Simulation of Variable Fluid-Properties Plate Heat Exchanger for Educational Purposes

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ABSTRACT

In this thesis a novel computer based model is developed which accurately simulates the operation of a plate heat exchanger (PHE). The model allows for the variation of *all* relevant fluid properties as the temperatures of the fluids vary through the PHE. It is set up in a spreadsheet in such a way that one can observe the variation of fluid properties and heat transfer parameters through the PHE during steady state operation. Although the model could be used for general purpose analysis of PHE's, it is intended to be used in an educational environment, where students can run "virtual lab sessions" with the model and so gain a better understanding of the overall and detailed operation of plate heat exchangers. The model is validated using experimental data representing a range of different PHE sizes, flow configurations, fluid types and flow conditions. Instructions have been provided on how it can be used in an educational environment to assist student to discover more about the general and detailed operation of a PHE.

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ABBREVIATIONS

AUT	Auckland University of Technology
CPM	Constant properties model
CPM1	Initial constant properties model
PHE	Plate heat exchanger
VPM	Variable properties model
VPM1	Initial variable properties model

NOMENCLATURE

A	Total heat transfer area in the PHE	m^2
A_{MN}	Heat transfer area for one element	m^2
A_c	Cross sectional area of a flow channel	
A_p	Heat transfer area for one plate	m^2
B	Coefficient for friction factor correlation	
C	Heat capacity rate	W/°C
$C1$	Heat capacity rate of fluid-1	W/°C
$C2$	Heat capacity rate of fluid-2	W/°C
C_{min}	Smaller heat capacity rate of fluid-1 or fluid-2	W/°C
C_{max}	Larger heat capacity rate of fluid-1 or fluid-2	W/°C
C_p	Specific heat capacity	J/kg°C
C_p1	Specific heat capacity of fluid-1	J/kg°C
C_p2	Specific heat capacity of fluid-2	J/kg°C
C_r	Heat capacity ratio	
c	Coefficient for friction factor correlation	
D	Coefficient for Nusselt number correlation	
d	Gap between plates in the PHE	m
d_h	Hydraulic diameter	m
f	Friction factor	
h	Convection (film) heat transfer coefficient	W/m^2°C
k	Thermal conductivity	W/m°C
L	Length of a flow channel	m
M	Total number of flow passages in the PHE	
m	Coefficient for Nusselt number correlation	
$\dot{m}$	Mass flow rate	kg/s
$\dot{m}1$	Mass flow rate of fluid-1	kg/s
$\dot{m}2$	Mass flow rate of fluid-2	kg/s

N	Number of vertical discretization elements	
n	Coefficient for Nusselt number correlation	
Nu	Nusselt number	
NTU	Number of transfer units	
P	Pressure	Pa
ΔP	Pressure drop	Pa
Pr	Prandtl number	
Re	Reynolds number	
$\dot{Q}$	Heat flow in PHE	W
$\dot{Q}_{max}$	Maximum possible heat transfer for a heat transfer situation	W
T	Temperature	°C
$T1$	Temperature of fluid-1	°C
$T2$	Temperature of fluid-2	°C
U	Overall heat transfer coefficient	W/m^2°C
w	Plate width (between gaskets)	m
x_W	Plate wall thickness	m
β	Plate, chevron angle	°
ε	Heat exchanger effectiveness	
ϕ	Plate, surface area enlargement factor	
μ	Fluid viscosity (dynamic)	kg/m.s
ρ	Fluid density	kg/m^3

Subscripts

AVG	Average
b	Bulk fluid
i	Discretization variable = flow channel
IN	At inlet port of PHE for a fluid
j	Discretization variable = vertical discretization
L	Left
M	Per flow channel
MN	Per element

OUT	At outlet port of PHE for a fluid
R	Right
W	Wall

Suffixes

1	relating to Fluid-1
2	relating to Fluid-2

CHAPTER 1

INTRODUCTION

1.1 Introduction

This thesis describes the development of a novel computer based model that can be used by students, in an educational environment, to accurately simulate the operation of a plate heat exchanger (PHE). The model is novel, firstly because it allows for the variation of *all* relevant fluid properties as the temperatures of the fluids vary through the PHE (which to the best of our knowledge had not been allowed for in the open literature at the onset of this investigation). Secondly it is set up in a spreadsheet in such a way that one can observe the variation of fluid properties and heat transfer parameters through the PHE during steady state operation. The model is intended to be used in an educational environment, where students can run “virtual lab sessions” with the model and so gain a better understanding of the overall and detailed operation of plate heat exchangers. This chapter provides an introduction to heat transfer and heat exchangers, a literature survey of current methods of PHE analysis and the objectives for this research project.

1.2 Background

Undergraduate students in mechanical engineering and related courses currently study heat transfer and the application of heat transfer theory to various engineering situations. One device that is closely studied in this area is the heat exchanger. A heat

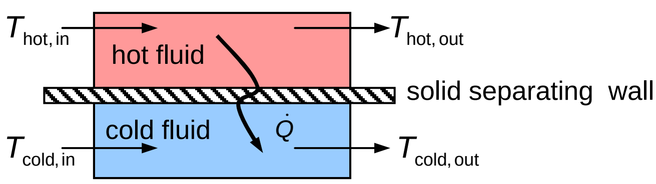
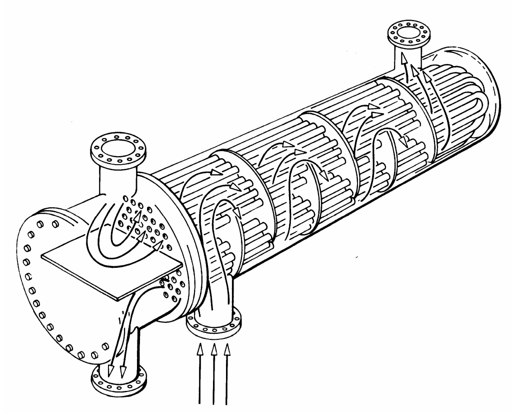
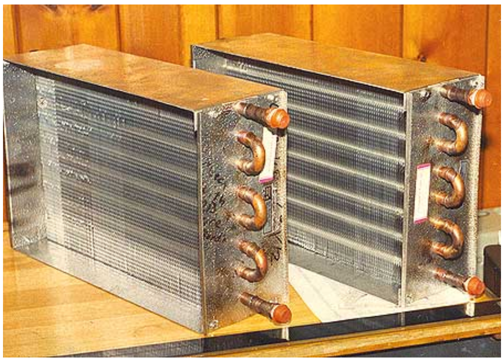


Figure 1.1 Heat exchanger operation

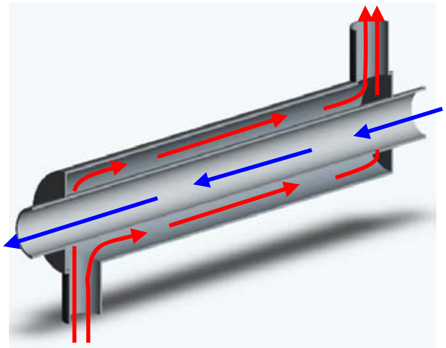


Shell & Tube [1]

Plate (PHE)
(courtesy Alfa Laval)



Cross flow



Double pipe

Figure 1.2 Different types of heat exchanger

exchanger, as schematically shown in Figure 1.1, is a device that allows the transfer of heat from a hotter fluid to a cooler fluid, while (in most cases) keeping the two fluids separate. To achieve this, a number of different designs are possible (See Figure 1.2) and most allow good thermal contact between the two fluids while keeping them separate from one another via a thin solid wall that has minimal thermal resistance. The solid separating wall can be in the form of a tube as in double-pipe, shell & tube and some cross-flow heat exchangers or in the form of a flat, corrugated plate as in the plate heat exchanger and other types of cross-flow heat exchanger.

Heat exchangers are commonly found in a wide range of industrial and commercial applications and so it is important that engineering students not only study the design of these devices but also that they gain a good understanding of their function and operation. There are two general types of heat exchanger problem encountered by engineers in practise: the “thermal design” and the “thermal rating” type of problem. In the thermal design type of problem the heat duty (heat transferred in the heat exchanger) is specified or known. This heat duty can be determined from knowing the mass flow rate, specific heat capacity and required inlet and outlet temperatures of at least one of the fluids. The engineer must then determine any unknown parameters for the other fluid flowing in the heat exchanger and use thermal design calculations to determine the size of the heat exchanger (heat transfer area) required to perform that heat duty.

In the thermal rating type of problem the performance of an existing heat exchanger (of known size, dimensions and heat transfer area) is determined when operating under specified inlet conditions (ie mass flow rates, specific heat capacities and inlet temperatures are known for both hot and cold fluids). Thermal rating calculations then enable the engineer to calculate the heat transferred in the heat exchanger (heat duty) and hence determine the outlet temperatures of the hot and cold fluids.

Also it should be noted that there is a linkage between the thermal and hydraulic performance of a heat exchanger. For a fluid flowing in a heat exchanger, experiencing a heat flow to or from the surface of the solid separating wall (tube or plate) there is a direct relationship between the convection heat transfer (film) coefficient, h (between fluid and wall) and the drag forces experienced by the fluid flowing across the wall of the flow passages. The convection heat transfer coefficient is used in determining the overall heat transfer coefficient, U while the drag forces are used in determining the pressure drop experienced by each fluid as it passes through the heat exchanger. Turbulent flow in the heat exchanger flow passages maximises convection and overall heat transfer coefficients (h and U), which is beneficial. But turbulent flow also maximises drag forces, pressure drops and hence pumping costs, which is not beneficial. So it is instructional for the student, whether the problem is one of thermal design or one of thermal rating, to also calculate the fluid pressure drops through the heat exchanger.

The focus of this thesis is one of education and improving students’ knowledge of how a heat exchanger operates, rather than giving them tools to enable them to design a heat exchanger. Therefore the model proposed by this thesis only considers the “thermal rating" (performance analysis) type of heat exchanger problem. The approach normally used in heat transfer textbooks for the rating of a heat exchanger is the traditional one of effectiveness-NTU ($\varepsilon - NTU$) [2]. This method is entirely valid, but it tends to become a dry, step-by-step procedure for the students, and as such it does not really add to an in-depth understanding of the overall functioning of the heat exchanger. Nor does it help them grasp the underlying principles that govern the operation of the heat exchanger.

The effectiveness, ε and the number of transfer units, NTU are parameters that are used to determine unknowns in the $\varepsilon - NTU$ method but they can also be used to give an idea of the performance of the heat exchanger. The effectiveness ε is the ratio of actual heat transfer in the heat exchanger to the maximum possible heat transfer between the two fluids (given an infinitely large heat exchanger), and is given by:

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} \tag{1.1}$$

The number of transfer units, NTU is a dimensionless parameter that measures the capacity of the heat exchanger to exchange heat. More specifically it is a measure of the heat exchanger's ability to change the temperature of the "minimum" fluid (the fluid that changes temperature most easily, has the lowest heat capacity rate), and is given by:

$$NTU = \frac{UA}{C_{min}} \tag{1.2}$$

So, no matter what calculation procedure is used, it would still be instructional for the student to determine these values for a particular rating situation so they can compare them with the results for other different rating situations for the same heat exchanger. Further discussion on the significance of ε and NTU can be found in standard heat transfer texts [1, 2].

Also, the $\varepsilon - NTU$ method, like many engineering analyses, makes a number of assumptions to simplify the equations derived. These assumptions include: uniform overall heat transfer coefficient, constant fluid properties, no axial conduction in the solid wall separating the hot and cold fluids and no heat losses to the environment, etc.

Furthermore, this method, on its own, does not allow for the determination of the temperature profiles through the heat exchanger (unless the heat exchanger is assumed to be a simple, pure counter-current or parallel flow heat exchanger, in which case the profiles are simple logarithmic ones).

It would be useful for the student to be able to investigate the effect of some of these assumptions on the results obtained from these methods and also to “see inside” the heat exchanger and find out how the various parameters (temperature, pressure drop, fluid properties, Reynolds number, etc) vary through the heat exchanger during steady state operation. It is felt that this would increase and add to the students overall appreciation of heat transfer theory and its application to heat exchangers. This thesis proposes an accurate software virtual model of a plate heat exchanger that allows for the variation of fluid properties through the heat exchanger and that will fulfil the goal of allowing students to become more familiar with the function and operation of a plate heat exchanger.

It is difficult in modern engineering education programmes to schedule enough laboratory or experimental time for students to become reasonably familiar with the overall operation of heat exchangers, let alone for them to gain an understanding of their detailed operation. One way of overcoming this problem is to set up a computer model that as nearly as possible simulates the operation of a real heat exchanger. If this model could be used in conjunction with a real, laboratory based heat exchanger then students could use the real heat exchanger over a short period of time (say, one lab session) to verify the validity of the computer model. They could then use the computer model, in their own time, to further investigate the operation (general and detailed) and underlying thermodynamic principles of operation of a heat exchanger. Students could “dissect” the computer model to further investigate the inner workings of that real-world heat exchanger. The computer model would be set up to allow the application or relaxing of various standard assumptions so that the student could investigate the validity of those assumptions that are normally made in traditional heat exchanger calculations. The model would allow students to vary different operating parameters and see the effect on heat exchanger performance; eg vary flow rates, inlet temperatures and fluid properties, to see the effects on heat transfer rate, pressure drop, effectiveness and NTU values. The model proposed by this thesis will achieve these requirements.

The purpose of a heat exchanger is to transfer heat from a hot fluid to a cold fluid and so the temperatures of the fluids change as they flow through the heat exchanger. Most fluid properties that relate to heat transfer and pressure drop (eg viscosity, density etc) are also temperature dependent. This means that the fluid properties will change as the fluids flow through the heat exchanger and this in turn will have an impact on heat transfer and fluid pressure drops. As discussed in section 1.4 most current heat exchanger analysis simplifies the situation by assuming that the fluid properties remain constant through the heat exchanger. However, the software model proposed by this thesis will allow for all of the fluid properties to vary with temperature through the heat exchanger. In this way the results will be more accurate and students will be able to compare the results from the variable properties model with calculations made assuming constant properties to find out if the constant properties assumption is valid or for what fluids and conditions it is valid.

1.3 Plate Heat Exchangers

The current study has been based on a plate heat exchanger. Plate heat exchangers (PHE's) are commonly used for the heating and cooling of fluids in a wide range of industries, including food and beverage, pharmaceutical, and chemical process industries.

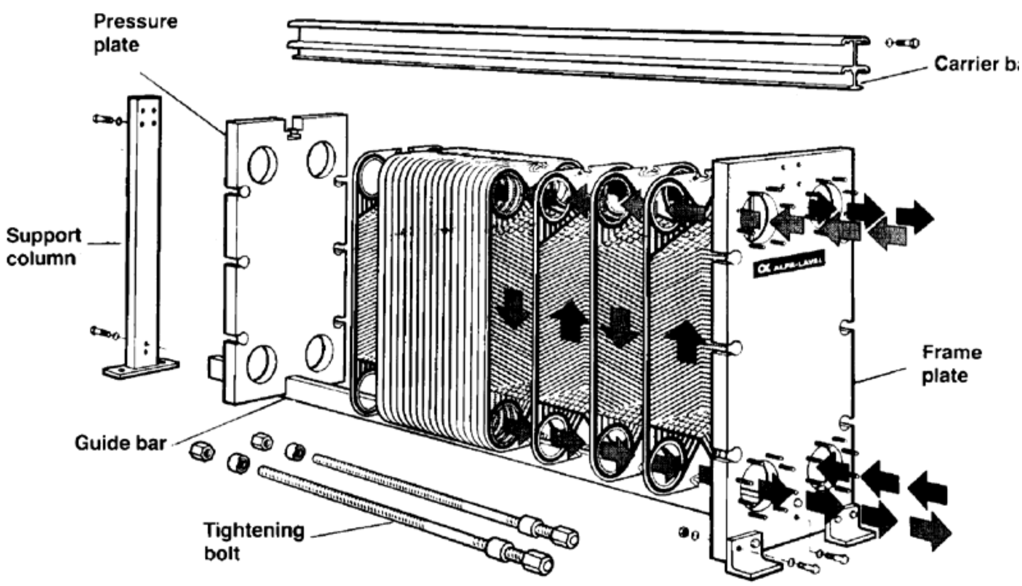


Figure 1.3 Exploded view of a plate heat exchanger (courtesy Alfa Laval)

In this type of heat exchanger the two fluid streams are separated by flat plates, held together in a press on a frame and sealed by gaskets, giving flat rectangular flow passages (See Figure 1.3). The hot and cold streams flow in alternate passages giving very efficient heat transfer. The plates are corrugated in various patterns to give increased turbulence and to give the plate strength (rigidity). The narrow plate gap and high turbulence give these heat exchangers high overall heat transfer coefficients (U) at low fluid velocities and low pressure drops.

PHE's are commonly used because of their high heat transfer capacity, compact size, the ability to be easily cleaned and the facility of adding or removing plates to easily adjust the heat transfer area. For these reasons, plate heat exchangers are a good candidate for student study and this is why they have been chosen for the current study.

The flat plates in a PHE consist of four flow ports, a gasket arrangement and a corrugated profile (see Figure 1.4). By adjusting the gasket arrangement and optionally

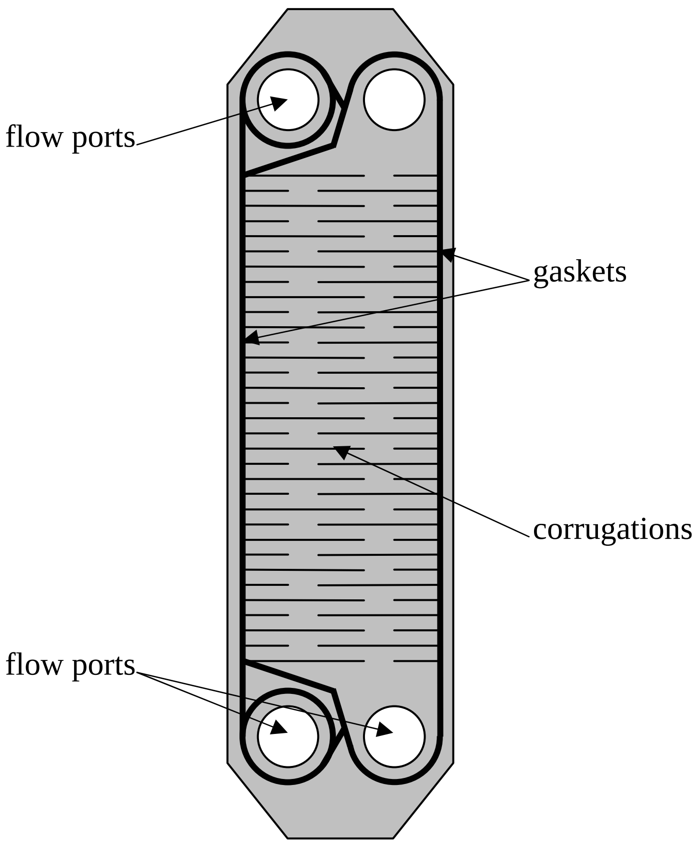


Figure 1.4 Detail of PHE plate

blanking off selected flow ports on individual plates and then arranging the plates in a specific order it is possible to change the way that the fluids pass through the PHE.

Many different flow configurations are possible with a PHE. Of these the most common are the parallel, series and combination flow configurations, shown schematically in Figure 1.5.

In the case of the parallel flow configuration the cold fluid enters the heat exchanger as a single stream which divides up and flows in a parallel fashion along every second

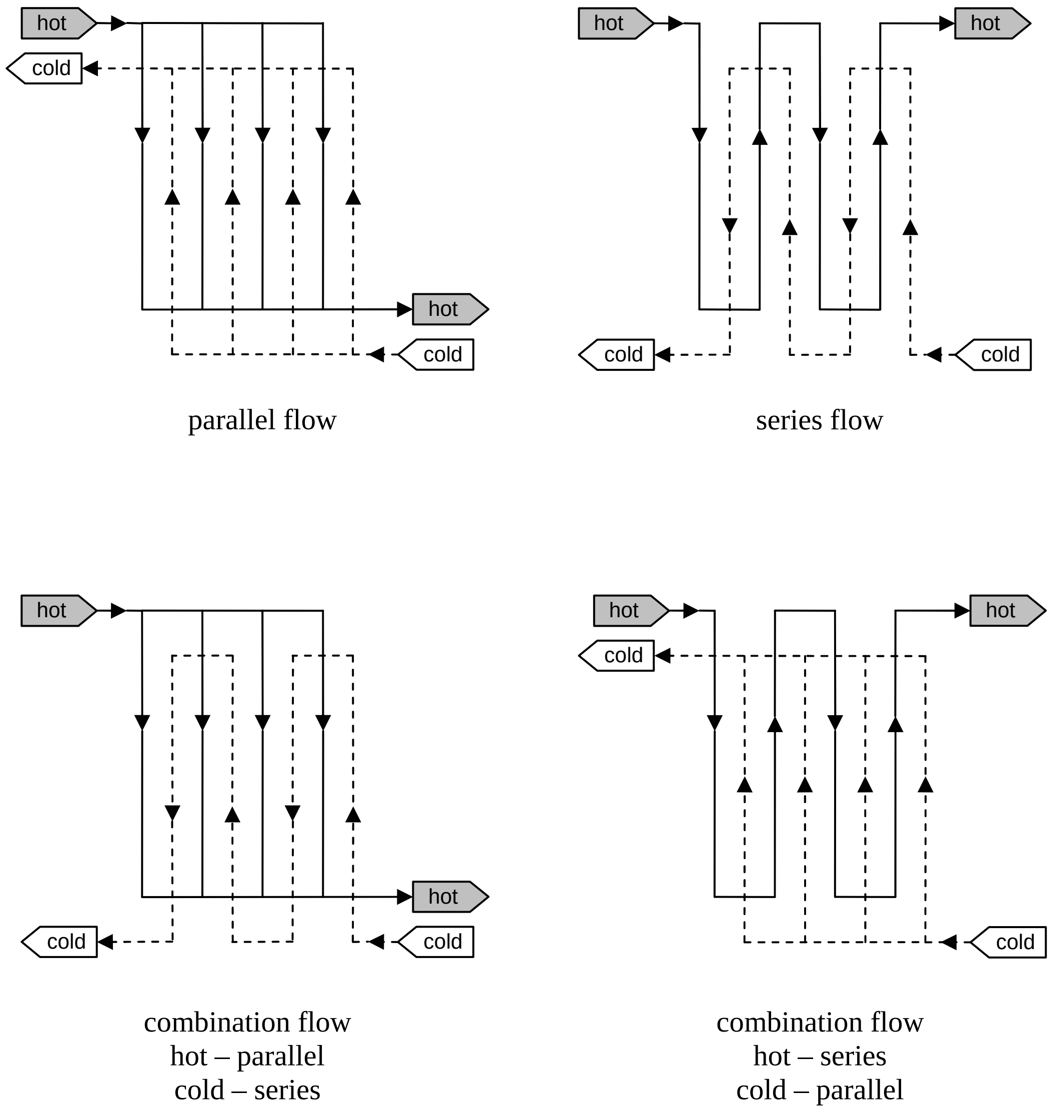


Figure 1.5 Common PHE flow configurations

flow passage. At the end of the cold flow passages the fluid flows combine together into a single stream and leave the opposite extremity of the heat exchanger. The hot fluid flows through the PHE in a similar fashion, but flows in the passages in-between each cold flow passage. This configuration gives excellent heat transfer and low pressure drops.

In the case of the series flow configuration the cold fluid enters the heat exchanger as a single stream and remains as a single stream as it flows up and down through the PHE in alternate cold flow passages. The hot fluid flows through the PHE in a similar fashion, but the single stream flows up and down in alternate hot flow passages. This configuration has much higher pressure drops than the parallel configuration but residence times in the PHE for each fluid are maximised.

In the case of the combination flow configuration, one fluid flows through the PHE in a series fashion while the other fluid flows through the PHE in a parallel fashion. This configuration is often used in pasteurisation of fluids whereby the cold fluid flows in series through the PHE and the hot fluid flows in parallel. This allows maximum residence time for the fluid to be pasteurised combined with good heat transfer characteristics from the hot fluid.

To increase the applicability of the virtual model, proposed by this thesis, it is set up in each of these three basic configurations: series, parallel and combination. The one combination model covers both combination cases shown in Figure 1.5.

1.4 Literature Survey

There are a large number of studies in the literature relating to the mathematical modelling of the steady state, thermal and hydraulic design and performance of PHE's. Some studies focus on the calculation methods; the setting of the governing equations that describe the operation of the PHE or the distribution of fluid temperature through the PHE, and the mathematical methods used to solve these equations. An early, pioneering work by Wolf [3] presents a general solution, using the matrix eigenvalue method, to the system of differential equations governing the distribution of temperature in the fluids flowing through parallel flow, multi-channel heat exchangers. It can be applied to PHE's but its application is not discussed and the method presented does not address the specification of boundary conditions. Zaleski [4] expands on this eigenvalue solution method and additionally considers the boundary conditions, but only for two specific configurations of PHE. Marano and Jechura [5] further expand on the eigenvalue solution method by widening its applicability to other types of heat exchangers and by describing the computer implementation of the solution method. Four different PHE's with different flow configurations are presented as examples in the study, and the computer method can be applied to a range of different PHE configurations. Zaleski and Klepacka [6] build on Zaleski's earlier work [4] by proposing a numerical solution to the eigenvalue method whereby the temperature profiles in each flow channel are approximated by a linear combination of exponential functions. Ribeiro and Andrade [7] use this numerical solution to propose an algorithm for the steady state simulation of PHE's [6]. The algorithm involves Gaussian elimination as part of the solution and is implemented on a computer using the FORTRAN programming language. The algorithm is validated against a range of experimental data with good results. All the works mentioned assume steady state operation and constant fluid properties through the PHE.

The matrix eigenvalue method of solving the governing equations for heat transfer in a PHE is well established and not beyond the capabilities of undergraduate students. However, it is a mathematically intensive method and the setting of boundary conditions for some PHE flow configurations makes solution more difficult. In fact many of the solution schemes require a numerical solution component. It is felt that implementing this solution method in the proposed software model may well detract from its central purpose of giving the student a better insight into the operation of a PHE. Especially when there are other perhaps simpler, more intuitive solution methods available.

An early work by McKillop & Dunkley [8] derives a system of differential equations to describe the temperature distribution in a PHE and applies a numerical integration technique to solve those equations. However, the three PHE's used in the study have unique flow configurations which limits the applicability of the results, and the differential equations derived are greatly simplified by assuming both fluids are water and fluid properties remain constant through the PHE (similar terms are cancelled in the equations to simplify them). Jackson and Troupe [9] also apply a numerical integration technique (Runge-Kutta) to solve the system of governing differential equations for heat transfer in a PHE. They correctly point out that this also allows for the determination of temperature profiles through the PHE but this study also assumes a constant overall heat transfer coefficient (and hence constant fluid properties) through the PHE. This study did produce several $\varepsilon - NTU$ plots that can be used in the thermal design or thermal rating of PHE's. Again the solution method is a well established one but is also mathematically intensive for the student.

Another early work by Buonopane [10] uses the corrected log mean temperature difference method to present a design methodology for parallel and series flow PHE's. Like the other early works, average values of fluid properties and heat transfer coefficients through the PHE are used. Although this work presents a design method for PHE's it also contains 40 data for a PHE set up with different numbers of plates, two different flow configurations (series and parallel) and a range of different flow rates and temperatures. This data can be used for validation purposes.

None of the studies mentioned so far consider the hydraulic performance of the PHE. ie they consider the thermal design or thermal performance only. It is desired to include a simple hydraulic performance (pressure drop) calculation in the proposed software model.

Another approach to PHE performance analysis is to use dimensional analysis, which in relation to heat exchangers is the effectiveness-NTU ($\varepsilon - NTU$) method. Wetter [11] describes a steady state simulation model of a PHE using the $\varepsilon - NTU$ method but it does not allow for the determination of the fluid temperature profiles through the PHE. Also the only fluid considered is air which limits its applicability. Another study by Kandlikar and Shah [12] produces closed form $\varepsilon - NTU$ relations for a wide range of PHE flow configurations. However, the relations only apply to PHE's with an infinite number of thermal plates. They found in practise that good results are achieved if the number of plates exceeds around 40 plates. This study also assumes constant fluid properties but is limited in its applicability to PHE's with large numbers of plates. The dimensional analysis method on its own is not suitable for the proposed software model as it does not allow the calculation of fluid temperature profiles though the PHE. However, as mentioned in section 1.2 the key parameters in the effectiveness-NTU method, namely effectiveness, ε and the number of transfer units, NTU are useful performance parameters the student needs to become familiar with in terms of PHE performance. Hence they are calculated and used in the proposed software.

Other recent studies describe the use of novel approaches to predict the performance of a heat exchanger. One study uses the finite element method to analyse the performance of a shell and tube condenser [13]. This method allows for the variation in heat transfer coefficients through the heat exchanger. Results are in good agreement with experimental data. Ranganayakulu and Seetharamu [14] also used the finite element method to predict the degradation of performance in a cross-flow tube-fin heat exchanger due to the combined effects of longitudinal heat conduction in the heat exchanger walls and temperature/flow non-uniformities. Another study looks at the application of artificial neural networks to simulate heat exchanger performance [15]. The method requires a significant amount of experimental data (to train the network) but it is successfully used to predict the heat transfer coefficients in a finned-tube heat exchanger. A further study uses a thermal network method to model a plate-fin tubular exchanger, operating as a condenser [16]. The phase changes in the refrigerant in this condenser are handled by using an effective specific heat which varies through the exchanger. Good agreement is achieved between the model and experimental results. Another study by Ribando et al. [17] uses a unique numerical lumped approach that can be used in the thermal design and thermal performance analysis (rating) of heat exchangers. In this study the lumped approach is applied to shell & tube and crossflow heat exchangers. The assumptions used in this study include constant fluid properties and constant overall heat transfer coefficient. This study also promotes the use of this approach in an educational environment due to its mathematical simplicity and ease of understanding. Although none of these studies relate specifically to PHE's, the methods described could well be applied to PHE's. In fact the lumped numerical approach [17] has been selected for use in the proposed software model to predict the performance of PHE's and is described in detail in Chapters 2 and 3. The reasons for its selection include firstly that it lends itself to be modified to allow for the variation of properties through the PHE, and secondly it uses simple mathematical techniques and an easily understood formulation of the governing heat transfer equations. This means undergraduate students can focus more on the physical heat transfer situation and not become too bogged down in mathematically intensive solution methods.

A number of recent studies focus on the dynamic simulation of PHE's. One reason for developing a dynamic model is to assist in the development of a proper control strategy for PHE's [18]. It has also been noted [19] that a dynamic model can give better understanding of the steady state situation. Although this thesis does not consider the dynamic analysis of the PHE it is felt to be worthwhile to survey the literature in this area to see if any findings are applicable to the steady state operation of a PHE. Das and Roetzel [18, 20, 21] and Das and Murugesan [22] describe in detail aspects of the development of a dynamic simulation of a PHE and subsequent comparison to experimental results. Their method of analysis of the heat transfer situation involves the formulation of the governing partial differential equations and boundary conditions, which are solved by taking Laplace transforms. The solution in the time domain is achieved by a numerical inverse Laplace transform technique (Fourier series approximation). Added to this, the method of analysis includes allowance for what is known as the *phase lag effect* and the *dispersion* effect. The phase lag effect relates to the fact that in a parallel flow PHE as a fluid enters the exchanger a portion of it immediately flows into the first flow channel, but there is an increasing time delay involved for the portions of fluid entering flow channels 2, 3, 4…etc. Added to this there are two common configurations of PHE; the U-type configuration and the Z-type configuration. In the U – type configuration all four flow connections are at one end of the PHE and so the fluid in the flow passages closer to this end have less total distance to travel through the PHE than the fluid in the passages furthest from the connection end. In the Z-type configuration there are two connections at each end of the PHE and so each fluid stream, irrespective of flow channel, has the same distance to travel though the PHE. The phase lag effect is accentuated in U – type configurations. Also, most virtual models of PHE's assume plug flow within the flow channel; however, it has been suggested that some back mixing occurs as the fluid flows along a PHE flow channel and in these studies this has been allowed for by introducing a dispersion term in the energy equation. It is a conclusion of these studies that it is very important to include the effects of phase lag and dispersion in the transient analysis of PHE's [18, 20-22]. However, there is little mention in the literature of these effects being considered in the steady state analysis of PHE's and so these effects are not considered in the proposed steady state software model. These studies of dynamic response also assumed constant fluid properties through the PHE.

Some studies focus on the flow passages within the PHE, analysing the flow patterns between the corrugated plates and so developing equations for the heat transfer coefficient and friction factor within the flow channel. Blomerius et al [23] investigated flow patterns and heat transfer in corrugated flow passages by numerically solving the Navier-Stokes and energy equations. Their study focussed on the effect of the size of the plate corrugations on resulting flow patterns, friction and heat transfer. They only considered plates with corrugations at one fixed angle (45°). No Nusselt number or friction factor correlations are reported. Other studies in this area also tend to look at specific corrugation patterns and their emphasis is on analysis of experimental results and techniques [24, 25]. Ciofalo et al. [24] does derive heat transfer and pressure drop correlations but they apply to a unique combination of plate types known as “corrugated-undulated” plates. Heggs [25] produces three dimensional maps of the local heat transfer coefficient from experimental data. A recent study utilises porous media flow theory applied to the flow passages in a PHE to develop new correlations for the Nusselt number and the friction factor [26]. The results look promising; however, correlation coefficients must be determined experimentally for each different plate.

An important aspect of any heat transfer and pressure drop analysis is the selection of the Nusselt number and friction factor correlations. Recent studies have performed experimental analyses on PHE’s with a common type of corrugated plate called a chevron plate [27-29]. The aim of these studies has been to develop Nusselt number and friction factor correlations that are widely applicable and that depend on the plate geometry. Earlier studies in this area are summarised by Muley and Manglik [27, 28] and by Talik et al. [30]. Muley & Mangliks studies appear to represent the state of the art in relation to heat transfer and pressure drop in typical PHE flow channels and covers a range of fluid types and plate geometries [27-29]. For this reason the correlations reported by Muley and Manglik [28, 29] are used to predict the convection heat transfer coefficients and friction factors in the proposed software model and are described in detail in Chapters 2 and 3. These correlations also include a viscosity correction factor that was first reported by Sieder and Tate [31] and allows for the change in viscosity between the wall and the bulk fluid in a convection heat transfer situation. Since that first study by Sieder and Tate other researchers have looked at the viscosity correction factor and the study by Buyukalaca and Jackson [32] provides a good summary of earlier studies in this area. The conclusion of Buyukalaca and Jackson is that the original viscosity correction factor first proposed by Sieder and Tate is probably the best one to use. The proposed variable properties software model uses the Sieder and Tate [31] viscosity correction factor and is described in detail in Chapter 3.

Some researchers have noted that most studies assume that in parallel flow heat exchangers the flow stream divides evenly between all available flow channels. Rao et al [33] note that the wide variation in results from the various reported heat transfer correlations could in fact be due to this assumption being incorrect. If the flow along different channels varies then the heat transfer coefficient between channels will also vary. The study suggests the flow distribution is in fact uneven and could be described by the flow distribution proposed by Bassiouny and Martin [34, 35]. This factor is definitely worthy of consideration, however, it is not incorporated into this current study. The study uses an eigenvalue method to solve the governing differential equations with allowance for the uneven flow distribution [33]. The study also assumes constant fluid properties through the PHE and does not allow for any viscosity correction factor to the convection heat transfer coefficient.

A number of works consider PHE performance from an optimisation perspective. Two representative studies consider the optimisation of the thermal and hydraulic design in an attempt to get the best thermal performance at minimum pressure drop or maximum thermal performance for a specified pressure drop, for example [36, 37]. Other studies seek to optimise the flow arrangement for a given heat duty [38, 39]. Although these studies are informative and describe useful equations and methodologies their main aim is one of design rather than performance analysis.

Very few of the studies mentioned in this section allow for variable fluid properties through the PHE. Most studies relating to heat transfer analysis and the allowance for variable properties do so for either simple or unique geometries or consider only one fluid property. Derevich and Smirnova [40] analyse heat transfer in an experimental heat pump consisting of a copper pipe wound around a steel tube. They allow for the variation in fluid properties in their calculations and determine fluid properties using equations of state. Pelletier et al [41] combine the finite element method with allowance for variation of fluid properties to determine convective heat transfer to fluids in rectangular heated enclosures. Hassanien [42] analyses flow and heat transfer performance of fluids flowing over heated flat plates, allowing for variation in fluid properties with temperature. Fluid properties are estimated using simple power law relationships. Kang and Christensen [43] analyse the effect of fluid property variation with temperature in heat transfer to the fluid flowing inside a spirally fluted tube. In this case they only consider viscosity variations. Ravi Kumar and Sarangi [44] consider the variation of heat capacity in PHE's used in unique conditions (cryogenic). All of these works conclude that allowing for fluid property variation with temperature has a significant effect on results compared to assuming constant properties. Most studies also concluded that the viscosity variation with temperature is a major influencing factor. If the fluid involved in heat transfer has a viscosity that varies greatly with temperature (oils, syrups etc) then the difference in results between allowing for variable properties and not is significant.

A very recent study by Gut & Pinto [39] develops a modelling framework for different configurations of PHE which allows for the variation of the overall heat transfer coefficient through the PHE. The study is very thorough and complete covering a wide range of flow configurations, and allowing for the calculation of pressure drops and for the behaviour of non-Newtonian fluids. The method of solution involves a complex "assembling algorithm" which attempts to allow for all possible flow configurations. The governing differential equations are solved using a "second order finite difference method". The focus of the study is to determine the influence of flow configuration on PHE performance and ultimately develop a method of configuration optimization. In the examples stated in the study the properties of the fluids used are determined by polynomial or similar equations. Although this study does allow for variation of fluid properties through the PHE it does not allow for the viscosity correction factor in the determination of the convection heat transfer coefficient.

Due to the fact that none of the above studies allow for the general case of analysing the performance of a PHE with variable fluid properties and a viscosity correction factor, the software model proposed in this thesis will incorporate the variation of fluid properties with fluid temperature through the PHE. Additionally, it will allow for the viscosity correction factor to be incorporated into the calculation of the convection heat transfer coefficient.

In summary, the software model proposed in this study uses a lumped numerical solution [17] to the governing heart transfer equations, incorporating variable fluid properties. State of the art empirical correlations [28, 29], incorporating a viscosity correction factor [31] are used to determine the heat transfer coefficients and pressure drops in the corrugated plate flow passages.

