REDUCING THE GAP BETWEEN THE SCHOOL AND UNIVERSITY MATHEMATICS: UNIVERSITY LECTURERS' PERSPECTIVE

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This paper deals with a very practical issue. In many countries there is a gap between school and university mathematics. The transition period from school to university can be hard for many students. Even students with good marks in school mathematics experience psychological difficulties at university and sometimes fail the first year university mathematics courses. Often the pass rate in the first year university mathematics is around 50%. Different parties – school teachers, university lecturers, first year university students, administrators, researchers – might have different views on the reasons for the gap and the ways to narrow or fill it. The purpose of this paper is to present responses of university lecturers worldwide to a short survey concerning the transition period between the school and university mathematics.

INTRODUCTION

This study presents and systematises the responses of 63 university lecturers from 24 countries. A cross-country approach was chosen to reduce the differences in cultures, curricula and education systems. The importance of the transition period from school to university is reflected in a number of publications for general public, professional education journals, books and research forums (e.g. Tall, 1997; Crawford, 1994; Barnard, 2003; Anthony, 2000; Leamson, 1999; Gruenwald, 2003). There are several regular international conferences devoted only to this issue. We just mention one of them – the biannual Pacific Rim conference "First Year in Higher Education" (www.qut.edu.au/talss/fye/home.htm). Apart from common reasons for the gap between school and university, the gap in mathematics has its own specific reasons. Unlike many other disciplines mathematics is a compulsory subject up to a quite high level at school – in some countries up to the final year. Numerous applications make this subject a must for other subjects and for everyday life. Mathematical thinking is a part of our culture. But different groups based on professional, social, cultural characteristics, race, ethnic, gender, etc. have different views of mathematics and its role in the society. Recent school reforms in many countries made significant changes in mathematics curricula. Extensive use of technology both at school and university also created many debates and discussions. Many university lecturers feel that there is a need to investigate the ways of reducing the gap between the school and university mathematics. This issue is timely in many countries. As an example we quote from the joint report "Tackling the Mathematics Problem" prepared by the Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society: "There is unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates….The serious problems perceived by those in higher education are:

(i) a serious lack of essential technical facility – the ability to undertake numerical and algebraic calculation with fluency and accuracy;

(ii) a marked decline in analytical powers when faced with simple problems requiring more than one step;

(iii) a changed perception of what mathematics is – in particular of the essential place within it of precision and proof.

This is no way restricted to those `new undergraduates´ who ten years ago would not have proceeded to higher education. The problem is more serious; it is not just the case that some students are less well-prepared, but many `high-attaining´ students are seriously lacking in fundamental notions of the subject." (www.lms.ac.uk/policy/tackling/report.html).

FRAMEWORKS

In this study, practice was selected as the basis for the research framework and, it was decided "to follow conventional wisdom as understood by the people who are stakeholders in the practice" (Zevenbergen & Begg, 1999). 63 university lecturers from 24 countries were surveyed using a combination of two nonprobability sampling methods - judgement and convenience. The results of the survey can be treated as a pilot study. The survey was sent to selected participants of international mathematics education conferences held in 2000-2002 and university lecturers who either teach university mathematics or write papers on mathematics education at university level or both. The response rate was 36% which shows the importance of the topic - it is a quite high rate of response for busy professionals. The questionnaire comprised three open ended questions: the reasons for the gap, the successful remedies that work at their universities and also their ideas on what else can be done to narrow the gap. We summarised the answers by categorising the participants' responses and calculating percentages. We also presented the most common answers, strategies and ideas. Such an exchange of best strategies that work at some universities and suggested ideas that are worth looking at can make readers try to implement them at their own institutions.

