

The Baire Property in Hit-and-Miss Topologies

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Introduction

When is the hyperspace of a given topological space X (hereditarily) Baire?

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This question was first considered by McCoy in 1975 for the case of the Vietoris topology. Since then, there has been a great progress towards its complete solution. In particular, the following people have made their contributions: Zsilinszky, Bouziad, Hola, Chaber, Pol, Cao, Garcia-Ferreira, Gutev, Tomita.

McCoy's theorems

What McCoy did in 1975 can be summarized as follows:

McCoy's First Theorem: If either X is T_1 and $(2^X, \tau_v)$ is Baire or $(\mathcal{K}(X), \tau_v)$ is Baire, then X is Baire.

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McCoy's Last Theorem: If X is a quasi-regular and Baire space having a countable pseudo-base, then $(2^X, \tau_v)$ is Baire. Further, if X is quasi-regular and $(\mathcal{K}(X), \tau_v)$ is Baire, then X^n is a Baire space for all $n < \omega$.

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McCoy's Last Theorem: If X is a quasi-regular and Baire space having a countable pseudo-base, then $(2^X, \tau_v)$ is Baire. **Further, if X is quasi-regular and $(\mathcal{K}(X), \tau_v)$ is Baire, then X^n is a Baire space for all $n < \omega$.**

Thus, if we take a metric Baire space X whose square X^2 is not Baire, then $(\mathcal{K}(X), \tau_v)$ is not Baire. In 2007, Cao, Gutev and Garcia-Ferreira showed this is also true for $(2^X, \tau_v)$.

Main techniques of McCoy

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Given a finite sequence U_0, U_1, \dots, U_n of open sets of X , let

$$[U_0, \dots, U_n] = \left(\prod_{i \leq n} U_i \right) \times \left(\bigcup_{i \leq n} U_i \right)^{\omega \setminus (n+1)}.$$

Then, τ^* is a topology on X^ω having the family of all sets of the above form as a base. This topology is called the **pinched-cube topology** by Piotrowski, Rosłanowski and Scott in 1983.

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Recently, Zsilinszky, Cao and Tomita modified McCoy's techniques to investigate Baireness of the Wijsman topology.

Recent applications

Given a metric space (X, d) , Zsilinszky modified the pinched-cube topology on X^ω so that a basic open set having the form

$$[U_0, \dots, U_n]_B = \left(\prod_{i \leq n} U_i\right) \times (X \setminus B)^{\omega \setminus (n+1)},$$

where B is a finite union of closed balls. Then, he applied this topology to characterize Baireness of 2^X with the Wijsman topology for an almost locally separable metric space (X, d) .

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Recently, Cao and Tomita extended the method they developed on Tychonoff cube X^ω , and solved a problem posed by Zsilinszky in 2006.

The hit-and-miss topology

Given a space X , $E \subseteq X$ and $\mathcal{V} \subseteq \tau(X)$, let

$$E^+ = \{A \in 2^X : A \subseteq E\},$$

$$\mathcal{V}^- = \{A \in 2^X : A \cap V \neq \emptyset \text{ for all } V \in \mathcal{V}\}.$$

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Let $\Delta \subseteq 2^X \cup \{\emptyset\}$. Then the **upper Δ -topology** τ_Δ^+ on 2^X is generated by $\{(X \setminus E)^+ : E \in \Delta\}$. The **lower Vietoris topology** τ^- is generated by $\{\mathcal{V}^- : \mathcal{V} \in [\tau(X)]^{<\omega}\}$. The Δ -topology τ_Δ is just $\tau_\Delta^+ \vee \tau^-$.

The proximal hit-and-miss topology

Let (X, \mathcal{U}) be a Hausdorff uniform space, and $E \subseteq X$. Let

$$E^{++} = \{A \in 2^X : U(A) \subseteq E \text{ for some } U \in \mathcal{U}\}.$$

The **upper proximal Δ -topology** $\tau_{p\Delta}^+$ on 2^X is generated by $\{(X \setminus E)^{++} : E \in \Delta\}$. The proximal Δ -topology $\tau_{p\Delta}$ on 2^X is just $\tau_{p\Delta}^+ \vee \tau^-$.

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When Δ varies, we obtain various hypertopologies. For example, τ_Δ is the Vietoris topology and $\tau_{p\Delta}$ is the proximal topology when $\Delta = 2^X$; τ_Δ is the ball topology and $\tau_{p\Delta}$ is the proximal ball topology when Δ is the collection of proper closed balls of a metric space (X, d) .

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Let (X, \mathcal{U}) be a uniform space, and $U \in \mathcal{U}$. Let

$$H(U) = \{(A, B) : B \subseteq U(A), A \subseteq U^{-1}(B)\}.$$

The Hausdorff uniformity $H(\mathcal{U})$ on 2^X is generated by $\{H(U) : U \in \mathcal{U}\}$.