1.5 Spreadsheet Model

It is decided to set the model up in a Microsoft Excel® spreadsheet, a commonly available spreadsheet programme available on many computers, rather than using a programming language. The advantage of setting up and solving the equations in a spreadsheet is that it does not require any specialist programming knowledge and it allows the students to quickly examine the underlying formulae that make up the spreadsheet. It also allows the students to see all of the parameters (node temperatures, fluid properties etc) as they vary through the PHE, displayed at once on the spreadsheet and they can easily plot the variation of different parameters through the PHE using Microsoft Excel's® charting capability. Furthermore, the student can alter the spreadsheet easily to incorporate other desired calculations or modifications.

Microsoft Excel® also has built in iteration capabilities that can be controlled in terms of numbers of iterations and a maximum change parameter between successive iterations.

A disadvantage of setting the model up in a spreadsheet is that a new spreadsheet or model must be created for each new PHE configuration ie if the number of plates changes or the flow configuration changes then a new spreadsheet model must be set up. Once the new spreadsheet model is set up then the various input parameters (fluid flow rates, fluid properties, inlet temperatures) can be changed to see the effect these have on heat duty, outlet temperatures and performance parameters, ε and NTU. However, this is acceptable as the intended purpose of the model is to *rate* an existing, fixed configuration PHE.

1.6 Objectives

The objective of this study is to set up software models that simulate the performance of a plate heat exchanger. These models will be used in an educational environment to help students understand the overall and detailed operation of a plate heat exchanger. Specifically:

1. Set up a constant properties model (CPM) that will simulate the performance of a PHE under the assumption that fluid properties remain constant through the PHE. This model will be set up in three common flow configurations and will allow the

user to observe temperature changes through the PHE. The model will allow students to investigate and alter underlying formulae. The model will also output standard performance parameters (overall heat transfer coefficients, effectiveness, NTU, fluid pressure drops).

2. Validate the CPM against the standard $\varepsilon - NTU$ method that is described in text books [2].

3. Set up a variable properties model (VPM) that will simulate the performance of a PHE allowing for the more realistic situation of fluid properties varying through the PHE. This model will also be set up in three common flow configurations and will allow the user to observe temperature and other parameter changes (fluid properties, Reynolds number, heat transfer coefficients, friction factors etc) through the PHE. The model will allow students to investigate and alter underlying formulae. The model will also output standard performance parameters (overall heat transfer coefficients, effectiveness, NTU, fluid pressure drops).

4. Experimentally validate the VPM with experimental data obtained from an available PHE heat transfer rig.

5. Experimentally validate the VPM with experimental data representing a range of flow configurations, fluid types and flow conditions, available from the open literature.

6. Outline procedures for a range of suggested “virtual” experiments using a real PHE and the different software models to investigate various aspects of heat exchanger performance. These experiments to include the investigation of the constant properties assumption.

7. Provide operational notes and flow charts describing the usage of the software models.

CHAPTER 2

CONSTANT PROPERTIES MODEL

2.1 Introduction

As mentioned in Chapter 1 there are two types of heat exchanger problem encountered in practise: the "thermal design" type of problem and the "thermal rating" type of problem. The focus of this thesis is one of education and improving students' knowledge and understanding of how a heat exchanger works, rather than giving them computerised tools to enable them to *design* a heat exchanger. Therefore this thesis only considers the "thermal rating" type of heat exchanger problem.

The thermal rating of a heat exchanger involves predicting the thermal performance of an existing heat exchanger under specified operating conditions (Figure 2.1). In this situation the physical characteristics of the heat exchanger are known and fixed (heat transfer area, wall thickness, flow passage dimensions, flow configuration and number of plates or tubes etc). The mass flow rates, fluid properties and *inlet* temperatures are specified for the hot and cold fluids. The overall heat transfer coefficient may be

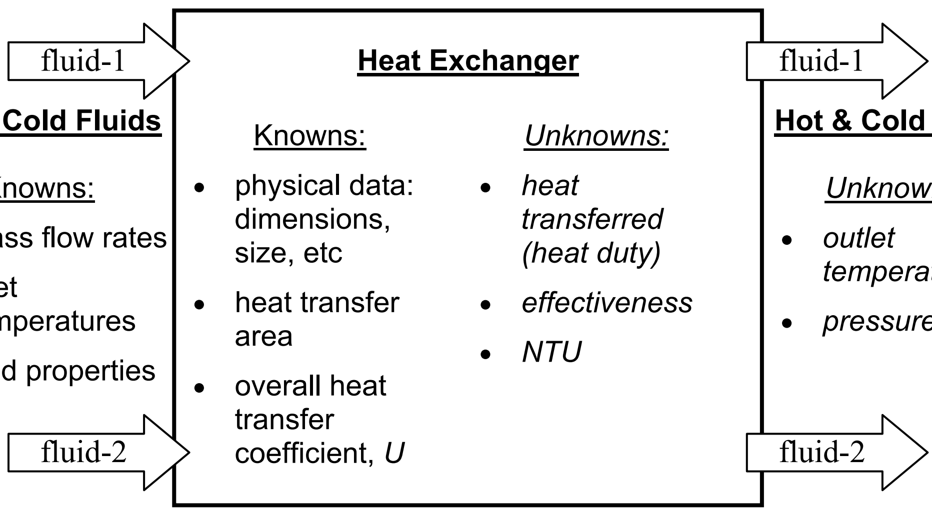


Figure 2.1 The thermal rating of a heat exchanger

specified (or calculated separately using the convection heat transfer (film) coefficients and wall properties). From these "knowns", thermal rating calculations can be carried out to determine the "unknowns": the heat transferred (heat duty) and the *outlet* temperatures of the hot and cold fluids. Also, the performance parameters for the heat exchanger, ε and NTU, can be determined for the specified operating conditions.

As mentioned in Chapter 1 there is a direct relationship between the convection heat transfer coefficients and the pressure drops for the fluids flowing through the heat exchanger. As fluid mass flow rates increase through a given heat exchanger the overall heat transfer coefficient increases and the fluid pressure drops increase. The increase in overall heat transfer coefficient is beneficial as this means an increase in heat transferred, however an increase in pressure drop is not so beneficial. If the pressure requirement is too high it could exceed the maximum allowable pressure for that heat exchanger. This is especially important with plate heat exchangers (PHE's) as they have relatively low maximum allowable pressures due to their flat plate and gasket design. Also, increased pressure drops mean increased pumping costs. So, the pressure drop of the fluids flowing through the heat exchanger is an important operational parameter and additional *hydraulic* calculations enable the pressure drop of the hot and cold fluids through the heat exchanger to be determined. Although the emphasis of the models developed in this thesis is one of *thermal* performance, it is instructional for the student to see the effect of changing input parameters on this one aspect of *hydraulic* performance, namely the pressure drop of the fluids flowing through the heat exchanger.

In this chapter, two constant-properties-models (CPM's) are developed to perform these rating calculations for a PHE. These two models are to be implemented in the educational software proposed by this thesis. Both models use a discretization and calculation procedure similar to that used for shell & tube and cross-flow heat exchangers with fluid constant properties [17].

The initial CPM1, hereafter referred to as the CPM1, predicts the performance of existing PHE's by solving the heat balance equation and the overall heat transfer equation as applied to a small element of the heat exchanger. It requires the user to input *arbitrary (fixed) values* of the fluid properties and overall heat transfer coefficient before it calculates the heat duty, outlet temperatures and performance parameters.

Because the focus of this initial CPM1 is on verifying the heat transfer calculations, it does not calculate fluid pressure drops.

The second constant properties model, referred to as the CPM, also calculates the heat duty, outlet temperatures and performance parameters in a similar way to the initial model. However, it incorporates tables of fluid property data (variation of fluid properties with temperature) and also incorporates simple, well known correlations to determine the average convection heat transfer (film) coefficient and average friction factor for each fluid. This allows the second CPM to *automatically* calculate and display the fluid properties at the average fluid temperature for each of the two fluid streams. It also allows the second model to automatically calculate and display the average overall heat transfer coefficient and the pressure drop for each fluid stream.

The second CPM is more helpful to students in becoming familiar with the operation of a heat exchanger because it automatically calculates and displays the correct fluid properties, overall heat transfer coefficient and fluid pressure drops. The student can then vary any of the input parameters (flow rates, inlet temperatures, or even heat exchanger physical parameters) and see how the heat duty, pressure drop and outlet temperatures change as a result. Also, this second CPM can be used by the student as a comparison against the more accurate variable-properties-model described in Chapter 3. In this way the student can investigate the effect of one of the assumptions commonly made in the rating of PHE's, namely that properties remain constant through the heat exchanger.

The thermal aspects (heat duty and outlet temperatures) of the two models are verified against calculations using the traditional $\varepsilon - NTU$ approach.

2.2 Initial Constant-Properties-Model (CPM1) Formulation

Consider a typical plate heat exchanger. As mentioned in Chapter 1, in this type of heat exchanger the two fluid streams are separated by flat plates, held together on a frame and sealed by gaskets, giving flat rectangular flow passages. The hot and cold streams flow in alternate flow passages. There are many different flow configurations possible for a PHE, of which the series, parallel and combination flow configurations are the most common (see Figure 1.5). Versions of the two CPM's are developed for each of these three possible flow configurations. Generally the explanations and diagrams used throughout this chapter relate to a parallel flow configuration as a general case. Where distinction between the different configurations is required then additional explanation and diagrams are provided.

2.2.1 Discretization of the Plate Heat Exchanger

Consider a PHE with $M+1$ plates. This gives M flow passages and if it is assumed that M is an even number then each fluid will have the same number of flow passages through the PHE (see Figure 2.2). To develop a set of consistent equations the PHE is divided into small elements with an a *i,j* numbering scheme, where *i* refers to the flow passage and depends on the number of plates (total flow passages = *M*) and *j* refers to the number of elements that the flow passage is vertically discretized into (total = *N*).

For the parallel flow configuration shown in Figure 2.2 the first fluid, fluid-1 is shown

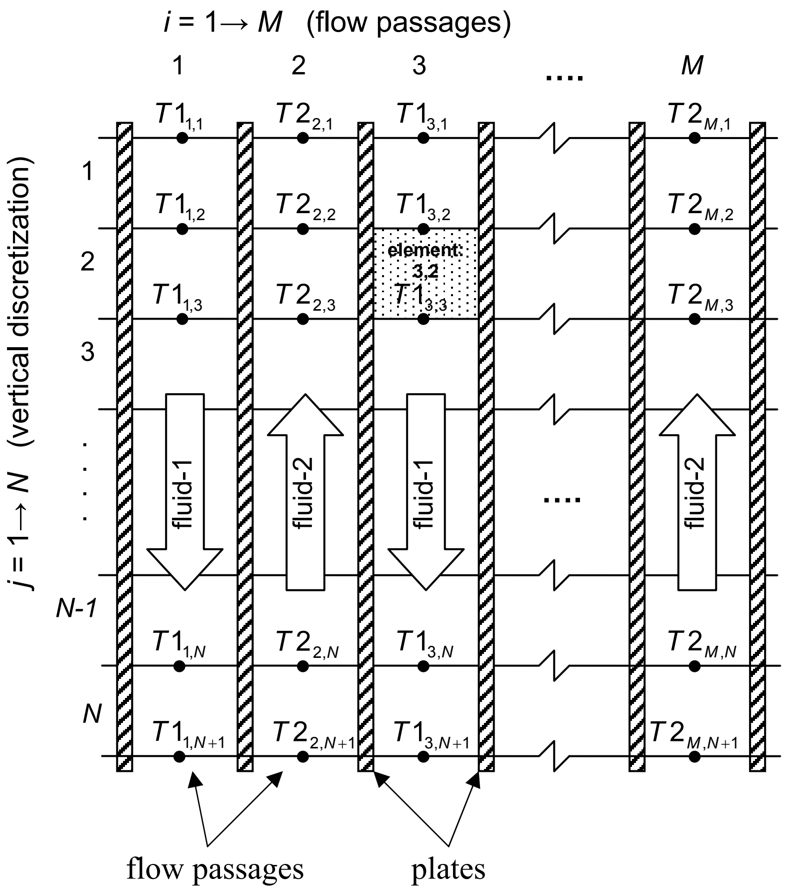


Figure 2.2 Discretization of plate heat exchanger

flowing vertically downwards in the odd numbered flow passages (i = 1, 3, 5…) and the second fluid, fluid-2 is shown flowing vertically upwards in the even numbered flow passages (i = 2, 4, 6…). Fluid node temperatures are defined as being at element edges. For example, for element 3,2 fluid-1 enters the element at a temperature of $T1_{3,2}$ and leaves the element at a temperature of $T1_{3,3}$.

Throughout the remainder of this thesis when a fluid is referred to as flowing *down* a flow passage in the PHE it is flowing in the direction where j (vertical discretization) is *increasing*. Conversely, when a fluid is referred to as flowing *up* a flow passage in the PHE it is flowing in the direction that j is *decreasing*.

2.2.2 Assumptions

In the development of the initial CPM1 the following assumptions are made:

- The user adds an appropriate value for the overall heat transfer coefficient, U.
- The user adds appropriate values for fluid properties.
- The overall heat transfer coefficient and the fluid properties remain constant throughout the heat exchanger.
- A simple average temperature difference, rather than the log mean temperature difference, is used in the heat transfer equation for each element (assumes element size is small). (This assumption is implemented to give the student insight into the fact that for small temperature differences there is little difference between the log-mean temperature difference and the average temperature difference. Also, using a simple average temperature difference greatly simplifies the setting up of the numerical solution used in this model).
- No heat loss to the environment.
- Steady state conditions.
- Element volumes are constant through the heat exchanger.
- No axial conduction.
- Plug flow in the flow channels
- No change of phase (both fluids remain as liquids throughout the PHE)

Furthermore, for a parallel flow configuration:

- The inlet temperature to each flow passage for a fluid is the same as the inlet temperature to the heat exchanger for that fluid.
- The mass flow rate of each fluid entering the PHE divides equally between the available flow passages for each fluid.
- Perfect mixing of the fluid from the outlets of the flow passages as they recombine to leave the PHE in a single flow stream.

Also, to simplify the models and number of equations derived, it is assumed that fluid-1 is the *hot* fluid and fluid-2 is the *cold* fluid. It is found that, in the utilisation of the models, if this restriction is not adhered to then the only consequence is that the heat flows are displayed as negative numbers rather than positive ones.

With these assumptions, it is now possible to develop a set of equations that will enable the calculation of the fluid node temperatures throughout the PHE for the different flow configurations. Once these have been determined then the fluid outlet temperatures from the heat exchanger, the heat transferred in the heat exchanger and the performance parameters: effectiveness and NTU, can also be determined.

2.2.3 Fluid Node Temperatures

If each plate in the PHE has a length L and width w (between the gaskets) and heat transfer area of $A_p = Lw$, then the total heat transfer area A of the heat exchanger is given by:

$$A = (M - 1)A_p \tag{2.1}$$

(Note: If there are $M + 1$ plates, then two of the plates, at either end of the heat exchanger are not involved in heat transfer, hence heat transfer occurs across $M - 1$ plates)

Now for a parallel flow configuration, let fluid-1 enter the top of the PHE with a mass flow rate of $\dot{m}1$, inlet temperature $T1_{IN}$ and specific heat capacity C_p1. The flow divides evenly between the $M/2$ flow passages (which for this analysis is assumed to be all the odd numbered passages: 1, 3, 5 … $(M - 1)$) and enters each passage with a heat capacity rate of $C1_M$. The heat capacity rate of fluid-1 entering the exchanger is $C1 = \dot{m}1 \times C_p 1$ so the heat capacity rate of fluid-1 in each flow passage, $C1_M$ is given by

$$C1_M = \frac{C1}{(M/2)} \qquad \text{(parallel flow configuration)} \qquad (2.2)$$

Similarly, fluid-2 enters the bottom of the PHE with a mass flow rate of $\dot{m}2$, inlet temperature $T2_{IN}$ and specific heat capacity $C_p 2$. The flow divides evenly between the $M/2$ flow passages (which for this analysis is assumed to be all the even numbered

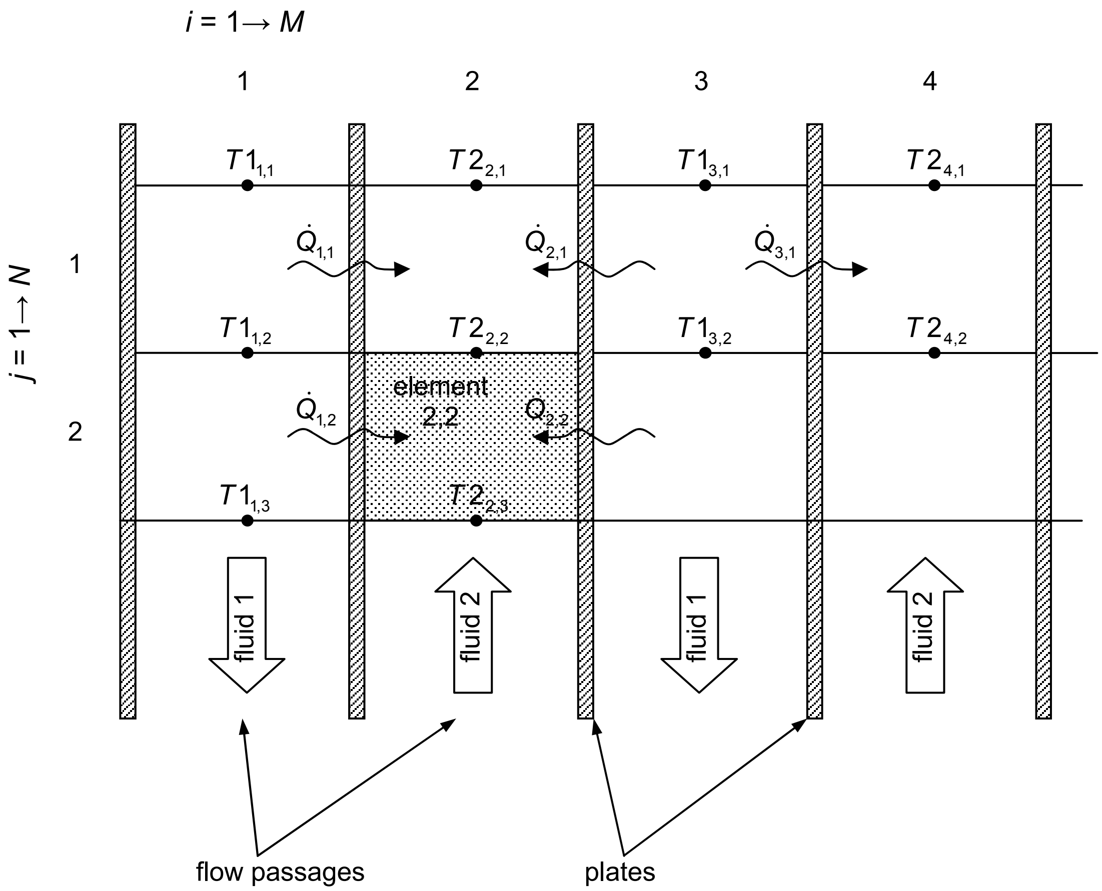


Figure 2.3 PHE –details of heat flows

passages: 2, 4, 6 … M) and enters each flow passage with a heat capacity rate of $C2_M$. The heat capacity rate of fluid-2 entering the PHE is $C2 = \dot{m}2 \times C_{\mathrm{p}}2$, so the heat capacity rate of fluid-2 in each flow passage, $C2_M$ is given by:

$$C2_M = \frac{C2}{(M/2)} \qquad \text{(parallel flow configuration)} \qquad (2.3)$$

In the case of a series flow configuration, as fluid-1 enters the PHE it does not divide up, rather it *all* flows in a series fashion along all the odd numbered passages (ie from 1 to 3 to 5 … to $(M-1)$). In this case the heat capacity rate of fluid-1 in each passage, $C1_M$ is the same as the heat capacity rate of fluid-1 entering the PHE, $C1$.

$$C1_M = C1 \qquad \text{(series flow configuration)} \qquad (2.4)$$

Similarly, for fluid-2 in a series flow configuration, the heat capacity rate in each flow passage, $C2_M$ is the same as the heat capacity rate of fluid-2 entering the PHE, $C2$.

$$C2_M = C2 \qquad \text{(series flow configuration)} \qquad (2.5)$$

Referring to Figure 2.3 the heat flow $\dot{Q}_{i,j}$ is defined as the heat flowing between the two adjacent elements i,j and $i+1,j$. Also if the overall heat transfer coefficient for the PHE is U then any element i,j has a UA value of UA_{MN} where:

$$UA_{\mathrm{MN}} = \frac{UA}{(M-1)\times N} \qquad (2.6)$$

In a PHE any one flow passage is exchanging heat with the two flow passages on either side of it. However, in the first or left-most flow passage and in the last or right-most flow passage heat is being exchanged with only one other flow passage. For this reason different equations are developed to calculate the node temperatures for the first flow passage and for the last flow passage as well as for fluid-1 and for fluid-2 in the remaining flow passages.

Consider a small element adjacent to the left boundary (flow passage 1, element $1, j$) in the PHE (see Figure 2.3). If fluid-1, of heat capacity rate $C1_M$, is flowing *down* this flow passage then it enters the element at temperature $T1_{1,j}$ and leaves at temperature $T1_{1,j+1}$ so that the heat flow, $\dot{Q}_{1,j}$, from that element is given by:

$$\dot{Q}_{1,j} = C1_M \left(T1_{1,j} - T1_{1,j+1}\right) \tag{2.7}$$

Assuming no axial conduction, this heat transfers from element $1, j$ across the plate to the adjacent element, $2, j$. If the element size is small then the heat transfer equation can be written using a simple average temperature difference rather than the log-mean temperature difference, with little error:

$$\dot{Q}_{1,j} = UA_{MN}\left(\frac{T1_{1,j} + T1_{1,j+1}}{2} - \frac{T2_{2,j} + T2_{2,j+1}}{2}\right) \tag{2.8}$$

From equations (2.7) and (2.8) it can be shown that the outlet temperature, $T1_{1,j+1}$ from the element is given by:

$$T1_{1,j+1} = \frac{T1_{1,j}\left(C1_M - \dfrac{UA_{MN}}{2}\right) + \dfrac{UA_{MN}}{2}\left(T2_{2,j} + T2_{2,j+1}\right)}{\left(C1_M + \dfrac{UA_{MN}}{2}\right)} \tag{2.9}$$

Now consider a small element adjacent to the right boundary (flow passage M, element M,j) in the PHE. Fluid-2, of heat capacity rate $C2_M$, flows *up* this flow passage and enters the element at temperature $T2_{M,j+1}$ and leaves at temperature $T2_{M,j}$ so that the heat flow, $\dot{Q}_{M-1,j}$, to that element is given by:

$$\dot{Q}_{M-1,j} = C2_M \left(T2_{M,j} - T2_{M,j+1}\right) \tag{2.10}$$

Assuming no axial conduction, and using the average temperature difference, the heat transfer equation between this element M, j and the adjacent element, $M-1, j$ can be written:

$$\dot{Q}_{M-1,j} = UA_{MN}\left(\frac{T1_{M-1,j} + T1_{M-1,j+1}}{2} - \frac{T2_{M,j} + T2_{M,j+1}}{2}\right) \tag{2.11}$$

From equations (2.10) and (2.11) it can be shown that the outlet temperature $T2_{M,j}$ from the element is given by:

$$T2_{M,j} = \frac{T2_{M,j+1}\left(C2_M - \frac{UA_{MN}}{2}\right) + \frac{UA_{MN}}{2}\left(T1_{M-1,j} + T1_{M-1,j+1}\right)}{\left(C2_M + \frac{UA_{MN}}{2}\right)} \tag{2.12}$$

Now, consider an element located at position *i,j* for fluid-1. Fluid-1, of heat capacity rate $C1_M$, flows *down* the flow passage and enters the element at temperature $T1_{i,j}$ and leaves at temperature $T1_{i,j+1}$. This element is exchanging heat with the two adjacent elements and so the total heat flow from that element is equal to the sum of $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$ and is given by:

$$\dot{Q}_{i-1,j} + \dot{Q}_{i,j} = C1_M\left(T1_{i,j} - T1_{i,j+1}\right) \tag{2.13}$$

The heat transfer equation can now be written for $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$:

$$\dot{Q}_{i-1,j} = UA_{MN}\left(\frac{T1_{i,j} + T1_{i,j+1}}{2} - \frac{T2_{i-1,j} + T2_{i-1,j+1}}{2}\right) \tag{2.14}$$

$$\dot{Q}_{i,j} = UA_{MN}\left(\frac{T1_{i,j} + T1_{i,j+1}}{2} - \frac{T2_{i+1,j} + T2_{i+1,j+1}}{2}\right) \tag{2.15}$$

From equations (2.13) – (2.15) it can be shown that the outlet temperature $T1_{i,j+1}$ from the element is given by:

$$T1_{i,j+1} = \frac{T1_{i,j}\left(C1_M - UA_{MN}\right) + \frac{UA_{MN}}{2}\left(T2_{i-1,j} + T2_{i-1,j+1} + T2_{i+1,j} + T2_{i+1,j+1}\right)}{\left(C1_M + UA_{MN}\right)} \tag{2.16}$$

Finally, consider an element located at position *i,j* for fluid-2. Fluid-2, of heat capacity rate $C2_M$, flows *up* the flow passage and enters the element at temperature $T2_{i,j+1}$ and leaves at temperature $T2_{i,j}$. This element is exchanging heat with the two adjacent elements and so the total heat flow to that element is equal to the sum of $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$ and is given by:

$$\dot{Q}_{i-1,j} + \dot{Q}_{i,j} = C2_M \left(T2_{i,j} - T2_{i,j+1} \right) \tag{2.17}$$

The heat transfer equation can now be written for $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$:

$$\dot{Q}_{i-1,j} = UA_{MN} \left(\frac{T1_{i-1,j} + T1_{i-1,j+1}}{2} - \frac{T2_{i,j} + T2_{i,j+1}}{2} \right) \tag{2.18}$$

$$\dot{Q}_{i,j} = UA_{MN} \left(\frac{T1_{i+1,j} + T1_{i+1,j+1}}{2} - \frac{T2_{i,j} + T2_{i,j+1}}{2} \right) \tag{2.19}$$

From equations (2.17) – (2.19) it can be shown that the outlet temperature $T2_{i,j}$ from the element is given by:

$$T2_{i,j} = \frac{T2_{i,j+1}\left(C2_M - UA_{MN}\right) + \frac{UA_{MN}}{2}\left(T1_{i-1,j} + T1_{i-1,j+1} + T1_{i+1,j} + T1_{i+1,j+1}\right)}{\left(C2_M + UA_{MN}\right)} \tag{2.20}$$

Now, in a series flow configuration it is possible for fluid-1 to be flowing *up* some of the flow passages and for fluid-2 to be flowing *down* some of the flow passages. Following a similar derivation as outlined above it can be shown that for fluid-1 flowing *up* a flow passage the outlet temperature, $T1_{i,j}$ from an element *i,j* is given by an equation very similar to equation (2.16)

$$T1_{i,j} = \frac{T1_{i,j+1}\left(C1_M - UA_{MN}\right) + \frac{UA_{MN}}{2}\left(T2_{i-1,j} + T2_{i-1,j+1} + T2_{i+1,j} + T2_{i+1,j+1}\right)}{\left(C1_M + UA_{MN}\right)} \tag{2.21}$$

Similarly for fluid-2 flowing *down* a flow passage the outlet temperature, $T2_{i,j+1}$ from an element *i,j* is given by an equation very similar to equation (2.20).

$$T2_{i,j+1} = \frac{T2_{i,j}\left(C2_M - UA_{MN}\right) + \frac{UA_{MN}}{2}\left(T1_{i-1,j} + T1_{i-1,j+1} + T1_{i+1,j} + T1_{i+1,j+1}\right)}{\left(C2_M + UA_{MN}\right)} \qquad (2.22)$$

2.2.3.1 Boundary Conditions

For a parallel flow configuration it is assumed that the inlet temperature to each flow passage is the same as the inlet temperature to the heat exchanger for that fluid, so for fluid-1 flowing *down* the PHE and fluid-2 flowing *up* the PHE, temperatures at the boundaries are given by:

for fluid-1 and i = 1, 3, 5… $(M-1)$

$$T1_{i,1} = T1_{IN} \qquad \text{(parallel flow configuration)} \qquad (2.23)$$

and for fluid-2 and i = 2, 4, 6… M

$$T2_{i,N+1} = T2_{IN} \qquad \text{(parallel flow configuration)} \qquad (2.24)$$

For a series flow configuration the inlet temperature to the left-most and right-most flow channels is the same as the fluid-1 and fluid-2 inlet temperature respectively. And, the inlet temperatures to the remaining flow channels depend on the outlet temperature of the flow channel that immediately precedes it. So,

left-most flow channel:

$$T1_{1,1} = T1_{IN} \qquad \text{(series flow configuration)} \qquad (2.25)$$

right-most flow channel:

$$T2_{M,N+1} = T2_{IN} \qquad \text{(series flow configuration)} \qquad (2.26)$$

for fluid-1 and $i = 1, 5, 9 \ldots$

$$T1_{i+2,N+1} = T1_{i,N+1} \qquad \text{(series flow configuration)} \qquad (2.27)$$

and also for fluid-1 and $i = 3, 7, 11\ldots$

$$T1_{i+2,1} = T1_{i,1} \qquad \text{(series flow configuration)} \qquad (2.28)$$

Now for fluid-2 and $i = 4, 8, 12\ldots$

$$T2_{i,N+1} = T2_{i+2,N+1} \qquad \text{(series flow configuration)} \qquad (2.29)$$

and also for fluid-2 and $i = 2, 6, 10\ldots$

$$T2_{i,1} = T2_{i+2,1} \qquad \text{(series flow configuration)} \qquad (2.30)$$

For a parallel flow configuration, the equations developed for the temperatures at the nodes of each element (equations (2.9), (2.12), (2.16) and (2.20)) produce a set of $M \times N$ equations and with the known boundary conditions, equations (2.23) and (2.24), there are $M \times N$ unknowns. Similarly a series or combination flow configuration will produce $M \times N$ equations with $M \times N$ unknowns.

These equations can be solved by a number of standard mathematical methods. In this thesis the equations are set up in a Microsoft Excel® spreadsheet and the iteration capabilities of the spreadsheet are used to solve the equations. The setting up of the equations in Microsoft Excel® is described in detail in section 2.2.5.

2.2.4 Rating Calculations

Once the fluid node temperatures are determined, the fluid outlet temperatures and heat transferred in the heat exchanger can be calculated.

In the case of parallel flow configuration it is assumed that the fluid flow divides evenly amongst the available flow passages. If perfect mixing is assumed when the fluid from the flow passages recombines into a single flow stream then the fluid outlet temperatures can be calculated as a simple arithmetic average of the flow passage outlet temperatures for each fluid:

$$T1_{\mathrm{OUT}} = \frac{\sum_{i=1,3...}^{M-1} T1_{i,N+1}}{M/2} \qquad \text{(parallel flow configuration)} \quad (2.31)$$

$$T2_{\mathrm{OUT}} = \frac{\sum_{i=2,4,...}^{M} T2_{i,1}}{M/2} \qquad \text{(parallel flow configuration)} \quad (2.32)$$

In the case of a series flow configuration the outlet temperature of fluid-1 is the temperature of the fluid leaving the $i = (M-1)$ flow channel. Depending on the number of flow channels fluid-1 could leave the $i = (M-1)$ flow channel at $j = 1$ or at $j = N+1$. Similarly the outlet temperature for fluid-2 is the temperature of the fluid leaving the $i = 2$ flow channel. And again depending on the number of flow channels fluid-2 could leave the $i = 2$ flow channel at $j = 1$ or at $j = N+1$. So

$$T1_{\mathrm{OUT}} = T1_{M-1,1} \text{ or } T1_{M-1,N+1} \qquad \text{(series flow configuration)} \quad (2.33)$$

and

$$T2_{\mathrm{OUT}} = T2_{2,1} \text{ or } T2_{2,N+1} \qquad \text{(series flow configuration)} \quad (2.34)$$

Now, the total heat transferred, $\dot{Q}$ in the heat exchanger can be determined in several different ways.

The heat transferred in the heat exchanger is equal to the heat loss from the hot fluid:

$$\dot{Q} = C1(T1_{\mathrm{IN}} - T1_{\mathrm{OUT}}) \quad (2.35)$$

The heat transferred in the heat exchanger is also equal to the heat gain of the cold fluid:

$$\dot{Q} = C2(T2_{\mathrm{OUT}} - T2_{\mathrm{IN}}) \quad (2.36)$$

And the heat transferred is equal to the sum of all the heat transferred in the elements of the heat exchanger (taking all the $\dot{Q}_{i,j}$ quantities as positive quantities):

$$\dot{Q} = \sum_{i=1}^{M-1}\sum_{j=1}^{N}\dot{Q}_{i,j} \tag{2.37}$$

The results of equations (2.35), (2.36) and (2.37) should be identical and comparing them allows an initial check on the integrity of the fluid node temperature calculations.

The various performance parameters can also be determined using the standard formulas [2]:

$$NTU = \frac{UA}{C_{\min}} \tag{2.38}$$

effectiveness

$$\varepsilon = \frac{\dot{Q}}{C_{\min}\left(T1_{\mathrm{IN}} - T2_{\mathrm{IN}}\right)} \tag{2.39}$$

heat capacity ratio

$$C_{\mathrm{r}} = \frac{C_{\min}}{C_{\max}} \tag{2.40}$$

where $C_{\min}$ is the lesser of the two heat capacity rates, $C1$ and $C2$.

2.2.5 Spreadsheet Implementation

The equations for determining the fluid node temperatures and the equations used to rate the heat exchanger are now set up in a Microsoft Excel® spreadsheet. Since the model will ultimately be used in conjunction with a PHE set up in a heat transfer rig at the Auckland University of Technology (AUT) it is decided to initially set up the spreadsheet CPM based on this PHE. It is a 21-plate PHE with a parallel flow configuration. Further details of this PHE and the experimental rig are contained in Chapter 4 where experimental results are generated for comparison with the variable-properties-model.

The CPM is set up with the input data and rating equations in the upper region of the spreadsheet and the fluid node temperature calculations in the lower region of the spreadsheet. The fluid node temperatures are set up so that each spreadsheet column represents a flow passage and each row represents the vertical elements that each flow passage is discretized into (see Figure 2.4). Details on how to use the spreadsheet are given in Chapter 6.

Now, because the columns and rows of the spreadsheet represent the flow passages and vertical discretization respectively, the spreadsheet structure needs to be modified if the number of plates changes, or if it is decided to change the vertical discretization, or if the flow configuration changes. In other words a new spreadsheet model would need to be set up for each individual PHE depending on the number of plates, vertical discretization and flow configuration. Although this may seem a disadvantage at first, the aim of this software model is to rate an existing, specific PHE and give students an appreciation for the overall/detailed operation of that exchanger. So once the spreadsheet is set up for a specific heat exchanger the various inputs (fluid flows, fluid properties and inlet temperatures) can be varied as desired to gain appreciation for the operation of that specific PHE. And, if required, the structure of the spreadsheet can be easily modified to incorporate more or less plates, different vertical discretizations and different flow configurations.

The CPM1 is also set up in series flow and combined flow configurations.

The flow chart of the operation of the spreadsheet for the initial CPM1 is shown in Figure 2.5.

