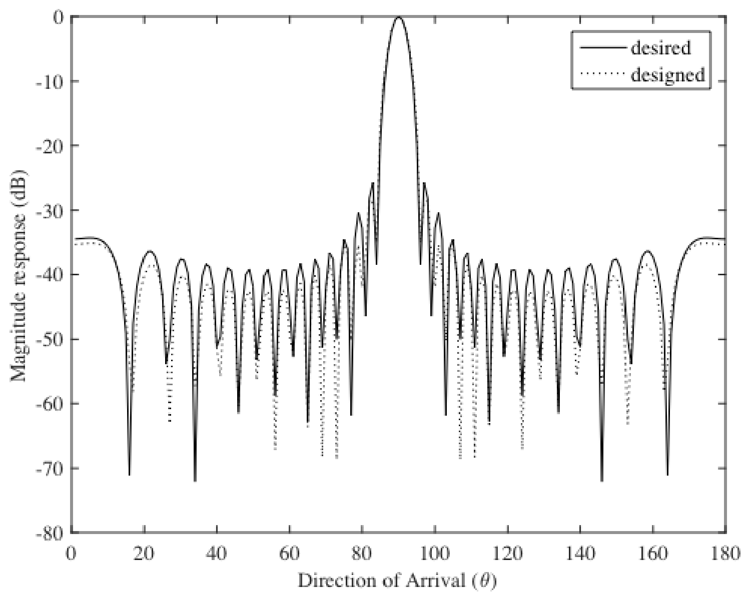

Figure A.1 The difference of desired response and designed response in linear array

Figure A.1 shows a perfect performance when α becomes smaller and β becomes larger as compared with the result of β =0.1. The computation time is 0.083200s. α should keep smaller to ensure the main beam is narrower.

In this result, the designed response is very close to the desired one. It uses nine antenna elements to achieve the performance of 100 antenna elements.

Table A-1 The result of w coefficients in design of linear array

n	d_n/λ	w_n
1	0	0.7087
2	0.62	0.8803
3	2.68	0.8192
4	5.91	0.7246
5	6.03	0.6051
6	6.66	0.4757
7	7.97	0.3500
8	9.34	0.2344
9	10.88	0.3285

This design illustrates the effectiveness of CS in the design of one-dimensional linear array.

Section A.2 Design of sparse rectangular planar array using CS

Consider a uniform 10×10 rectangular array, the weights according to the antenna elements should have 100 values. From Figure 4.8, θ is from [-30°, 90°], ϕ is from [0°, 90°]. As $|Ax-b|\le\sigma$, σ is the upper limit of the tolerable difference between desired and designed response. σ needs to be small enough. Set σ =0.05. Figure A.2 shows the weights distribution after optimization by SPGL1. This result uses 34 number of weight coefficients to achieve the performance of a planar antenna array with 100 elements. The difference between the designed response and desired response is small. The computation time of the whole design is 0.253410s.

Table A-2 shows the result of weight coefficients obtained by the CS method. The result is related to 33 number of antenna elements after remove the zero or nearly zero values of weight coefficients.

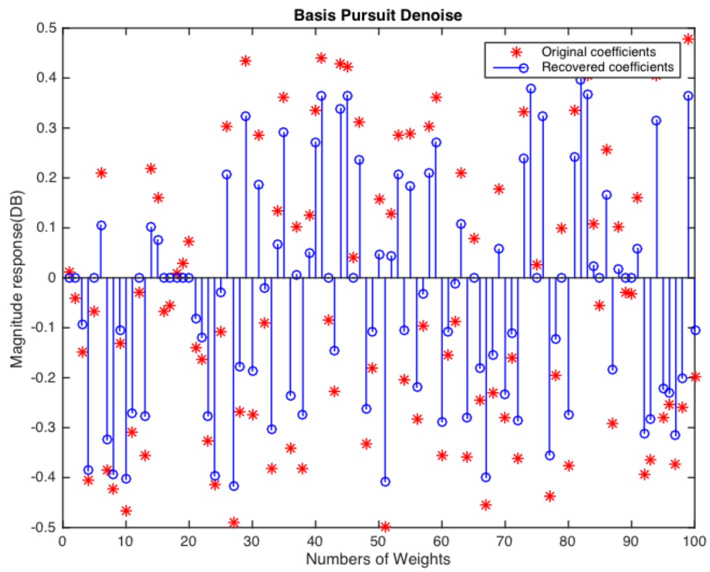

Figure A.2 The difference of recovery weights and original weights of rectangular antenna array

Table A-2 Weights after sparseness optimization

n	w_n	n	w_n	n	w_n	n	w_n
1	0.203	10	0.174	19	0.401	28	0.473
2	0.289	11	0.302	20	0.334	29	0.114
3	0.172	12	0.478	21	0.359	30	0.265
4	0.432	13	0.142	22	0.109	31	0.190
5	0.343	14	0.392	23	0.177	32	0.419
6	0.189	15	0.294	24	0.305	33	0.489
7	0.236	16	0.315	25	0.467	34	0.140
8	0.357	17	0.147	26	0.402		
9	0.105	18	0.230	27	0.324		

Figure A.3 shows the desired response of the rectangular planar array. The desired response is an equal spaced rectangular array with d = 0.5λ. Figure A.4 shows the difference between designed and desired response. From Figure A.4, the difference is negligible. Figure A.5 shows the response of the designed antenna array.

