Free-body and flexural motion of a floating elastic plate under wave maker forcing

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Outline

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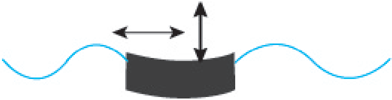
Numerical simulations

Summary

Elastic plate floating on water

Modelling regimes

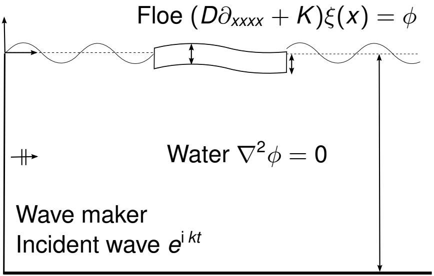
Zero-thickness *VS* **Lateral forcing**



Need the velocity potential along the draft: vertical distance between the waterline and the bottom of the plate.

Elastic plate floating on water

Mathematical Model



Green's function

$$\nabla^2 G(x, z | x_0, z_0) = \delta(x - x_0)\delta(z - z_0)$$

$$\partial_z G = \frac{\omega^2}{g} G \quad \text{on the surface}$$

$$\partial_n G = 0 \quad \text{on other boundary}$$

$$G = \frac{1}{2\,\mathrm{i}} \sum_{n=0}^{\infty} \frac{e^{\mathrm{i}\,k_n(x+x_0)} + e^{\mathrm{i}\,k_n|x-x_0|}}{k_n c_n} w_n(z) w_n(z_0)$$

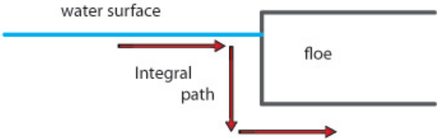
$w_n(z) = \cosh k_n(z + h)$ are eigenfunctions.

Surge motion

The boundary integral equation

$$\epsilon\phi = \phi_{\text{Incident}} - \int_{\text{boundary}} \{(\partial_{n_0} G)\phi - G(\partial_{n_0}\phi)\}\, ds_0,$$

$\epsilon = \theta/2\pi$, where θ is the angle of the corner.



Amplitude of the surge

$$u \propto \int_{-d}^{0} (\phi(b,z) - \phi(a,z))\, dz$$

Green's function

Log-like singularity of the Green's function

$$G \sim \log\left(|x - x_0| + |z - z_0|\right) \quad \text{as } x \to x_0, z \to z_0, \text{ for } (x_0, z_0) \text{ on the corner.}$$

Replace the singular part with a known singular series

$$\sum_{n=1}^{\infty} e^{-n\gamma|x-x_0|} \cosh \frac{\mathrm{i}\, n\pi}{h}(z+h) \cosh \frac{\mathrm{i}\, n\pi}{h}(z_0+h)$$

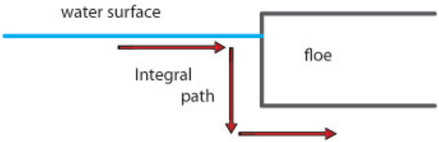
Separating of singular part

Separating the Green's function

$$G = \widetilde{G} + \log \mathcal{R}$$

$\widetilde{G}$ is bounded

The integrals involving $\log \mathcal{R}$ can be evaluated analytically.



Modes of the motion

Expansion of the surface deflection

$$\xi(x) = \sum_{m=0}^{M} \xi_m X_m(x)$$

where X_m are the eigenfunctions of

$$X'''' - \alpha_m^4 X = 0, \quad \text{for edges } X'' = 0, X''' = 0$$

$$X_m \propto \{\cos \alpha_m x, \cosh \alpha_m x, \sin \alpha_m x, \sinh \alpha_m x\}$$

Heave and pitch

- ξ_0 represents heave motion
- ξ_1 represents pitch motion

Computation method

Expansion of the potential

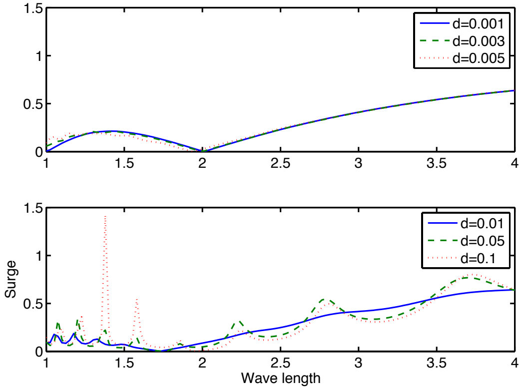
$$\phi(a,z) = \sum_{n=1}^{N} c_n(a)\mathcal{C}_{2n}(z), \quad \phi(b,z) = \sum_{n=1}^{N} c_n(b)\mathcal{C}_{2n}(z)$$

where $\mathcal{C}_{2n}(z)$ is the Gegenbauer polynomial with a weighting function.

Solving the BIE

1. Formulate a system of equations with X_n and $\mathcal{C}_{2n}$
2. Solve for the system of equations for $\{\xi_n\}$ and $\{c_{2n}\}$

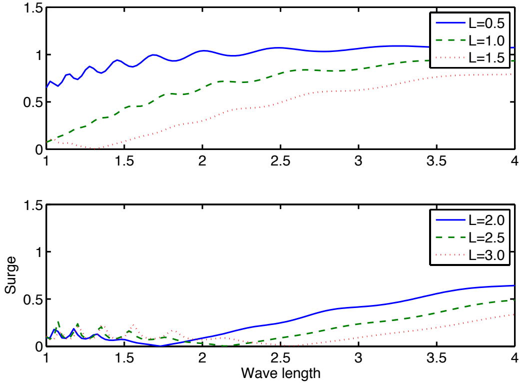
Surge motion



Amplitude of the surge motion of various thicknesses ranging

- from very thin 1mm
- to thick 100mm

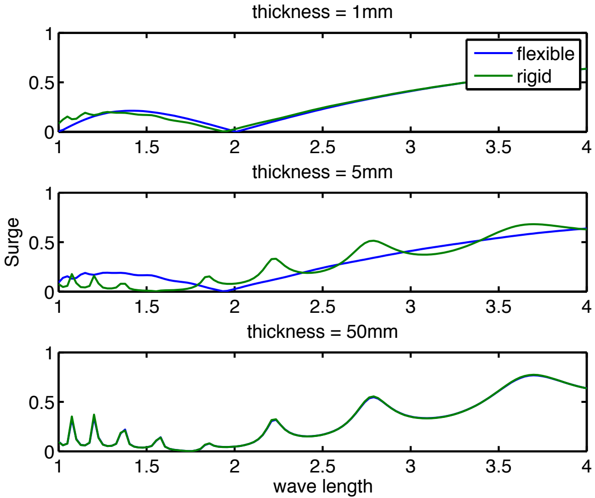
Surge motion



Amplitude of the surge motion of various length ranging

- from short 0.5m
- to long 3m

Surge motion



Comparison between flexible and stiff plates

- very thin 1.0mm
- medium thickness 5.0mm
- thick 50mm

Summary

- Complete description of the hydro-elastic motions of a finite floe.
- Include the draft of the plate and compute the surge motion.
- Analytical treatment of the singularities at the corners of the plate.
- Reduction of computation using the orthogonal polynomials and eigenfunctions.