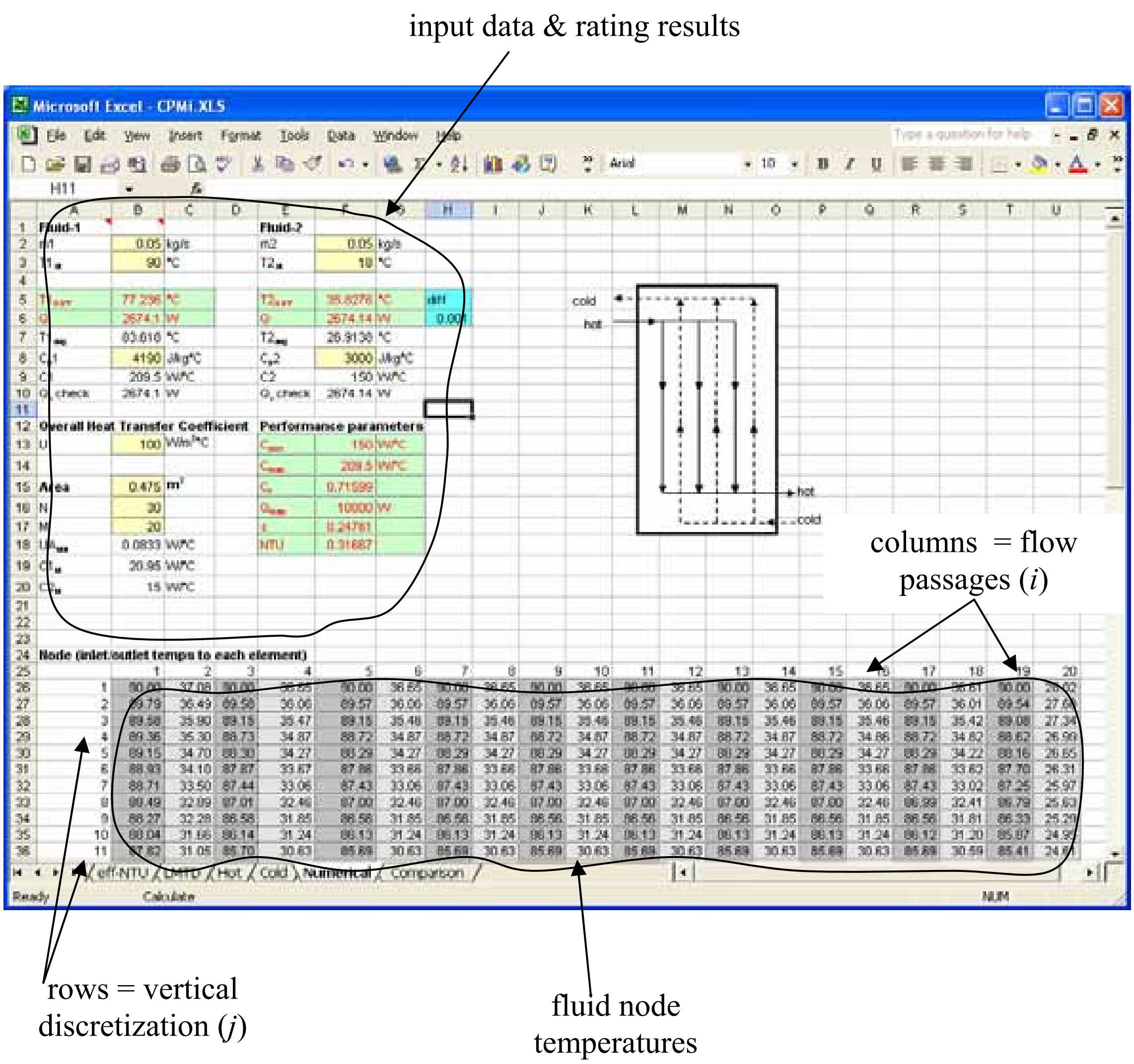


Figure 2.4 Spreadsheet implementation, initial constant-properties-model (CPM1)

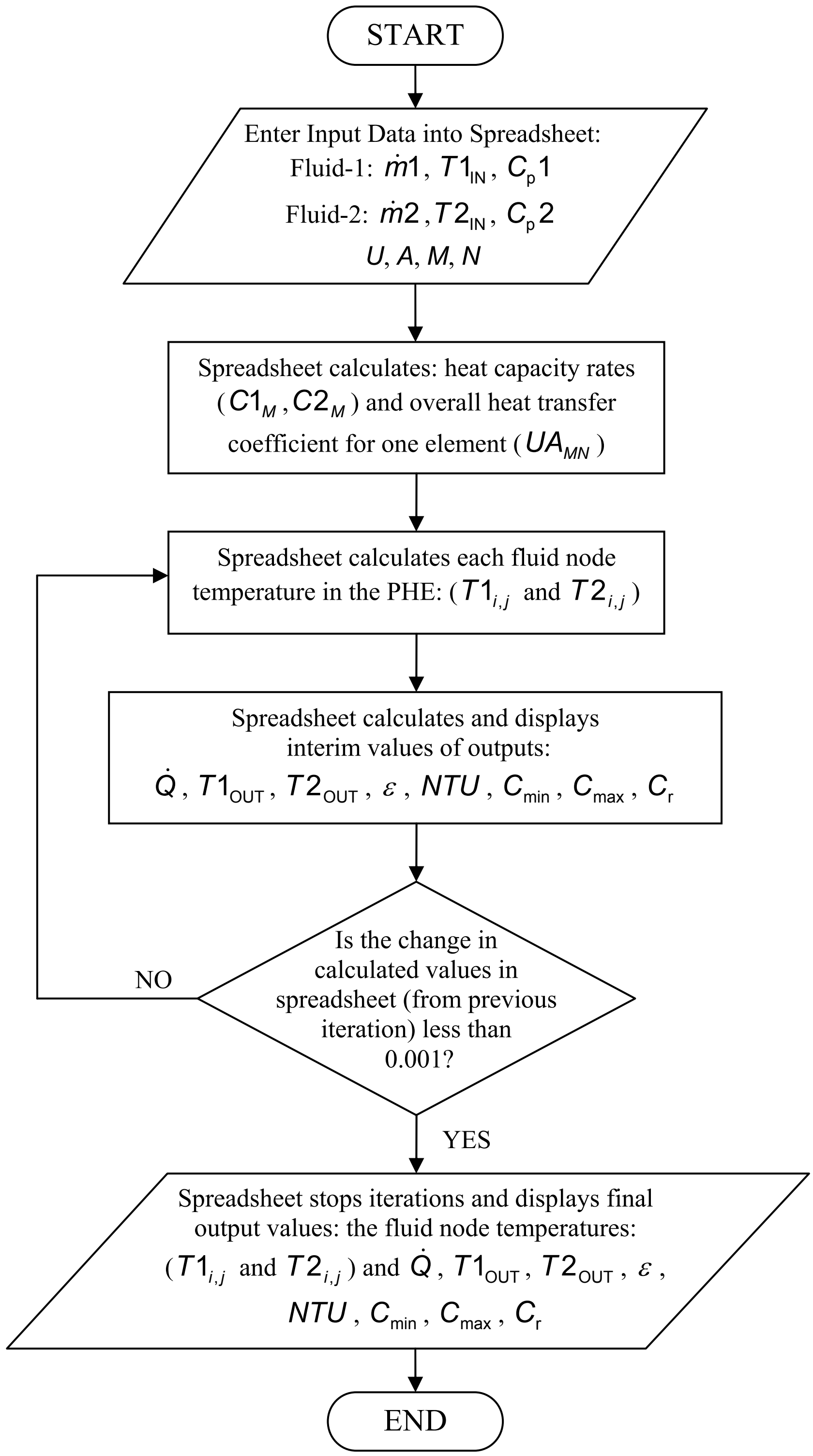


Figure 2.5 Flow chart of spreadsheet operation, initial constant-properties-model (CPM1)

2.2.6 Validation/Results

The basis of the validation of the initial CPM1 is to compare the rating results or outputs from the CPM1 with those achieved using the standard $\varepsilon - NTU$ method. This gives an initial check that the underlying equations and calculation method used to build the initial CPM1 are correct.

As described in the previous section, the CPM1 is initially set up to simulate the PHE in the experimental heat transfer rig at AUT. This PHE has 21 plates and is set up for a parallel flow configuration. Dummy data is entered into the CPM to represent a range of fluids, flow rates and conditions. This same dummy data is used to generate results using the $\varepsilon - NTU$ method.

The results are summarised in Tables 2.1 and 2.2 which show a range of different flow rates for hot (fluid-1) and cold (fluid-2) fluids and the resulting outlet temperatures and performance parameters determined using the CPM1 and the $\varepsilon - NTU$ method. Figures 2.6 and 2.7 show fluid outlet temperatures determined using the CPM1 compared to the outlet temperatures determined using the $\varepsilon - NTU$ method.

Good agreement is achieved between the results of the CPM and the results of the $\varepsilon - NTU$ method. The maximum percentage error for these results is no more than 1 to 2 %. Note: The percentage error for all validation results presented in this thesis is defined as a percentage difference compared to the full scale (or maximum) value.

It is considered unnecessary to set up comparisons between the series and combination flow configurations since the equations used to develop these configurations are substantially the same as those for the parallel configuration.

It is necessary to develop this initial CPM1 to prove in principle that the equations used are correct and also to prove the validity and workability of the spreadsheet implementation. However this initial model has several weak areas. Firstly, the property data has to be entered in manually and there is no check that it is valid for the fluid or temperatures entered. Secondly, the overall heat transfer coefficient is also entered manually and there is no check or relationship to the actual flow conditions in the heat exchanger. Finally, the method does not include a calculation for pressure drop and this parameter (ΔP) is also useful for students wanting to understand and analyse the performance of an existing heat exchanger.

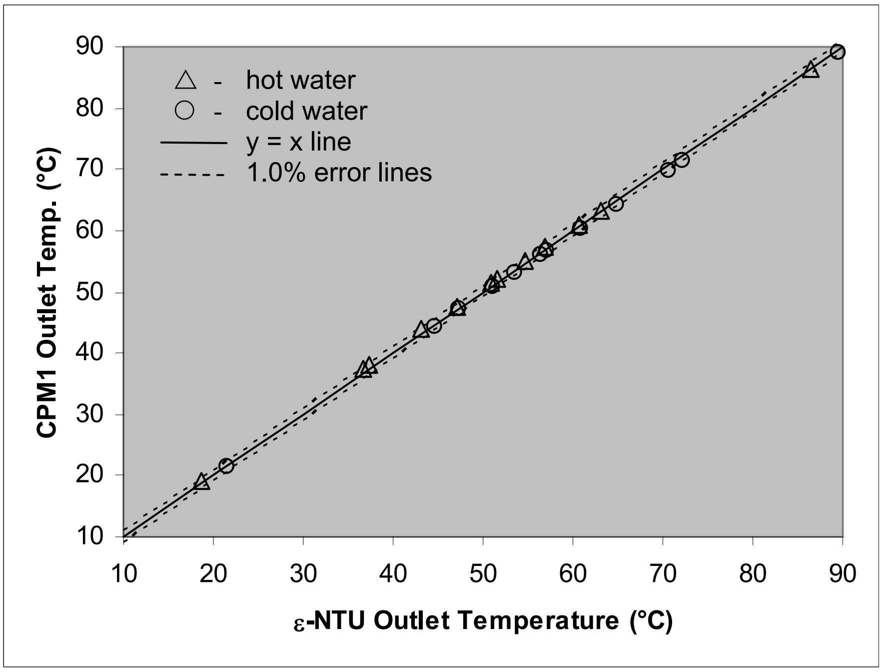


Figure 2.6 Comparison between the initial CPM1 and $\varepsilon - NTU$ results for water versus water in a PHE

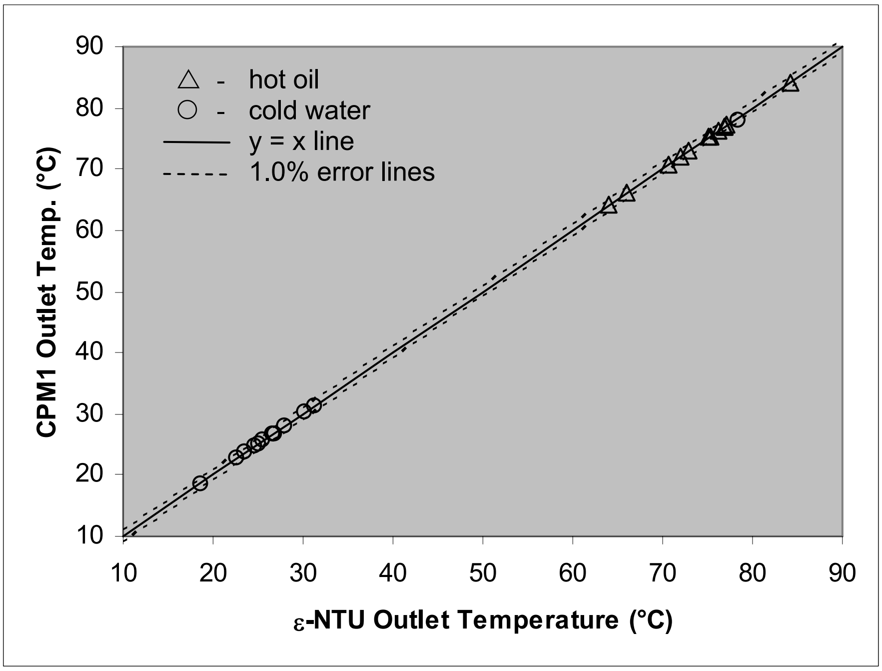


Figure 2.7 Comparison between the initial CPM1 and $\varepsilon - NTU$ results for cold water versus hot oil in a PHE

Table 2.1 Validation of initial CPM1: water versus water results

m1 (kg/s) (input) (hot)	0.05	1	0.05	0.25	0.5	1	0.5	1.5	2	1.5	1	2
m2 (kg/s) (input) (cold)	0.05	0.05	1	0.25	0.5	0.5	1	1.5	1.5	2	1	2
$T1_{in}$ (°C) (input)	90	90	90	90	90	90	90	90	90	90	90	90
$T1_{out}$(CPM1) (°C)	**38.19**	**86.46**	**19.00**	**43.88**	**47.70**	**63.38**	**37.38**	**55.01**	**61.01**	**51.33**	**52.05**	**57.28**
$T1_{out}$(ε-NTU) (°C)	**37.36**	**86.45**	**18.60**	**43.22**	**47.14**	**63.04**	**36.72**	**54.62**	**60.69**	**50.90**	**51.59**	**56.93**
ΔT1%	**2.23**	**0.02**	**2.16**	**1.54**	**1.20**	**0.54**	**1.80**	**0.72**	**0.52**	**0.83**	**0.89**	**0.61**
$T2_{in}$ (°C) (input)	18	18	18	18	18	18	18	18	18	18	18	18
$T2_{out}$(CPM1) (°C)	**69.87**	**89.15**	**21.55**	**64.23**	**60.40**	**71.44**	**44.34**	**53.07**	**56.80**	**47.11**	**56.04**	**50.80**
$T2_{out}$(ε-NTU) (°C)	**70.71**	**89.52**	**21.57**	**64.89**	**60.97**	**72.12**	**44.67**	**53.47**	**57.22**	**47.43**	**56.50**	**51.15**
ΔT2%	**1.18**	**0.41**	**0.09**	**1.03**	**0.93**	**0.94**	**0.74**	**0.74**	**0.74**	**0.67**	**0.81**	**0.68**
U (W/m^2°C) (input)	1200	2300	2200	4100	6500	8100	7800	12800	13800	13800	10100	15000
NTU	2.7273	5.2273	4.9940	1.8636	1.4773	1.8409	1.7706	0.9697	1.0455	1.0417	1.1477	0.8523
C_r	0.9988	0.0497	0.0501	0.9976	0.9976	0.4982	0.5006	0.9976	0.7473	0.7527	0.9976	0.9976
ε (CPM1)	0.7205	0.9882	0.9861	0.6420	0.5889	0.7422	0.7308	0.4871	0.5389	0.5371	0.5284	0.4556
ε (ε-NTU)	0.7320	0.9934	0.9917	0.6513	0.5968	0.7517	0.7400	0.4926	0.5447	0.5430	0.5347	0.4604
Q (CPM1) (W)	**10841**	**14871**	**14857**	**48306**	**88618**	**111690**	**110100**	**219912**	**243263**	**243359**	**159013**	**274218**
Q (ε-NTU) (W)	**11015**	**14948**	**14941**	**49003**	**89799**	**113108**	**111483**	**222377**	**245917**	**246013**	**160933**	**277109**
ΔQ%	**1.58**	**0.52**	**0.56**	**1.42**	**1.32**	**1.25**	**1.24**	**1.11**	**1.08**	**1.08**	**1.19**	**1.04**

Table 2.2 Validation of initial CPM1: water versus oil results

m1 (kg/s) (input) - hot oil	0.05	1	0.05	0.25	0.5	1	0.5	1.5	2	1.5	1	2
m2 (kg/s) (input) - cold water	0.05	0.05	1	0.25	0.5	0.5	1	1.5	1.5	2	1	2
$T1_{in}$ (°C) (input)	90	90	90	90	90	90	90	90	90	90	90	90
$T1_{out}$(CPM1) (°C)	**66.10**	**84.22**	**64.22**	**70.70**	**73.04**	**77.01**	**72.09**	**76.37**	**77.16**	**75.43**	**75.23**	**76.96**
$T1_{out}$(ε-NTU) (°C)	**65.95**	**84.15**	**64.10**	**70.60**	**72.97**	**76.95**	**72.02**	**76.33**	**77.11**	**75.38**	**75.18**	**76.92**
ΔT1%	**0.22**	**0.08**	**0.19**	**0.14**	**0.10**	**0.07**	**0.10**	**0.06**	**0.06**	**0.07**	**0.07**	**0.06**
$T2_{in}$ (°C) (input)	18	18	18	18	18	18	18	18	18	18	18	18
$T2_{out}$(CPM1) (°C)	**30.12**	**77.79**	**18.65**	**27.84**	**26.68**	**31.30**	**22.59**	**24.98**	**26.77**	**23.60**	**25.56**	**24.68**
$T2_{out}$(ε-NTU) (°C)	**30.20**	**78.45**	**18.66**	**27.89**	**26.72**	**31.36**	**22.60**	**25.00**	**26.80**	**23.62**	**25.59**	**24.70**
ΔT2%	**0.24**	**0.84**	**0.02**	**0.17**	**0.14**	**0.19**	**0.08**	**0.10**	**0.12**	**0.08**	**0.11**	**0.09**
U (W/m^2°C) (input)	100	850	100	380	650	1000	670	1500	1900	1600	1100	1900
NTU	0.4481	1.9318	0.4481	0.3390	0.2886	0.2273	0.2974	0.2220	0.2109	0.2368	0.2442	0.2109
C_r	0.5072	0.0968	0.0254	0.5096	0.5120	0.9766	0.2560	0.5120	0.6826	0.3840	0.5120	0.5120
ε (CPM1)	0.3320	0.8303	0.3580	0.2681	0.2355	0.1848	0.2488	0.1892	0.1784	0.2024	0.2051	0.1811
ε (ε-NTU)	0.3340	0.8395	0.3598	0.2694	0.2366	0.1856	0.2498	0.1899	0.1790	0.2031	0.2059	0.1817
Q (CPM1) (W)	**2533**	**12494**	**2732**	**10278**	**18145**	**27803**	**19166**	**43738**	**54970**	**46783**	**31606**	**55815**
Q (ε-NTU) (W)	**2548**	**12633**	**2745**	**10329**	**18224**	**27927**	**19241**	**43892**	**55171**	**46945**	**31726**	**56003**
ΔQ%	**0.60**	**1.09**	**0.48**	**0.49**	**0.43**	**0.44**	**0.39**	**0.35**	**0.37**	**0.35**	**0.38**	**0.34**

2.3 Second Constant-Properties-Model (CPM) Formulation

The second CPM is developed because of the limitations found in the initial CPM1. The second CPM is developed using the initial CPM1 as a foundation. It assumes the same flow discretization scheme as in the initial model; however the following modifications are made. Firstly, tables of fluid property data (variation of properties with temperature) are added on a separate worksheet (in the same workbook) and equations are entered to automatically determine the fluid properties at the average fluid temperatures. Secondly, simple, well known correlations are added to determine the average convection heat transfer coefficient (h) and the friction factor (f) for each fluid flowing through the PHE. This second modification is made so that the overall heat transfer coefficient, U and the fluid pressure drops can be determined automatically. The calculation of the overall heat transfer coefficient and the pressure drop also requires the addition of physical heat exchanger data to the model (ie the dimensions of the flow passages and plate thickness).

The assumptions made in the second CPM are substantially the same as in the initial CPM1 except for the first two assumptions (see section 2.2.2) which now become:

- Model calculates overall heat transfer coefficient, U depending on physical heat exchanger data and tabular fluid property data (at average fluid temperatures) using an empirical correlation.
- Model calculates values for fluid properties depending on average fluid temperatures in the heat exchanger

Note: Average fluid temperatures are defined by equations (2.41) and (2.42).

2.3.1 Calculation of Fluid Properties

In this second CPM, as in the initial CPM1, it is assumed that the fluid properties remain constant through the heat exchanger. However, in the second CPM the fluid properties are calculated at an appropriate average temperature. This appropriate average temperature is the average fluid temperature for each fluid as it flows through the heat exchanger.

If fluid-1 enters the heat exchanger at a temperature of $T1_{IN}$ and leaves at $T1_{OUT}$ then the average fluid temperature for fluid-1 is:

$$T1_{AVG} = \frac{T1_{IN} + T1_{OUT}}{2} \tag{2.41}$$

Similarly the average fluid temperature for fluid-2 is:

$$T2_{AVG} = \frac{T2_{IN} + T2_{OUT}}{2} \tag{2.42}$$

Initially, the values of $T1_{OUT}$ and $T2_{OUT}$ are not known, so the iteration capabilities of the spreadsheet are used to converge towards the correct values for $T1_{OUT}$, $T2_{OUT}$.

When it is required to determine fluid property data at some average fluid temperature the most suitable form for a computer program is an equation relating property values to temperature. However, fluid property versus temperature data is often available in textbooks and datasheets in a tabular form and so a suitable polynomial curve fit technique must be used to generate polynomial equations relating the desired fluid property to temperature.

Although a spreadsheet can easily use these polynomial equations, a spreadsheet also lends itself to calculations using the raw tabular data. By using a simple linear interpolation technique on the tabular data it is possible to get accurate property data at any temperature (within the temperature range of the tabular data). This also saves the user from having to determine the polynomial equation for the properties by curve fitting techniques. And, if in fact the property data is available in equation form then this also can be easily incorporated into the spreadsheet

In the second CPM the fluid properties are entered into the spreadsheet in tabular form and a simple linear interpolation technique is used to determine fluid property values at any temperature. For example, consider the property data given in tabular form as in Table 2.3.

Table 2.3 Property data

Temperature, T	Property, P
T_1	P_1
T_2	P_2
…	…
T_k	P_k
T_{k+1}	P_{k+1}
…	…

If the property, P is required at some average temperature T_{AVG} and $T_k < T_{AVG} < T_{k+1}$ then linear interpolation gives:

$$P = P_k + \frac{(T_{AVG} - T_k)}{(T_{k+1} - T_k)}(P_{k+1} - P_k) \tag{2.43}$$

Equation (2.43) is used in the second CPM to determine fluid properties at the relevant fluid average temperature.

2.3.2 Calculation of the Convection and Overall Heat Transfer Coefficients

There are many empirical correlations reported in the literature for determining the convection heat transfer (film) coefficient and friction factor for PHE's and these have been briefly discussed in Chapter 1.

For this second CPM a simpler form of the correlations for Nusselt number and friction factor will be used. (The simpler form of the Nusselt number correlation does not include a viscosity correction term). The correlation for the Nusselt number in a flow channel with Reynolds number, Re and fluid Prandtl number, Pr, is generally accepted to be of the form [2]:

$$\mathrm{Nu} = D\,\mathrm{Re}^m\,\mathrm{Pr}^n \tag{2.44}$$

where D, m and n are constants and vary depending on the specific PHE and the flow conditions. The model allows D, m and n to be set as required and can be set the same for both fluids or differently for each fluid.

The convection heat transfer coefficient can then be calculated from the definition of the Nusselt number:

$$h = \frac{\mathrm{Nu}\,k}{d_\mathrm{h}} \tag{2.45}$$

Where k is the thermal conductivity of the fluid (W/m°C) and d_h is the hydraulic diameter of the flow passage (m).

If the convection heat transfer coefficient for fluid-1 in a flow channel is $h1$ and the convection heat transfer coefficient for fluid-2 in the adjacent flow channel is $h2$ and the flow channels are separated by a plate of wall thickness x_w with thermal conductivity k_w (see Figure 2.8) then the overall heat transfer coefficient, U is given by:

$$U = \left(\frac{1}{h1} + \frac{x_\mathrm{w}}{k_\mathrm{w}} + \frac{1}{h2}\right)^{-1} \tag{2.46}$$

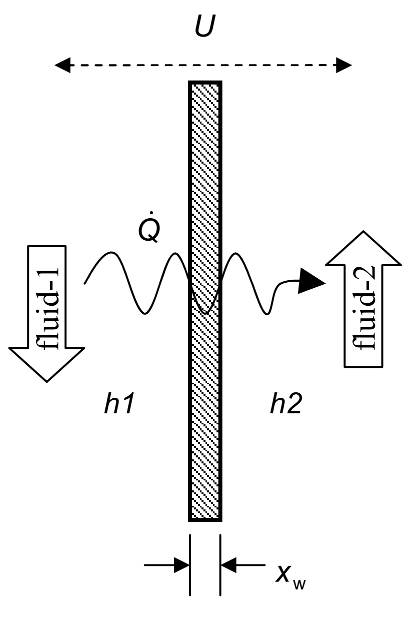


Figure 2.8 Overall heat transfer coefficient, U

2.3.3 Calculation of the Friction Factor and the Pressure Drop

The correlation for the friction factor in a flow channel with Reynolds number, Re is generally considered to be of the form [2]:

$$f = B\,\mathrm{Re}^{c} \tag{2.47}$$

where B and c are constants depending on the specific PHE. The model allows B and c to be set as required and can be set the same for both fluids or differently for each fluid.

For a flow channel of length L, hydraulic diameter d_h and cross-sectional area A_c with fluid-1 flowing in the channel with a mass velocity of $\dot{m}1/MA_c$, density $\rho 1$ and friction factor $f1$, the pressure drop in that flow channel, $\Delta P1_{\text{flow channel}}$ can then be calculated:

$$\Delta P1_{\text{flow channel}} = f1\frac{2L\left[\dot{m}1/\left(MA_c\right)\right]^2}{\rho 1\, d_h} \tag{2.48}$$

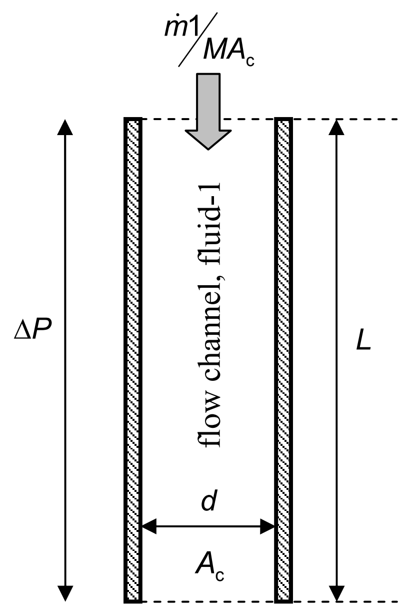


Figure 2.9 Pressure drop in a flow channel

and similarly for fluid-2:

$$\Delta P2_{\text{flow channel}} = f2\frac{2L\left[\dot{m}1/(MA_{\text{c}})\right]^2}{\rho 2\ d_{\text{h}}} \tag{2.49}$$

The overall pressure drop in a PHE from inlet pipe to outlet pipe is made up of four components: the core pressure drop in the flow channels as per equations (2.48) and (2.49), the pressure drop in the internal distribution manifolds, the pressure drop due to elevation change and the pressure drop in the short manifold pipes that connect to the rest of the plant pipe work. It is generally considered that the bulk of this overall pressure drop occurs in the core or flow passages. Since the purpose of this thesis is mainly to look at the thermal rating of a PHE and the linkage between thermal performance and hydraulic performance it is decided that calculating core pressure drops alone would suit this purpose. So for the purposes of this thesis only the core pressure drops will be considered.

For a parallel flow configuration, with constant fluid properties, the pressure drop along each flow channel for a specific fluid is equal and equal to the overall core pressure drop for that fluid in the PHE. So for fluid-1,

$$\Delta P1 = \Delta P1_{\text{flow channel}} = f1\frac{2L\left[\dot{m}1/(MA_{\text{c}})\right]^2}{\rho 1\ d_{\text{h}}} \qquad \text{(parallel flow configuration)} \tag{2.50}$$

and similarly for fluid-2,

$$\Delta P2 = \Delta P2_{\text{flow channel}} = f2\frac{2L\left[\dot{m}1/(MA_{\text{c}})\right]^2}{\rho 2\ d_{\text{h}}} \qquad \text{(parallel flow configuration)} \tag{2.51}$$

For a series flow configuration the pressure drop for a fluid flowing through the PHE is equal to the sum of all the pressure drops in each flow channel. For each fluid there are $M/2$ flow channels and the pressure drop is the same in each flow channel under the assumption of constant fluid properties, so for fluid-1:

$$\Delta P1 = f1 \frac{L\left[\dot{m}1/A_c\right]^2}{\rho 1\, d_h\, M} \qquad \text{(series flow configuration)} \qquad (2.52)$$

and similarly for fluid-2

$$\Delta P2 = f2 \frac{L\left[\dot{m}1/A_c\right]^2}{\rho 2\, d_h\, M} \qquad \text{(series flow configuration)} \qquad (2.53)$$

2.3.4 Fluid Node Temperatures

The fluid node temperatures in the second CPM are calculated for a parallel flow configuration PHE, as in the initial CPM1, using equations (2.9), (2.12), (2.16) and (2.20) along with the known boundary conditions, equations (2.23) and (2.24). Together these produce a set of $M \times N$ equations with $M \times N$ unknowns. Similarly a series or combination flow configuration will produce $M \times N$ equations with $M \times N$ unknowns. These equations are solved, as in the initial CPM1, by setting them up in a Microsoft Excel® spreadsheet and using the iteration capabilities of the spreadsheet. This is described in more detail in section 2.3.6.

2.3.5 Rating Calculations

The various rating calculation outputs are calculated as in the initial CPM1. The outlet temperatures $T1_{OUT}$ and $T2_{OUT}$ are determined using equations (2.31) and (2.32) respectively for a parallel flow configuration and equations (2.33) and (2.34) respectively for a series flow configuration. The total heat transferred (heat duty) $\dot{Q}$ is determined using equations (2.35) – (2.37). The performance parameters: effectiveness, ε and NTU are determined using equations (2.38) and (2.39) respectively. Additionally, the pressure drops for the fluids are determined using equations (2.50) – (2.53) depending on flow configuration.

2.3.6 Spreadsheet Implementation

The spreadsheet implementation of the second CPM is similar to the spreadsheet implementation of the initial CPM1. Again, the implementation is based on the PHE in the heat transfer rig at AUT: a 21-plate PHE with a parallel flow configuration.

New areas of the spreadsheet are used to input the additional data (plate dimensions etc) and calculations, see Figure 2.10. An additional worksheet is added to include the tables of property data, see Figure 2.11. Details on how to use the spreadsheet are given in Chapter 6

This second CPM is also set up in series and combination flow configurations, although these are not used in the validation phase.

The flow chart of the operation of the spreadsheet for the second CPM is shown in Figure 2.12.

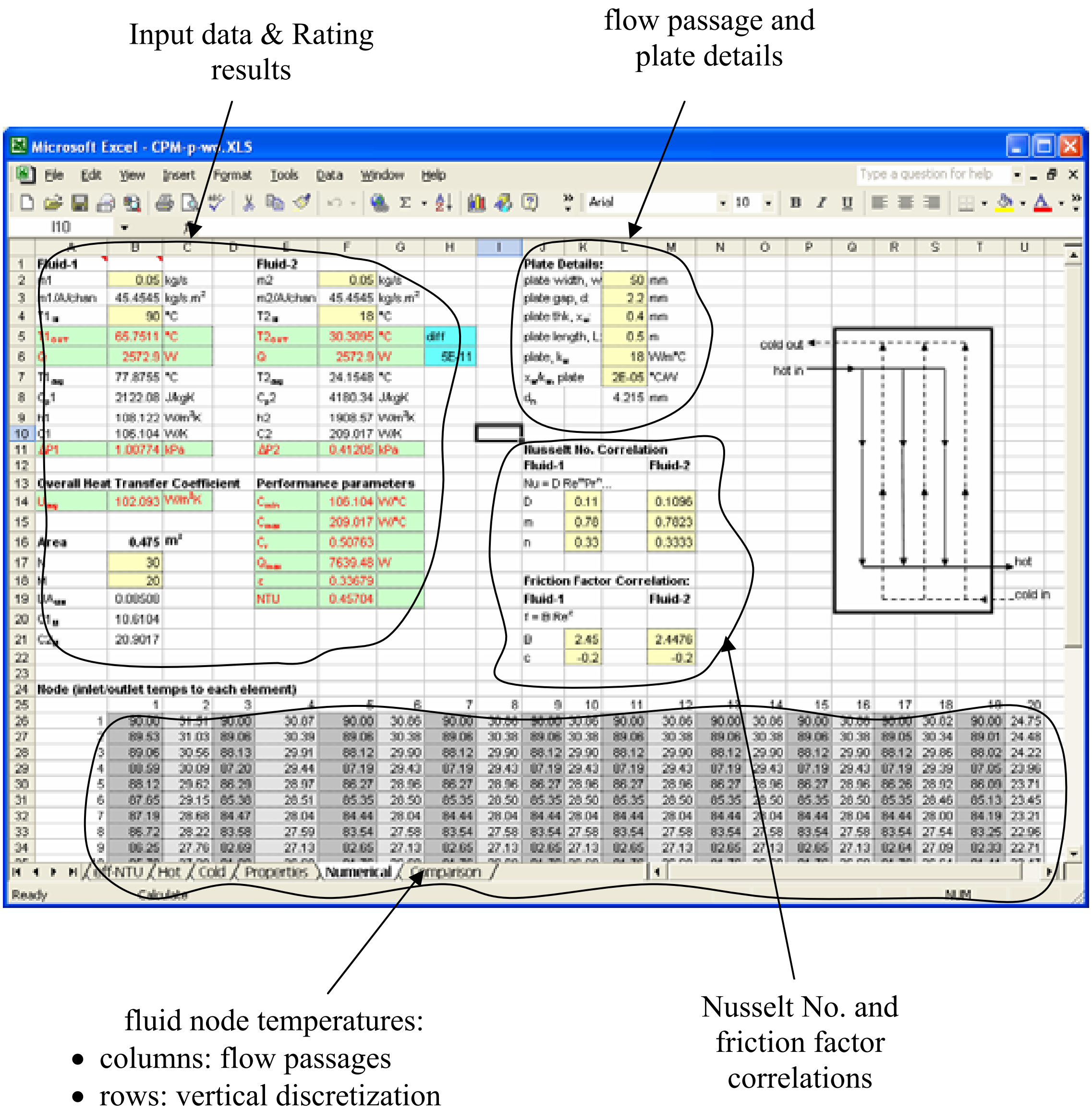
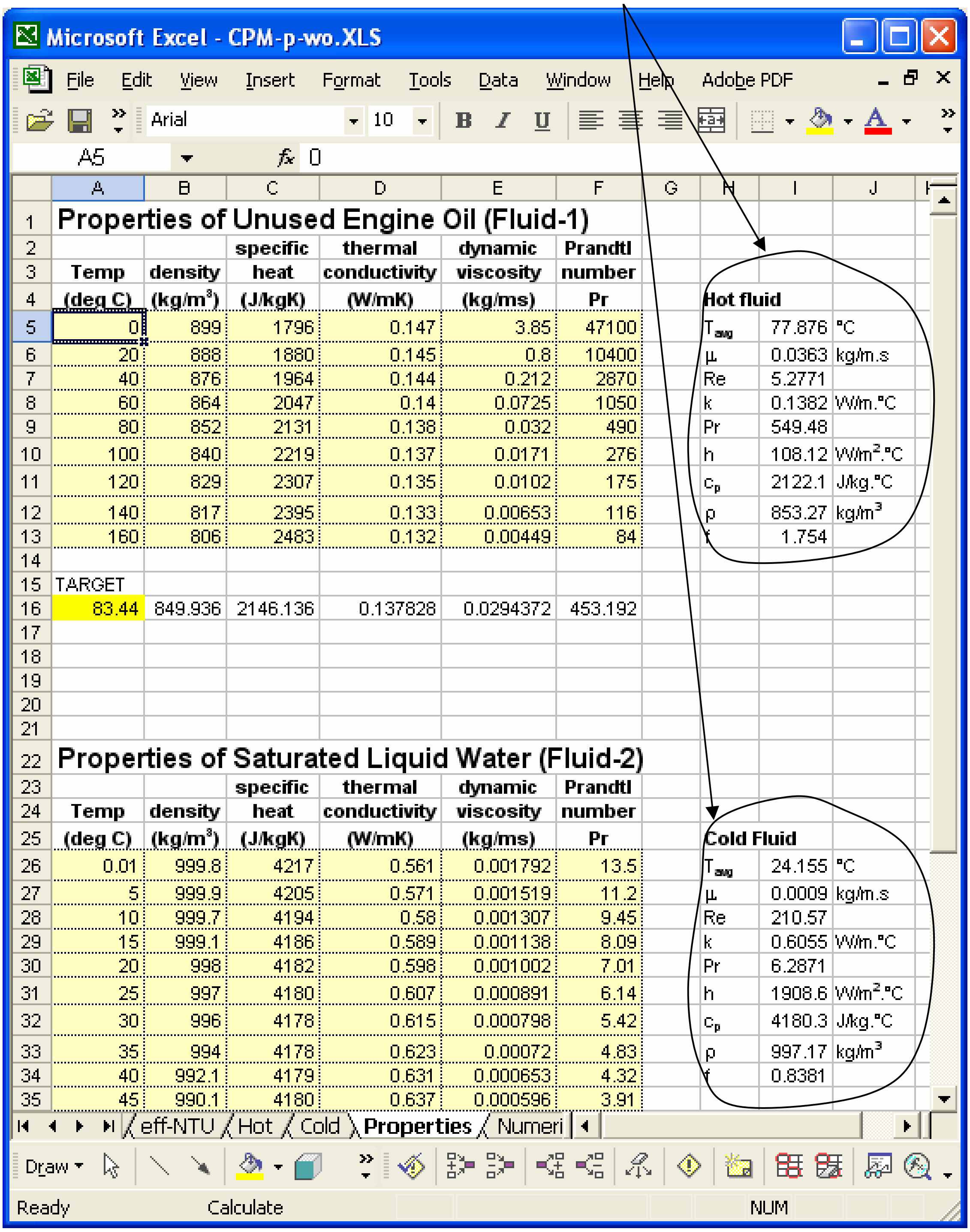


Figure 2.10 Spreadsheet implementation of second constant-properties-model, CPM

Fluid properties and parameters calculated at the average fluid temperature for the fluid



Microsoft Excel - CPM-p-wo.XLS

File Edit View Insert Format Tools Data Window Help Adobe PDF

Properties of Unused Engine Oil (Fluid-1)

Temp (deg C)	density (kg/m³)	specific heat (J/kgK)	thermal conductivity (W/mK)	dynamic viscosity (kg/ms)	Prandtl number Pr
0	899	1796	0.147	3.85	47100
20	888	1880	0.145	0.8	10400
40	876	1964	0.144	0.212	2870
60	864	2047	0.14	0.0725	1050
80	852	2131	0.138	0.032	490
100	840	2219	0.137	0.0171	276
120	829	2307	0.135	0.0102	175
140	817	2395	0.133	0.00653	116
160	806	2483	0.132	0.00449	84
TARGET					
83.44	849.936	2146.136	0.137828	0.0294372	453.192

Hot fluid		
T_{avg}	77.876	°C
μ	0.0363	kg/m.s
Re	5.2771	
k	0.1382	W/m.°C
Pr	549.48	
h	108.12	W/m².°C
c_p	2122.1	J/kg.°C
ρ	853.27	kg/m³
f	1.754	

Properties of Saturated Liquid Water (Fluid-2)

Temp (deg C)	density (kg/m³)	specific heat (J/kgK)	thermal conductivity (W/mK)	dynamic viscosity (kg/ms)	Prandtl number Pr
0.01	999.8	4217	0.561	0.001792	13.5
5	999.9	4205	0.571	0.001519	11.2
10	999.7	4194	0.58	0.001307	9.45
15	999.1	4186	0.589	0.001138	8.09
20	998	4182	0.598	0.001002	7.01
25	997	4180	0.607	0.000891	6.14
30	996	4178	0.615	0.000798	5.42
35	994	4178	0.623	0.00072	4.83
40	992.1	4179	0.631	0.000653	4.32
45	990.1	4180	0.637	0.000596	3.91

Cold Fluid		
T_{avg}	24.155	°C
μ	0.0009	kg/m.s
Re	210.57	
k	0.6055	W/m.°C
Pr	6.2871	
h	1908.6	W/m².°C
c_p	4180.3	J/kg.°C
ρ	997.17	kg/m³
f	0.8381	

eff-NTU / Hot / Cold / Properties / Numeri

Ready Calculate NUM

Figure 2.11 Property data on spreadsheet

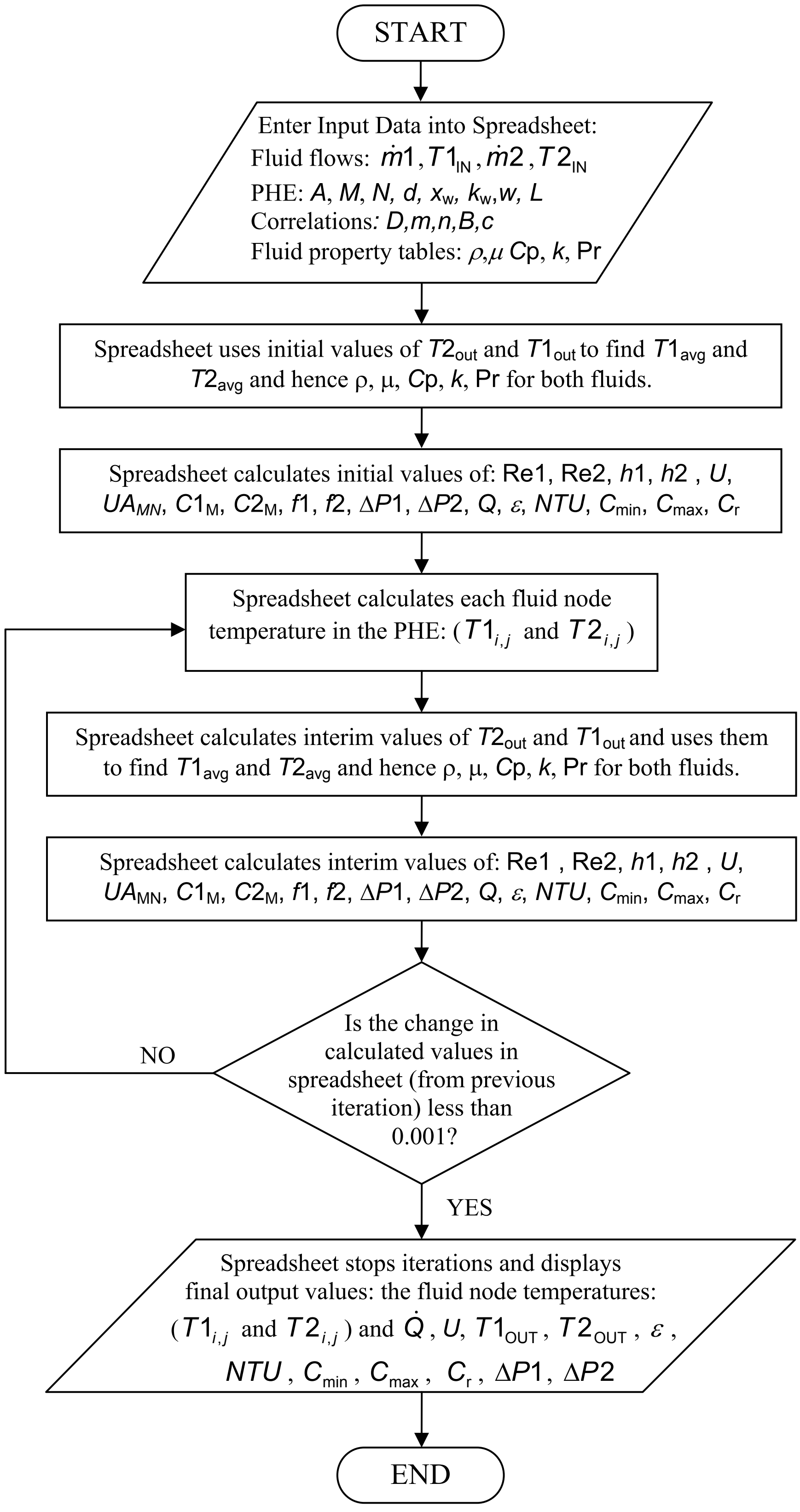


Figure 2.12 Flow chart of spreadsheet operation, second constant-properties-model, CPM

2.3.7 Validation/Results

The basis of the validation of the second CPM is, like the CPM1, to compare the rating results or outputs from the CPM with those achieved using the standard $\varepsilon - NTU$ method.