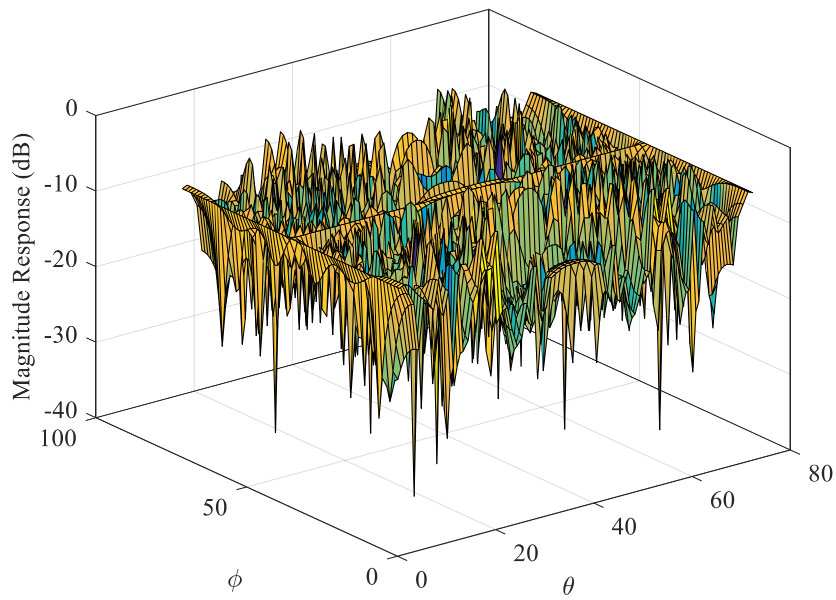

Figure A.3 The desired response of a rectangular planar array

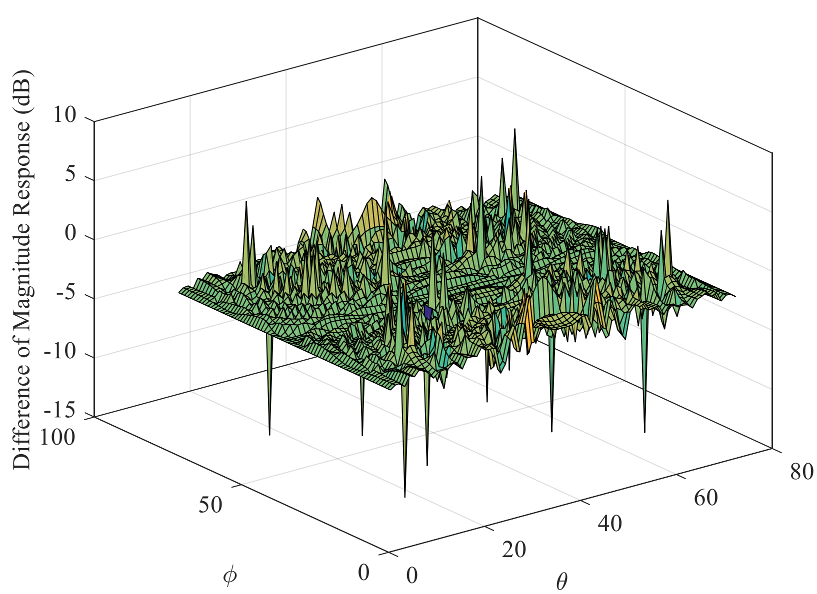

Figure A.4 The difference of desired array response and designed array response

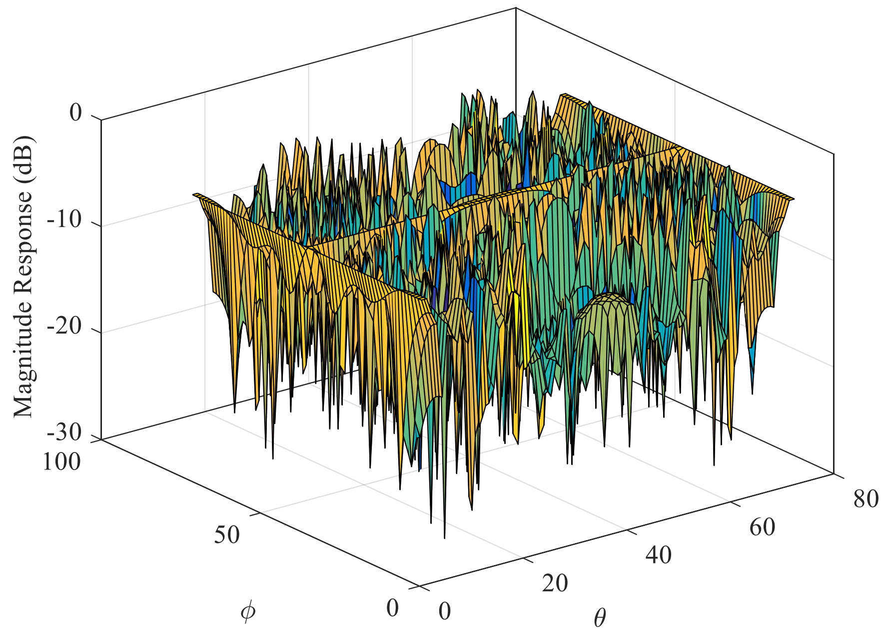

Figure A.5 The designed response of planar array

Section A.3 Sparse rectangular planar array design using GA

In this part, GA method is used to optimize the weight coefficients in array. Consider a uniform 10×10 rectangular array with element spacing of d=0.5 λ . With the GA method, the best solution of weight coefficients distribution is shown in Figure A.6.

The weights coefficients show in Table A-3. If the population is 40, crossover set to 90% and generation set to 500. This result of weight is less than the result shows in Table 4-2. However, the computation time is 85.082869s, 9s longer as compared to the result in Table 4-2.