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It can be shown that the upper Hausdorff uniformity topology on 2^X is the same as the proximal topology; and the lower Hausdorff uniformity topology on 2^X is generated by $\{\mathcal{V}^- : \mathcal{V} \in \mathbb{L}\}$, where \mathbb{L} is some collection of locally finite families of open sets.

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This topology can also be split into two parts: the lower part is τ^- ; and the upper part $\tau_{w_d}^+$ is generated by

$$\left\{ \{A \in 2^X : d(A, x) > \varepsilon\} : x \in X, \varepsilon > 0 \right\}.$$

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The Wijsman topology

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Although the Wijsman topology is also hit-and-miss, to work with the Baire property, it is easier to consider a closely related topology, namely the ball topology.

- $(2^X, \tau_{w_d}^+)$ is Baire if and only if $(2^X, \tau_b^+)$ is Baire.

The upper topologies – I

Theorem 1: Let X be a T_1 -space, and \mathcal{N} the family of closed nowhere dense sets in X .

(i) Suppose that for any $A \in 2^X$ and $B \in \mathcal{N}$ with $A \cap B = \emptyset$, there exists an $E \in \Delta$ such that $B \subseteq E$ and $A \cap E = \emptyset$, that is, Δ separates elements in \mathcal{N} from arbitrary elements in 2^X . If $(2^X, \tau_{\Delta}^+)$ is Baire, then X is Baire.

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(ii) If X is Baire and Δ is a π -base for 2^X , then $(2^X, \tau_{\Delta}^+)$ is Baire.

Furthermore, if (X, \mathcal{U}) is a Hausdorff uniform space, then

(iii) $(2^X, \tau_{\Delta}^+)$ is a Baire space if and only if $(2^X, \tau_{p\Delta}^+)$ is Baire.

The upper topologies – II

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- A T_1 topological space X is Baire if and only if $(2^X, \tau_v^+)$ is Baire.
- For a Hausdorff uniform space (X, \mathcal{U}) , the following are equivalent:
 - (i) (X, \mathcal{U}) is Baire;
 - (ii) $(2^X, \tau_p^+)$ is Baire;
 - (iii) $(2^X, \tau_{H(\mathcal{U})}^+)$ is Baire;
 - (iv) $(2^X, \tau_v^+)$.

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 - (iii) $(2^X, \tau_{H(\mathcal{U})}^+)$ is Baire;
 - (iv) $(2^X, \tau_v^+)$.
- For a metric space, $(2^X, \tau_{w_d}^+)$ is Baire if and only if $(2^X, \tau_{pb}^+)$ is Baire, if and only if $(2^X, \tau_b^+)$ is Baire..

Quasi-Urysohn families

We shall call a family $\Delta \subseteq 2^X \cup \{\emptyset\}$ **quasi-Urysohn** provided that whenever $B \in \Sigma(\Delta)$ and $W_i \in \tau(X) \setminus \{\emptyset\}$ are disjoint for each $i \leq n$, there is $D \in \Sigma(\Delta)$ such that $B \subseteq \text{int}D \subseteq D$, and $W_i \cap (X \setminus D) \neq \emptyset$ for each $i \leq n$.

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Which families of closed subsets are quasi-Urysohn?

- $\{\emptyset\}$ is quasi-Urysohn.
- If X is quasi-regular, then 2^X is quasi-Urysohn.
- The family of all closed proper balls in a metric (X, d) is quasi-Urysohn.

A generic theorem

Theorem 2. Let X be a Hausdorff space. Suppose that Δ is a quasi-Urysohn family. If X^ω is Baire, then $(2^X, \tau_\Delta)$ is Baire.

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Corollary 2.1. Let X be a Hausdorff space. If X^ω is Baire, then $(2^X, \tau^-)$ is Baire ($\Delta = \{\emptyset\}$).

Corollary 2.2. [Cao and Tomita, 07] Let X be a quasi-regular space. If X^ω is Baire, then $(2^X, \tau_v)$ is Baire ($\Delta = 2^X$).

Corollary 2.2. [Cao and Tomita, ??] Let (X, d) be a metric space. If X^ω is Baire, then $(2^X, \tau_{w_d})$ is Baire ($\Delta = \{ \text{proper closed balls} \}$).

Sketch of the proof

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Suppose that σ is a strategy for β in the Choquet game played in the hyperspace $(2^X, \tau_\Delta)$ with the initial step

$$\sigma(\emptyset) = \left(\bigcap_{i \leq n_0} U_0(i)^- \right) \cap (X \setminus B_0)^+,$$

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We construct a strategy θ for β in X^ω inductively by letting the initial step as follows:

Sketch of the proof continued

$$\theta(\emptyset) = \prod_{i < n_0} U_0(i) \times \prod_{i \geq n_0} X.$$

Suppose that the second player α responds by

$$\Pi_0 = \prod_{i < n_0} V_0(i) \times \prod_{i < m_0} W_0(i) \times \prod_{i \geq m_0 + n_0 - 1} X.$$

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Then, in the hyperspace,

$$\left(\bigcap_{i < n_0} V_0(i)^- \right) \cap (X \setminus B_0)^+ \subseteq \sigma(\emptyset).$$

Using the strategy σ , we assume that β 's next move is

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Since Δ is quasi-Urysohn, we can require $B_0 \subseteq \text{int} B_1$.