Again the validation is based on the PHE in the heat transfer rig at AUT. Dummy data is entered into the CPM to represent a range of flow rates and conditions. This same dummy data is used to generate results using the $\varepsilon - NTU$ method. In this case the properties of water and engine oil are entered into the Property sheet of the CPM. The properties of water and oil are obtained from a standard heat transfer text [2].

The results are summarised in Tables 2.4 and 2.5 which show a range of different flow rates for hot (fluid-1) and cold (fluid-2) fluids and the resulting outlet temperatures and performance parameters determined using the CPM and the $\varepsilon - NTU$ method. Figures 2.13 and 2.14 show fluid outlet temperatures determined using the CPM compared to the outlet temperatures determined using the $\varepsilon - NTU$ method.

Again, good agreement is achieved between the results of the CPM and the results of the $\varepsilon - NTU$ method. The maximum percentage error for these results is no more than 1 to 2 %.

This second CPM incorporates a number of improvements over the initial CPM1 as outlined at the beginning of section 2.3. However, as discussed in Chapter 1, it is noticed in the literature that there are very few design or rating calculations for PHE's that allow for variable fluid properties through the heat exchanger. Most authors consider the fact that most relevant fluid properties do not vary that greatly with temperature and so the assumption of constant fluid properties is probably a valid one. However some fluid properties, namely viscosity, do vary significantly with temperature for some fluids.

The spreadsheet solution lends itself to the calculation of fluid properties for each cell in the discretized heat exchanger so the next step in the development of the spreadsheet model is to modify it to allow for the variation of ALL fluid properties through the heat exchanger.

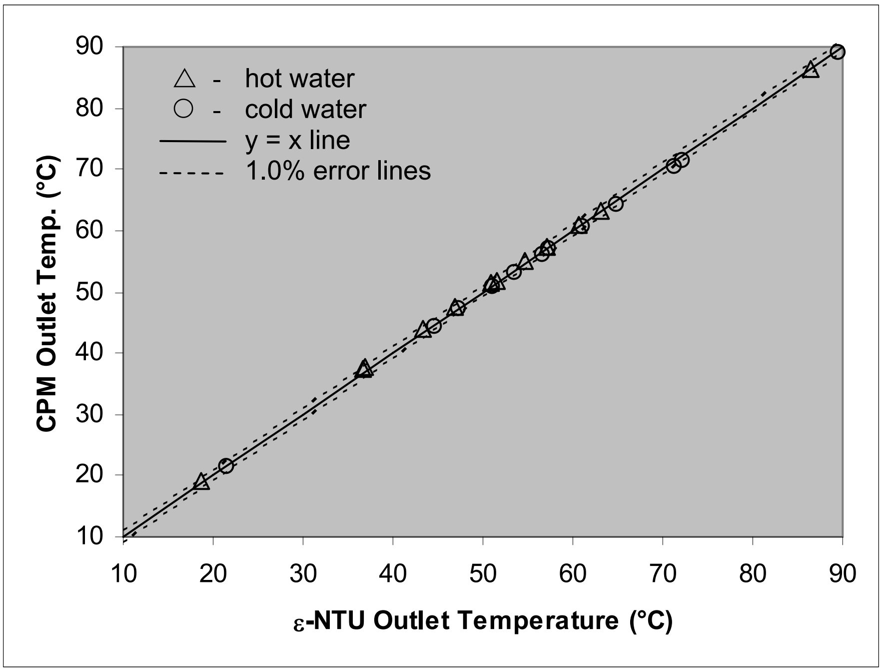


Figure 2.14 Comparison between the second CPM and $\varepsilon - NTU$ results for water versus water in a PHE

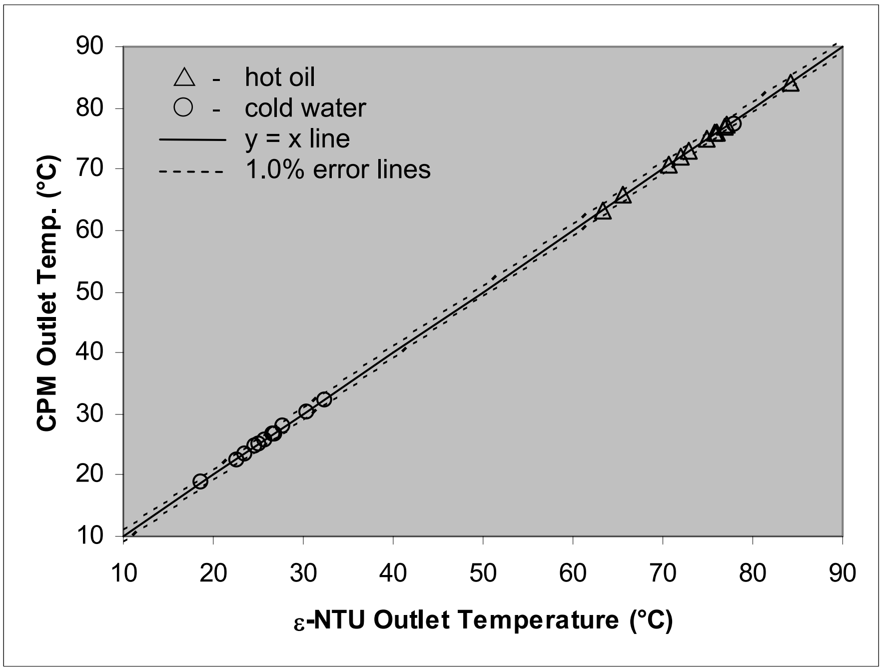


Figure 2.13 Comparison between the second CPM and $\varepsilon - NTU$ results for water versus oil in a PHE

Table 2.4 Validation of CPM: water versus water results

m1 (kg/s) (input) (hot)	0.05	1	0.05	0.25	0.5	1	0.5	1.5	2	1.5	1	2
m2 (kg/s) (input) (cold)	0.05	0.05	1	0.25	0.5	0.5	1	1.5	1.5	2	1	2
$T1_{in}$ (°C) (input)	90	90	90	90	90	90	90	90	90	90	90	90
$T1_{out}$(CPM) (°C)	**37.69**	**86.46**	**19.04**	**43.96**	**47.54**	**63.34**	**37.33**	**54.98**	**60.95**	**51.34**	**51.95**	**57.35**
$T1_{out}$(ε-NTU) (°C)	**36.85**	**86.44**	**18.63**	**43.29**	**46.97**	**63.00**	**36.67**	**54.59**	**60.63**	**50.92**	**51.49**	**57.01**
ΔT1%	**2.30**	**0.02**	**2.21**	**1.53**	**1.21**	**0.54**	**1.81**	**0.72**	**0.52**	**0.83**	**0.89**	**0.60**
ΔP1 (kPA)	**0.36**	**75.77**	**0.37**	**6.51**	**22.59**	**77.36**	**22.85**	**161.85**	**269.93**	**162.45**	**78.26**	**270.94**
$T2_{in}$ (°C) (input)	18	18	18	18	18	18	18	18	18	18	18	18
$T2_{out}$(CPM) (°C)	**70.39**	**89.11**	**21.55**	**64.14**	**60.57**	**71.51**	**44.39**	**53.13**	**56.88**	**47.08**	**56.16**	**50.76**
$T2_{out}$(ε-NTU) (°C)	**71.24**	**89.49**	**21.57**	**64.81**	**61.14**	**72.19**	**44.72**	**53.53**	**57.30**	**47.40**	**56.62**	**51.11**
ΔT2%	**1.19**	**0.42**	**0.10**	**1.03**	**0.93**	**0.94**	**0.74**	**0.74**	**0.74**	**0.67**	**0.82**	**0.67**
ΔP2 (kPA)	**0.38**	**0.37**	**92.24**	**6.99**	**24.50**	**24.06**	**87.83**	**179.26**	**178.11**	**304.27**	**85.97**	**302.15**
U (CPM) (W/m^2°C)	1245	2272	2170	4078	6561	8129	7825	12830	13850	13769	10162	14951
NTU	2.8315	5.1615	4.9301	1.8544	1.4917	1.8475	1.7759	0.9724	1.0497	1.0406	1.1553	0.8499
C_r	0.9984	0.0497	0.0500	0.9979	0.9975	0.4983	0.5010	0.9968	0.7473	0.7522	0.9971	0.9966
ε (CPM)	0.7276	0.9877	0.9855	0.6409	0.5912	0.7432	0.7315	0.4879	0.5400	0.5369	0.5300	0.4550
ε (ε-NTU)	0.7394	0.9930	0.9912	0.6501	0.5991	0.7527	0.7407	0.4934	0.5459	0.5428	0.5364	0.4598
Q (CPM) (W)	**10949**	**14871**	**14839**	**48211**	**88939**	**111837**	**110253**	**220169**	**243673**	**242984**	**159456**	**273741**
Q (e-NTU) (W)	**11126**	**14950**	**14925**	**48905**	**90128**	**113259**	**111640**	**222640**	**246337**	**245633**	**161386**	**276622**
ΔQ%	**1.60**	**0.53**	**0.58**	**1.42**	**1.32**	**1.26**	**1.24**	**1.11**	**1.08**	**1.08**	**1.20**	**1.04**

Table 2.5 Validation of CPM: water versus oil results

m1 (kg/s) (input) - hot oil	0.05	1	0.05	0.25	0.5	1	0.5	1.5	2	1.5	1	2
m2 (kg/s) (input) - cold water	0.05	0.05	1	0.25	0.5	0.5	1	1.5	1.5	2	1	2
$T1_{in}$ (°C) (input)	90	90	90	90	90	90	90	90	90	90	90	90
$T1_{out}$(CPM) (°C)	**65.75**	**84.26**	**63.37**	**70.76**	**72.93**	**76.09**	**72.11**	**76.09**	**77.21**	**75.80**	**74.96**	**76.88**
$T1_{out}$(ε-NTU) (°C)	**65.60**	**84.20**	**63.24**	**70.67**	**72.85**	**76.03**	**72.04**	**76.04**	**77.17**	**75.75**	**74.90**	**76.84**
ΔT1%	**0.23**	**0.08**	**0.21**	**0.13**	**0.10**	**0.09**	**0.10**	**0.07**	**0.06**	**0.06**	**0.08**	**0.06**
ΔP1 (kPA)	**1.01**	**209.34**	**1.02**	**17.79**	**61.68**	**213.30**	**61.78**	**442.48**	**740.78**	**442.76**	**213.82**	**741.32**
$T2_{in}$ (°C) (input)	18	18	18	18	18	18	18	18	18	18	18	18
$T2_{out}$(CPM) (°C)	**30.31**	**77.35**	**18.67**	**27.81**	**26.73**	**32.27**	**22.57**	**25.13**	**26.75**	**23.46**	**25.71**	**24.73**
$T2_{out}$(ε-NTU) (°C)	**30.39**	**78.01**	**18.68**	**27.86**	**26.77**	**32.34**	**22.59**	**25.16**	**26.79**	**23.48**	**25.74**	**24.76**
ΔT2%	**0.25**	**0.84**	**0.02**	**0.17**	**0.14**	**0.21**	**0.08**	**0.10**	**0.12**	**0.08**	**0.12**	**0.09**
ΔP2 (kPA)	**0.41**	**0.38**	**92.92**	**7.51**	**26.19**	**25.86**	**92.02**	**189.84**	**189.16**	**319.76**	**91.40**	**318.88**
U (CPM) (W/m^2°C)	102	832	104	378	654	1090	667	1539	1896	1554	1125	1919
NTU	0.4570	1.8911	0.4672	0.3376	0.2909	0.2478	0.2969	0.2274	0.2098	0.2297	0.2495	0.2124
C_r	0.5076	0.0967	0.0253	0.5101	0.5112	0.9746	0.2553	0.5128	0.6846	0.3845	0.5123	0.5133
ε (CPM)	0.3368	0.8243	0.3699	0.2672	0.2371	0.1982	0.2484	0.1931	0.1776	0.1972	0.2089	0.1822
ε (ε-NTU)	0.3389	0.8334	0.3717	0.2685	0.2382	0.1991	0.2494	0.1938	0.1782	0.1979	0.2097	0.1828
Q (CPM) (W)	**2572**	**12406**	**2819**	**10257**	**18246**	**29819**	**19099**	**44729**	**54900**	**45659**	**32216**	**56316**
Q (e-NTU) (W)	**2588**	**12542**	**2833**	**10307**	**18326**	**29960**	**19174**	**44890**	**55101**	**45814**	**32341**	**56507**
ΔQ%	**0.61**	**1.09**	**0.49**	**0.49**	**0.44**	**0.47**	**0.39**	**0.36**	**0.36**	**0.34**	**0.39**	**0.34**

CHAPTER 3

VARIABLE PROPERTIES MODEL

3.1 Introduction

The constant-properties-model (CPM) works well under the assumption of constant fluid properties through the plate heat exchanger (PHE). In reality however, the fluid properties do change as the temperature of the fluid changes through the heat exchanger. One of the main goals of this thesis is to develop a model which allows for this variation in fluid properties through the heat exchanger. This variable-properties-model (VPM) will therefore provide more accurate and realistic results.

Also, the purpose of the VPM is one of education, to improve the student's knowledge and understanding of how a PHE works. So this VPM, like the CPM, will *rate* an existing PHE (rather than *design* a new PHE) and so will give insight to the student on the effects of variable properties when rating or analysing the performance of a PHE.

This VPM will not only be closer to reality but by comparing the results of this model with the CPM, it will allow the student to discover whether the assumption of constant fluid properties is valid or not, or perhaps in what situations it is valid or not valid. A number of fluids, notably oils and syrups, show a marked variation in fluid properties with temperature, so this model will have valuable application in industries (food, oil, petrochemical etc) dealing with heat transfer in such fluids.

The VPM presented in this chapter allows for the variation of fluid properties with temperature through the heat exchanger. In the latter part of this chapter refinements are made to this VPM by including a viscosity correction factor in the model that makes adjustments to the convection heat transfer coefficient as a result of the variation in fluid viscosity between the wall and bulk fluid in each element through the PHE. The initial variable properties model is referred to as VPM1 and the refined variable properties model as VPM.

3.2 Initial Variable-Properties-Model (VPM1) Formulation

The VPM1 uses the same discretization scheme as the CPM as described in section 2.2.1, whereby the PHE is divided up into a number of small constant volume elements. Again versions of the VPM1 are developed for the three common flow configurations: parallel, series and combination. Generally the explanations and diagrams used throughout this chapter relate to a parallel flow configuration as a general case. Where distinction between the different configurations is required then additional explanation and diagrams are provided.

The variation of fluid properties through the heat exchanger is allowed for by calculating the fluid properties at the average temperature of each element. The heat balance and heat transfer equations are also applied element by element through the heat exchanger.

3.2.1 Assumptions

In the development of the equations for the VPM1 the following assumptions are made:

- The fluid properties change from element to element through the PHE as the temperature of the fluid in each element varies. It is assumed that the fluid properties remain constant, evaluated at the average fluid temperature, *within* each element.
- The convection heat transfer coefficient and friction factor change from element to element through the PHE as the temperature and fluid properties of each element varies. It is assumed that the convection heat transfer coefficient and friction factor remain constant, evaluated at the average fluid temperature, *within* each element.
- The use of a simple average temperature difference in the heat transfer equation, rather than the log mean temperature difference, for each element. (As previously discussed, this assumption is implemented to give the student insight into the fact that for small temperature differences there is little difference between the log-mean temperature difference and the average temperature difference. Also, using a simple average temperature difference greatly simplifies the setting up of the numerical solution used in this model).
- No heat loss to the environment.
- Steady state conditions.

- Element volumes are constant through the heat exchanger.
- No axial conduction.
- Plug flow in the flow channels
- No change of phase (both fluids remain as liquids throughout the PHE)

Furthermore, for a parallel flow configuration:

- The inlet temperature to each flow passage for a fluid is the same as the inlet temperature to the heat exchanger for that fluid.
- The mass flow rate of each fluid entering the PHE divides equally between the available flow passages for each fluid.
- Perfect mixing of the fluid from the outlets of the flow passages as they recombine to leave the PHE in a single flow stream.

Also, to simplify the models and number of equations derived, it is assumed that fluid-1 is the *hot* fluid and fluid-2 is the *cold* fluid. It is found that in the utilisation of the models if this restriction is not adhered to then, as with the CPM, the only consequence is that the heat flows are displayed as negative numbers rather than positive ones.

3.2.2 Fluid Properties as a Function of Temperature

In the VPM1 the fluid properties are determined at the *average fluid temperature in each element* in the heat exchanger. Referring to Figure 3.1, if fluid-1 flows down the flow passage then it enters an element i,j in the PHE at a temperature of $T1_{i,j}$ and leaves at $T1_{i,j+1}$ and so the average fluid temperature for that element is:

$$T1_{i,j\,\mathrm{avg}} = \frac{T1_{i,j} + T1_{i,j+1}}{2} \tag{3.1}$$

For fluid-2, if it is flowing up the flow passage, then it enters an element i,j in the PHE at a temperature of $T2_{i,j+1}$ and leaves at $T2_{i,j}$ and the average fluid temperature for the element is:

$$T2_{i,j\,\mathrm{avg}} = \frac{T2_{i,j+1} + T2_{i,j}}{2} \tag{3.2}$$

Initially, the outlet temperature of each element is not known and the model relies on the iteration capabilities of the spreadsheet to converge towards the correct values of $T1_{i,j+1}$ and $T2_{i,j}$ for each element.

The calculation of the fluid properties at the *average element temperature* is achieved using tables of fluid property data and interpolation, as used in the CPM. If the value of the property P, is required at temperature T_{AVG} and $T_k < T_{\text{AVG}} < T_{k+1}$ in a property table (Table 2.3), then linear interpolation gives the value of P as:

$$P = P_k + \frac{\left(T_{\text{AVG}} - T_k\right)}{\left(T_{k+1} - T_k\right)}\left(P_{k+1} - P_k\right) \tag{3.3}$$

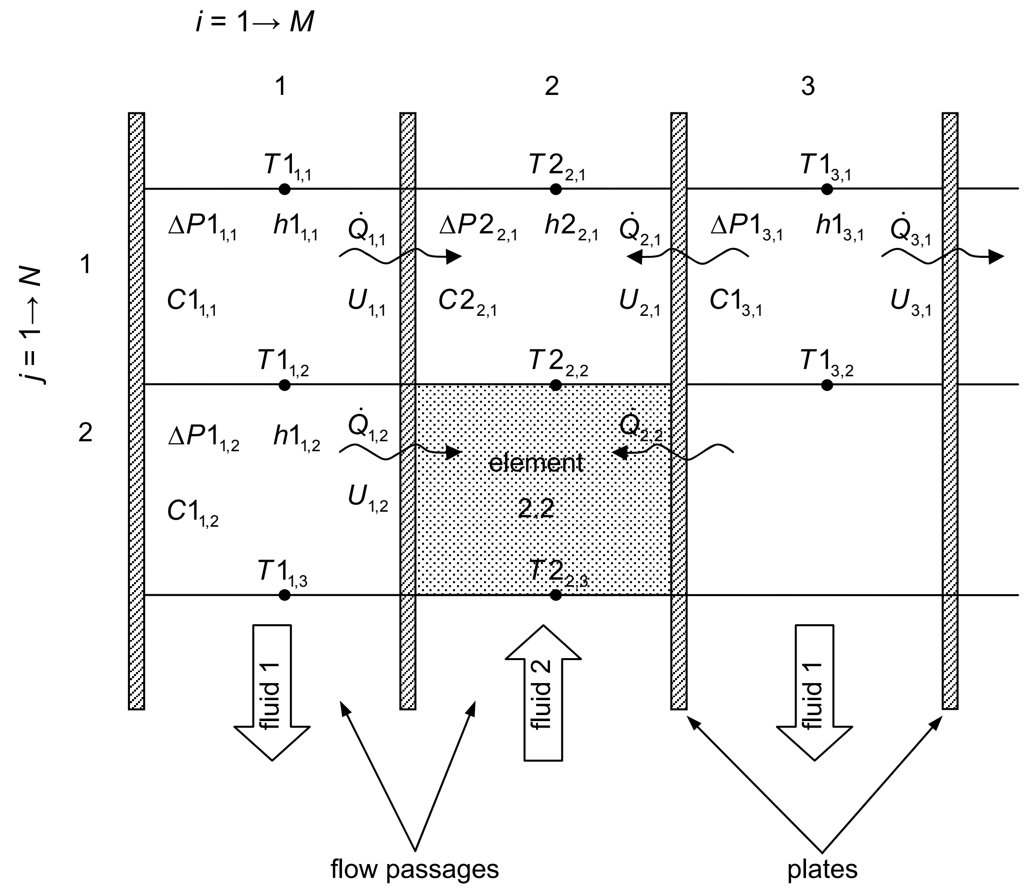


Figure 3.1 Discretization – variable fluid properties and parameters

3.2.3 Convection and Overall Heat Transfer Coefficients

In the VPM1 the convection and overall heat transfer coefficients are also calculated for each element through the PHE. The general form of the correlation for the Nusselt number is applied to each element, i, j in the discretized PHE to give:

$$\mathrm{Nu}_{i,j} = D\mathrm{Re}_{i,j}^{m}\,\mathrm{Pr}_{i,j}^{n} \tag{3.4}$$

As previously discussed, D, m and n are considered to be constants. Although the values of D, m and n are dependent on the physical characteristics of the particular PHE which remain unchanged for a specific PHE, they also depend on the flow conditions which may vary through the PHE. Most empirical correlations published in the literature [28-30] are valid over a stated range of Reynolds numbers and obviously it is possible in a variable properties scenario for the flow conditions to change such that in one part of the PHE the correlation is valid, whereas in another part the correlation is "invalid". Invalid meaning that the errors involved in its use are larger. It should be noted however that there are literally dozens of different correlations in the literature [28-30], as is pointed out in Chapter 1, and Muley and Manglik [28, p.110] note "The predictions from most of these equations are observed to disagree considerably with each other and present rather a wide performance envelope". So, in the VPM1 it is assumed that the coefficients of the Nusselt number correlation remain constant throughout the heat exchanger, even if flow conditions do vary. However they can be set separately for each fluid and in this way if one fluid is water-like and the other fluid is significantly different from water (eg oil or thick syrup etc) then appropriate correlation coefficients can be set for each fluid.

The Reynolds number and the Prandtl number, used in the Nusselt number correlation are calculated for each element, using the fluid properties determined at the average element temperature ($T1_{i,j\,\mathrm{avg}}$ or $T2_{i,j\,\mathrm{avg}}$).

The convection heat transfer coefficient between the walls and the fluid in each element is $h_{i,j}$ and is determined from the definition of the Nusselt number (equation 2.33) to give:

$$h_{i,i} = \frac{\mathrm{Nu}_{i,j}\, k_{i,j}}{d_{\mathrm{h}}} \tag{3.5}$$

So for fluid-1 the convection heat transfer coefficient is $h1_{i,j}$ and for fluid-2 it is $h2_{i,j}$.

When considering the heat transfer between adjacent elements i, j and $i+1, j$ (Figure 3.1) the heat transferred is $\dot{Q}_{i,j}$ and the overall heat transfer coefficient, $U_{i,j}$, is given by:

$$U_{i,j} = \left(\frac{1}{h_{i,j}} + \frac{x_{\mathrm{w}}}{k_{\mathrm{w}}} + \frac{1}{h_{i+1,j}} \right)^{-1} \tag{3.6}$$

3.2.4 Friction Factor and Pressure Drop

In this model the general form of the correlation for the friction factor is applied to each element, i, j in the discretized PHE to give:

$$f_{i,j} = B\mathrm{Re}_{i,j}^{c} \tag{3.7}$$

As previously discussed the constants B and c are assumed to remain constant through the heat exchanger and are dependent on the physical characteristics of the PHE. As discussed above for the Nusselt number correlation the coefficients B and c can vary with flow conditions however they are assumed to remain constant throughout the VPM1. Like the Nusselt number correlations they can be set separately for each fluid.

Now, applying equations (2.48) and (2.49) to an element i, j in the discretized PHE the pressure drop across the element for fluid-1, $\Delta P1_{i,j}$ is

$$\Delta P1_{i,j} = f1_{i,j} \frac{2\frac{L}{N}\left[\dot{m}1/(MA_{\mathrm{c}})\right]^{2}}{\rho 1_{i,j}\, d_{\mathrm{h}}} \tag{3.8}$$

and the pressure drop across an element for fluid-2, $\Delta P2_{i,j}$ is:

$$\Delta P2_{i,j} = f2_{i,j}\frac{2\frac{L}{N}\left[\dot{m}2/(MA_{c})\right]^{2}}{\rho 2_{i,j}\, d_{h}} \tag{3.9}$$

For a parallel flow configuration, the pressure drop for each fluid stream through the heat exchanger can be determined by summing the element pressure drops in each flow channel and averaging across the flow channels. So for fluid-1:

$$\Delta P1 = \frac{\sum_{i=1,3\ldots}^{M-1}\sum_{j=1}^{N}\Delta P1_{i,j}}{M/2} \qquad \text{(parallel flow configuration)} \tag{3.10}$$

and similarly for fluid-2:

$$\Delta P2 = \frac{\sum_{i=2,4\ldots}^{M}\sum_{j=1}^{N}\Delta P2_{i,j}}{M/2} \qquad \text{(parallel flow configuration)} \tag{3.11}$$

For a series flow configuration the pressure drop for each fluid stream through the heat exchanger can be determined by summing the element pressure drops in each flow channel then summing all the flow channel pressure drops together, so for fluid-1:

$$\Delta P1 = \sum_{i=1,3\ldots}^{M-1}\sum_{j=1}^{N}\Delta P1_{i,j} \qquad \text{(series flow configuration)} \tag{3.12}$$

and similarly for fluid-2:

$$\Delta P2 = \sum_{i=2,4\ldots}^{M}\sum_{j=1}^{N}\Delta P2_{i,j} \qquad \text{(series flow configuration)} \tag{3.13}$$

The overall pressure drop in a PHE from inlet pipe to outlet pipe is made up of four components: the core pressure drop in the flow passages (as per equations (3.10) – (3.13)), the pressure drop in the internal distribution manifolds, the pressure drop due to elevation change and the pressure drop in the short manifold pipes that connect to the rest of the plant pipe work. For the purposes of this thesis, as previously discussed, only the core pressure drops will be considered.

3.2.5 Fluid Node Temperatures

As discussed previously, the PHE consists of $M+1$ plates of which $M-1$ are involved in heat transfer. If each plate in the PHE has a length L and width w (between the gaskets) and heat transfer area of $A_\mathrm{p} = Lw$, then the total heat transfer, A is given by:

$$A = (M-1)A_\mathrm{p} \tag{3.14}$$

And, if each plate is discretized into N elements then the heat transfer area of a single element, A_{MN} (transferring heat with one other element), is given by:

$$A_{MN} = \frac{A_\mathrm{p}}{N} = \frac{A}{(M-1)N} \tag{3.15}$$

Now, for a parallel flow configuration, let fluid-1 enter the top of the PHE with a mass flow rate of $\dot{m}1$ and inlet temperature $T1_\mathrm{IN}$. Assuming the flow divides evenly between the $M/2$ flow passages (which, as discussed previously, is assumed to be all the odd numbered passages: 1, 3, 5 … $(M-1)$), it enters the first element of each passage with a mass flow rate of $\dot{m}1/(M/2)$.

The fluid-1 properties for each element are determined at the average fluid temperature for that element, $T1_{i,j\,\mathrm{avg}}$. So the heat capacity rate, $C1_{i,j}$ for any element i,j with fluid-1 of heat capacity $C_\mathrm{p}1_{i,j}$ is given by:

$$C1_{i,j} = C_\mathrm{p}1_{i,j}\frac{\dot{m}1}{M/2} \qquad \text{(parallel flow configuration)} \tag{3.16}$$

Similarly, fluid-2 enters the bottom of the PHE and so enters the last element of the all the even numbered flow passages with a mass flow rate $\dot{m}2/(M/2)$ and a temperature of $T2_\mathrm{IN}$. Like wise the heat capacity rate, $C2_{i,j}$ for any element i,j with fluid-2 of heat capacity $C_\mathrm{p}2_{i,j}$ is given by:

$$C2_{i,j} = C_\mathrm{p}2_{i,j}\frac{\dot{m}2}{M/2} \qquad \text{(parallel flow configuration)} \tag{3.17}$$

In the case of a series flow configuration, as fluid-1 enters the PHE it does not divide up, rather it *all* flows in a series fashion along the odd numbered passages with a mass flow rate of $\dot{m}1$. In this case the heat capacity rate $C1_{i,j}$ for any element i,j with fluid-1 of heat capacity $C_p1_{i,j}$ is given by.

$$C1_{i,j} = C_p1_{i,j}\dot{m}1 \qquad \text{(series flow configuration)} \qquad (3.18)$$

Similarly, for fluid-2 in a series flow configuration, the mass flow rate through each even numbered flow channel is $\dot{m}2$ and the heat capacity rate $C2_{i,j}$ for any element i,j with fluid-2 of heat capacity $C_p2_{i,j}$ is given by:.

$$C2_{i,j} = C_p2_{i,j}\dot{m}2 \qquad \text{(series flow configuration)} \qquad (3.19)$$

As explained for the CPM, in a PHE any one flow passage is exchanging heat with the two flow passages on either side of it. However, in the left-most flow passage and in the right-most flow passage, heat is being exchanged with only one other flow passage. For this reason different equations are developed to calculate the node temperatures for the first flow passage and for the last flow passage as well as different equations for fluid-1 and for fluid-2 in the remaining flow passages.

Consider a small element adjacent to the left boundary (flow passage 1, element $1,j$) in the PHE (see Figure 3.1). If fluid of heat capacity rate $C1_{1,j}$ is flowing down this flow passage it enters the element at temperature $T1_{1,j}$ and leaves at temperature $T1_{1,j+1}$ so that the heat flow, $\dot{Q}_{1,j}$, from that element is given by:

$$\dot{Q}_{1,j} = C1_{1,j}\left(T1_{1,j} - T1_{1,j+1}\right) \qquad (3.20)$$

Assuming no axial conduction, this heat transfers from element $1,j$ across the plate to the adjacent element, $2,j$. The overall heat transfer coefficient between elements $1,j$ and $2,j$ is $U_{1,j}$, and if the element size is small then the heat transfer equation can be written using a simple average temperature difference rather than the log-mean temperature difference, with little error:

$$\dot{Q}_{1,j} = U_{1,j} A_{MN}\left(\frac{T1_{1,j} + T1_{1,j+1}}{2} - \frac{T2_{2,j} + T2_{2,j+1}}{2}\right) \tag{3.21}$$

From equations (3.20) and (3.21) it can be shown that the outlet temperature, $T1_{1,j+1}$ from the element is given by:

$$T1_{1,j+1} = \frac{T1_{1,j}\left(C1_{1,j} - \frac{U_{1,j} A_{MN}}{2}\right) + \frac{U_{1,j} A_{MN}}{2}\left(T2_{2,j} + T2_{2,j+1}\right)}{\left(C1_{1,j} + \frac{U_{1,j} A_{MN}}{2}\right)} \tag{3.22}$$

Now, consider a small element adjacent to the right boundary (flow passage M, element M, j) in the PHE (see Figure 3.1). Fluid of heat capacity rate $C2_{M,j}$ flows up this flow passage and enters the element at temperature $T2_{M,j+1}$ and leaves at temperature $T2_{M,j}$ so that the heat flow, $\dot{Q}_{M-1,j}$, to that element is given by:

$$\dot{Q}_{M-1,j} = C2_{M,j}\left(T2_{M,j} - T2_{M,j+1}\right) \tag{3.23}$$

The overall heat transfer coefficient between elements M, j and $M-1, j$ is $U_{M-1,j}$ and using a simple average temperature difference the heat transfer equation between element M, j across the plate to the adjacent element, $M-1, j$ can be written:

$$\dot{Q}_{M-1,j} = U_{M-1,j} A_{MN}\left(\frac{T1_{M-1,j} + T1_{M-1,j+1}}{2} - \frac{T2_{M,j} + T2_{M,j+1}}{2}\right) \tag{3.24}$$

From equations (3.23) and (3.24) it can be shown that the outlet temperature, $T2_{M,j}$ from the element is given by:

$$T2_{M,j} = \frac{T2_{M,j+1}\left(C2_{M,j} - \frac{U_{M-1,j} A_{MN}}{2}\right) + \frac{U_{M-1,j} A_{MN}}{2}\left(T1_{M-1,j} + T1_{M-1,j+1}\right)}{\left(C2_{M,j} + \frac{U_{M-1,j} A_{MN}}{2}\right)} \tag{3.25}$$

Now, consider an element located at position *i,j* for fluid-1. Fluid of heat capacity rate $C1_{i,j}$ flows down the flow passage and enters the element at temperature $T1_{i,j}$ and leaves at temperature $T1_{i,j+1}$. This element is exchanging heat with the two adjacent elements and so the total heat flow from that element is equal to the sum of $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$ and is given by:

$$\dot{Q}_{i-1,j} + \dot{Q}_{i,j} = C1_{i,j}\left(T1_{i,j} - T1_{i,j+1}\right) \tag{3.26}$$

The heat transfer equations can now be written for $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$, remembering that for $\dot{Q}_{i-1,j}$ the overall heat transfer coefficient is $U_{i-1,j}$ and for $\dot{Q}_{i,j}$ the overall heat transfer coefficient is $U_{i,j}$:

$$\dot{Q}_{i-1,j} = U_{i-1,j} A_{MN}\left(\frac{T1_{i,j} + T1_{i,j+1}}{2} - \frac{T2_{i-1,j} + T2_{i-1,j+1}}{2}\right) \tag{3.27}$$

$$\dot{Q}_{i,j} = U_{i,j} A_{MN}\left(\frac{T1_{i,j} + T1_{i,j+1}}{2} - \frac{T2_{i+1,j} + T2_{i+1,j+1}}{2}\right) \tag{3.28}$$

From equations (3.26) - (3.28) it can be shown that the outlet temperature $T1_{i,j+1}$ from the element is given by:

$$T1_{i,j+1} = \frac{T1_{i,j}\left[C1_{i,j} - \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right] + \frac{U_{i-1,j}A_{MN}}{2}\left(T2_{i-1,j} + T2_{i-1,j+1}\right) + \frac{U_{i,j}A_{MN}}{2}\left(T2_{i+1,j} + T2_{i+1,j+1}\right)}{\left[C1_{i,j} + \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right]} \tag{3.29}$$

Finally, consider an element located at position i, j for fluid-2. Fluid of heat capacity rate $C2_{i,j}$ flows up this flow passage and enters the element at temperature $T2_{i,j+1}$ and leaves at temperature $T2_{i,j}$. This element is exchanging heat with the two adjacent elements and so the total heat flow to that element is equal to the sum of $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$ and is given by:

$$\dot{Q}_{i-1,j} + \dot{Q}_{i,j} = C2_{i,j}\left(T2_{i,j} - T2_{i,j+1}\right) \tag{3.30}$$

The heat transfer equation can now be written for $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$:

$$\dot{Q}_{i-1,j} = U_{i-1,j}A_{MN}\left(\frac{T1_{i-1,j} + T1_{i-1,j+1}}{2} - \frac{T2_{i,j} + T2_{i,j+1}}{2}\right) \tag{3.31}$$

$$\dot{Q}_{i,j} = U_{i,j}A_{MN}\left(\frac{T1_{i+1,j} + T1_{i+1,j+1}}{2} - \frac{T2_{i,j} + T2_{i,j+1}}{2}\right) \tag{3.32}$$

From equations (3.30) - (3.32) it can be shown that the outlet temperature $T2_{i,j}$ from the element is given by:

$$T2_{i,j} = \frac{T2_{i,j+1}\left[C2_{i,j} - \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right] + \frac{U_{i-1,j}A_{MN}}{2}\left(T1_{i-1,j} + T1_{i-1,j+1}\right) + \frac{U_{i,j}A_{MN}}{2}\left(T1_{i+1,j} + T1_{i+1,j+1}\right)}{\left[C2_{i,j} + \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right]} \tag{3.33}$$

Now, in a series flow configuration it is possible for fluid-1 to be flowing *up* some of the flow passages and for fluid-2 to be flowing *down* some of the flow passages. Following a similar derivation as outlined above it can be shown that for fluid-1 flowing *up* a flow passage the outlet temperature, $T1_{i,j}$ from an element *i,j* is given by an equation very similar to equation (3.29).

$$T1_{i,j} = \frac{T1_{i,j+1}\left[C1_{i,j} - \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right] + \frac{U_{i-1,j}A_{MN}}{2}\left(T2_{i-1,j} + T2_{i-1,j+1}\right) + \frac{U_{i,j}A_{MN}}{2}\left(T2_{i+1,j} + T2_{i+1,j+1}\right)}{\left[C1_{i,j} + \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right]} \tag{3.34}$$

Similarly for fluid-2 flowing *down* a flow passage the outlet temperature, $T2_{i,j+1}$ from an element *i,j* is given by an equation very similar to equation 3.33.

$$T2_{i,j+1} = \frac{T2_{i,j}\left[C2_{i,j} - \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right] + \frac{U_{i-1,j}A_{MN}}{2}\left(T1_{i-1,j} + T1_{i-1,j+1}\right) + \frac{U_{i,j}A_{MN}}{2}\left(T1_{i+1,j} + T1_{i+1,j+1}\right)}{\left[C2_{i,j} + \frac{A_{MN}}{2}\left(U_{i-1,j} + U_{i,j}\right)\right]} \tag{3.35}$$

3.2.6 Boundary Conditions

For a parallel flow configuration it is assumed that the inlet temperature to each flow passage is the same as the inlet temperature to the heat exchanger for that fluid, as described for the CPM, so for fluid-1 flowing *down* the PHE and fluid-2 flowing *up* the PHE, temperatures at the boundaries are given by equations (2.23) and (2.24).