From the result designed by GA, the computation time is quite long. This method requires better computation facilities. Besides, this method cannot guarantee a good result in a short time.

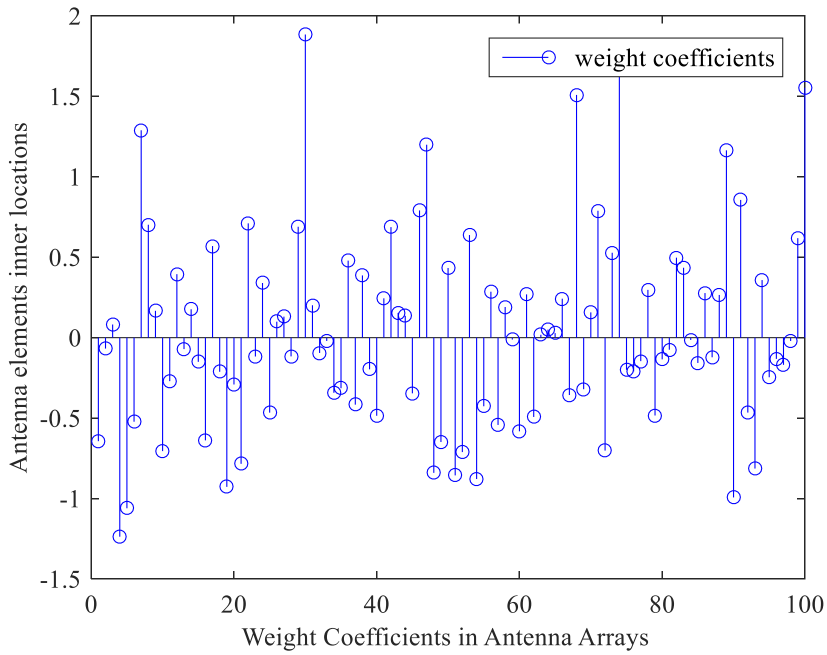

Figure A.6 Weight location distribution in planar antenna array.

Table A-3 Result of weight coefficients after GA selection

n	w_n	n	w_n	n	w_n	n	w_n
1	1.289	10	0.569	19	0.244	28	0.190
2	0.167	11	0.710	20	0.687	29	0.269
3	0.395	12	0.340	21	0.153	30	0.242
4	0.176	13	0.131	22	0.136	31	1.509
5	0.569	14	0.689	23	0.791		
6	1.289	15	1.884	24	1.199		
7	0.167	16	0.201	25	0.433		
8	0.395	17	0.479	26	0.638		
9	0.176	18	0.387	27	0.286		

Figure A.7 shows the designed response of sparse rectangular planar array using GA. The desired response can be found in Figure A.3. The difference between desired and designed response is small. The disadvantage of this method is related to the long computation time issue.