Theoretical framework was based on the Piaget concept of cognitive conflict and David Tall's works on advanced mathematical thinking (Piaget, 1985; Tall, 1991, 1995, 1997, 2001). The idea of this framework is expressed in the following extract: "Advanced mathematics, by its very nature, includes concepts which are subtly at variance with naïve experience. Such ideas require an immense personal reconstruction to build the cognitive apparatus to handle them effectively. It involves a struggle with inevitable conflicts which require resolution and reconstruction" (Tall, 1991). The main reason for the gap between the school and university mathematics according to the participants' responses as we can see later is the difference in thinking. Another extract supports this: "The formal presentation of material to students in university mathematics courses – including mathematics majors, but even more for those who take mathematics as a service subject – involves conceptual obstacles that make the pathway very difficult for them to travel successfully. And the changes in technology, that render routine tasks less needful of labour, suggest that the time for turning out students whose major achievement is in reproducing algorithms in appropriate circumstances is fast passing and such an approach needs to move to one which attempts to develop much more productive thinking" (Tall, 1991). Many students are exposed to a formal deductive approach in teaching mathematics for the first time in their life only at university and therefore in their first year experience a tremendous cognitive conflict. "At school the accent is on computations and manipulation of symbols to "get an answer", using graphs to provide imagery to suggest properties. At university there is a bifurcation between technical mathematics that follows this style (with increasingly sophisticated techniques) and formal mathematics, which seeks to place the theory on a systematic, axiomatic basis" (Tall, 1997). "During the difficult transition from pre-formal mathematics to a more formal understanding of mathematical processes there is a genuine need to help students gain insight into the concepts" (Tall, 1991). It is not an easy task and requires transition from one stage to another in the Piagetian stage theory where the previous knowledge conflicts with new ideas (Piaget, 1985).

THE STUDY

The questionnaire given to the university lecturers included the following 3 questions:
Question 1. What do you think are the reasons for the gap between the school and university mathematics?
Question 2. What is your Department doing to reduce the gap?
Question 3. In your opinion what else can be done to make the transition period smoother?

The Participants' Responses

Below are the main statistics and the most common participants' responses to the questionnaire. The numbers after quotations correspond to the participants' names on the list arranged in a random order to protect privacy.

Question 1. What do you think are the reasons for the gap between the school and university mathematics?

Reason 1. Higher level of thinking at university mathematics (72%)
Different emphasis: on calculations, techniques, algorithms, manipulations at school versus on theory, proof, conceptual understanding at university. This difference is reflected in textbooks and assessment.
"The teaching style in schools encourages rote learning of disjointed 'facts' and algorithms which are not underpinned by understanding of the meaning of them or of the fundamental relationship between them" (20)
"They learn maths almost without theoretical explanation and only calculation in high school days" (18)
"The school mathematics is aimed to coach for a formal solution of as many exercises as possible, with only superficial understanding the theory, under the everyday instructor supervision. On the contrary, university mathematics is aimed to give in-depth theoretical understanding through the formal delivering lectures with the minimal instructor supervision" (15)
"High school math is very mechanical and situational. University math is more theoretical and eventually becomes proof-oriented. The material is qualitatively different and we expect more out of the students" (19)
"Most of our students have not seen formal proof before entering university" (22)
"There is a major jump in thinking level into the abstract world of proofs" (39)
"…'recipes' for doing standard problems. The result of this is that many students don't **have** to understand the ideas behind the problems, just **do** them, and others don't even realise that there **is** more understanding to have. Being able to perform the right steps is what maths is about" (51)

Just to illustrate the above most common respondents' concerns we would like to give here two real examples from the Bursary mathematics exams in 1996 and 1993 respectively.

Example 1. (Bursary Mathematics with Statistics, 1996)
Question. "Show that the equation $x^2 - \sqrt{x} - 1 = 0$ has a solution between x = 1 and x = 2.
Model Solution. If f(x) = $x^2 - \sqrt{x} - 1$ then f(1) = - 1 < 0 and f(2) = 1.58 > 0.
So graph of f crosses x-axis between 1 and 2."

Comments. The suggested solution is based on the special case of the Intermediate Value Theorem which has 2 conditions: the continuity of f(x) on [a,b] and the condition f(a) x f(b) < 0. But only the second condition is checked and the first is ignored as if it was 'not essential'. It was a written exam and all working must be shown. The fact that the condition of continuity of the function f(x) was not required by the examiners to get a full mark for solving the question was very dangerous. The message was clear – the manipulations are important but the property of the function is not. No wonder that students don't pay attention to all conditions of the theorem and properties of the functions – it is simply not required. Some years ago one of the authors of this paper gave the following provocative question to 5 average students doing Bursary mathematics course: "Show that the equation $\frac{x^2+x+1}{x-1.5}=0$ has a solution between x = 1 and x = 2." All 5 students quickly 'showed' that by misusing the Intermediate Value Theorem. They only checked f(1) = - 6 < 0 and f(2) = 14 > 0 for the function f(x) = $\frac{x^2+x+1}{x-1.5}$. The function was deliberately chosen a bit more complex than a simple hyperbola f(x) = $\frac{1}{x-1.5}$ to provoke the students to calculations which they did! It is hard to blame them for that if they were taught in a way that only calculations/manipulations/transformations/techniques are important in mathematics.