For a series flow configuration the inlet temperature to the left-most and right-most flow channels is the same as the fluid-1 and fluid-2 inlet temperature respectively. And, the inlet temperatures to the remaining flow channels depend on the outlet temperature of the flow channel that immediately precedes it. This is identical to the situation for the CPM and equations (2.25), (2.27) and (2.28) apply for fluid-1 and equations (2.26), (2.29) and (2.30) apply for fluid-2.

For a parallel flow configuration, the equations developed for the temperatures at the nodes of each element (equations (3.22), (3.25), (3.29) and (3.33)) produce a set of $M \times N$ equations and with the known boundary conditions (2.23) and (2.24), there are $M \times N$ unknowns. Similarly a series or combination flow configuration will produce $M \times N$ equations with $M \times N$ unknowns.

These equations are implemented in the VPM1 spreadsheet and the iteration capabilities of the spreadsheet are used to solve the equations. The setting up of the equations in Microsoft Excel® is described in section 3.2.8

3.2.7 Rating Calculations

Once the fluid node temperatures are determined, the fluid outlet temperatures and the heat transferred in the heat exchanger can be calculated.

In the case of a parallel flow configuration, it is assumed that the fluid flow divides evenly amongst the available flow passages. As for the CPM, if perfect mixing is assumed when the fluid from the flow passages recombines into a single flow stream the fluid outlet temperatures can be determined as a simple arithmetic average of the flow for each fluid and are given by equations (2.31) and (2.32).

In the case of a series flow configuration the outlet temperature of fluid-1 and fluid-2 are the outlet temperatures from the flow passages at the extreme right and extreme left, respectively, of the PHE as for the CPM and so are given by equations (2.33) and (2.34).

Now, the total heat transferred, $\dot{Q}$ in the heat exchanger can be determined in the same way that it is determined in the CPM: the heat lost from the hot, fluid-1, equation (2.35); the heat gained by the cold, fluid-2, equation (2.36) or the sum of all the heat transferred in the elements of the heat exchanger, equation (2.37).

The overall performance parameters, NTU, ε and C_r, can be determined in the same way as in the CPM using Equations (2.38) - (2.40). These equations require values for U and C_{min} which in this VPM1 vary with temperature through the heat exchanger. Therefore the average values for these two parameters can calculated in much the same way as for the CPM. The average fluid temperatures, $T1_{AVG}$ and $T2_{AVG}$ are determined using equations (2.41) and (2.42). The relevant average fluid properties are then determined at these average temperatures and equations (2.44) - (2.46) are used to find U_{avg}. The average specific heat capacities of the two fluids are then used to determine $C_{min,avg}$.

$$C1_{\mathrm{avg}} = \dot{m}1\, C_{\mathrm{p}}1_{\mathrm{avg}} \tag{3.36}$$

and
$$C2_{\mathrm{avg}} = \dot{m}2\, C_{\mathrm{p}}2_{\mathrm{avg}} \tag{3.37}$$

$C_{\mathrm{min,\,avg}}$ is the smaller of $C1_{\mathrm{avg}}$ and $C2_{\mathrm{avg}}$. Thus overall values of the performance parameters, NTU, ε and C_{r} can be determined.

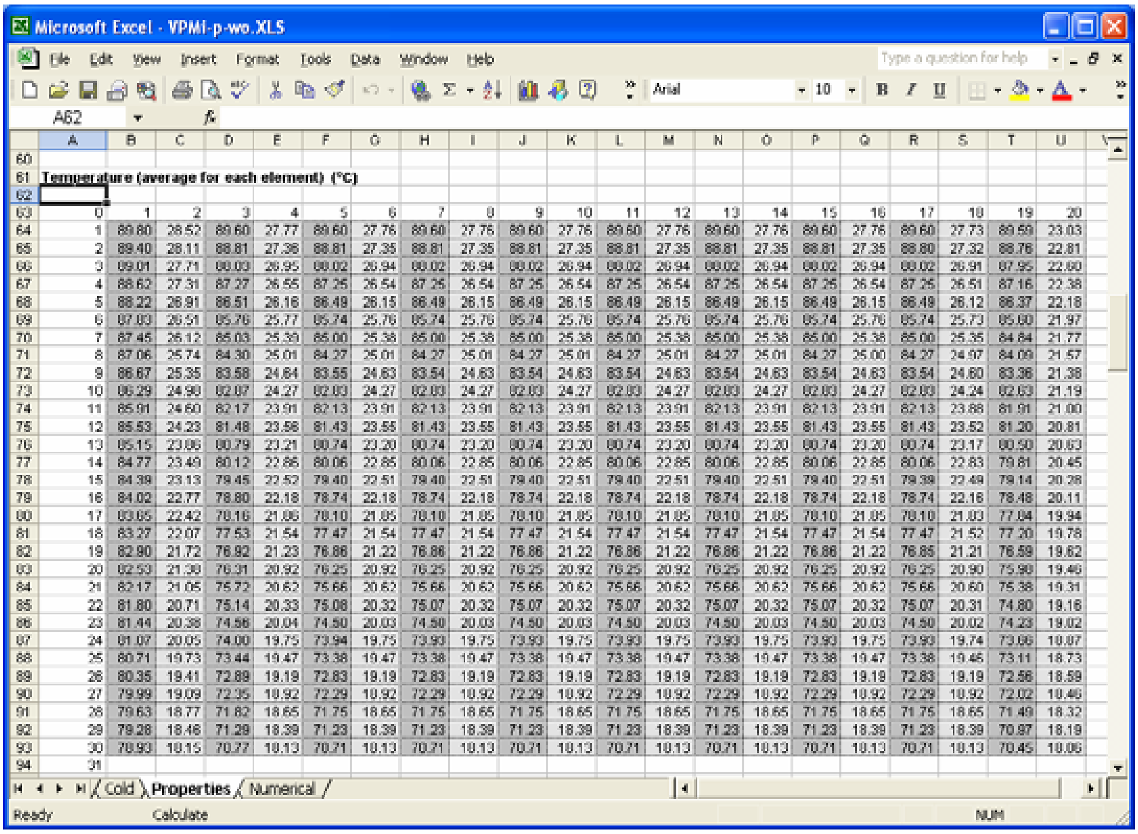
3.2.8 Spreadsheet Implementation

The equations for determining the fluid node temperatures and the equations used to rate the PHE are set up in a Microsoft Excel® spreadsheet in much the same way as for the CPM. Again, the implementation is based on the PHE in the heat transfer rig at AUT: a 21-plate PHE with a parallel flow configuration. New areas of the spreadsheet are used to implement the additional calculations and to store the various fluid properties and parameters for each element that the heat exchanger is discretized into.

The Properties sheet consists of an area for the tabular property data, as for the CPM (Figure 2.10). Below this is placed a matrix of average temperatures for each element (see Figure 3.2) and also matrices of property data (density, specific heat capacity, thermal conductivity, dynamic viscosity and Prandtl number) for each element in the discretized heat exchanger (see for example Figure 3.3, showing density data).

The Numerical calculation sheet contains the main calculation area and fluid node temperatures as in the CPM (Figure 2.10) but now includes matrices of Reynolds Number ($\mathrm{Re}_{i,j}$), convection heat transfer coefficients ($h1_{i,j}$ and $h2_{i,j}$), heat capacity rates ($C1_{i,j}$ and $C2_{i,j}$), overall heat transfer coefficients ($U_{i,j}$), element heat transfer ($Q_{i,j}$), friction factor ($f1_{i,j}$ and $f2_{i,j}$), and pressure drop ($\Delta P1_{i,j}$ and $\Delta P2_{i,j}$) for each element. Figure 3.4 shows the matrix of Reynolds numbers as an example.

The flow chart for the operation of the spreadsheet is shown in Figure 3.5.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
60																					
61	**Temperature (average for each element) (°C)**																				
62																					
63	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
64	1	89.80	28.52	89.60	27.77	89.60	27.76	89.60	27.76	89.60	27.76	89.60	27.76	89.60	27.76	89.60	27.76	89.60	27.73	89.59	23.03
65	2	89.40	28.11	88.81	27.36	88.81	27.35	88.81	27.35	88.81	27.35	88.81	27.35	88.81	27.35	88.81	27.35	88.80	27.32	88.76	22.81
66	3	89.01	27.71	88.03	26.95	88.02	26.94	88.02	26.94	88.02	26.94	88.02	26.94	88.02	26.94	88.02	26.94	88.02	26.91	87.95	22.60
67	4	88.62	27.31	87.27	26.55	87.25	26.54	87.25	26.54	87.25	26.54	87.25	26.54	87.25	26.54	87.25	26.54	87.25	26.51	87.16	22.38
68	5	88.22	26.91	86.51	26.16	86.49	26.15	86.49	26.15	86.49	26.15	86.49	26.15	86.49	26.15	86.49	26.15	86.49	26.12	86.37	22.18
69	6	87.83	26.51	85.76	25.77	85.74	25.76	85.74	25.76	85.74	25.76	85.74	25.76	85.74	25.76	85.74	25.76	85.74	25.73	85.60	21.97
70	7	87.45	26.12	85.03	25.39	85.00	25.38	85.00	25.38	85.00	25.38	85.00	25.38	85.00	25.38	85.00	25.38	85.00	25.35	84.84	21.77
71	8	87.06	25.74	84.30	25.01	84.27	25.01	84.27	25.01	84.27	25.01	84.27	25.01	84.27	25.01	84.27	25.00	84.27	24.97	84.09	21.57
72	9	86.67	25.35	83.58	24.64	83.55	24.63	83.54	24.63	83.54	24.63	83.54	24.63	83.54	24.63	83.54	24.63	83.54	24.60	83.36	21.38
73	10	86.29	24.98	82.87	24.27	82.83	24.27	82.83	24.27	82.83	24.27	82.83	24.27	82.83	24.27	82.83	24.27	82.83	24.24	82.63	21.19
74	11	85.91	24.60	82.17	23.91	82.13	23.91	82.13	23.91	82.13	23.91	82.13	23.91	82.13	23.91	82.13	23.91	82.13	23.88	81.91	21.00
75	12	85.53	24.23	81.48	23.56	81.43	23.55	81.43	23.55	81.43	23.55	81.43	23.55	81.43	23.55	81.43	23.55	81.43	23.52	81.20	20.81
76	13	85.15	23.86	80.79	23.21	80.74	23.20	80.74	23.20	80.74	23.20	80.74	23.20	80.74	23.20	80.74	23.20	80.74	23.17	80.50	20.63
77	14	84.77	23.49	80.12	22.86	80.06	22.85	80.06	22.85	80.06	22.85	80.06	22.85	80.06	22.85	80.06	22.85	80.06	22.83	79.81	20.45
78	15	84.39	23.13	79.45	22.52	79.40	22.51	79.40	22.51	79.40	22.51	79.40	22.51	79.40	22.51	79.40	22.51	79.39	22.49	79.14	20.28
79	16	84.02	22.77	78.80	22.18	78.74	22.18	78.74	22.18	78.74	22.18	78.74	22.18	78.74	22.18	78.74	22.18	78.74	22.16	78.48	20.11
80	17	83.65	22.42	78.16	21.86	78.10	21.85	78.10	21.85	78.10	21.85	78.10	21.85	78.10	21.85	78.10	21.85	78.10	21.83	77.84	19.94
81	18	83.27	22.07	77.53	21.54	77.47	21.54	77.47	21.54	77.47	21.54	77.47	21.54	77.47	21.54	77.47	21.54	77.47	21.52	77.20	19.78
82	19	82.90	21.72	76.92	21.23	76.86	21.22	76.86	21.22	76.86	21.22	76.86	21.22	76.86	21.22	76.86	21.22	76.85	21.21	76.59	19.62
83	20	82.53	21.38	76.31	20.92	76.25	20.92	76.25	20.92	76.25	20.92	76.25	20.92	76.25	20.92	76.25	20.92	76.25	20.90	75.98	19.46
84	21	82.17	21.05	75.72	20.62	75.66	20.62	75.66	20.62	75.66	20.62	75.66	20.62	75.66	20.62	75.66	20.62	75.66	20.60	75.38	19.31
85	22	81.80	20.71	75.14	20.33	75.08	20.32	75.07	20.32	75.07	20.32	75.07	20.32	75.07	20.32	75.07	20.32	75.07	20.31	74.80	19.16
86	23	81.44	20.38	74.56	20.04	74.50	20.03	74.50	20.03	74.50	20.03	74.50	20.03	74.50	20.03	74.50	20.03	74.50	20.02	74.23	19.02
87	24	81.07	20.05	74.00	19.75	73.94	19.75	73.93	19.75	73.93	19.75	73.93	19.75	73.93	19.75	73.93	19.75	73.93	19.74	73.66	18.87
88	25	80.71	19.73	73.44	19.47	73.38	19.47	73.38	19.47	73.38	19.47	73.38	19.47	73.38	19.47	73.38	19.47	73.38	19.46	73.11	18.73
89	26	80.35	19.41	72.89	19.19	72.83	19.19	72.83	19.19	72.83	19.19	72.83	19.19	72.83	19.19	72.83	19.19	72.83	19.19	72.56	18.59
90	27	79.99	19.09	72.35	18.92	72.29	18.92	72.29	18.92	72.29	18.92	72.29	18.92	72.29	18.92	72.29	18.92	72.29	18.92	72.02	18.46
91	28	79.63	18.77	71.82	18.65	71.75	18.65	71.75	18.65	71.75	18.65	71.75	18.65	71.75	18.65	71.75	18.65	71.75	18.65	71.49	18.32
92	29	79.28	18.46	71.29	18.39	71.23	18.39	71.23	18.39	71.23	18.39	71.23	18.39	71.23	18.39	71.23	18.39	71.23	18.39	70.97	18.19
93	30	78.93	18.15	70.77	18.13	70.71	18.13	70.71	18.13	70.71	18.13	70.71	18.13	70.71	18.13	70.71	18.13	70.71	18.13	70.45	18.06
94	31																				

Figure 3.2 Example of matrix of average element temperatures

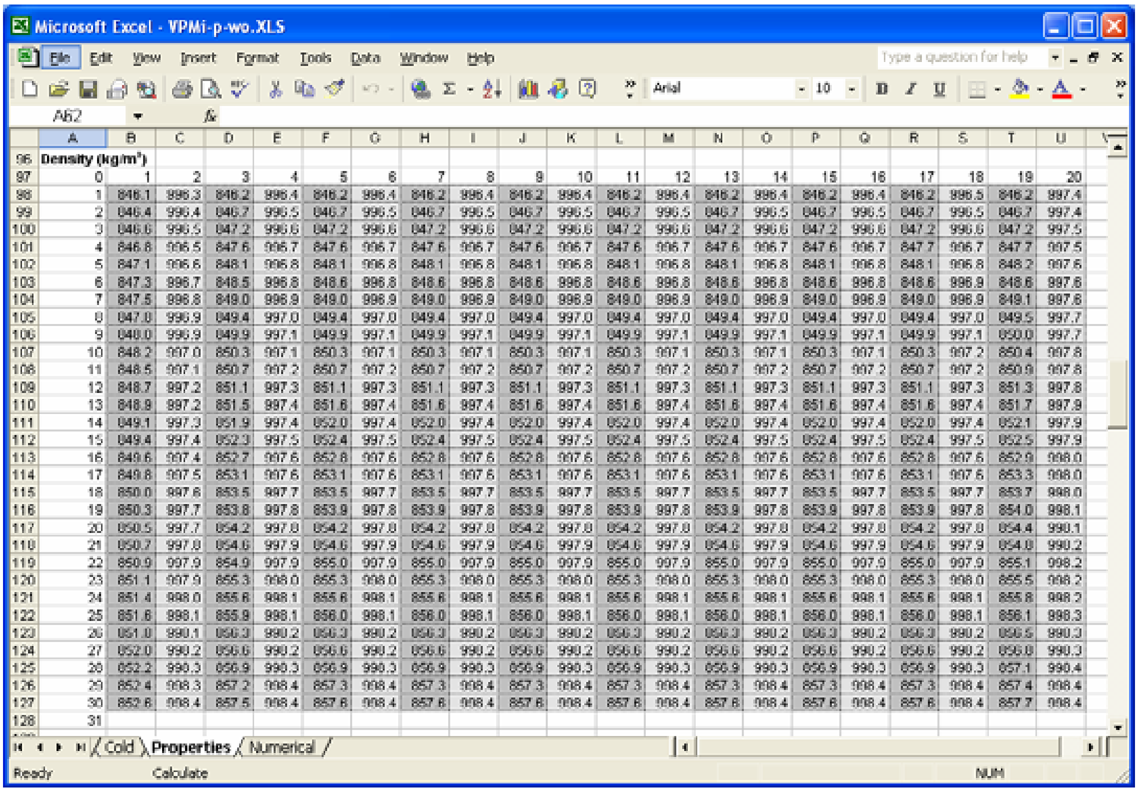


Figure 3.3 Example of matrix of fluid densities in each element

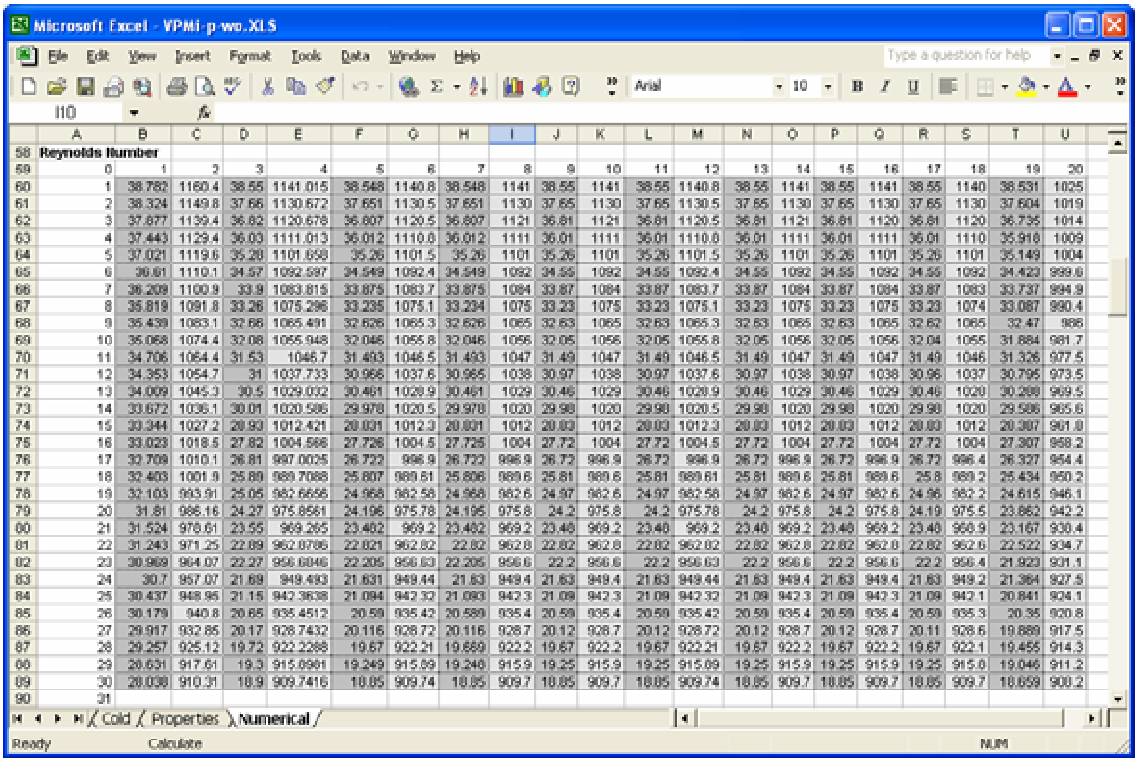


Figure 3.4 Example of matrix of Reynolds numbers in each element

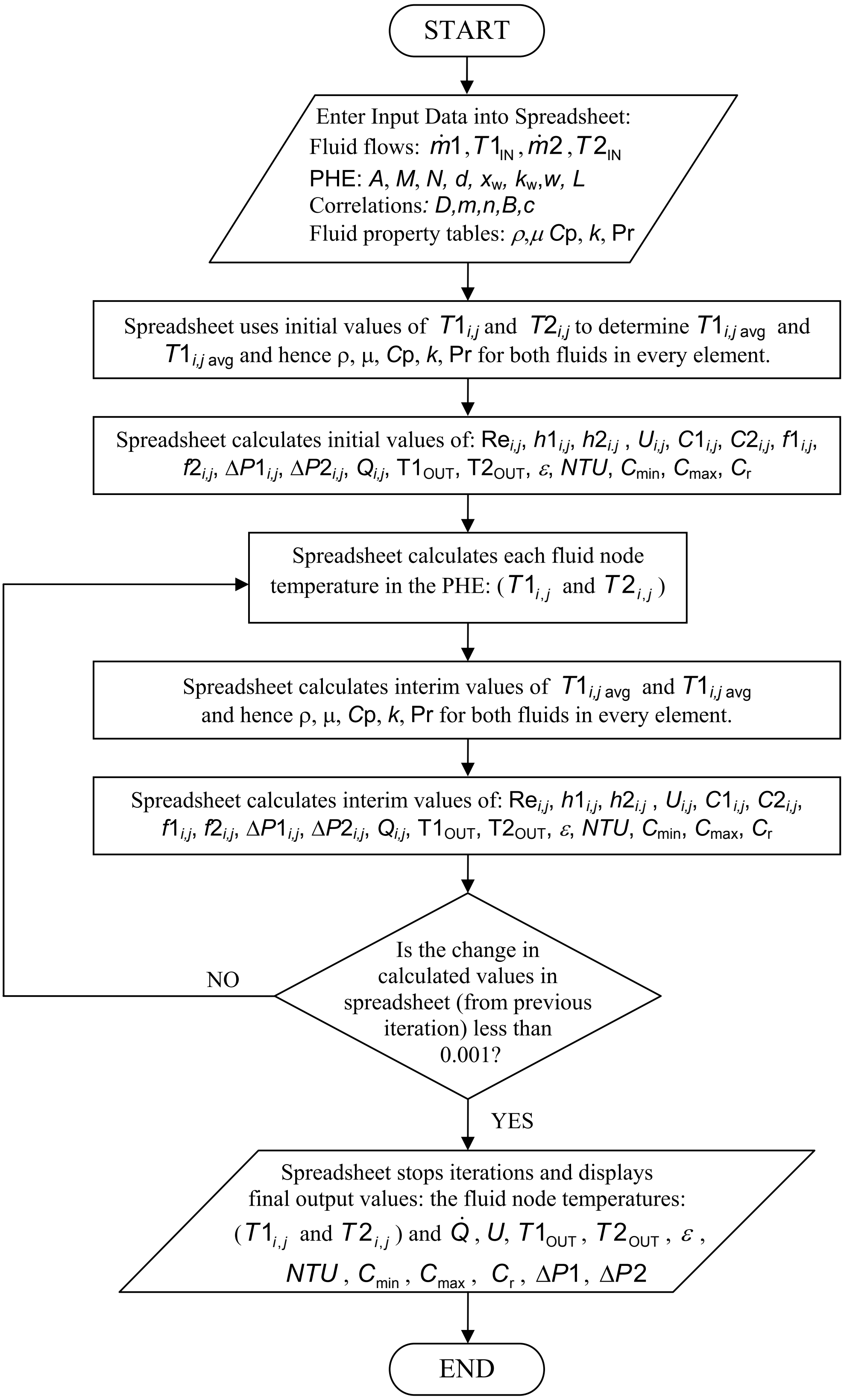


Figure 3.5 Flowchart of spreadsheet operation, initial variable-properties-model, VPM1

3.2.9 Validation

Since this initial VPM1 is really a stepping stone towards the final VPM it is not directly validated. However, it is compared to the results from the CPM and the results from the final VPM in Chapter 6. The final, refined VPM is carefully validated in Chapter 5.

3.3 Refinement of the Variable Properties Model

The VPM allows for variation of all fluid properties with temperature through the heat exchanger, element by element. However, as discussed in Chapter 1, a number of researchers have noted [31, 32] that the dynamic viscosity, in particular, is strongly dependent on temperature for many fluids. For this reason several Nusselt number correlations have been developed [28-30] that include a correction term for the convection heat transfer due to the viscosity change between the fluid at the wall and the bulk fluid. These correlations are usually of the form [2].

$$\mathrm{Nu} = D\mathrm{Re}^m \mathrm{Pr}^n \left(\frac{\mu_\mathrm{b}}{\mu_\mathrm{w}} \right)^{0.14} \tag{3.38}$$

where μ_b and μ_w are the dynamic viscosity of the fluid evaluated at the bulk fluid

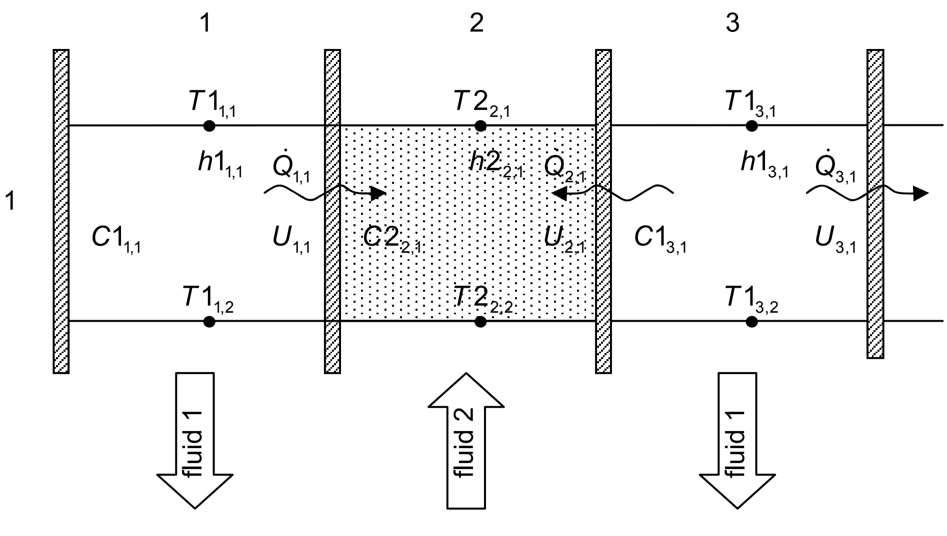


Figure 3.6 Convection heat transfer coefficients, VPM1

temperature and evaluated at the wall temperature respectively. This allows a more accurate value for the convection heat transfer coefficient, h to be determined.

For the initial VPM1, the convection heat transfer coefficient $h_{i,j}$ has been calculated for each element i,j through the heat exchanger. But, it is assumed that it remains as a single constant value *within* each element. Now, as discussed previously, apart from the elements in the first and last flow passages, all elements in the PHE are exchanging heat with *two* adjacent elements. So, for element i,j, the overall heat transfer coefficients, $U_{i-1,j}$ and $U_{i,j}$ need to be determined to find the two heat flows $\dot{Q}_{i-1,j}$ and $\dot{Q}_{i,j}$ respectively, see equations (3.27), (3.28), (3.31) and (3.32). And, currently in the initial VPM1, the one value $h_{i,j}$ is used in element i,j to determine $U_{i-1,j}$ as well as to determine $U_{i,j}$ (see Figure 3.6 and equation (3.6)).

However, each element in the heat exchanger is bounded by two walls and these walls will be at different temperatures, so, if the more realistic correlation of general equation (3.38) is to be implemented, *two* convection heat transfer coefficients for each element will need to be determined. One, $h_{\mathrm{L}\,i,j}$ in relation to heat transfer with element $i-1,j$ to the left of element i,j (a component of $U_{i-1,j}$) and one, $h_{\mathrm{R}\,i,j}$ in relation to heat transfer with element $i+1,j$ to the right of element i,j (a component of $U_{i,j}$). See Figure 3.7

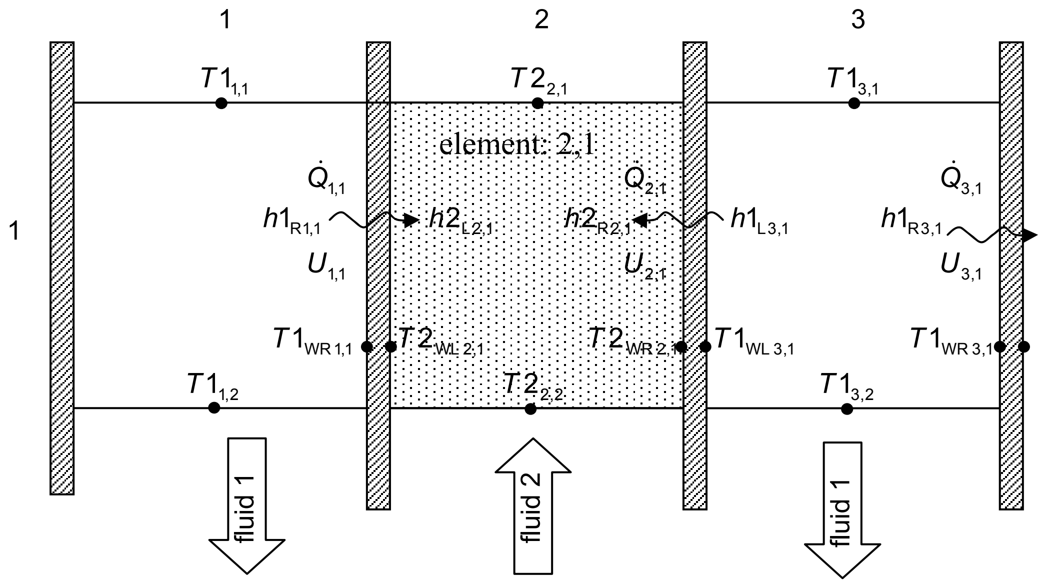


Figure 3.7 Refined VPM, wall temperatures and convection coefficients

In order to use the correlation of general equation (3.38) the viscosity μ_{w} is evaluated for that the fluid at the wall temperature, so the wall temperatures of each element must also be calculated in the model. Once the wall temperatures have been determined then the correlation of general equation (3.38) can be used for each element to determine the convection and overall heat transfer coefficients. So, the only modification to the initial VPM1 is in terms of how the overall heat transfer coefficient is calculated. Once the overall heat transfer coefficient has been determined for each element, the calculations for the node temperatures and performance parameters are the same as previously described for the initial VPM1.

3.3.1 Wall Temperatures

When using the more accurate correlation of general equation (3.38) an iterative approach is normally used. The wall temperature is guessed, fluid properties are determined, convection heat transfer and overall heat transfer coefficients are calculated and so the heat transferred is determined. The heat transferred is then used to calculate the wall temperature and compared with the initial guess… this process being repeated until convergence occurs. This same basic procedure is used in the VPM.

Firstly, equations need to be developed so that the wall temperatures can be determined. For each element in the PHE, apart from elements in the first and last flow channels, two wall temperatures (left and right) need to be determined. For elements in the first and last flow channel only one wall temperature needs to be determined.

Referring to Figure 3.7 and ignoring which fluid is in the flow channel, the convection heat transfer equations from one element i, j to the element immediately to its *right*, $i+1, j$ can be written:

for element i, j

$$\dot{Q}_{i,j} = h_{\mathrm{R}\,i,j} A_{MN} \left(T_{\mathrm{WR}\,i,i} - T_{i,j\,\mathrm{avg}} \right) \tag{3.39}$$

and also for element $i+1, j$

$$\dot{Q}_{i,j} = h_{\mathrm{L}\,i+1,j} A_{MN} \left(T_{i+1,j\,\mathrm{avg}} - T_{\mathrm{WL}\,i+1,i} \right) \tag{3.40}$$

Where A_{MN}, $T1_{i,j\,\mathrm{avg}}$ and $T2_{i,j\,\mathrm{avg}}$ are given by equations (3.15), (3.1) and (3.2) respectively. The overall heat transfer equation for this situation can also be written:

$$\dot{Q}_{i,j} = U_{i,j}\, A_{MN}\left(T_{i,j\,\mathrm{avg}} - T_{i+1,j\,\mathrm{avg}}\right) \qquad (3.41)$$

By combining equations (3.39) and (3.41), an expression for the wall temperature $T_{\mathrm{WR}\,i,j}$ to the right of element i,j can be obtained:

$$T_{\mathrm{WR}\,i,j} = T_{i,j\,\mathrm{avg}} - \frac{U_{i,j}}{h_{\mathrm{R}\,i,j}}\left(T_{i,j\,\mathrm{avg}} - T_{i+1,j\,\mathrm{avg}}\right) \qquad (3.42)$$

Similarly, if the convection heat transfer equations and overall heat transfer equation are written for heat transfer from element i,j to the element immediately to its *left*, $i-1,j$ an expression for the wall temperature $T_{\mathrm{WL}\,i,j}$ can be obtained:

$$T_{\mathrm{WL}\,i,j} = T_{i,j\,\mathrm{avg}} - \frac{U_{i-1,j}}{h_{\mathrm{L}\,i,j}}\left(T_{i,j\,\mathrm{avg}} - T_{i-1,j\,\mathrm{avg}}\right) \qquad (3.43)$$

3.3.2 Convection and Overall Heat Transfer Coefficients

Once the wall temperatures have been determined for each element then the dynamic viscosity for the relevant fluid at the wall temperature can be determined and used in the correlation of general equation (3.38) to determine the convection heat transfer coefficients for each element: $h_{\mathrm{L}\,i,j}$ and $h_{\mathrm{R}\,i,j}$ in an iterative fashion. General equation (3.38) is now written in a form that can be used in each element. To determine $h_{\mathrm{L}\,i,j}$ in element i,j the Nusselt number correlation for convection heat transfer between the bulk fluid and the *left* wall is:

$$\mathrm{Nu}_{\mathrm{L},i,j} = D\,\mathrm{Re}_{i,j}^{m}\,\mathrm{Pr}_{i,j}^{n}\left(\frac{\mu_{\mathrm{b}}}{\mu_{\mathrm{w,L}}}\right)_{i,j}^{0.14} \qquad (3.44)$$

and

$$h_{\mathrm{L}\,i,j} = \frac{\mathrm{Nu}_{\mathrm{L},i,j}\ k_{i,j}}{d_{\mathrm{h}}} \tag{3.45}$$

To determine $h_{\mathrm{R}\,i,j}$ in element i,j the Nusselt number correlation for convection heat transfer between the bulk fluid and the *right* wall is:

$$\mathrm{Nu}_{\mathrm{R},i,j} = D\mathrm{Re}_{i,j}^{m}\,\mathrm{Pr}_{i,j}^{n}\left(\frac{\mu_{\mathrm{b}}}{\mu_{\mathrm{w,R}}}\right)_{i,j}^{0.14} \tag{3.46}$$

and

$$h_{\mathrm{R}\,i,j} = \frac{\mathrm{Nu}_{\mathrm{R},i,j}\ k_{i,j}}{d_{\mathrm{h}}} \tag{3.47}$$

Finally, the overall heat transfer coefficients for each element can be determined. By referring to equation (3.6) and considering the heat transfer between adjacent elements i,j and $i+1,j$ (Figure 3.7) the heat transferred is $\dot{Q}_{i,j}$ and the overall heat transfer coefficient, $U_{i,j}$, is given by:

$$U_{i,j} = \left(\frac{1}{h_{\mathrm{R}\,i,j}} + \frac{x_{\mathrm{w}}}{k_{\mathrm{w}}} + \frac{1}{h_{\mathrm{L}\,i+1,j}}\right)^{-1} \tag{3.48}$$

The overall heat transfer coefficient is used iteratively to determine the correct wall temperatures.

In this way, using equations (3.42) – (3.48) the heat transfer between two elements can be more accurately determined, by allowing for the sometimes significant viscosity changes between wall and bulk fluid.