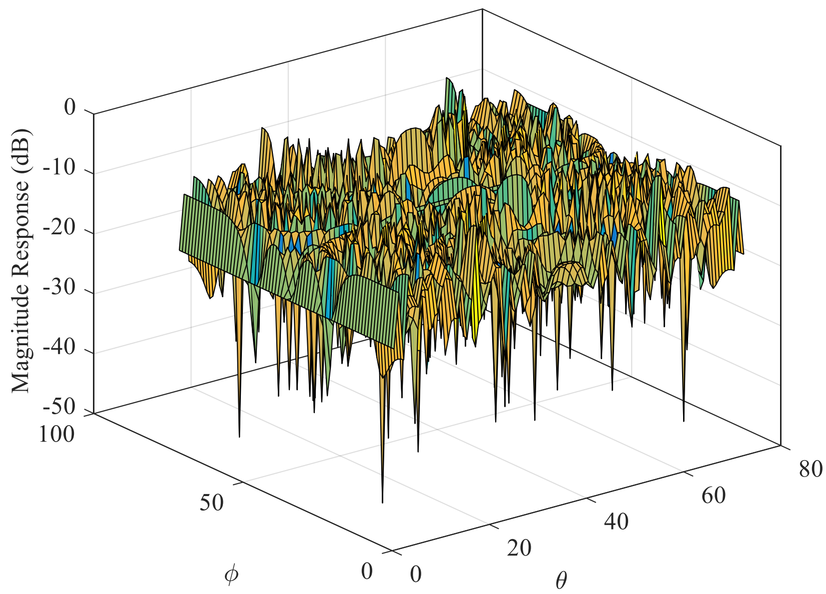

Figure A.7 The designed response of planar array by GA

With the population size = 20, crossover still set to 90% and generation=500.

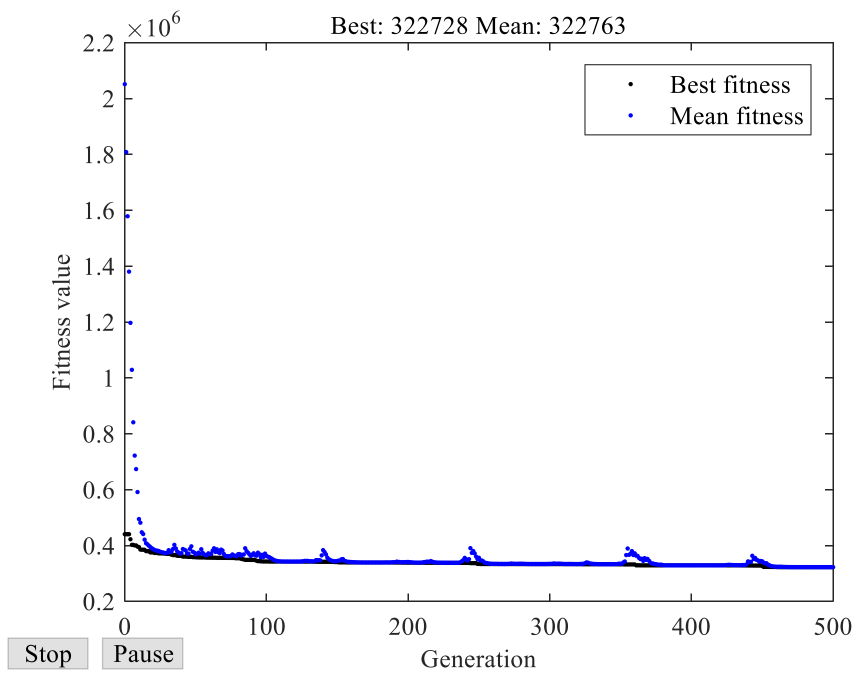

Figure A.8 The weight coefficients fitness optimization.

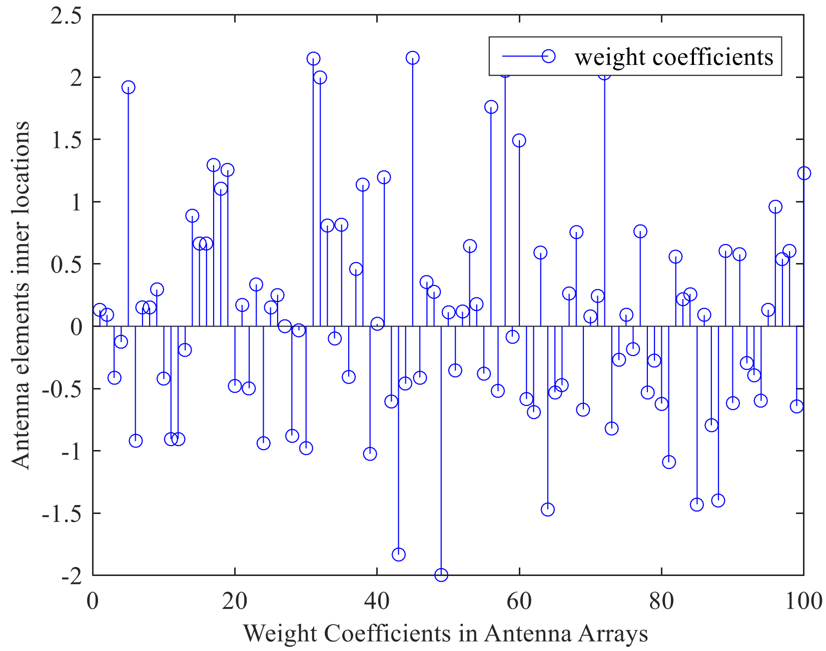

Figure A.9 Weight coefficients distribution in antenna array.