Sketch of the proof continued

Next, we construct $\theta(\Pi_0)$ as follows

$$\theta(\Pi_0) = \prod_{i < n_0} U_1(i) \times \prod_{i < m_0} W_0(i) \times \prod_{n_0 \leq i < n_1} U_1(i) \times \prod_{i \geq m_0 + n_1 - 1} X.$$

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Here, the special “**splitting trick**” is applied. The process can be carried on inductively. We can construct θ for all possible **legal partial plays** Π_0, \dots, Π_n of α .

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Here, the special “**splitting trick**” is applied. The process can be carried on inductively. We can construct θ for all possible **legal partial plays** Π_0, \dots, Π_n of α .

At the end, since θ cannot be a winning strategy for β in X^ω , there must be a full play $\{\Pi_n : n < \omega\}$ for α with nonempty intersection. Then, we collect a coordinate from each column corresponding to $U_n(i)$. Finally, we can close it up by putting these coordinates together and taking the closure.

Sufficient conditions

In the light of Theorem 2, we may want to know **for which classes of spaces X , must X^ω be Baire?** Some of them are listed below:

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In the light of Theorem 2, we may want to know **for which classes of spaces X , must X^ω be Baire?** Some of them are listed below:

- Baire spaces having a countable π -base;
- Metric hereditarily Baire spaces;
- Separable metric Baire spaces;
- Weakly α -favorable spaces;
- Metric almost locally separable Baire spaces;
- Čech-complete spaces;
- Baire spaces having a countable-in-itself π -base;
- Almost locally $uK - U$ Baire spaces** (D. Fremlin, T. Natkaniec and I. Reclaw, *Fund. Math.* **165** (2000), 239-247; or L. Zsilinszky, *Fund. Math.* **183** (2004), 115-121.)

A short summary

Let X be a quasi-regular space belonging to any class such that X^ω is Baire. Then $(2^X, \tau_v)$ is Baire. Conversely, in 2007, Cao and Tomita constructed a metric Baire space such that $(2^X, \tau_v)$ is Baire, but X^ω is not Baire.

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For uniform or metric spaces, the Baireness of proximal hypertopologies is equivalent to that of the corresponding non-proximal versions of hypertopologies.

For a metric space (X, d) , belonging to any class such that X^ω is Baire. Then $(2^X, \tau_{w_d})$ is Baire. **There is a non-Baire metric space whose Wijsman hyperspace is Baire.** The Baireness of Wijsman topology is equivalent to that of ball topology.

Some questions

Question 1. Given a metric Baire space (X, d) , must $(2^X, \tau_{w_d})$ be Baire?

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There is a metric space (X, d) such that $(2^X, \tau_{w_d})$ is Baire, but $(2^X, \tau_v)$ is not Baire.

Question 2. Is there a metric Baire space whose Vietoris hyperspace $(2^X, \tau_v)$ is Baire, but whose Wijsman hyperspace $(2^X, \tau_{w_d})$ is not Baire?

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Question 2. Is there a metric Baire space whose Vietoris hyperspace $(2^X, \tau_v)$ is Baire, but whose Wijsman hyperspace $(2^X, \tau_{w_d})$ is not Baire?

Question 3. Let X be a metrizable space. Suppose that $(2^X, \tau_{w_d})$ is Baire for all compatible metric d . Must $(2^X, \tau_v)$ be Baire?

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Question 4. Let (X, d) be a Baire metric space. Must $(2^X, \tau(d_H))$ be Baire? If the answer is “no”, when is $(2^X, \tau(d_H))$ Baire?

One possible direction towards this question is to work on the locally finite topology.

Connections with orders

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First, if 2^X is ordered by the reverse inclusion: $A \sqsubseteq B$ if and only if $B \subseteq A$. Then \mathcal{V}^- is a **lower set** in sense that if $A \in \mathcal{V}^-$, then $B \in \mathcal{V}^-$ for any $A \sqsubseteq B$. On the other hand U^+ is an **upper set** in sense that if $A \in U^+$, then $B \in U^+$ for any $A \sqsubseteq B$.

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So, it would be interesting to look at topologies on partially ordered sets that arise as the joint of a topology of a lower sets and a topology of an upper sets.

Connections with orders cont.

Further, some important topologies in the domain theory and computational metric space theory, such as the Lawson topology and the formal ball topology have the previous mentioned nature. It is known that completeness property plays an important role in the computing theory. It may be interesting to explore the Baire property of these topologies as well.

Connections with orders cont.

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Thank You for Your Attention!