3.3.3 Further Refinements to the VPM

It is stated previously that the values of the Nusselt number and friction factor coefficients; D, m and n in equations (3.4), (3.38), (3.44) and (3.46) and B and c in equation (3.7) depend on the physical characteristics of the *particular* PHE and the flow conditions within the PHE. The large range of possible correlations (and hence different values of the coefficients D, m, n, B and c) has also been discussed earlier in this chapter and in Chapter 1. However, Muley and Manglik [28, 29] studied heat transfer in PHE's fitted with a very common type of PHE plate called the *chevron* plate and proposed that these coefficients depend on the geometry of the chevron pattern, specifically the chevron angle β and the surface area enlargement factor ϕ. See Figure 3.8. Correlations are proposed for the Nusselt number and the isothermal friction factor, for a range of Reynolds numbers, to be of the form [28]:

$$\mathrm{Nu} = p1(\beta)q1(\phi)\mathrm{Re}^{r1(\beta)}\,\mathrm{Pr}^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14} \tag{3.45}$$

and

$$f = p2(\beta)q2(\phi)\mathrm{Re}_{i,j}^{r2(\beta)} \tag{3.46}$$

Where p, q and r are functions and are specified in the study [28].

This refinement has also been added to the VPM to extend its applicability and allow the student to study the effect of plate geometry on the performance of an existing heat exchanger. However, the values of the coefficients D, m, n, B and c can still be entered manually to suit any desired correlations of the standard forms ie forms of equations

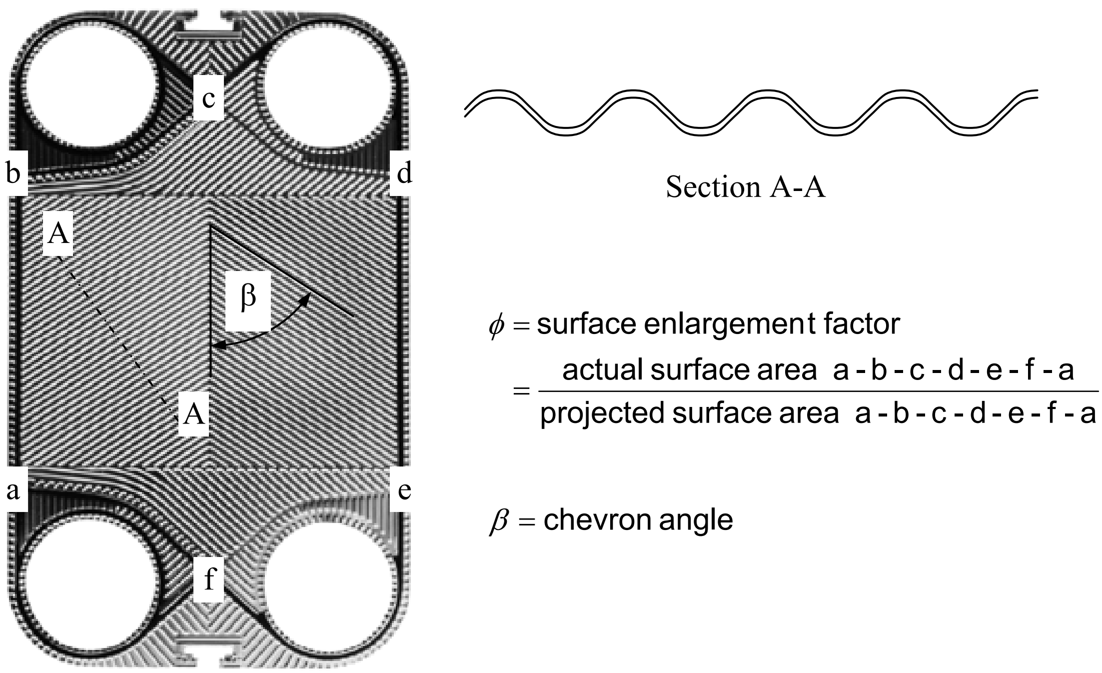


Figure 3.8 Typical plate with chevron pattern, showing β and ϕ

(3.7), (3.44) and (3.46). In this way the student can also study the effects of different correlations on the PHE performance.

3.3.4 Spreadsheet Implementation

The VPM spreadsheet is modified to allow for the additional calculations required in the refinements mentioned above. Additional areas on the Property sheet are used to store the matrices of viscosities determined at the wall temperatures, $\mu_{w,Li,j}$ and $\mu_{w,Ri,j}$ in each element. Additional areas on the Numerical calculation sheet are used to store calculations for the wall temperatures, $T_{WL\,i,j}$ and $T_{WR\,i,j}$ and convection heat transfer coefficients $h_{L\,i,j}$ and $h_{R\,i,j}$. The functions $p1$, $q1$ and $r1$, and $p2$ & $q2$ [28] for equations (3.45) and (3.46) are also entered into the spreadsheet to determine the correlation coefficients. Figure 6.9 shows part of the final VPM spreadsheet

The flow chart for the operation of the refined VPM is shown in Figure 3.9

The VPM now incorporates all of the features that enable it to be used as a constructive investigation tool for students, a tool that can be used to discover how a PHE operates, in general and in detail.

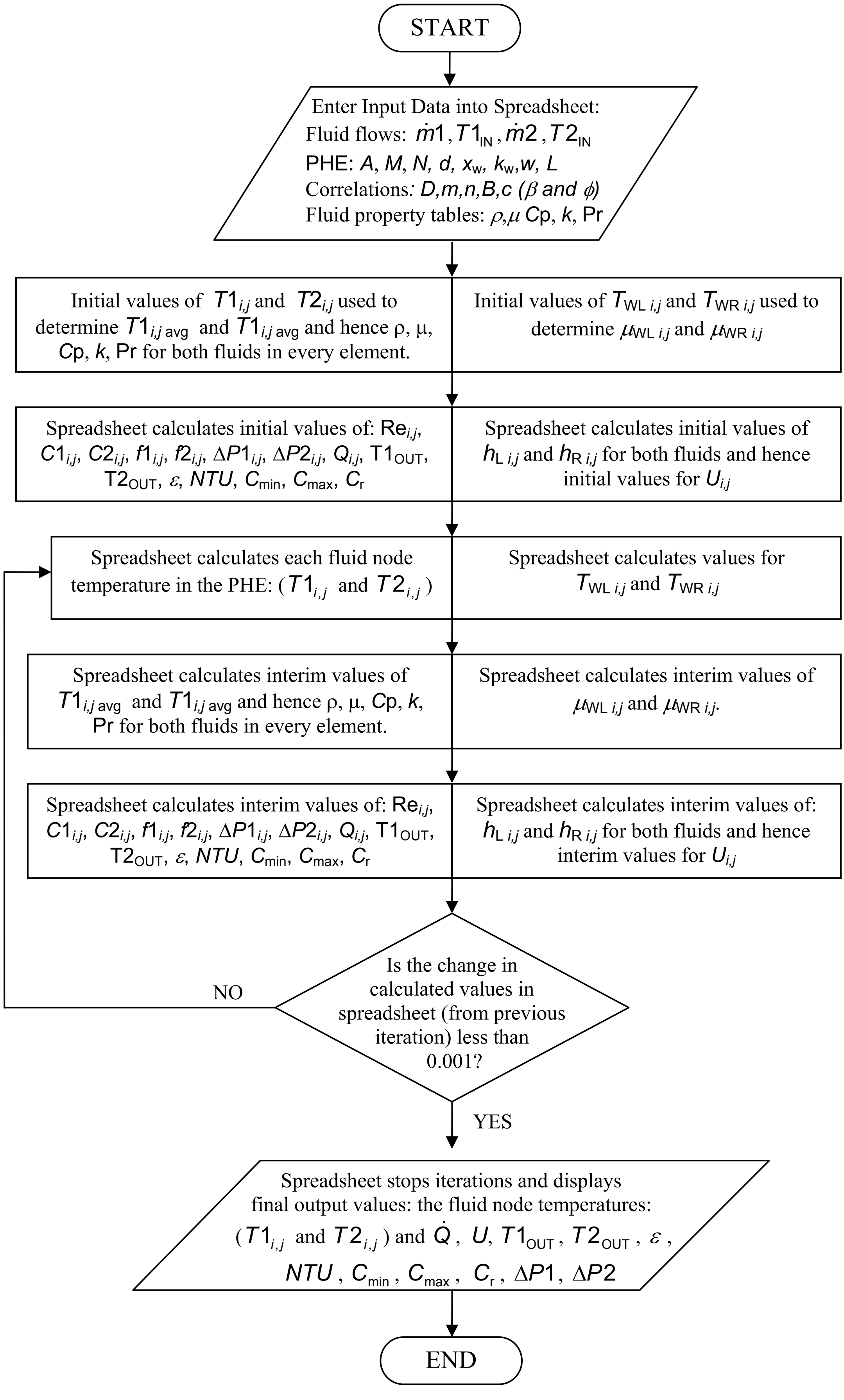


Figure 3.9 Flowchart of spreadsheet operation, refined VPM

CHAPTER 4

EXPERIMENTAL VALIDATION

4.1 Introduction

The major part of the validation of the variable properties model (VPM) will be presented in Chapter 5 using experimental data that is available in the literature. These data are for a range of different plate heat exchanger (PHE) configurations and is for a number of different fluids. However, in this chapter the results of some experiments conducted at the Auckland University of Technology (AUT) using a hot water versus cold water PHE are presented. It is felt that some experimental validation is necessary using the available heat transfer rig rather than completely depending on available data in the literature. The available experimental rig is somewhat limited in that there is no facility for pressure drop measurements. Also the water flow rates are restricted by the fact that they are supplied directly from the building's utility water supply, which could not be altered easily. Nevertheless some experimental data is generated to support the validation of the VPM.

4.2 Experimental Setup

The experimental rig for the water versus water PHE set up at AUT is shown in Figure 4.1 and schematically in Figure 4.2. It consists of a cold water supply and a hot water supply to an Alfa Laval P-20 plate heat exchanger. The primary measurements made with the apparatus are flow rates and temperatures (inlet and outlet) of both hot and cold flow streams.

The cold fluid supply to the PHE is from a utility cold water supply fed directly to the cold inlet of the heat exchanger via a ball valve. The ball valve allows the flow rate of the cold water through the heat exchanger to be manually adjusted.

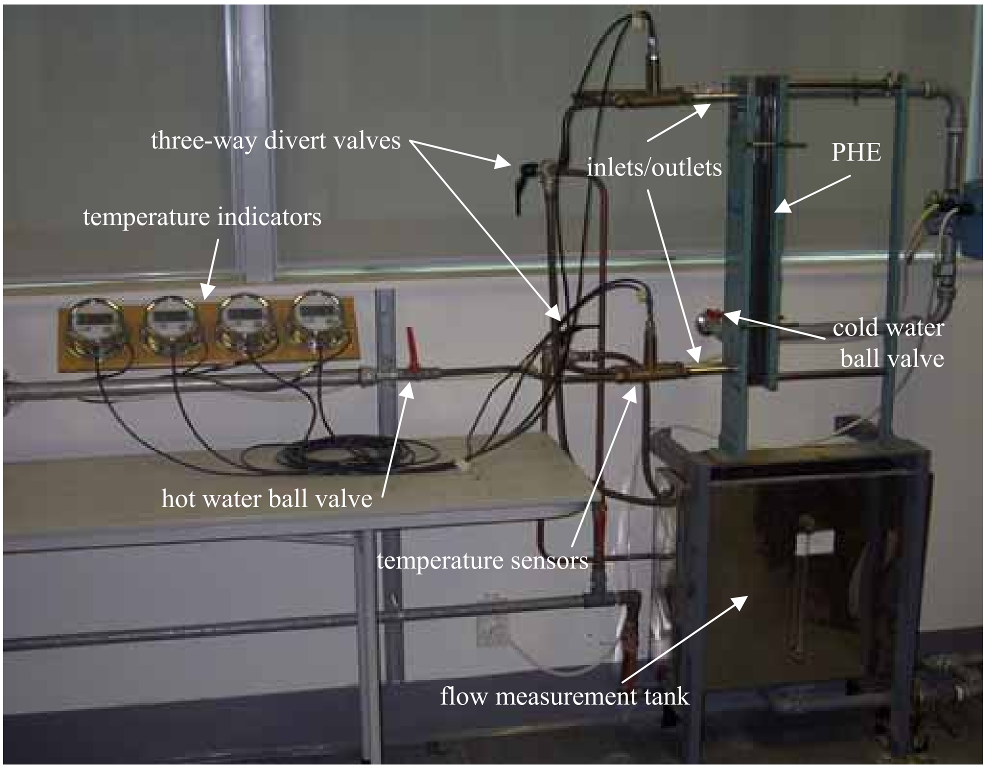


Figure 4.2 Experimental setup

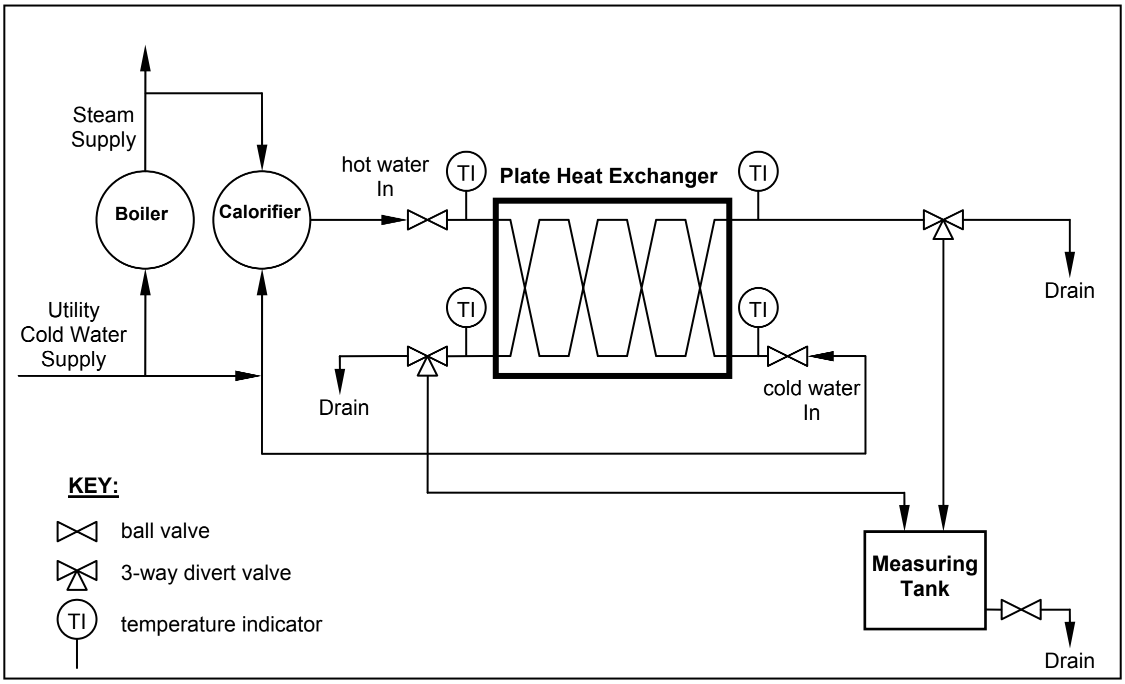


Figure 4.1 Schematic of experimental rig

The hot fluid supply to the PHE is hot water drawn from a steam-heated calorifier. Cold water from the utility cold water supply is supplied to the calorifier where it is heated using steam from a small electrode boiler. The heated water is then piped directly, at the utility supply pressure, to the hot inlet of the heat exchanger via another ball valve. This ball valve allows the flow rate of the hot water through the PHE to be manually adjusted.

The outlet of the cold fluid from the heat exchanger is piped via a three-way divert valve to a 20 litre measuring tank. The three-way divert valve allows the outlet cold fluid from the heat exchanger to be fed either to drain or to the measuring tank. In a similar way the outlet of the hot fluid from the PHE is piped via another three-way divert valve to the measuring tank. This allows the outlet hot water to be directed either to the measuring tank or to drain. The measuring tank is fitted with a drain valve to allow drainage once it is full and flow rate measurement is complete.

4.3 Plate Heat Exchanger

The plate heat exchanger used in the experimental validation is an Alfa Laval model P-20 commonly used in the dairy industry. It consists of 21, stainless steel (AISI Type

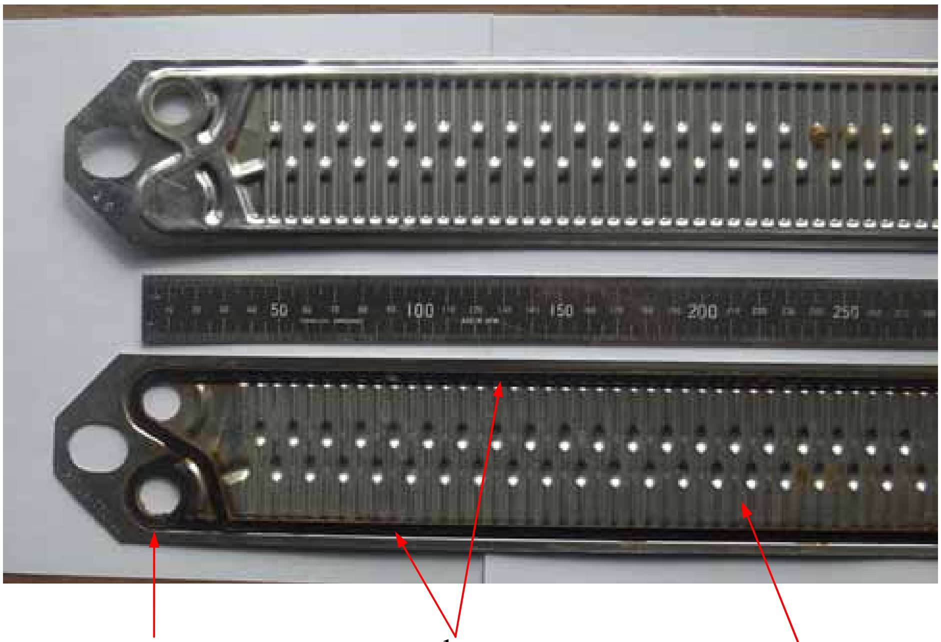


Figure 4.3 Detail of stainless steel plates from the PHE

316) plates with gaskets arranged to give a U-type, single-pass, parallel flow arrangement. Figure 4.3 shows two plates from the PHE. The gaskets and corrugated surface can be clearly seen.

The U-type flow arrangement within the plate heat exchanger means that the inlet and outlet ports are on the same end of the PHE. The parallel flow arrangement is as per Figure 1.5 in Chapter 1.

The physical parameters of the stainless steel plates are given in Table 4.1. The definition of these physical parameters is clarified in Figure 4.4 (refer also to Figure 3.8).

Table 4.1 Alfa Laval P-20 plate characteristics

Plate Characteristics	
Length, L (mm)	500 mm
Width between gaskets, w (mm)	50 mm
Thickness, x_w (mm)	0.4 mm
Distance between plates, d (mm)	2.2 mm
Chevron angle, β	90°
Area ratio, ϕ (estimate)	1.5

4.4 Experimental Measurement

Flow rate measurements for the hot and cold fluid streams are achieved using a simple measuring tank and stopwatch technique. The measuring tank is fitted with a sight glass to facilitate the reading of volume in the tank. This tank sight glass is calibrated with two level marks to indicate an increase in volume of 20 litres.

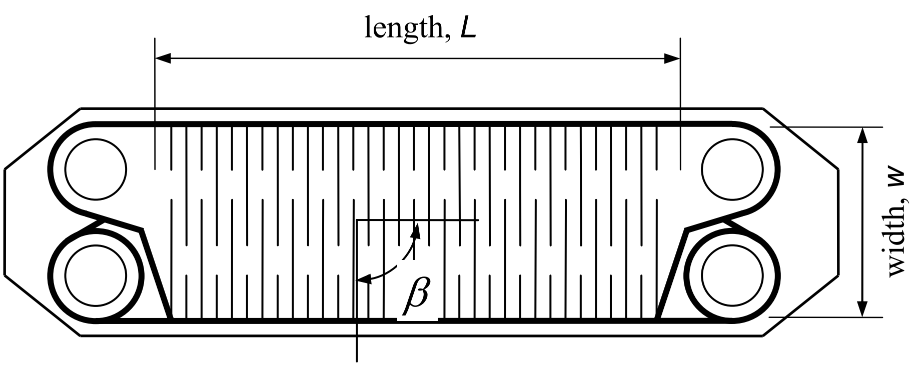


Figure 4.4 Plate parameters

To measure the cold fluid flow rate for a particular experimental "run", the three-way divert valve on the hot fluid stream is adjusted to direct the hot water from the hot outlet of the PHE to drain. The three-way divert valve on the cold fluid stream is adjusted to direct the cold water from the cold outlet of the PHE to the measuring tank. As the level of water in the sight glass passes the first mark a stop watch is started. When the level of water passes the second (20 litre) mark the stop watch is stopped and the time recorded. The flow rate for the hot stream is measured in a similar way.

For a particular experimental run the ball valves on the hot and cold inlets to the heat exchanger are adjusted to new positions and the times for 20 litres of flow are recorded. The flow rates of the hot and cold streams for any particular experimental run are measured at least another two times during the period of time allowed for the system to reach steady state conditions (around 30 minutes).

Temperature measurements on the hot and cold inlet and outlet streams are carried out using PT100 RTD sensors connected to digital readouts (Servotech, model AT400). The temperature probes are placed in brass pockets, close to the PHE inlet and outlet ports.

There is no provision for the measurement of fluid pressures on the inlet or outlet flow streams of the PHE.

4.5 Experimental Procedure

The steps involved in performing an experimental run are as follows:

1. The boiler and calorifier are turned on and allowed to come up to an operational state.

2. The drain valve on the measurement tank is closed and the two three-way divert valves are adjusted to allow the hot and cold outlets from the PHE to flow to drain.

3. The hot water ball valve is opened and adjusted to give the desired flow (low, medium or full flow)

4. The cold water ball valve is opened and also adjusted to give the desired flow (low, medium or full flow)

5. The flows and temperatures are allowed to stabilise for a few minutes and then the hot and cold water flow rates are measured (as described above in section 4.4)

6. The temperatures of all four flow streams (hot in and out, cold in and out) are noted every 10 minutes or so until there is little or no change in their values. This usually takes 20 – 30 minutes. During this time period the flow rates of the hot and cold streams are measured at least another two times.

7. Once the temperatures are stable, the final readings are noted down along with the flow rate readings.

8. This procedure is repeated for different combinations of hot and cold flow rates.

4.6 Experimental Results

The raw results achieved using the experimental rig and procedure described in this chapter are contained in Table 4.2. This represents a range of operating conditions whereby the hot and cold water is run through the PHE at a combination of low, medium and high flow rates.

Table 4.2 Raw experimental results

	Hot Water			**Cold Water**		
Run #	$T1_{IN}$ (°C)	$T1_{OUT}$ (°C)	Time (s) (for 20 l)	$T2_{IN}$ (°C)	$T2_{OUT}$ (°C)	Time (s) (for 20 l)
1	68.7	37.8	68	14	61	108
2	68	26.1	69	13.8	47.5	58
3	61.2	38.8	56.5	13.7	55.8	116
4	60.1	26.1	59	13.7	43.3	53
5	81.6	18.8	115	13.7	41.8	53
6	80.1	28.8	113	14.1	62.5	109.5

To convert the volume flow rates into mass flow rates it is necessary to estimate the density of the water in the measuring tank. It is noted that since the volume of the hot and cold water is measured at the *outlet* of the PHE the temperature of the water in the measuring tank should be approximately equal to the outlet temperature from the PHE for that flow stream. There is some variation in the temperature drop between the PHE outlet and the measuring tank but on average it is found to be no more than around 5% of the PHE outlet temperature. So, it is decided to calculate the water density in the measuring tank at 95% of the outlet temperature for the particular flow stream.

The density of water at the specified temperature is determined from standard thermodynamic tables for the properties of saturated water available in most heat transfer textbooks [2].

The mass flow rates achieved for the six experimental runs are summarised in Table 4.3

Table 4.3 Mass flow rates

Run #	Hot water flow rate (kg/s)	Cold water flow rate (kg/s)
1	0.292	0.182
2	0.289	0.341
3	0.352	0.170
4	0.338	0.374
5	0.174	0.374
6	0.176	0.180

It is important that the VPM proposed by this thesis is validated against PHE's operating under a range of different configurations and conditions. The results obtained from an experimental rig that was made available at AUT, as described in this chapter, along with data from the literature for a range of PHE flow configurations, fluid types and operating conditions will be used to validate the VPM in Chapter 5.

CHAPTER 5

VALIDATION RESULTS

5.1 Introduction

If the variable properties model (VPM) of a plate heat exchanger (PHE) is to be used to accurately simulate the performance of a real PHE it is necessary to validate the model using experimental data from known PHE's with different fluids and different flow configurations. This will confirm that the VPM can be used in a wide range of PHE heat transfer situations. However, although experimental data covering a wide range of situations is available in the literature it is also necessary to validate the VPM using experimental data obtained from a specific PHE in the heat transfer rig made available at the Auckland University of Technology (AUT). The reasons for this are, firstly, it gives us a better feel for the operation of the VPM with our own PHE and secondly the educational aspects of the VPM, as discussed in Chapter 6, suggest comparison of the VPM against a PHE of known configuration that the students at AUT have access to.

This chapter, therefore, outlines the validation of the VPM against four different sources of experimental data representing different fluids and flow configurations. Firstly the VPM is validated against data obtained from the water-versus-water PHE set up in a parallel flow configuration at AUT, as described in Chapter 4. Secondly, it is validated against experimental data from a water-versus-water PHE set up with varying numbers of plates in both series and parallel flow configurations [10]. Thirdly, it is validated against experimental data for a combination flow configuration PHE used for the pasteurisation and cooling of orange juice [45]. Finally it is validated against experimental data for a large, parallel flow PHE used for the pasteurisation of milk [7].

Although the data generated specifically for this thesis, using the apparatus and procedure described in Chapter 4, is somewhat limited, it is felt that, due to the existence of so much additional data reported in the literature it is not necessary to attempt to reproduce those experiments in this thesis. Also the advantage of using the reported data is that it covers a wide range of sizes and configurations of PHE as well as a range of different fluid types which may not be possible to achieve with a single experimental rig.

The basis of the following validations is to modify the VPM to exactly match the configuration and flow conditions of the PHE in each of the reported experiments. Then input the physical data of each specific PHE into the VPM along with inlet temperatures and flow rates for both hot and cold fluids. Finally, compare the outlet temperatures predicted by the VPM with the actual experimental outlet temperatures. It is possible to compare other performance parameters as well, such as overall heat transfer coefficient, U, number of transfer units, NTU, and effectiveness, ε, however these are generally not reported in the selected published literature. Nevertheless, the outlet temperatures serve as a perfectly acceptable means of comparing the accuracy of the VPM with the actual experimental data.

5.2 Water versus Water Validation

This part of the validation uses results from PHE's set up to exchange heat between hot water and cold water in two different flow configurations: parallel and series.

5.2.1 Parallel Configuration

This first part of the validation uses the data generated by the PHE at AUT as recorded in Chapter 4. This PHE exchanges heat between hot water and cold water in a parallel flow configuration and has 21 plates. Since the VPM was originally set up to simulate this specific PHE, very little modification of the model is required.

The inputs to the VPM, as described in the flow chart in Figure 3.9, are entered into the model. This includes the physical characteristics of the PHE (plate dimensions, number of plates etc), the properties of water, the coefficients for the Nusselt number and friction factor correlations and the process data inputs (inlet temperatures and mass flow rates for the hot and cold streams).

The properties of water versus temperature (specific heat, heat capacity, density, thermal conductivity and Prandtl number) are obtained from standard thermodynamic tables for the properties of saturated liquid water [2].

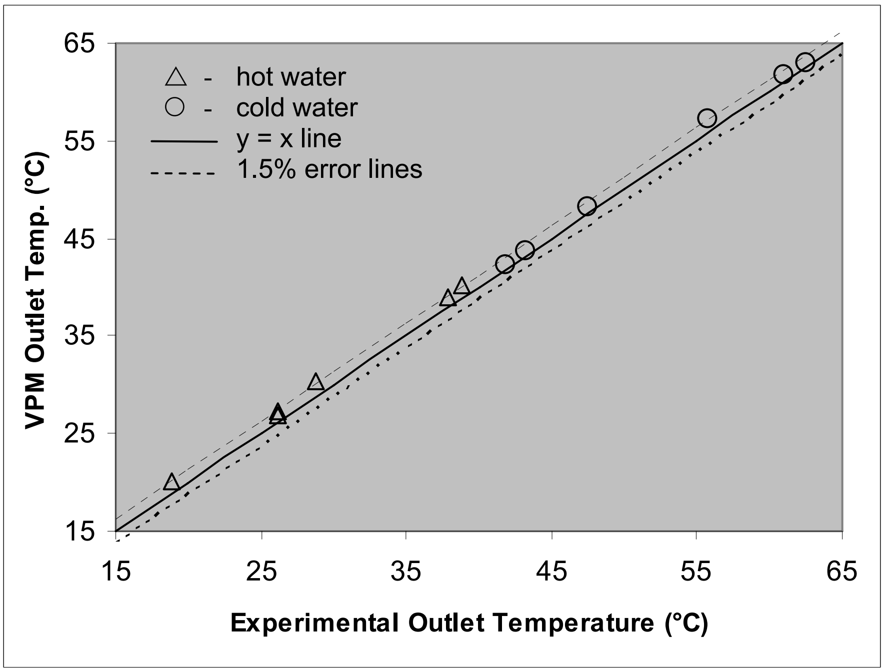


Figure 5.1 Comparison between the VPM and experimental results for a water-versus-water PHE at AUT

The coefficients for the Nusselt number correlation used in the VPM are chosen to be $\beta = 90$ and $\phi = 1.5$. Although the value of the chevron angle, $\beta = 90$ is strictly outside the limits of the correlation as originally reported [28] this is found to give good results with this PHE. Also, it is difficult to accurately determine the value for the surface area enlargement factor, ϕ but it is estimated to be 1.5 and this is also found to give good results for this PHE.

A comparison between the experimental and VPM results for the outlet temperatures is given in Figure 5.1. The results show excellent agreement and most results from the VPM are within 1.5% error of the experimental values.

5.2.2 Parallel and Series Configurations

This part of the validation uses experimental data for a water-versus-water PHE with different flow rates, temperatures and numbers of plates for both series and parallel flow configurations [10]. Of the 40 data points presented, only 17 data are used as the remaining data relates to the PHE set up with an *even* number of plates (and hence an *odd* number of flow passages). In this situation there is one more flow passage within the PHE for either the hot or the cold fluid and the investigation does not clarify which fluid (hot or cold) uses that additional flow passage. Without this clarification it is difficult to set up an accurate model for validation and so these data are ignored.

The data presented is for a PHE with both series and parallel flow configurations with the number of plates ranging from 3 to 19 plates. Since the VPM is set up for a PHE with a specific number of plates and a specific flow configuration, it is necessary to set up a separate model for virtually every data point presented in this investigation.

The series and parallel versions of the VPM are modified for each different case depending on the number of plates. Each new VPM that is set up is thoroughly checked for errors before final results are recorded.

Again, the properties of water versus temperature are obtained from standard thermodynamic tables for the properties of saturated liquid water [2].

The coefficients for the Nusselt number and friction factor correlations are not given in the investigation but a picture of the plate is provided in the paper clearly showing horizontal corrugations and from this the values for β and ϕ are estimated to be: $\beta = 90°$ and $\phi = 1.21$.

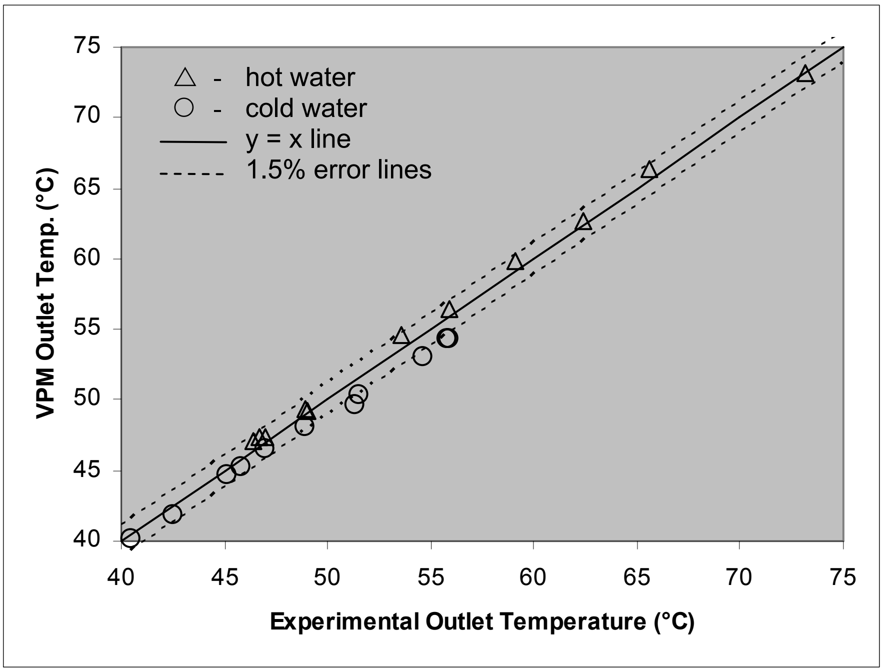


Figure 5.2 Comparison between the VPM and experimental results for a water versus water PHE [10]: PARALLEL flow configuration

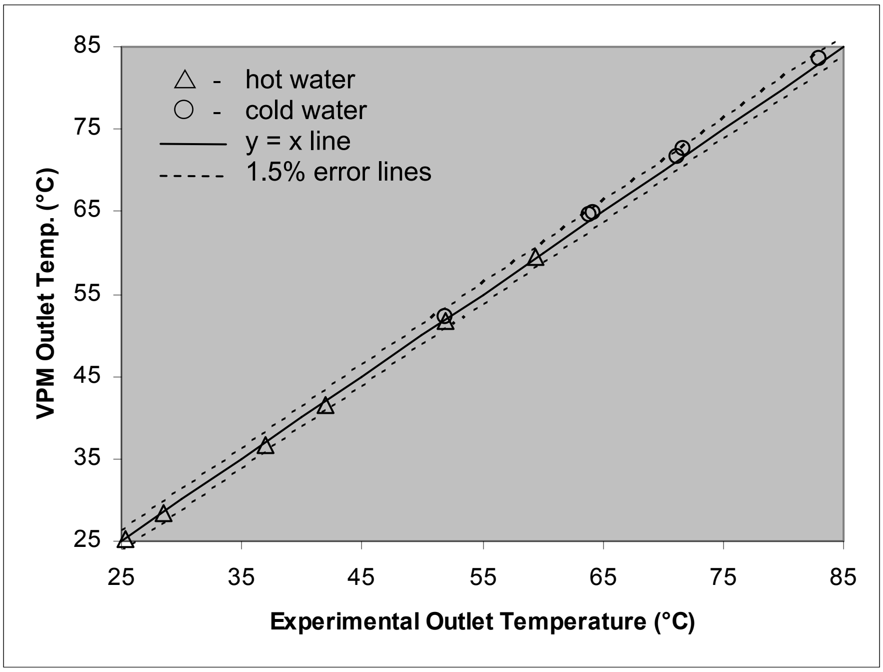


Figure 5.3 Comparison between the VPM and experimental results for a water versus water PHE [10]: SERIES flow configuration

A comparison between the experimental and VPM results for the outlet temperatures is given in Figures 5.2 and 5.3. The results show excellent agreement and most results from the VPM are within 1.5 % error of the experimental values. The results for the parallel configuration data show slightly more scatter than the results from the series configuration data. A reason for this could be due to one of the assumptions made for the parallel flow configuration case, namely that the flow divides equally between all the flow channels. With varying, unequal flow division it is possible to get this greater scatter of results.

5.3 Orange Juice Validation

This part of the validation uses experimental data for a PHE used to pasteurise and cool orange juice [45]. This PHE is set up with two separate sections, one with 17 plates for pasteurising the orange juice with hot water and the other with 13 plates for cooling the hot pasteurised orange juice with cold water. In both sections the orange juice is arranged to flow in a series configuration and the hot and cold water arranged to flow in a parallel configuration. In this investigation these combination configurations of the PHE did not change during the experimentation and the 40 experimental data reported therefore represent a range of different temperatures and flow rates for hot water, cold water and orange juice. Since the number of plates in each section of the PHE are not changed (apart from 3 data points for the pasteuriser, which are ignored) one VPM is set up for the pasteuriser section and one VPM is set up for the cooling section.

Again, the properties of water versus temperature are obtained from standard thermodynamic tables for the properties of saturated liquid water [2].

The properties of orange juice are not available in tabular form, however an equation to determine the viscosity of orange juice is available [48] and equations to determine the specific heat, density and thermal conductivity are also available [46]. The properties sheet on the VPM is modified to use these formulae to calculate the required properties at the average element temperatures. One difficulty in using these equations is that they require the concentration of orange juice to be known. Unfortunately the investigation [45] does not give any information in this regard. Nevertheless, it is found that a water concentration of between 85 to 90% gives reasonable results. This also highlights the flexibility of the VPM in that it can be easily modified to handle property data in the form of equations, as well as in tabular form.

The investigation determines a correlation for the Nusselt number, in the usual form, as

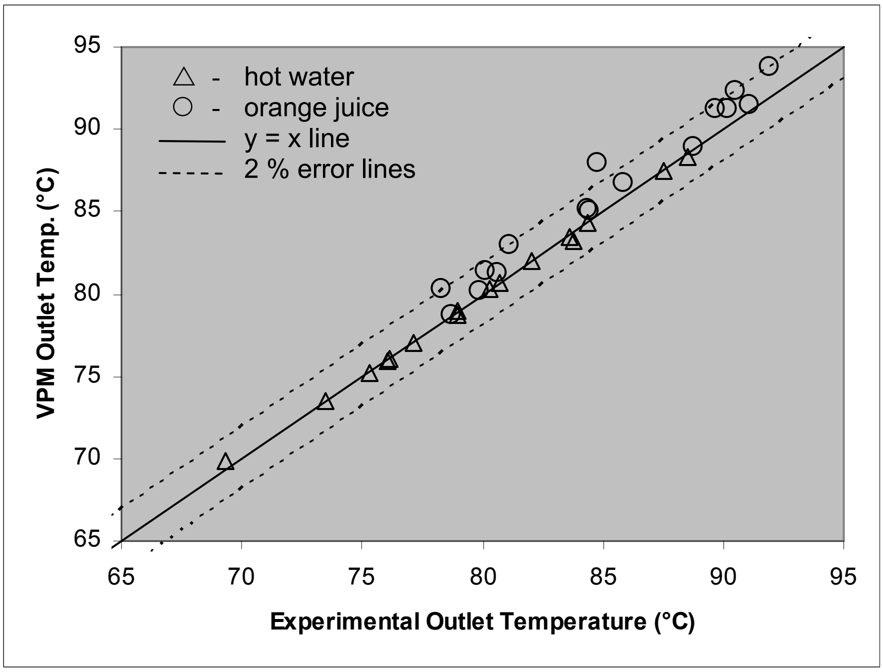


Figure 5.4 Comparison between the VPM and experimental results for an orange juice versus water PHE [45]: PASTEURISATION

a function of the Reynolds number and the Prandtl number [45]. However it is found that the VPM gives more accurate results using the Nusselt number correlation already programmed into the VPM and using the following coefficients : $\beta = 70°$ and $\phi = 1.45$.