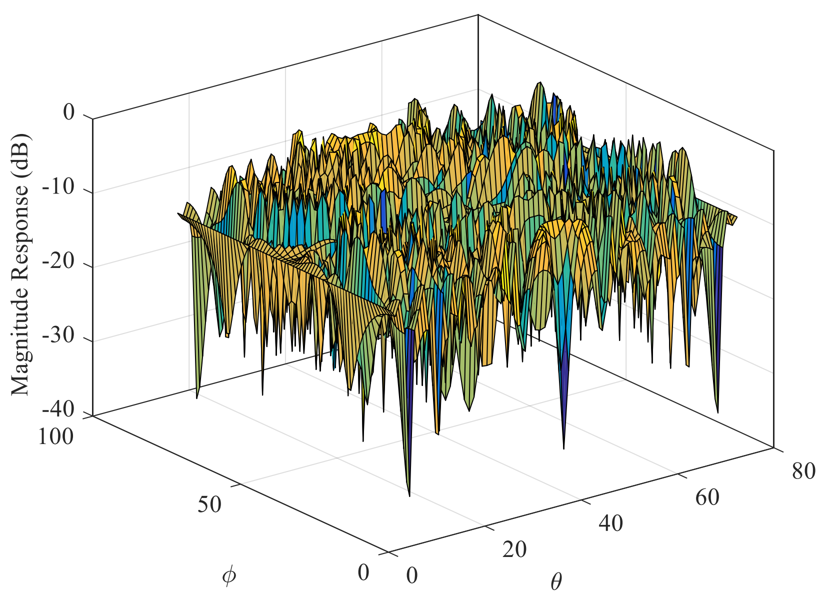

Figure A.10 The designed response of planar array by GA

The iteration time of this process is 92.932510s.

With the population remained as 40, crossover is 90%, generation change to 50.

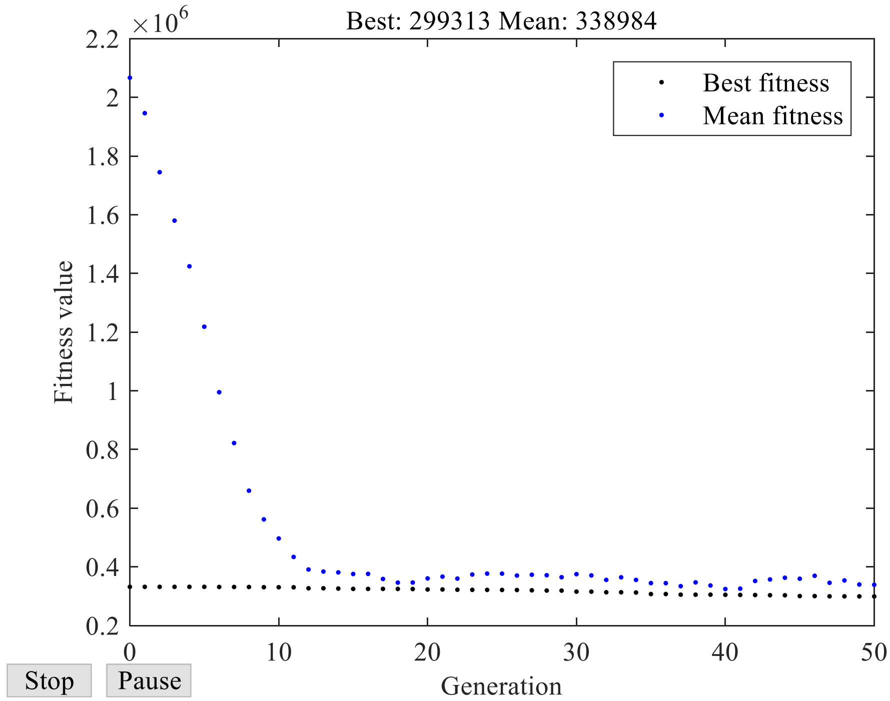

Figure A.11 The weight coefficients fitness optimization.

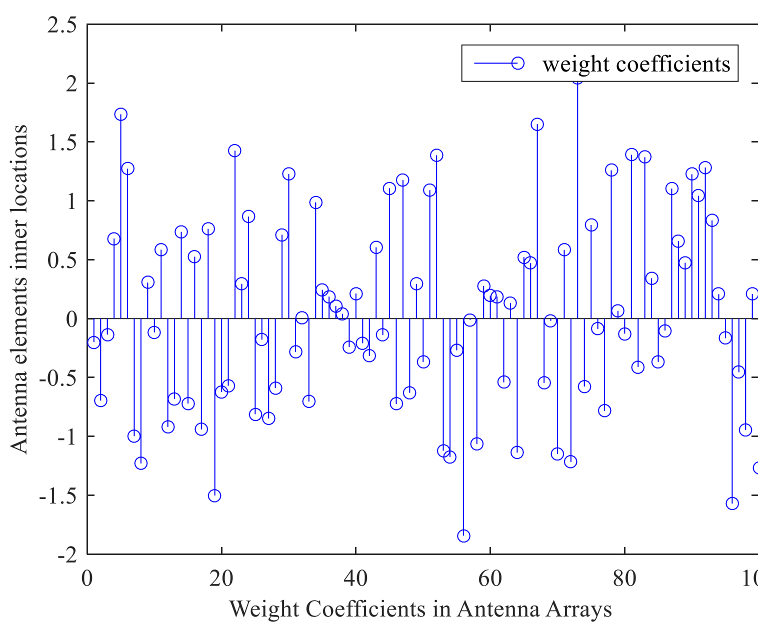

Figure A.12 Weight coefficients distribution in antenna array.