Example 2. (Bursary Mathematics with Calculus, 1993)
Question. "Solve the equation $\log_2 (9x-1) - \log_2(x+2) = 3$.
Model Solution.

$$\log_2 \frac{9x-1}{x+2} = 3$$

$$\frac{9x-1}{x+2} = 2^3$$

$$9x-1 = 8(x+2)$$

$$x = 17$$

Comments. No checking of the validity of the answer ignoring the domain of the log function! If it was 'not essential' then the following solution is also true?!

$$\log_2(9x-10) - \log_2(2x-3) = 0$$

$$\log_2 \frac{9x-10}{2x-3} = 0$$

$$\frac{9x-10}{2x-3} = 1$$

$$9x-10 = 2x-3$$

$$x = 1$$

Again the message to a student is clear – you can get a full mark for a question if you do only calculations.

The above two examples also illustrate another concern of the respondents that is expressed later in the paper - lack of communication between school and university in particular in setting school mathematics exams. It is hard to imagine a university lecturer who would accept the model solutions for examples 1-2 above as complete solutions.

Reason 2. Emphasis on passing the exam at school (37%)
"Assessment culture at school means that many students do not learn to understand, they learn to pass exams" (21)
"School mathematics is not truly 'learned' and stored in long term memory but is quickly lost after the final examination is safely passed" (20)
Reason 3. School syllabus is too broad (34%)
"The high school curriculum is too thick for students to understand whole, so students pick up some maths classes so they looses the maths understanding" (31)
"The amount of maths taught at secondary level is too big to enable students to really understand. The result is that they are trying to remember only instead of understand" (27)
Reason 4. Too optimistic assumptions and expectations of university lecturers (33%)
"… we often expect that (a) all students learn in the same way we did – and that's the best way (b) what students learned at school was the same as we did, and with the same depth" (51)
Reason 5. Different ways of teaching/learning (30%)
"Students are not prepared to assume responsibility for their learning – rather, they expect continuation of spoon-feeding from high school" (14)
"Many students have problems adjusting to studying at university. In particular they are used to a teacher planning their study for them and at university they have to do this themselves" (55)
Reason 6. Lack of mathematics background of mathematics teachers in schools and lack of teaching skills of university lecturers (26%)
"The lack of well qualified teachers of maths in schools means that students do not all get a good background in maths in schools" (21)
"University staff are appointed because of their knowledge and research record in many cases, not because of their teaching skills, and this puts an extra onus on students to find their own way to understand content" (38)
Reason 7. Lack of communication between school and university (17%)
"We at Uni level … aren't keeping up with what is happening at schools" (48)
"We have almost no communication with the schools. The university and school sectors have almost no overlap" (51)
"Our … lack of knowledge and understanding of what is currently taught at school" (52)
Reason 8. Changes in environment (15%)
"For students, coming to university everything is new: new people, environment, social contexts, norms, expectations. Drastic decline in the amount of personal attention students get from their teachers, compared to high school. Large classes create intimidating situations" (14)
"Some students cannot cope with the freedom they have being away from home for the first time" (26)

Question 2. What is your Department doing to reduce the gap?

Remedy 1. Personal approach (55%)
- Learning support centres
- Small classes
- Individual consultations
- Streaming after diagnostic tests
- Extra tutorials

Remedy 2. Bridging courses (52%)
- Different levels
- Different length

- Different emphasis – e.g. "courses concentrating on mathematical thinking (proof) rather than just pushing content" (21) (10%)
  Some participants indicated that often the bridging courses don't fill the gap. The two main reasons are:
  a) The courses are too short in duration. Many bridging courses are just a few weeks or, at most, months long. During that time some students are not able to master the material usually covered at school during several years.
  b) The mathematical background of the students often is so poor that the emphasis in the bridging courses is on the basics of mathematics: rules, techniques, manipulations, and algorithms. There is no time to teach students higher level of thinking (proofs, reasoning, etc). So the gap in thinking is not filled.