A comparison between the experimental and VPM results for the outlet temperatures is given in Figures 5.4 and 5.5. In both cases the scatter for the orange juice outlet temperatures is greater than the scatter for the water outlet temperatures. This is probably due to several factors: firstly the orange juice properties are not known as accurately as the water properties, and there is a natural variability depending on variety of orange etc, secondly the exact concentration of the orange juice is not known, thirdly the orange juice may not behave as Newtonian liquid, and finally the Nusselt number correlation may not be as accurate for the orange juice (as compared to water). Nevertheless the results are mostly within 2% error of the experimental values for the pasteurisation unit and mostly within 5% error for the cooling unit. The outlet temperatures for the water predicted by the VPM are more accurate in both cases. For the pasteurisation unit the water outlet temperatures are mostly within 0.2% error of the experimental values while for the cooling section they are within 1.5% error of the experimental values.

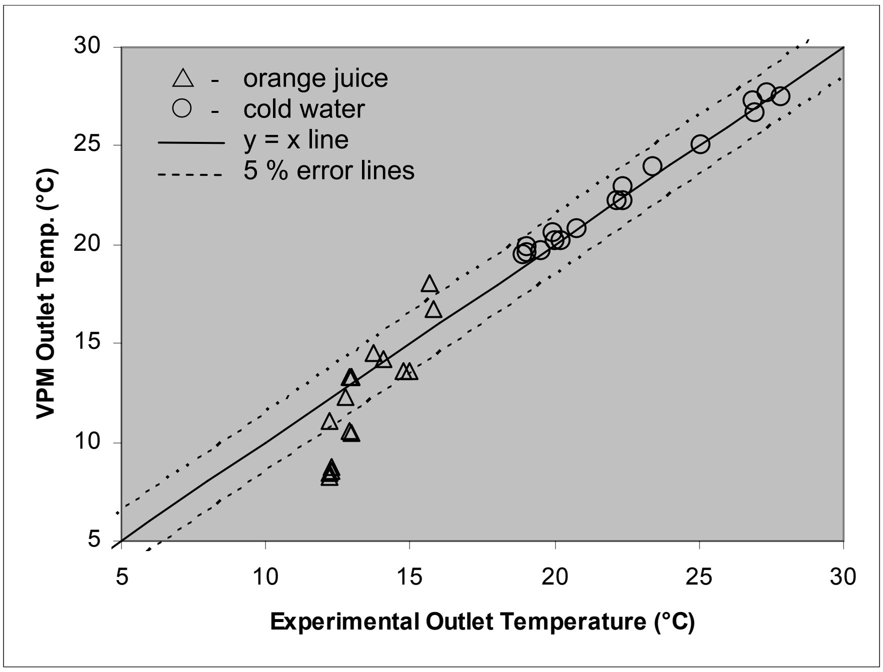


Figure 5.5 Comparison between the VPM and experimental results for an orange juice versus water PHE [45]: COOLING

5.4 Milk Validation

Finally, data for a parallel flow configuration PHE used in the pasteurisation of milk [7] is used to validate the VPM. Although only one data point is presented, this investigation also presents a model that predicts the temperature profiles along the various flow channels within the PHE. Since the VPM can also produce temperature profiles of any selected channel, the temperature profiles generated by the VPM are compared with those generated by the model in this investigation [7]. The PHE used in this investigation is an APV model HXB5 with 52 plates.

Again, the properties of water versus temperature are obtained from standard thermodynamic tables for the properties of saturated liquid water [2].

The properties of milk are obtained from charts of experimental data from several different sources. Tabular data is generated from these charts for the density [47], for the viscosity [49], for the specific heat capacity [51] and for the thermal conductivity [50]. There is no clear indication as to what composition of milk is being processed in this investigation [7], however, it is assumed to be whole milk.

Some data is supplied in the investigation regarding the shape of the corrugations on each plate in the PHE and from this data values of $\beta = 90°$ and $\phi = 1.4$ are determined for the Nusselt number correlation used in the VPM. Unfortunately only one data point is provided so it is not possible to fully confirm the validity of these selections.

Table 5.1 Comparison between results of the VPM, another model [7] and experimental results for a milk pasteurisation PHE

Stream	**Pressure drop (kPa)**		**Outlet temperatures (°C)**			**% error in temp. (VPM/Actual)**
	model [7]	VPM	Actual	model [7]	VPM	
milk	7.6	38.7	88	83.5	86.4	1.8
water	6.69	42.9	70	73.5	71.1	1.6

A comparison between the experimental and the VPM results are presented in Table 5.1. The pressure drops do not compare well, but both these results are from models and no experimental benchmark is given. The predicted outlet temperatures from the VPM are within 2 % of the experimental values. The predicted outlet temperatures from the model in the investigation [7] have an error of around 5 %. The VPM gives a superior result, however, more data points would be required to confirm this.

Temperature profiles are presented in this investigation [7] for a selection of flow channels in the PHE. The temperature profiles predicted by the VPM are shown in Figure 5.6 for those same flow channels. Although a direct comparison between the profiles presented in the study [7, p. 64, Fig. 6], is not possible due to differing outlet temperatures and the sheer quantity of data points, a visual comparison of the profiles show that they are substantially similar. Figure 5.6 uses the same data markers and line types as the study [7] so that a visual comparison can be easily made. It is interesting to note from these temperature profiles that the temperature variation in the flow channels for one fluid at the extreme ends of the PHE are similar to each other (channel 1 and channel 51) but different from the temperature variation for that same fluid in flow channels away from the ends (channel 9 and 31). This is observed for the other fluid as well. The reason for this being that the channels at the extreme ends are exchanging

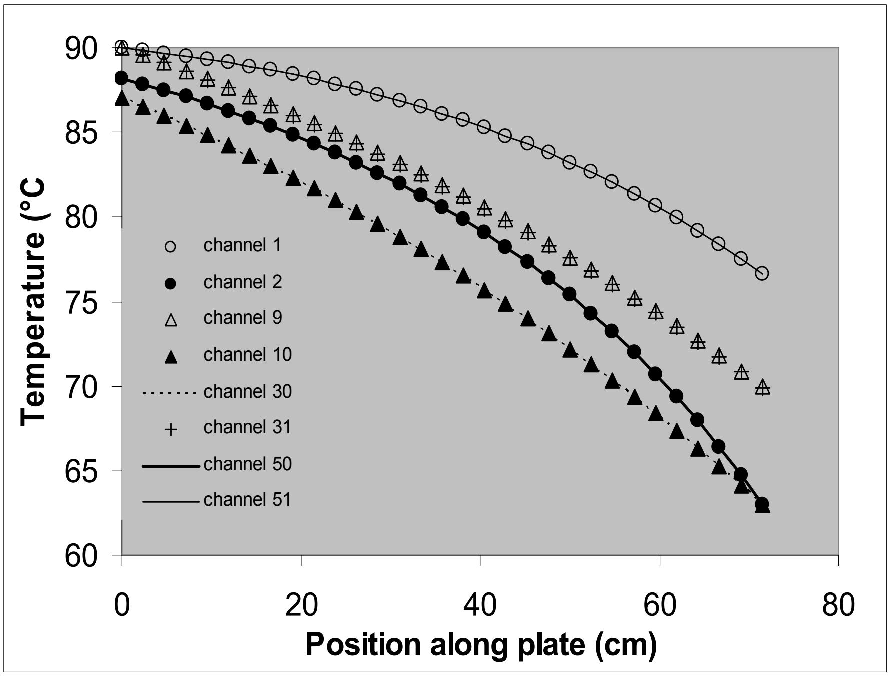


Figure 5.6 Temperature profiles determined by the VPM in a milk pasteurisation PHE

heat with only one other flow channel, whereas all other flow channels exchange heat with two other flow channels.

5.5 Summary

It is clear that the VPM is able to closely simulate the operation and outputs of a range of PHE's with different flow configurations, different flow rates and inlet temperatures, and different fluid types. This confirms that the VPM is a suitable model for predicting the performance of PHE's and students can use this model in the investigation of the operation and performance of any particular PHE.

CHAPTER 6

DISCUSSION AND CONCLUSIONS

6.1 Introduction

The validation results presented in the Chapter 5 clearly show that the VPM is a suitable tool for predicting the operation of a PHE operating with a range of different fluids and under a range of different flow configurations. In this chapter the results of the validation experiments are briefly discussed and then the significance of the VPM and the fact that it allows for the variation of fluid properties through the PHE, as compared to constant properties, is also discussed. The VPM is developed primarily as an educational tool to assist students in the understanding of the overall and detailed operation of a PHE. Different ways that the VPM can be used for this purpose are also discussed in this chapter. Finally, procedures or "user instructions" are presented for the detailed operation of the VPM.

6.2 Discussion of Validation Results

The results from the validation experiments confirm that the VPM is indeed an acceptably accurate tool for the thermal rating (prediction of performance) of a PHE. Results of each validation experiment are discussed in detail in Chapter 5.

Certainly for water versus water applications, in a range of different flow configurations and PHE sizes, the results of the VPM are well within acceptable experimental error. The error in the results of the VPM for other fluids like milk and orange juice are a little greater but still acceptable and in the case of the milk validation, the results from the VPM are more accurate than a model, which was set up specifically for that application [7].

This gives confidence that the model can be used to predict the performance of a wide range of existing PHE's, provided relevant fluid property data and relevant Nusselt number and friction factor correlations are available that apply to the specific situation. Although the VPM could be used in any situation its primary area of use, in relation to this study, is in conjunction with the PHE set up at AUT, or PHE's set up at other educational institutions, for the purposes of helping students understand the operation of a typical PHE. Different ways that the VPM can be used in this regard are detailed in section 6.4 of this chapter.

Unfortunately it has not been possible to confirm or validate experimentally the accuracy of the pressure drop calculations in the models developed. Although there are a number of studies that report thermal, heat transfer data for PHE's, and these have been used for validation in this thesis, none of these studies report accompanying pressure drop data. However the pressure drop calculation is included in the models as an *indication* of the likely changes in pressure drop for different heat transfer and flow conditions and to indicate the linkage between pressure drop and heat transfer in the PHE. The correlations and formulae used to calculate pressure drop are well known and proven [2] so although the calculations have not been experimentally validated there is confidence that they will be reasonably accurate.

6.3 Variable Properties versus Constant Properties

One of the main aims of this investigation is to develop a computer based, general purpose model of a PHE that allows for the *variation of fluid properties* as the temperature of the fluids vary. This in turn implies that other fluid property based parameters, such as Reynolds number, Prandtl Number, Nusselt Number, local heat transfer coefficients, friction factor and so on, will also vary through the PHE as the temperature of the fluid varies. As discussed in Chapter 1, most of the models used to simulate heat transfer in the literature assume that the fluid properties remain constant through the heat exchanger and are evaluated at the average relevant fluid temperature. This begs the question: is this assumption valid? To help answer this question, results of a comparison between the constant properties model (CPM) and the VPM are now presented and discussed. It should be noted that the purpose of this thesis is not to prove whether or not the constant properties assumption is a valid one or not, rather it is to develop a tool (the VPM and CPM) that could be used to investigate assumptions like this. This section therefore is an indication of how the models can be used to investigate such assumptions.

During the development of the VPM a constant properties model (CPM) was initially developed (as described in Chapter 2). This CPM was developed using the same methodology as the VPM (ie using a spreadsheet, iterative method etc) but it assumes that the fluid properties remain constant through the PHE. A series of virtual experiments are set up to compare the results of the CPM and the VPM operating under identical conditions. These are set up in an effort to see if there are significant differences between the results of the VPM and the CPM, thus allowing us to draw some conclusions regarding the validity of the constant properties assumption.

Also, during the development of the VPM an initial version of the VPM was developed (as discussed in Chapter 3 and referred to as the VPM1). The VPM1 allows for variable properties but uses a simpler form of the Nusselt number correlation (which does not allow for the effect that the variation in viscosity of the fluid through an element has on the convection heat transfer coefficient). The VPM1 assumes that the convection heat transfer coefficient in an element has a single value even though that element is exchanging heat with two other, different elements. So, the final version of the VPM allows for the variation of viscosity through an element by using a viscosity correction factor [31] and it determines two heat transfer convection coefficients for those elements that are exchanging heat with two other elements on either side of it. The

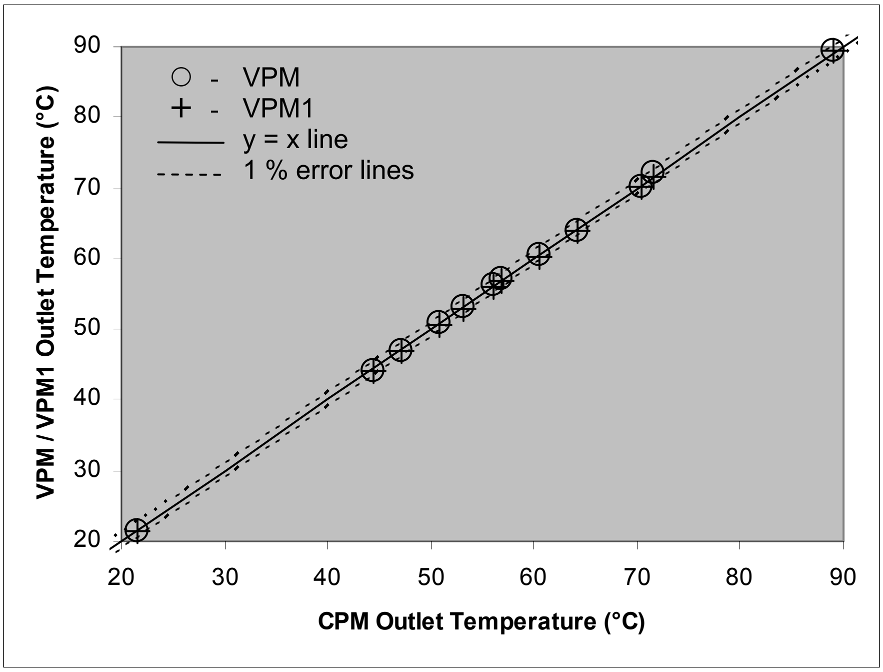


Figure 6.1 Comparison between the VPM and the CPM results for a water-versus-water PHE: COLD water

results of the initial VPM1 are presented together with the results for the CPM and VPM for comparison purposes.

6.3.1 Water versus water

The three models (CPM, VPM1 and VPM) are set up to generally simulate the PHE at AUT. They are set up in a parallel flow configuration, with 21 plates and with other physical parameters to generally match this PHE. The inlet temperature for the hot water is set at 90°C and the inlet temperature of the cold water is set at 18°C. The fluid properties of water [2] are entered into the Properties worksheet of the three spreadsheets (CPM, VPM1 and VPM).

A range of flow rates are entered into the models and the resulting outlet temperatures and other operating parameters are recorded. Table 6.1 shows these results for a range of flow rates.

A comparison between the results obtained from the CPM, VPM1 and VPM for the outlet temperatures is given in Figures 6.1 and 6.2. The outlet temperatures of the VPM1 and VPM (y-axis) are plotted against the outlet temperatures from the CPM (x-

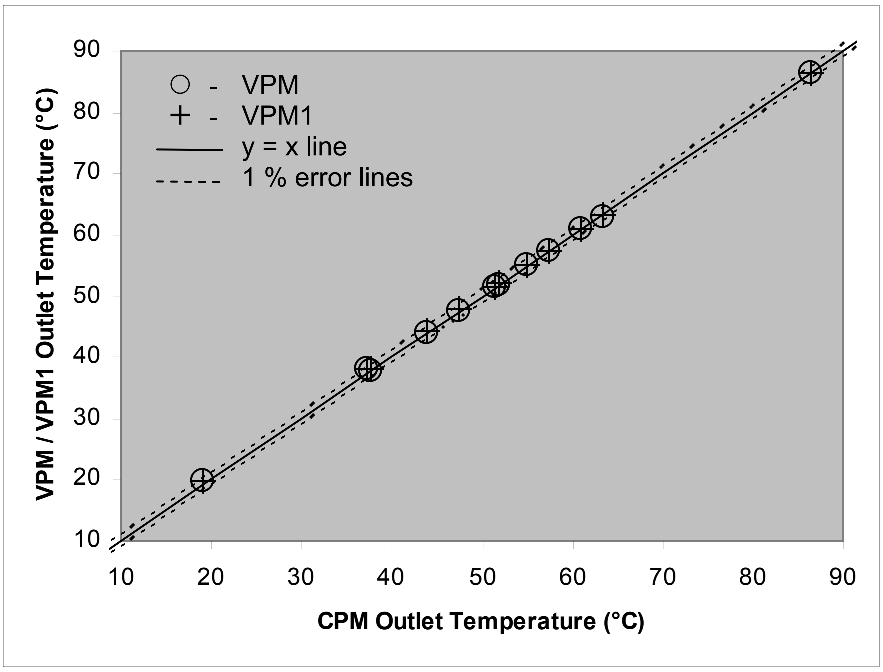


Figure 6.2 Comparison between the VPM and the CPM results for a water-versus-water PHE: HOT water

axis). The results show that for water versus water there is very little difference between the results of the VPM, the VPM1 and the CPM. This means that, in the case of water versus water in a PHE, the assumption of constant properties is in fact a valid one, according to these results. To verify this result, further virtual experiments could be set up to look at the effect of different flow configurations (series, combination), the effect of different numbers of plates, the effect of channel geometry (long narrow passages as compared to shorter wider passages) and so on, on any differences between the CPM and the VPM for water versus water duties.

It is proposed that the real differences in the results would be seen when one (or both) of the fluids in the heat exchanger has a wide variation of viscosity with temperature. Other researchers have confirmed that when considering variation of properties it is the viscosity that often has the widest variation with temperature and so the greatest impact on variation of conditions through a heat exchanger [31,32]. The next set of virtual experiments is set up with the PHE exchanging heat between water and oil. Oil has a wide variation of viscosity with temperature.

Table 6.1 VPM versus CPM results: water versus water

m1 (kg/s) - hot water		0.05	1	0.05	0.25	0.5	1	0.5	1.5	2	1.5	1	2
m2 (kg/s) - cold water		0.05	0.05	1	0.25	0.5	0.5	1	1.5	1.5	2	1	2
$T1_{out}$ (°C) (hot water)	CPM	37.69	86.46	19.04	43.96	47.54	63.34	37.33	54.98	60.95	51.34	51.95	57.35
	VPM1	37.96	86.45	19.75	44.19	47.74	63.23	37.94	55.10	60.98	51.53	52.09	57.45
	VPM	37.90	86.44	19.93	44.08	47.61	63.04	38.04	54.94	60.78	51.45	51.94	57.29
$T2_{out}$ (°C) (cold water)	CPM	70.39	89.11	21.55	64.14	60.57	71.51	44.39	53.13	56.88	47.08	56.16	50.76
	VPM1	70.12	89.37	21.52	63.91	60.37	71.69	44.09	53.01	56.82	46.93	56.01	50.66
	VPM	70.18	89.45	21.51	64.02	60.49	72.08	44.04	53.16	57.09	47.00	56.16	50.81
U (W/m^2°C)	CPM	1246	2272	2171	4079	6562	8129	7826	12830	13850	13770	10163	14951
	VPM1	1223	2593	1830	4024	6493	8246	7602	12762	13845	13652	10092	14888
	VPM	1227	2658	1768	4047	6535	8395	7542	12857	14009	13687	10164	15000
NTU	CPM	2.832	5.162	4.930	1.854	1.492	1.848	1.776	0.972	1.050	1.041	1.155	0.850
	VPM1	2.778	5.890	4.156	1.830	1.476	1.874	1.725	0.967	1.049	1.032	1.147	0.846
	VPM	2.790	6.038	4.016	1.840	1.486	1.908	1.711	0.974	1.062	1.034	1.156	0.853
Q (W)	CPM	10949	14871	14840	48211	88939	111838	110254	220169	243674	242985	159456	273742
	VPM1	10899	14937	14704	47987	88555	112275	109035	219483	243440	241844	158889	272999
	VPM	10911	14953	14666	48098	88826	113086	108825	220457	245084	242370	159514	274296
ε	CPM	0.7276	0.9877	0.9855	0.6409	0.5912	0.7432	0.7315	0.4879	0.5400	0.5369	0.5300	0.4550
	VPM1	0.7243	0.9920	0.9764	0.6379	0.5886	0.7461	0.7234	0.4864	0.5394	0.5344	0.5281	0.4538
	VPM	0.7251	0.9931	0.9739	0.6394	0.5904	0.7515	0.7220	0.4886	0.5431	0.5355	0.5302	0.4559
C_r	CPM	0.9984	0.0497	0.0500	0.9979	0.9975	0.4983	0.5010	0.9968	0.7473	0.7522	0.9971	0.9966
	VPM1	0.9984	0.0497	0.0500	0.9978	0.9975	0.4983	0.5010	0.9968	0.7473	0.7522	0.9971	0.9966
	VPM	0.9984	0.0497	0.0500	0.9978	0.9975	0.4983	0.5010	0.9968	0.7474	0.7522	0.9971	0.9966

6.3.2 Water versus oil

The three models (CPM, VPM1 and VPM) are again set up as for the water versus water virtual experiment. The inlet temperature for the hot oil is set at 90°C and the inlet temperature of the cold water is set at 18°C. The fluid properties of water and oil [2] are entered into the Properties worksheet of the three spreadsheets.

As for the previous virtual experiments a range of flow rates is entered into the models and the resulting outlet temperatures and other operating parameters are recorded. Table 6.2, at the end of this section, shows these results for a range of flow rates.

A comparison between the results obtained from the CPM, VPM1 and VPM for the outlet temperatures is given in Figures 6.3 and 6.4, plotted in the same manner as for the water versus water virtual experiment. Now some significant differences are observed. Firstly the variation in outlet temperatures between the CPM and VPM *for the oil* is greater than the variation in outlet temperatures *for the water*, in most cases. This confirms the earlier proposal that fluids with a large variation in viscosity with temperature will give different results if variation in fluid properties is allowed for. Secondly, the results of the VPM are significantly different from the CPM for most of

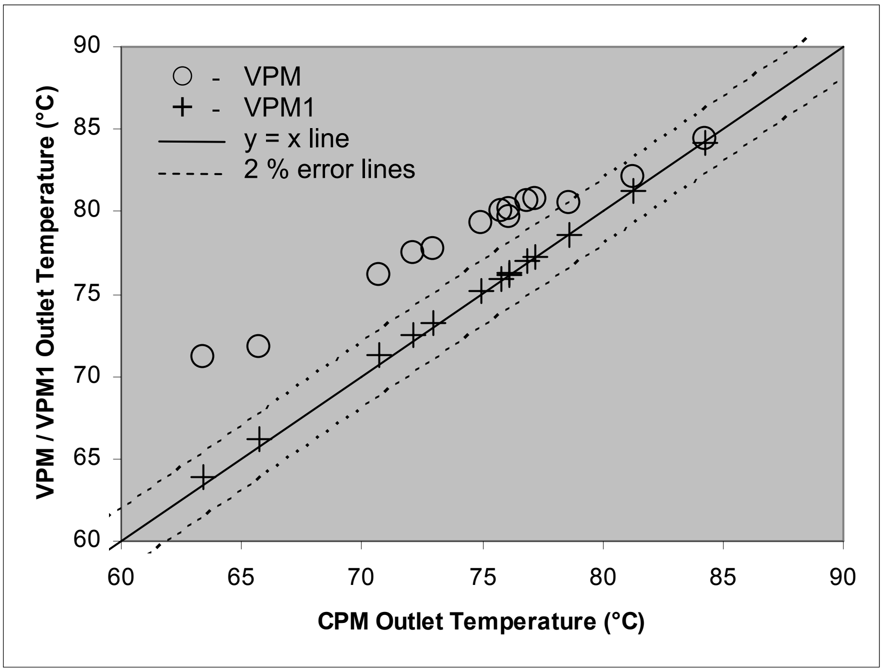


Figure 6.3 Comparison between the VPM and the CPM results for an oil - versus-water PHE: HOT oil

the cases, for the oil. (It is noted that the cases where the results are similar, for the oil, seem to occur when the PHE is operating with NTU values greater than 1 and effectiveness values great than 0.5, see data in Table 6.2). Finally, the results of the VPM are significantly different from the results of the initial VPM1 and in fact the VPM1 results are very close to the CPM results. This indicates that, according to these results, there is no real advantage in allowing for the variation in fluid properties through the heat exchanger in the simulation model unless the variation in viscosity and its effect on the convection heat transfer coefficient *within each element* is also allowed for.

The reason for this variation between the VPM and the CPM results is probably due to the fact that for fluids with a large viscosity variation with temperature there will be a significant variation between the viscosity of the fluid at the wall of an element and the viscosity in the in the "bulk fluid" of the element. This alone will have a significant impact on the convection heat transfer coefficient between the fluid and that wall… that is compared to determining the heat transfer coefficient using fluid properties simply evaluated at the bulk fluid temperature of the element. Added to this, each element, apart from the elements in the leftmost and rightmost channels, is bounded by two walls and these walls will be at different temperatures. So for a fluid with a large viscosity

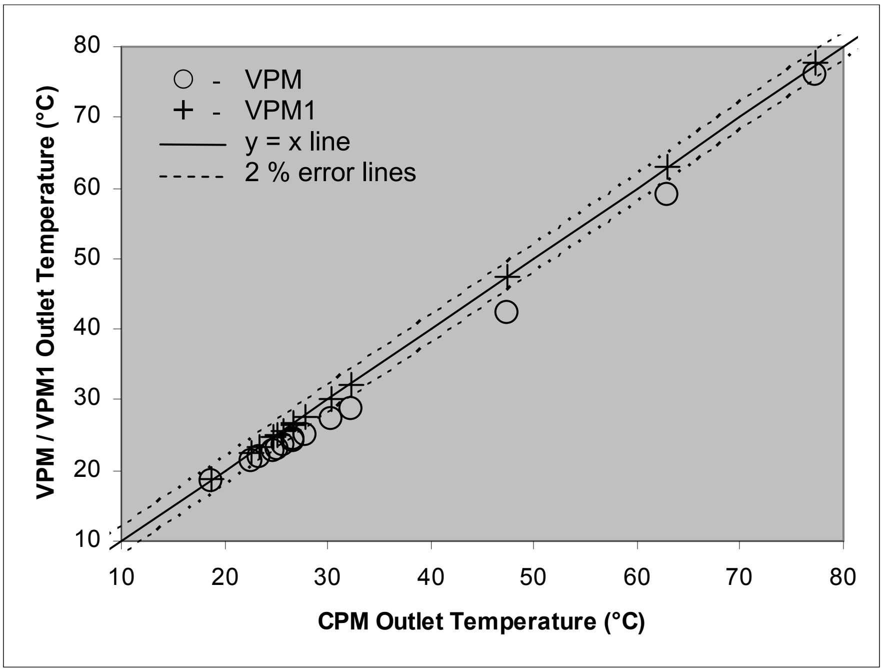


Figure 6.4 Comparison between the VPM and the CPM results for an oil - versus-water PHE: COLD water

variation with temperature the convection heat transfer coefficients between the fluid and the two walls will be different.

Now, with the VPM it is very easy to confirm proposed explanations like the one above because the various parameters (wall temperatures, viscosities, local convection heat transfer coefficients etc) are all stored in ranges of cells on the spreadsheet. So, immediately one can observe any differences by perusing these ranges of parameter values or by using the Chart Wizard built in to Microsoft Excel® it is possible to quickly plot charts of the desired parameters. For example, with the VPM set up as described in this section and with a hot oil flow rate of 0.05 kg/s and a cold water flow rate of 1 kg/s the variation of bulk fluid and wall temperatures for flow channels near the middle of the PHE are shown in Figures 6.5 and 6.6. As can be seen from these graphs the variation between the wall temperatures and the bulk fluid temperature is minimal for the water (less than 1°C) but is significantly large for the oil (~ 50 – 70°C!). This in turn has an effect on the viscosity, the viscosity ratio (μ/μ_w) and hence the convection heat transfer coefficient. However, at least for the channels shown, the differences between the *left and right wall temperatures* is minimal for both fluids. Setting up these charts using the Chart Wizard in Microsoft Excel® can be done quickly and simply.

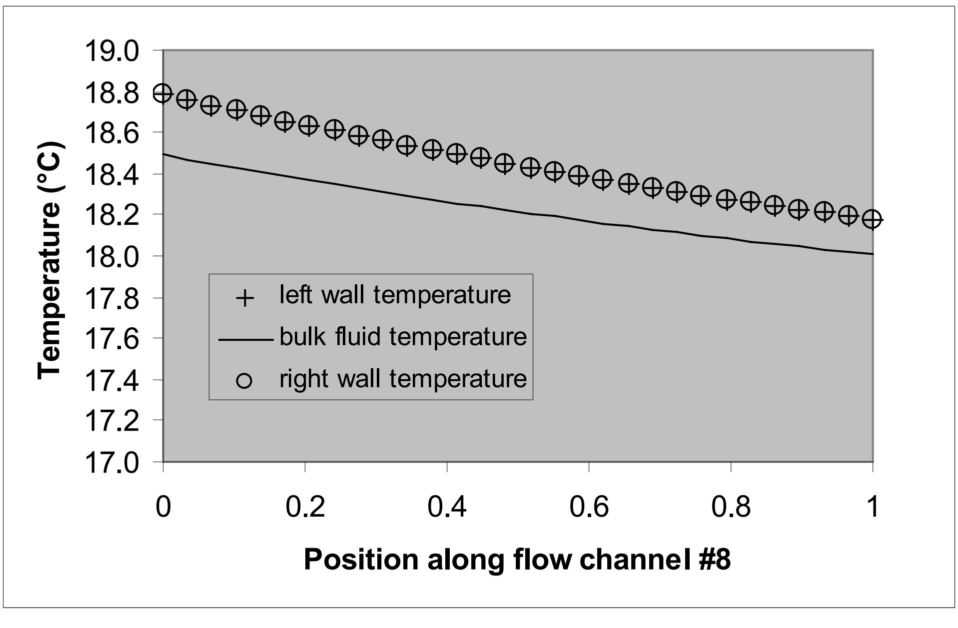


Figure 6.5 Variation in temperature of WATER along flow channel #8

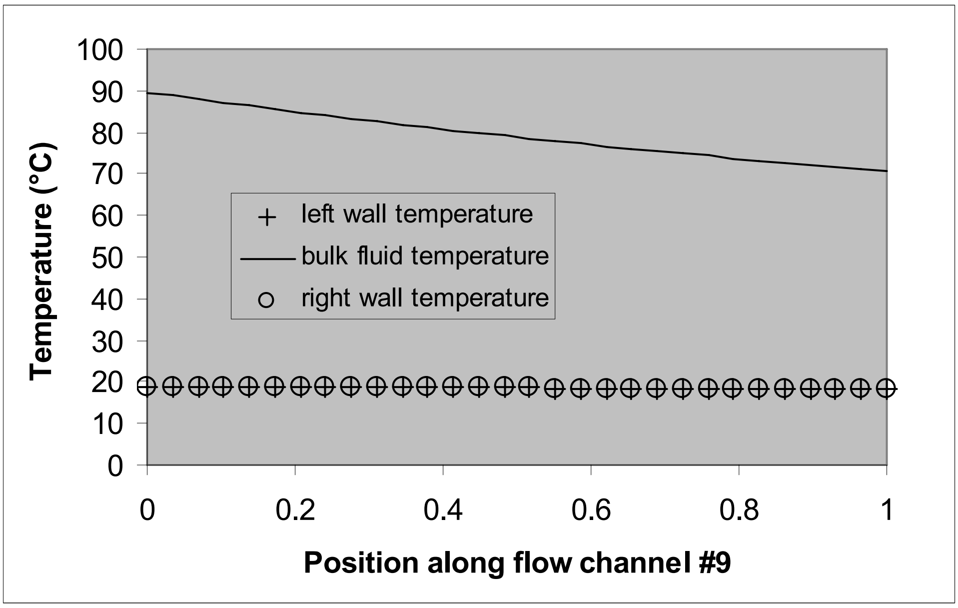


Figure 6.6 Variation in temperature of OIL along flow channel #9

This confirms that for certain fluids, for example, those that have a large variation in viscosity with temperature, the VPM will give different and more accurate results than the CPM. It also confirms that, for those fluids which do not have a wide variation of viscosity with temperature, making the assumption that fluid properties remain constant through the heat exchanger will probably give adequately accurate results.

Table 6.2 VPM versus CPM results: water versus oil (fluid-1)

m1 (kg/s) – hot oil		0.05	1	0.05	0.25	0.5	1	0.5	1.5	2	1.5	1	1
m2 (kg/s) – cold water		0.05	0.05	1	0.25	0.5	0.5	1	1.5	1.5	2	0.1	0.2
$T1_{out}$ (°C) (hot oil)	CPM	65.75	84.26	63.37	70.76	72.93	76.09	72.11	76.09	77.21	75.80	81.29	78.60
	VPM1	66.18	84.22	63.91	71.26	73.26	76.19	72.54	76.22	77.28	75.95	81.25	78.58
	VPM	71.75	84.39	71.17	76.14	77.78	79.65	77.51	80.11	80.78	80.01	82.05	80.54
$T2_{out}$ (°C) (cold water)	CPM	30.31	77.35	18.67	27.81	26.73	32.27	22.57	25.13	26.75	23.46	62.92	47.34
	VPM1	30.09	77.75	18.66	27.56	26.56	32.16	22.46	25.07	26.71	23.40	63.12	47.37
	VPM	27.32	76.01	18.48	25.11	24.28	28.65	21.21	23.09	24.33	21.86	59.01	42.37
U (W/m^2°C)	CPM	102	832	104	379	655	1090	667	1540	1896	1555	942	1020
	VPM1	100	855	102	368	641	1082	650	1526	1887	1538	953	1023
	VPM	71	786	68	255	442	765	442	1046	1306	1046	805	790
NTU	CPM	0.457	1.891	0.467	0.338	0.291	0.248	0.297	0.227	0.210	0.230	1.071	0.580
	VPM1	0.447	1.942	0.456	0.327	0.285	0.246	0.289	0.225	0.209	0.227	1.083	0.581
	VPM	0.316	1.786	0.305	0.226	0.196	0.174	0.196	0.154	0.144	0.154	0.915	0.449
Q (W)	CPM	2573	12406	2819	10257	18247	29820	19100	44730	54900	45660	18770	24513
	VPM1	2528	12497	2762	9995	17899	29603	18646	44319	54602	45174	18865	24550
	VPM	1948	12134	2009	7432	13126	22262	13409	31941	39722	32249	17143	20374
ε	CPM	0.3368	0.8243	0.3699	0.2672	0.2371	0.1982	0.2484	0.1931	0.1776	0.1972	0.6238	0.4074
	VPM1	0.3307	0.8303	0.3622	0.2602	0.2325	0.1967	0.2424	0.1913	0.1766	0.1951	0.6270	0.4081
	VPM	0.2534	0.8062	0.2615	0.1925	0.1697	0.1479	0.1734	0.1374	0.1280	0.1387	0.5698	0.3386
C_r	CPM	0.5076	0.0967	0.0253	0.5101	0.5112	0.9746	0.2553	0.5128	0.6846	0.3845	0.1938	0.3887
	VPM1	0.5078	0.0967	0.0253	0.5104	0.5114	0.9745	0.2555	0.5129	0.6847	0.3845	0.1939	0.3887
	VPM	0.5106	0.0966	0.0255	0.5129	0.5137	0.9712	0.2567	0.5149	0.6871	0.3861	0.1937	0.3879

6.4 Educational Uses

Although it appears from the discussion in section 6.3 that the advantage of using the VPM only applies when one or both of the fluids exhibits a viscosity (or other fluid property) that varies greatly with temperature, there are significant advantages in using the VPM in an educational environment. In this section, different ways that the VPM can be used in an educational environment are presented. The VPM can be used to give the student an overall appreciation and understanding of the operation of a PHE. It can also be used to give the student greater insight into the finer, detailed workings of a PHE.

6.4.1 Understanding Overall PHE Operation

A number of different student experiments and investigations are presented in this section to help the student gain a better understanding of the overall operation of a PHE. These investigations are varied and numerous and so only brief details are given in terms of how the investigation could be carried out.

6.4.1.1 Effect of Primary inputs: flow rates, temperatures, fluid properties

The aim of this student investigation is to verify to the student that the VPM is a valid model for a given, actual PHE, and then to help the student get a feel for how that PHE will perform under different operating conditions, by varying different input parameters in the VPM. In using the VPM the student is able to “try out” a wider range of parameter and configuration variations than is possible with a specific PHE. The student experiments presented here are written with our own PHE heat transfer rig (at AUT) in mind however they could equally apply to any PHE set up for student experiments in any educational institution.

The student carries out a laboratory experiment with the actual, physical PHE for a specified range of inlet conditions. The experimental procedure followed could be much the same as the experimental procedure described in Chapter 4. The VPM is set up to simulate the actual PHE and the student enters equivalent input conditions as those for the laboratory experiment. The student compares the results of the lab experiment with those from the VPM. This should confirm to the student the validity of using the VPM as a means of investigating PHE's (at least the specific PHE in the lab experiments).

In the second part of this investigation the student uses the VPM to investigate the effect of changing the various *primary* inputs (inlet temperatures, fluid flow rates and fluid properties) on the performance of the PHE (performance being in terms of the following outputs: $\dot{Q}$, U, NTU, ε, $T1_{OUT}$, $T2_{OUT}$, C_r and ΔP). Leading questions are posed, with suggestions on how the student can vary the parameters to answer those questions:

- For a given set of fluid inlet temperatures, does increasing the flow rate increase overall heat transfer coefficient and the heat transferred? Is there a limit to the heat transferred? What happens to the pressure drop as the overall heat transfer coefficient increases and why is this so?
- If the hot and cold fluid inlet temperatures are close together how does this affect performance? What if the inlet temperatures are far apart?
- Is there a difference in performance of the PHE if the mass flow rates of the hot and cold fluids are similar or very different?
- Is there some relationship between NTU and effectiveness for this heat exchanger?
- What factors seem to affect the value of NTU and effectiveness for a heat exchanger?
- If a high viscosity fluid (eg oil) is used as one fluid, how does this effect the performance of the PHE?
- Oils typically have specific heat values around half that of water… how does this affect the performance of the PHE?
- Is it possible to determine when a heat exchanger is operating at optimum performance?