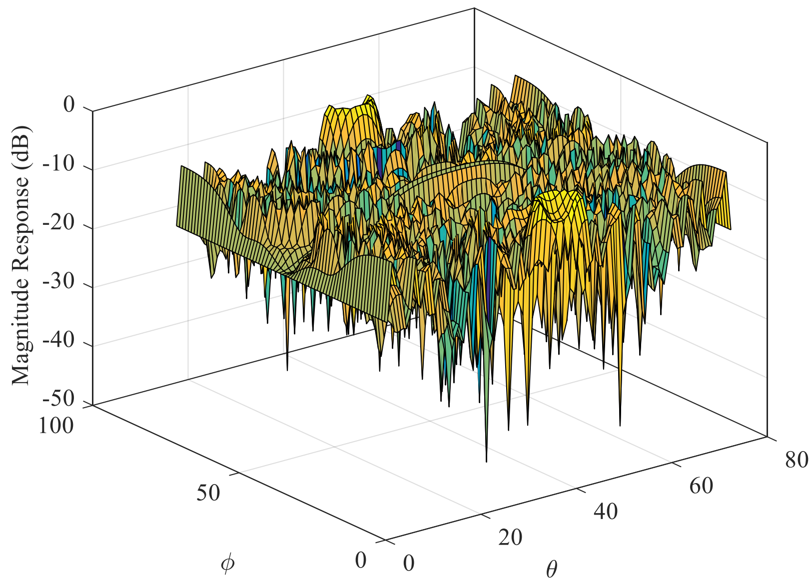

Figure A.13 The designed response of planar array by GA

The iteration time is 13.066175s.

With the population remained as 40, crossover change to 90%, generation change to 250.

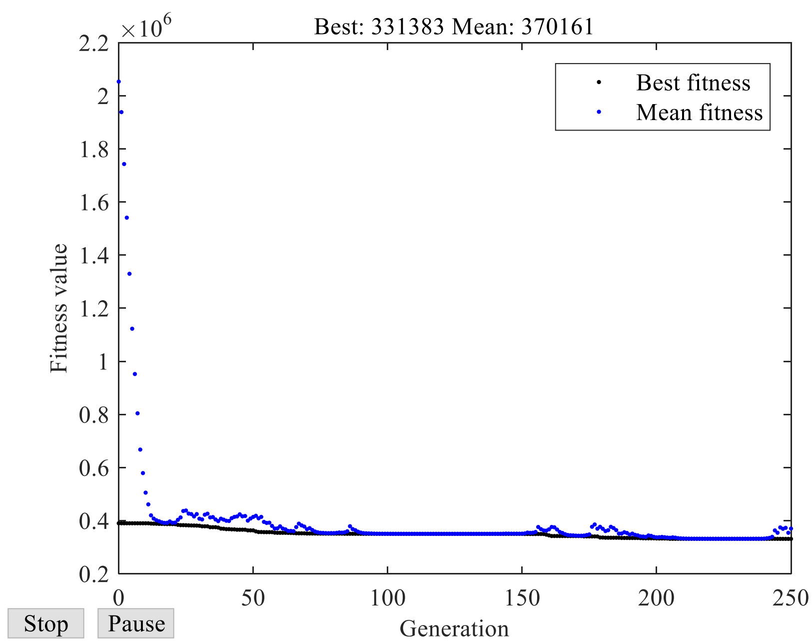

Figure A.14 The weight coefficients fitness optimization.

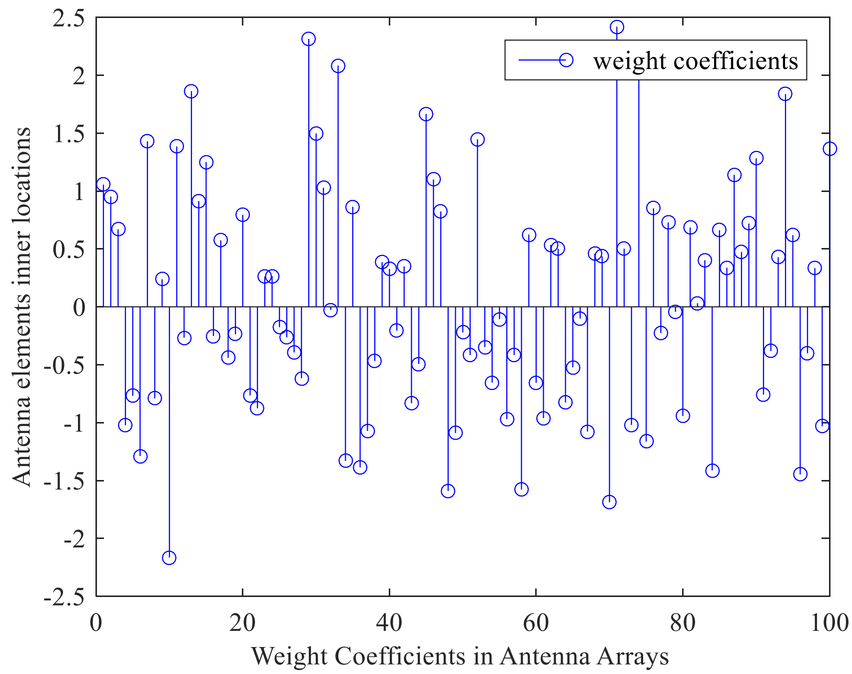

Figure A.14 Weight coefficients distribution in antenna array.

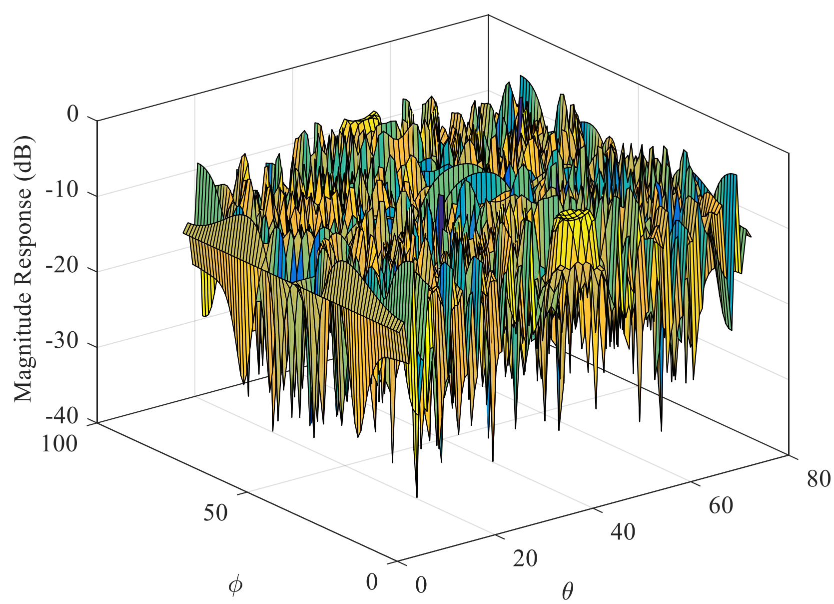

Figure A.15 The designed response of planar array by GA

The iteration time is 48.772931s. From the results, the iteration time is well related to generation numbers. However, GA still has longer computation time than CS.

If the population is 60, crossover is 90%, generation change to 500.

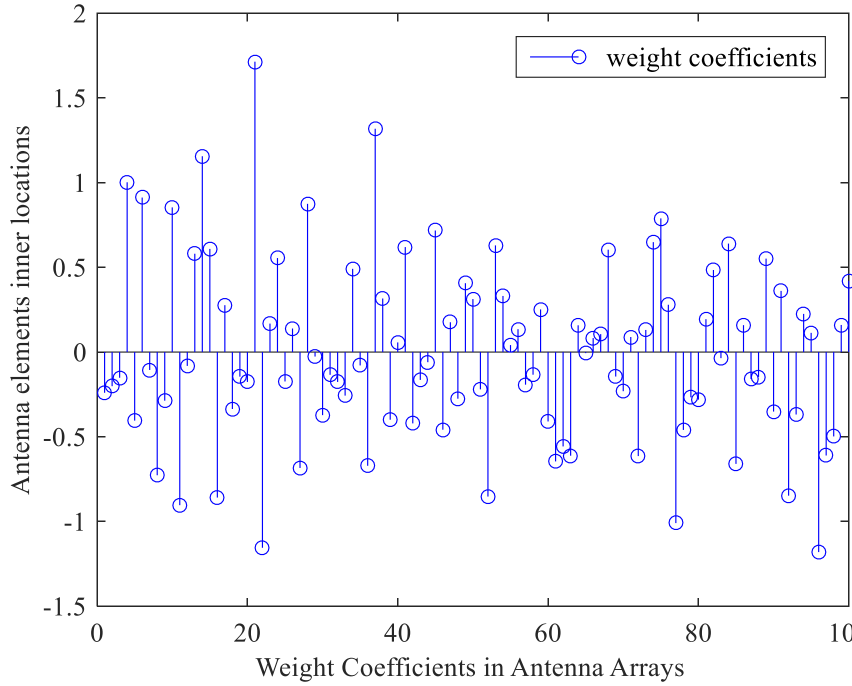

Figure A.16 Weight coefficients distribution in antenna array.