Many other questions could be asked along a similar line and could be tailored to whatever the specific heat transfer course or course of study is trying to emphasise.

6.4.1.2 Effect of Secondary inputs

In this student investigation the VPM is used in a similar way to the previous investigation but now to investigate how the performance of the PHE (outputs: $\dot{Q}$, U, NTU, ε, $T1_{OUT}$, $T2_{OUT}$, C_r and ΔP) is affected by the other *secondary* inputs. These secondary inputs being: plate parameters (β and ϕ, ie corrugation characteristics), plate spacing, plate size and thickness, Nusselt number coefficients and friction factor coefficients. Again, leading questions are posed, with suggestions on how the student can vary the parameters to answer those questions:

- For a given set of flow rates and inlet temperatures does changing the angle of corrugation, β on the plates affect performance? Take special note of heat duty and pressure drop. Is there an optimum corrugation angle for those conditions? (Vary β between 0 and 90°).
- Does varying the plate surface area enlargement factor, ϕ have any real effect on performance? (Vary ϕ between 1 and 1.5, 1 = flat plate).
- Does plate thickness have any real effect on performance? (Try halving the thickness, doubling the thickness).
- Does the plate spacing affect performance? (Try halving the spacing, doubling the spacing).
- Provide a range of Nusselt number coefficients (ones that have been published in the literature) and see what effect they have on predicted performance compared to the original VPM. Try different Nusselt number coefficients for the different fluids and observe what effect these have on performance.

6.4.1.3 Further Student Investigations

Additional student investigations could be set up with the following aims:

- To investigate the effect of changing the number of plates. Set up several VPM's with a range of different numbers of plates, say 3 plates, 11 plates, 21 plates and provide the same inputs (fluids, flow rates, temperatures etc) to each VPM and observe the effect on PHE performance. (Possible Question: Is there some point where adding more plates has no significant improvement in performance and is this related to the NTU and effectiveness?)

- To investigate the effect of different flow configurations. Set up several VPM's with different flow configurations (series, parallel, series-parallel and parallel-series) and again provide the same inputs to each VPM and observe the effect on PHE performance. (Possible question: what are the significant differences in performance between series and parallel, especially in terms of heat duty, $\dot{Q}$ and pressure drop, ΔP?)

- To investigate the constant properties assumption. Set up the CPM with the same configuration and inputs as the VPM, in a similar way to that described in section 6.3. The student produces results from a water versus water case for both the VPM and CPM and again for a water versus oil case. Then observe any differences and draw conclusions.

6.4.2 Understanding Detailed Operation of a PHE

In the previous section (6.4.1) different investigations are suggested to help the student gain understanding in the overall operation of a PHE. However, one of the significant advantages of the VPM is that it allows the student to see the inner workings of a PHE and so gain an even greater understanding of its operation. An advantage of setting up the VPM in a spreadsheet is the student can easily access and view the underlying formulae that are used in the spreadsheet and observe all of the parameters that influence the performance of the PHE, and how they vary through the PHE during steady state operation. The following suggested student investigations could be carried

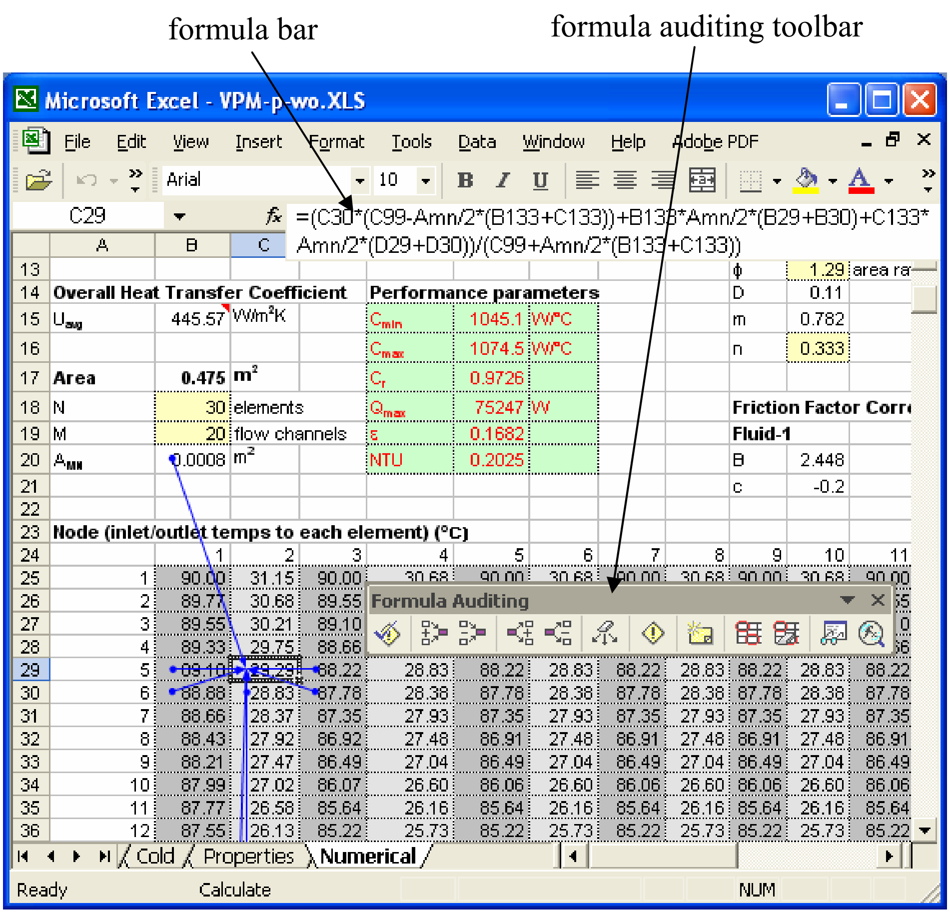


Figure 6.7 Investigating underlying formulae in the VPM

out on their own or in conjunction with the student investigations described in the previous section.

- To investigate the formulae used to set up the VPM. The student could be presented with the theory relating to the heat transfer equation and simple energy balances as applied to each element in the VPM and investigate how these are set up in each cell on the spreadsheet. The formula bar and the formula auditing toolbar in Microsoft Excel (see Figure 6.7) could be used to assist with this. The formula bar shows the equation and the formula auditing toolbar can be used to graphically show cell dependencies. Used together they can help the student "see" what parameters are used in each formula. In Figure 6.7 the blue lines indicate the dependent cells for the high-lighted cell.

- To investigate the variation of properties and parameters through the PHE during steady state operation. Most of the parameters relating to the properties and operation of the PHE are stored element by element in the spreadsheet. These parameters include node temperatures ($T1_{i,j}$ and $T2_{i,j}$), average temperatures ($T1_{i,j\ \mathrm{avg}}$ and $T2_{i,j\ \mathrm{avg}}$), wall temperatures ($T_{WL\ i,j}$ *and* $T_{WR\ i,j}$), Reynolds numbers ($\mathrm{Re}_{i,j}$), element overall heat transfer coefficients ($U_{i,j}$), element convection heat transfer coefficients for left and right walls ($h_{\mathrm{L}\ i,j}$ and $h_{\mathrm{R}\ i,j}$), element heat gain/loss ($C\Delta T_{i,j}$), heat capacity rate ($C1_{i,j}$ and $C2_{i,j}$), friction factor ($f1_{i,j}$ and $f2_{i,j}$), element pressure drop ($\Delta P1_{i,j}$ and $\Delta P2_{i,j}$) and of course fluid properties (density, specific heat, thermal conductivity, Prandtl number, bulk fluid viscosity and wall viscosities). Since these parameters are stored element by element in the spreadsheet it enables the student to observe the numerical values (see Figures 3.2 – 3.4) or to plot them as required.

  For example, the student could plot the variation of fluid temperature by distance through a specific flow channel (see Figures 6.5 and 6.6 for example) or the student could produce a 3-D plot of the variation of a parameter through all flow channels for a specific fluid as shown in Figure 6.8. Various conclusions can be drawn from these plots that help the student understand more about the inner workings of the PHE. For example, in Figure 6.8 it can be seen that the temperature variation through the flow channels in a parallel flow PHE is reasonable consistent apart from the first and last flow channels. This means for a parallel flow PHE with a large number of plates (say >50) the effect of the different temperature variation in

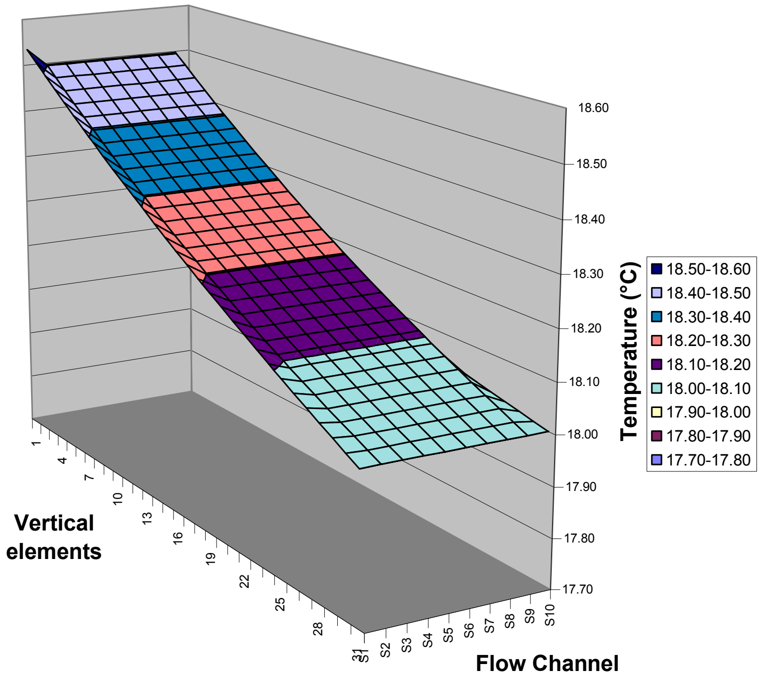


Figure 6.8 3-D (Surface) plot of temperature variation of a fluid through a parallel flow PHE

the first and last flow channels is going to be insignificant, while for a PHE with a small number of plates (say <10) the effect is going to be significant. Therefore for a parallel flow PHE with a large number of plates, it is going to behave similarly to a classic double pipe counter-flow heat exchanger and this is confirmed in the literature [12].

- To compare the usage of a simple average temperature difference in the heat transfer equation for each element of the discretized PHE versus the more correct log mean temperature difference. In this simple investigation the student could choose any four node temperatures representing the heat transfer situation between two adjacent elements in the PHE and calculate the average temperature difference and compare it to the log mean temperature difference. This should confirm that there is very little difference between the two for small temperature differences.

In summary, the ways that the VPM could be used are really only limited by the imagination of the tutor or user.

6.5 Instructions for VPM Operation

This part of the chapter gives instructions on how to use the VPM. It is effectively the "users manual" for the VPM software. For the purposes of this investigation the following points are assumed:

- The VPM and CPM are initially set up to generally simulate the PHE in the heat transfer rig at AUT. It is possible to modify the spreadsheet to allow for more or less plates, but the instructions for this procedure are not included here.
- The users are familiar with computers and have a working knowledge of Microsoft Excel

The purpose of presenting the manual in this section is to give a basic idea of how to use the software, hence it is written in an abbreviated style.

6.5.1 Getting Started

Requirements

Microsoft Excel (Excel 97, Excel 2000 or Excel XP are all acceptable).

A Pentium based computer that meets the requirements to run Microsoft Excel.

Software location

The VPM/CPM software is stored in three folders depending on the flow configuration. The folders are named "parallel", "series" and "combination" according to the flow configuration. Within each folder are two files, one with a filename beginning "VPM…" for the VPM software and one with a filename beginning "CPM…" for the CPM software.

Starting the VPM/CPM software

Start Microsoft Excel

Select File, Open from the menu and navigate to the folders where the software is located and open the desired version of the software (VPM or CPM with desired flow configuration)

Save a copy of the opened file to another work folder. (This way a "good" copy of the original software is always kept intact)

6.5.2 User Interface

The user interface for the VPM and CPM software consists of an Excel Workbook with a number of sheets. The "Numerical" worksheet contains the main calculation area and is where most of the user input data is entered. The cells are colour coded as an aid to usage: yellow for data input, green with red text for outputs and calculated performance parameters, light grey / grey for ranges of cells containing parameters calculated for each element through the PHE eg element node temperatures (See Figure 6.9). Most of the remaining cells with no colour (white) contain helpful information, headings and results of intermediate calculations. The blue coloured cells are part of the calculation procedure and are explained in the section on calculation procedures.

A graphic of a PHE is included on the Numerical worksheet which reminds the user of the flow configuration for this file.

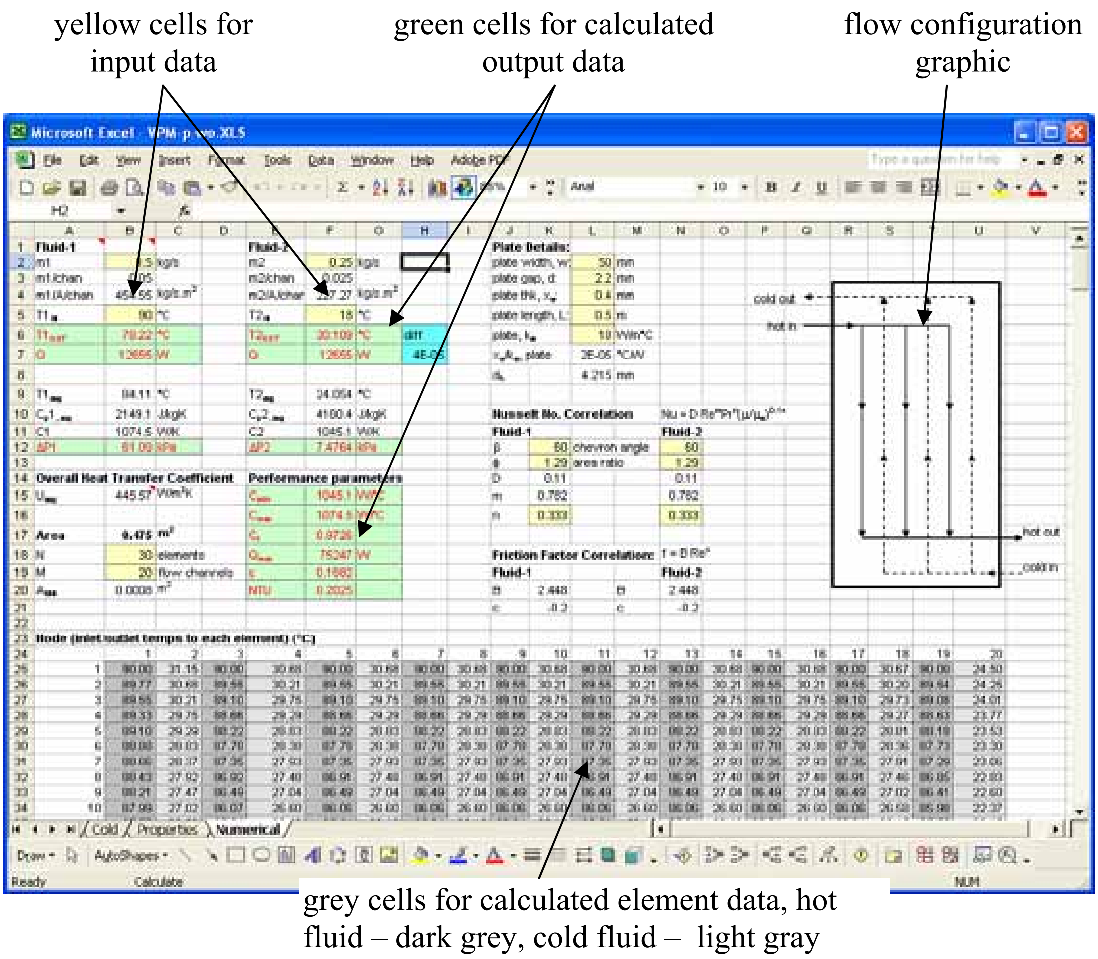


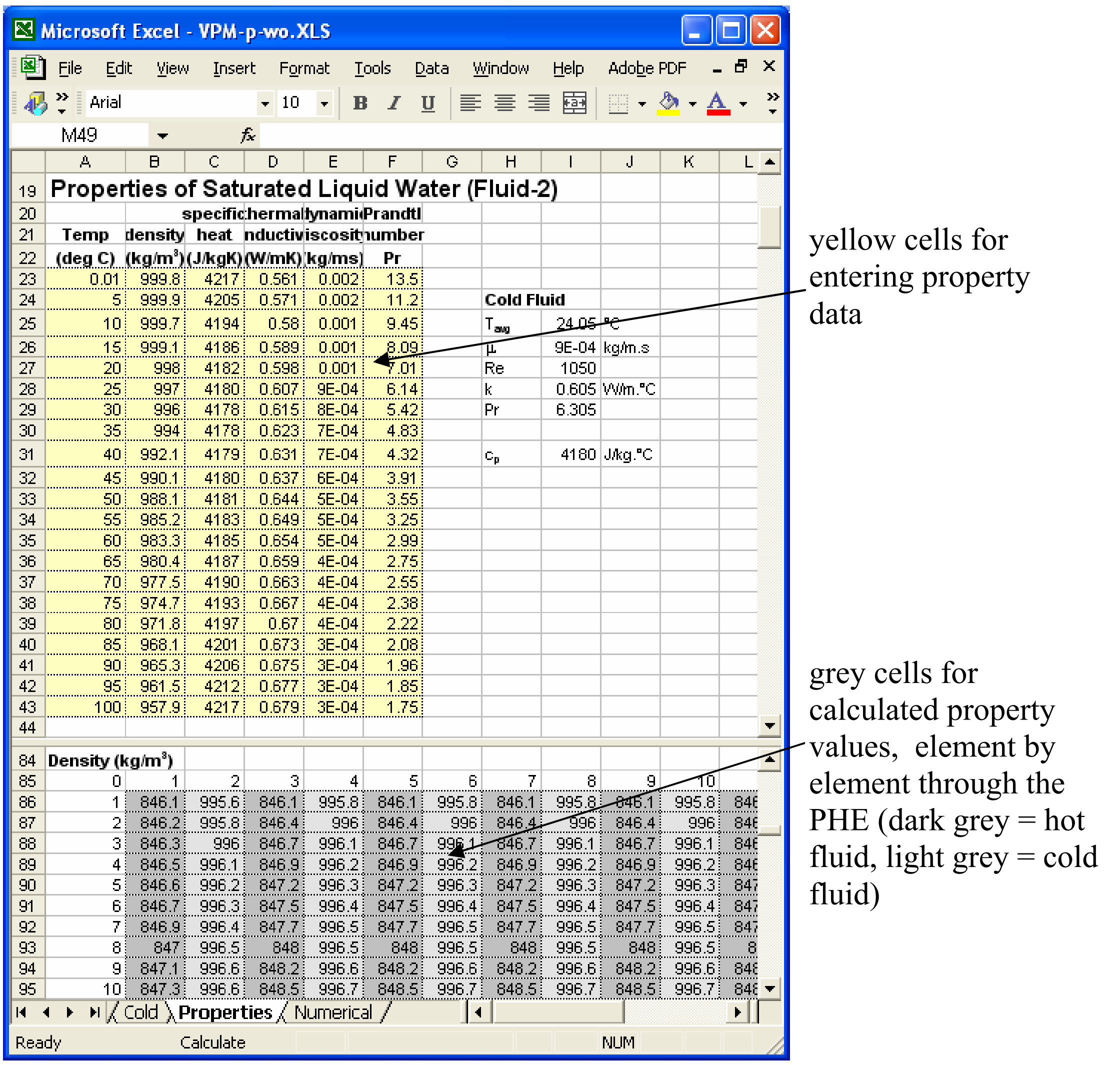
Figure 6.9 User Interface – colour coding of cells on Numerical worksheet

The red triangles in the corner of some cells indicate that that cell contains a helpful comment. Move the mouse pointer over such a cell and the helpful comment will pop-up.

The Properties worksheet contains data for the properties of the two fluids flowing through the PHE. The cells are again colour coded in a similar way to the Numerical worksheet, see Figure 6.10. Yellow cells for entering property data versus temperature, light grey cells for calculated property values in each element of the PHE.

6.5.3 Calculation Procedure and Entering Input Data

The VPM / CPM spreadsheet has been set up so that all the input data is entered and then the calculation procedure is initiated manually by pressing the F9 function key



Temp (deg C)	density (kg/m³)	specific heat (J/kgK)	thermal nductivi (W/mK)	dynamic viscosit (kg/ms)	Prandtl number Pr
0.01	999.8	4217	0.561	0.002	13.5
5	999.9	4205	0.571	0.002	11.2
10	999.7	4194	0.58	0.001	9.45
15	999.1	4186	0.589	0.001	8.09
20	998	4182	0.598	0.001	7.01
25	997	4180	0.607	9E-04	6.14
30	996	4178	0.615	8E-04	5.42
35	994	4178	0.623	7E-04	4.83
40	992.1	4179	0.631	7E-04	4.32
45	990.1	4180	0.637	6E-04	3.91
50	988.1	4181	0.644	5E-04	3.55
55	985.2	4183	0.649	5E-04	3.25
60	983.3	4185	0.654	5E-04	2.99
65	980.4	4187	0.659	4E-04	2.75
70	977.5	4190	0.663	4E-04	2.55
75	974.7	4193	0.667	4E-04	2.38
80	971.8	4197	0.67	4E-04	2.22
85	968.1	4201	0.673	3E-04	2.08
90	965.3	4206	0.675	3E-04	1.96
95	961.5	4212	0.677	3E-04	1.85
100	957.9	4217	0.679	3E-04	1.75

0	1	2	3	4	5	6	7	8	9	10
1	846.1	995.6	846.1	995.8	846.1	995.8	846.1	995.8	846.1	995.8
2	846.2	995.8	846.4	996	846.4	996	846.4	996	846.4	996
3	846.3	996	846.7	996.1	846.7	996.1	846.7	996.1	846.7	996.1
4	846.5	996.1	846.9	996.2	846.9	996.2	846.9	996.2	846.9	996.2
5	846.6	996.2	847.2	996.3	847.2	996.3	847.2	996.3	847.2	996.3
6	846.7	996.3	847.5	996.4	847.5	996.4	847.5	996.4	847.5	996.4
7	846.9	996.4	847.7	996.5	847.7	996.5	847.7	996.5	847.7	996.5
8	847	996.5	848	996.5	848	996.5	848	996.5	848	996.5
9	847.1	996.6	848.2	996.6	848.2	996.6	848.2	996.6	848.2	996.6
10	847.3	996.6	848.5	996.7	848.5	996.7	848.5	996.7	848.5	996.7

Figure 6.10 User Interface – colour coding of cells on Properties worksheet

(normally a spreadsheet automatically recalculates the entire spreadsheet whenever a value in a cell is changed). The spreadsheet has also been set up for iterations so that the spreadsheet recalculates all formulae until convergence occurs. Convergence is defined as the difference between successive values in any one cell being less than 0.001, from one iteration to the next. The maximum number of iterations has been set to 200. (Note: manual calculation and iteration parameters can be altered - from Excel menu select Tools, Options, Calculations).

So, overall method of usage is to enter all the input data into the VPM / CPM, including fluid properties and then press the F9 function key to initiate iterations. Iterations will stop when successive values are less than 0.001, which indicates that convergence has occurred.

The blue coloured cells labelled "diff" on the Numerical worksheet (see Figure 6.9) indicate the difference in values from one iteration to the next. In some cases more than 200 iterations are required and this is indicated if the iterations stop and the value in the blue "diff" cell is greater than 0.001. In this case just press the F9 function key for a second time to initiate another 200 iterations… again, the iterations will stop when the difference between successive values is less than 0.001.

Entering Data

Input data is entered into the yellow cells in the Numerical worksheet and the Properties worksheet and includes the following:

Fluid flow data: $\dot{m}1, T1_{IN}, \dot{m}2, T2_{IN}$

PHE physical data: A, M, N, d, x_w, k_w, w, L

Correlations: D, m, n, B, c (β and ϕ)

Fluid property tables (variation with temperature): ρ, μ Cp, k, Pr

Simply select the required cell, type in the new data and press the Enter key

Note: Normally, whichever is the hot fluid is set as "Fluid-1" and whichever is the cold fluid is set as "Fluid-2". If this is not adhered to, the calculations will still be correct, but the heat transferred in the PHE (heat duty) will display as a negative number.

Once all the data has been entered it is a good idea to save the file prior to initiating calculations. Remember to save the file in a different folder than the one containing the original VPM / CPM software. The calculations can then be initiated by pressing the F9 function key. In most cases convergence occurs (model gives results) after 10 – 15 seconds of iterations, although this depends on the number of flow channels (plates) in the specific model. At this point the output data can be observed (see next section) and/or further changes can be made to the input data and the output results recalculated using the F9 function key as described above.

6.5.4 Output and Results

The main output information from the VPM / CPM is shown on the Numerical worksheet, in the green coloured cells with red text. (See Figure 6.9). These outputs include: $\dot{Q}$, U, $T1_{\mathrm{OUT}}$, $T2_{\mathrm{OUT}}$, ε, NTU, $C_{\min}$, $C_{\max}$, C_{r}, $\Delta P1$, $\Delta P2$.

Also the grey coloured ranges of cells (matrices) show various parameters element by element through the PHE. The darker grey cells are for fluid-1 (the hot fluid) and the lighter grey cells are for fluid-2 (the cold fluid). Columns are flow channels and rows are the vertical discretization of the PHE. Parameters shown element by element on the Numerical worksheet of the VPM include: $T1_{i,j}$, $T2_{i,j}$, $\mathrm{Re}_{i,j}$, $C1_{i,j}$, $C2_{i,j}$, $U_{i,j}$, $Q_{i,j}$, $T_{\mathrm{WL}i,j}$, $T_{\mathrm{WR}i,j}$, $h_{\mathrm{L}i,j}$, $h_{\mathrm{R}i,j}$, $f1_{i,j}$, $f2_{i,j}$, $\Delta P1_{i,j}$, $\Delta P2_{i,j}$. Use the sliders on the right hand side of the Excel window to drag down and view the various parameter ranges (matrices).

The parameters shown element by element on the Properties worksheet include: $T1_{i,j\ \mathrm{avg}}$, $T2_{i,j\ \mathrm{avg}}$ and also ρ, μ, μ_{w}, Cp, k, Pr. Again, use the sliders on the right hand side of the Excel window to drag down and view the various parameter ranges (matrices).

Any of the input or output data can be copied and pasted to other Microsoft Windows® applications using normal Microsoft Windows® techniques. Also, any of the parameter variation data in the grey cells can be graphed using the normal Microsoft Excel® graphing techniques. The variation of fluid temperature through the PHE has been set up in two graphs already. The variation in temperature of Fluid-1 (normally set as the hot fluid) is found as a 3-D graph on the “Hot” worksheet. The variation in temperature of Fluid-2 (normally set as the cold fluid) is found as a 3-D graph on the “Cold” worksheet.

6.5.5 Troubleshooting

Sometimes if an item of input data is entered incorrectly (eg a negative number or outside the range of valid values for the VPM) then the output cells on the VPM spreadsheet will fill up with error messages generated by Excel. This situation is impossible to recover from, on the current spreadsheet, due to the fact that the spreadsheet is set up for iterations. The only way to resolve this situation is to close this file and open a fresh VPM file. If you have saved the file prior to initiating calculations then reopen the file and correct the incorrect data entry.

6.6 Conclusion and Recommendations

The focus of this thesis has been to develop a software model or simulation of a plate heat exchanger that takes into account the variation of properties through the heat exchanger and allows for the effect of the variation in viscosity between the wall and the bulk fluid on the convection heat transfer coefficient. The reason for developing this model has been to provide an accurate simulation of a PHE so that students can use it in an educational environment to investigate the overall and detailed operation of a typical item of heat transfer equipment, namely a plate heat exchanger.

This model has been successfully developed and validated using experimental data representing a range of different PHE sizes, flow configurations, fluid types and flow conditions. This means that it can be adapted to any particular PHE for further student investigation. Instructions have been also provided on how it can be used in an educational environment to assist students to discover more about the general and detailed operation of a PHE.

Recommendations for future development include:

1. Look at the viability of setting up the model in a true programming language (eg FORTRAN, Visual Basic). Advantages would be that the program could be made to be more robust (ie better error checking, less likely to crash, faster operation etc) and the one program could cover all flow configurations (rather than having separate spreadsheets for the different flow configurations). Disadvantages would be that it would be difficult to “see” the calculations and parameter matrices.

2. Automatic selection of Nusselt number and friction factor correlations depending on flow conditions. At present these are selected and input by the user.
3. Get student feedback on its ease of use, helpfulness etc. in an effort to improve its usability.
4. Allow for uneven flow distribution as proposed by Rao [33] and observe affect on performance.
5. Look at experimentally validating the pressure drop calculations.

REFERENCES

1. Mills AF. *Heat transfer*. Homewood: Irwin; 1992.

2. Çengel YA. *Heat transfer : a practical approach*. 2nd ed. Dubuque, Iowa: McGraw-Hill; 2002.

3. Wolf J. *General solution of the equations of parallel-flow multichannel heat exchangers*. International Journal of Heat and Mass Transfer 1963;7:901-919.

4. Zaleski T. *A general mathematical-model of parallel-flow, multichannel heat-exchangers and analysis of its properties*. Chemical Engineering Science 1984;39(7-8):1251-1260.

5. Marano JJ, Jechura JL. *Analysis of heat transfer in plate heat exchangers*. AIChE Symposium Series 1985;81(245):116-121.

6. Zaleski T, Klepacka K. *Approximate method of solving equations for plate heat-exchangers*. International Journal of Heat and Mass Transfer 1992;35(5):1125-1130.

7. Ribeiro CP, Andrade MHC. *An algorithm for steady-state simulation of plate heat exchangers*. Journal of Food Engineering 2002;53(1):59-66.

8. McKillop AA, Dunkley WL. *Plate heat exchangers: flow characteristics, heat transfer*. Industrial and Engineering Chemistry 1960;52(9):733-744.

9. Jackson BW, Troupe RA. *Plate heat exchanger design by the ε-NTU method*. Chemical Engineering Progress Symposium Series 1966;62(64):185-190.

10. Buonopane RA, Troupe RA, Morgan JC. *Heat transfer design method for plate heat exchangers*. Chemical Engineering Progress 1963;59(7):57-61.

11. Wetter M. *Simulation model: air-to-air plate heat exchanger*. California: Simulation Research Group Building Technologies Department, Environmental Energy Technologies Division, Lawrence Berkeley National Laboratory; 1999 January 1999. Report No.: LBNL-42354.

12. Kandlikar SG, Shah RK. *Asymptotic effectiveness-NTU formulas for multipass plate heat exchangers*. Transactions of the ASME Journal of Heat Transfer 1989;111:314-321.

13. Chaudhuri AR, Seetharamu KN, Sundararajan T. *Modelling of steam surface condenser using finite element methods*. Communications in Numerical Methods in Engineering 1997;13(12):909-921.

14. Ranganayakulu C, Seetharamu KN. *The combined effects of wall longitudinal heat conduction, inlet fluid flow nonuniformity and temperature nonuniformity in compact tube-fin heat exchangers: a finite element method*. International Journal of Heat and Mass Transfer 1999;42(2):263-273.

15. Diaz G, Sen M, Yang KT, McClain RL. *Simulation of heat exchanger performance by artificial neural networks*. Hvac&R Research 1999;5(3):195-208.

16. Kaga K, Yamada K, Kotoh S, Ogushi T. *Predicting the capacity of a heat exchanger by a thermal network method*. Heat Transfer - Asian Research 2002;31(2):128-140.

17. Ribando RJ, Oleary GW, CarlsonSkalak S. *General numerical scheme for heat exchanger thermal analysis and design*. Computer Applications in Engineering Education 1997;5(4):231-242.

18. Das SK, Spang B, Roetzel W. *Dynamic behavior of plate heat exchangers--experiments and modeling*. Journal of Heat Transfer 1995;117:859-864.

19. Shelden RA, Dunn IJ. *Dynamic simulation - modelling processes, the environment, the world*. Chemical Engineering Progress 2001 December 2001:44-48.

20. Das SK, Roetzel W. *Dynamic analysis of plate heat exchangers with dispersion in both fluids*. International Journal of Heat and Mass Transfer 1995;38(6):1127-1140.

21. Roetzel W, Das SK. *Hyperbolic axial dispersion model: concept and its application to a plate heat exchanger*. International Journal of Heat and Mass Transfer 1995;38(16):3065-3076.

22. Das SK, Murugesan K. *Transient response of multipass plate heat exchangers with axial thermal dispersion in fluid*. International Journal of Heat and Mass Transfer 2000;43(23):4327-4345.

23. Blomerius H, Holsken C, Mitra NK. *Numerical investigation of flow field and heat transfer in cross-corrugated ducts*. Journal of Heat Transfer 1999;121(2):314-321.

24. Ciofalo M, Di Piazza I, Stasiek JA. *Investigation of flow and heat transfer in corrugated-undulated plate heat exchangers*. Heat and Mass Transfer 2000;36(5):449-462.

25. Heggs PJ, Sandham P, Hallam RA, Walton C. *Local transfer coefficients in corrugated plate heat exchanger channels*. Chemical Engineering Research & Design 1997;75(A7):641-645.

26. Charre O, Jurkowski R, Bailly A, Meziani S, Altazin M. *General model for plate heat exchanger performance prediction*. Journal of Enhanced Heat Transfer 2002;9(5-6):181-186.

27. Muley A, Manglik RM. *Enhanced heat transfer characteristics of single-phase flows in a plate heat exchangers with mixed chevron plates*. Journal of Enhanced Heat Transfer 1997;4(3):187-201.

28. Muley A, Manglik RM. *Experimental study of turbulent flow heat transfer and pressure drop in a plate heat exchanger with chevron plates*. Journal of Heat Transfer-Transactions of the Asme 1999;121(1):110-117.

29. Muley A, Manglik RM, Metwally HM. *Enhanced heat transfer characteristics of viscous liquid flows in a chevron plate heat exchanger*. Journal of Heat Transfer-Transactions of the Asme 1999;121(4):1011-1017.

30. Talik AC, Swanson LW, Fletcher LS, Anand NK. *Heat transfer and pressure drop characteristics of a plate heat exchanger*. In: Proceedings of the 1995 ASME/JSME Thermal Engineering Joint Conference. Part 4 (of 4), Mar 19-24 1995; 1995; Maui, HI, USA: ASME, New York, NY, USA; 1995. p. 321-329.

31. Sieder EN, Tate GE. *Heat transfer and pressure drop of liquids in tubes*. Industrial and Engineering Chemistry 1936;28:1428-1935.

32. Buyukalaca O, Jackson JD. *The correction to take account of variable property effects on turbulent forced convection to water in a pipe*. International Journal of Heat and Mass Transfer [H.W. Wilson - AST] 1998;41(4-5):665 - 669.

33. Rao BP, Kumar PK, Das SK. *Effect of flow distribution to the channels on the thermal performance of a plate heat exchanger*. Chemical Engineering and Processing 2002;41(1):49-58.

34. Bassiouny MK, Martin H. *Flow distribution and pressure-drop in plate heat-exchangers. 2. Z-type arrangement*. Chemical Engineering Science 1984;39(4):701-704.

35. Bassiouny MK, Martin H. *Flow distribution and pressure-drop in plate heat-exchangers.1. U-type arrangement*. Chemical Engineering Science 1984;39(4):693-700.

36. Sitku G. *Algorithm of a computer program to determine the minimum number of plates in plate-type heat exchangers*. Energy Engineering 2002;99(4):74.

37. Wang LK, Sunden B. *Optimal design of plate heat exchangers with and without pressure drop specifications*. Applied Thermal Engineering 2003;23(3):295-311.

38. Zaleski T, Klepacka K. *Plate heat-exchangers - method of calculation, charts and guidelines for selecting plate heat-exchanger configurations*. Chemical Engineering and Processing 1992;31(1):49-56.

39. Gut JAW, Pinto JM. *Modeling of plate heat exchangers with generalized configurations*. International Journal of Heat and Mass Transfer 2003;46(14):2571-2585.

40. Derevich IV, Smirnova EG. *Calculating the parameters of heat transfer between countercurrent flows with variable thermophysical properties*. Theoretical Foundations of Chemical Engineering 2002;36(4):341-345.

41. Pelletier D, Ilinca F, Hetu JF. *Adaptive remeshing for convective heat-transfer with variable fluid properties*. Journal of Thermophysics and Heat Transfer 1994;8(4):687-694.

42. Hassanien IA. *Flow and heat transfer on a continuous flat surface moving in a parallel free stream with variable fluid properties*. Zeitschrift Fur Angewandte Mathematik Und Mechanik 1999;79(11):786-792.

43. Kang YT, Christensen RN. *The effect of fluid property variations on heat transfer in annulus side of a spirally fluted tube*. Journal of Enhanced Heat Transfer 2000;7(1):1-9.

44. Ravi Kumar KV, Sarangi S. *On the performance of high-NTU heat exchangers with variable heat capacity of the working fluid*. Heat Transfer Engineering 1991;12(1):37-42.

45. Kim HB, Tadini CC, Singh RK. *Heat transfer in a plate exchanger during pasteurization of orange juice*. Journal of Food Engineering 1999;42(2):79-84.

46. Telis-Romero J, Telis VRN, Gabas AL, Yamashita F. *Thermophysical properties of Brazilian orange juice as affected by temperature and water content*. Journal of Food Engineering 1998;38(1):27-40.

47. Kessler HG. *Food engineering and dairy technology*. Germany: Freising, Federal Republic of Germany : Verlag A., Kessler; 1981.

48. Ibarz A, Gonzalez C, Esplugas S. *Rheology of clarified fruit juices. 3. Orange juices*. Journal of Food Engineering 1994;21(4):485-494.

49. Fernandez-Martin F. *Influence of temperature and composition on some physical properties of milk and milk concentrates. II. Viscosity*. Journal of Dairy Research 1972;39(65):75-82.

50. Fernandez-Martin F, Montes F. *Influence of temperature and composition on some physical properties of milk and milk concentrates. III. Thermal Conductivity*. Milchwissenschaft 1972;27(12):772-776.

51. Fernandez-Martin F. *Influence of temperature and composition on some physical properties of milk and milk concentrates. I. Heat capacity*. Journal of Dairy Research 1972;39(65):65-73.