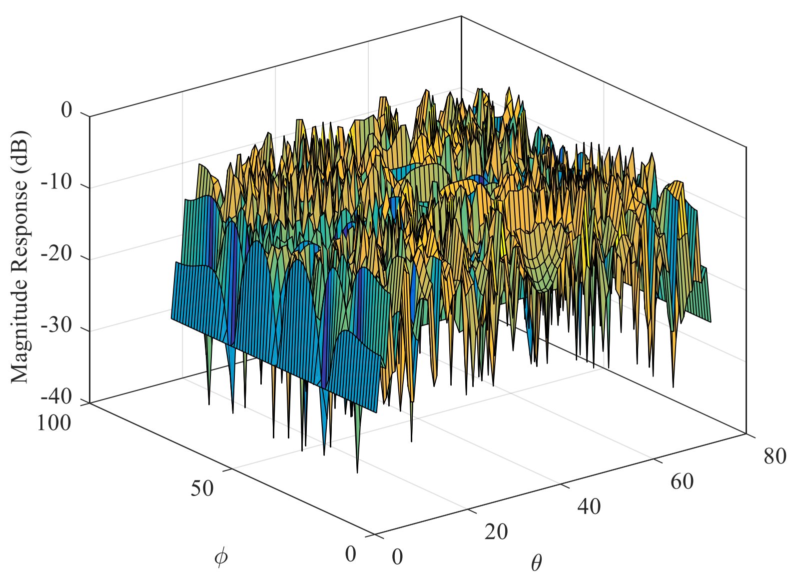

Figure A.17 The designed response of planar array by GA

The iteration time is 114.521620s. From this, the computation time does not quite depend on the population size.

If the population is 40, crossover is 85%, generation change to 500.

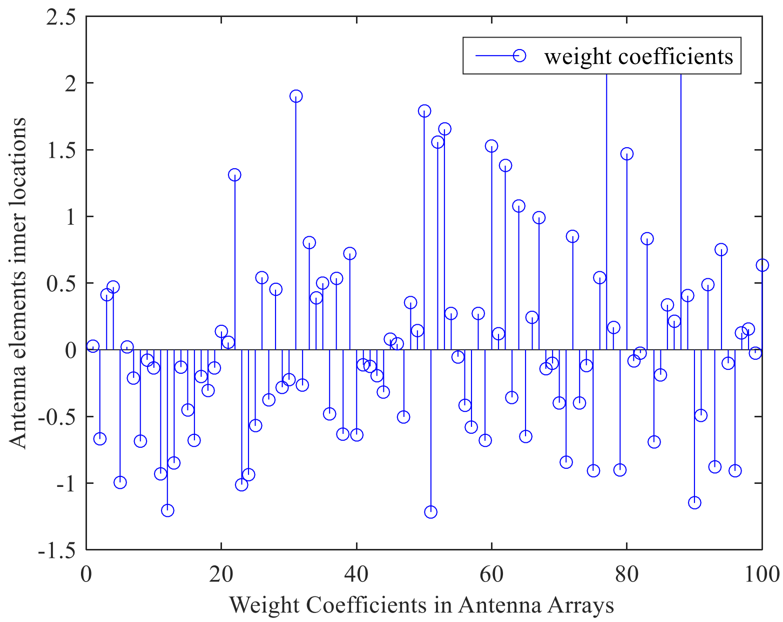

Figure A.18 Weight coefficients distribution in antenna array.

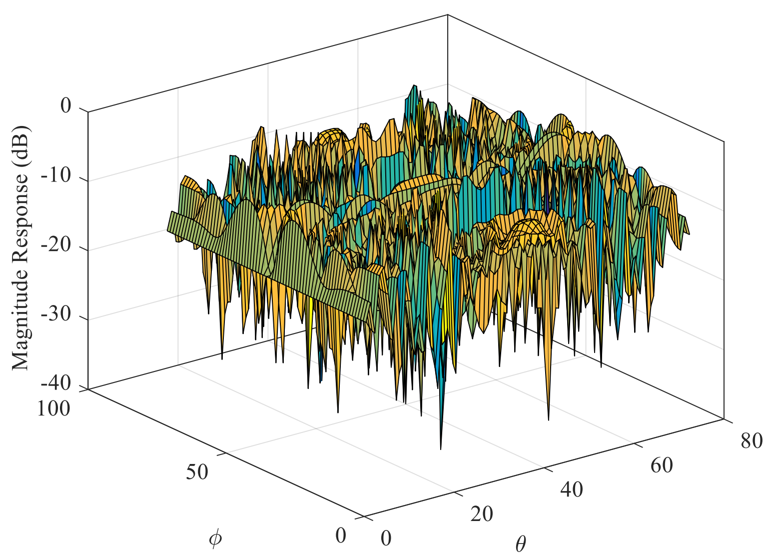

Figure A.19 The designed response of planar array by GA

The iteration time is 111.480991s.

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