Remedy 3. Developing different pedagogical strategies (32%)

- "We give one lecture on study skills for mathematics and problem solving techniques at the beginning of the year. Each member of our department acts as a mentor to our first year students" (55)
- "Setting weekly homework which is peer assessed in class following lecturer's working on board, thus trying to encourage an early engagement with new material and a revision of school material" (17)
  "A daily one-hour help session taught by current Masters students but this is basically a patch-up rather than developmental assistance" (20)
- "We aim to take things fairly slowly, with hand-out notes that have detailed explanations.
  We try to communicate to students just what is expected of them and how to go about achieving their goals in maths" (51)

Remedy 4. Improve communication between school and university (16%)

- "In the summer we offer workshops for teachers, and each year we invite one high school teacher to join our department as a "visiting master teacher" to teach lower level courses and engage with the college faculty in discussions about mathematics and pedagogy. As a consequence, many members of our department explore new methods of teaching; several have received national grants to support this work" (16)
- "In-service day for secondary school teachers. Summer school for year 10,11 students" (54)
- "Visit high schools and talk about university math courses, and the expectations that those courses place on students … do sessions for high school students (problem solving sessions and presentations on various math topics)" (14)
- "Many of our Department professors go to teach maths at school because they are able to tune proficiently themselves up to the children perception. Professor can imbue the minds of schoolchildren with interest to intriguing, challenging tasks, can give them a taste for non-standard solutions and half-open the curtain to what they will do in the university" (15)

Remedy 5. Change in assessment: weekly tests, oral exams, detailed feedback (16%)

- "We try to give them plenty of feedback on how they're progressing, with assignments and quizes. We try to listen and respond to as much feedback from students as possible/reasonable in terms of pace, timing of assessment or other administrative areas" (51)

Remedy 6. Lower the standard (12%)

- "Taking out or reducing the mathematics requirements in courses so students can pass!" (38)

Question 3. In your opinion what else can be done to make the transition period smoother?

Idea 1. Establish a system to monitor quality at schools and universities (60%)

- Better preparation of school teachers
- Improving school curriculum (less content, more proof, depth and rigour)
- Improving teaching skills of university lecturers - have tertiary teaching qualification

- “More deductive (although not necessarily formal) reasoning in high school would help” (19)
- “A bit more depth and rigour can be included in school mathematics so that the transition can be smoothened” (3)
- “Making it compulsory for all tertiary teachers of mathematics to have a tertiary teaching qualification as well as their mathematics qualification, hence making them aware of ideas about teaching and strategies for teaching” (21)

Idea 2. Extras: tutorials, courses, learning support, pastoral care, streaming, time (slower pace; ‘adjustment’ semester; summer school) (56%)

- “Involve the university learning centre as much as possible” (46)
- “Talk to first-year university students about how to study efficiently. Teach them how to read a maths textbook, i.e. how to do maths on their own” (14)
- “Differentiating among the newcomers, not the first day but after a month or so. Based on the student's own conception of her/his ability, motivation and background and on results from school and the first period at university the student should get an offer to choose among different strands, maybe leading to the same goal but with options for teaching/learning methods, time spent on the material and so on” (57)

Idea 3. Improve communication between school and university (38%)

- “Better contacts with secondary school teachers, in the hope that there could be changes from both sides and especially more information about what is expected from students and how to choose the best preparation for future studies” (26)
- “We must improve the network connecting university mathematics and school mathematics at all levels” (57)
- “Bring final year students into the university to see what it is like here” (36)

Idea 4. More attention to mathematics education at universities (18%)

- More research in didactics
- Establishing mathematics education units in mathematics departments
- “Set up a Centre with focus on maths/stats education” (54)
- “Including in mathematics departments a "Mathematics Education Group". Such a group might: legitimize pedagogical studies as a legitimate research area for tertiary teachers of mathematics hold regular educational seminars within the department that others could attend provide support for young faculty members who lack educational expertise mean that some maths ed journals are subscribed to, that would also perhaps influence the culture of the department” (21)

CONCLUSION

We deliberately leave making the conclusion of the study to the readers. Looking critically at the participants’ responses to the questionnaire one can reflect their own teaching practice, pick up successful strategies that work at some universities, try to implement some suggested ideas at their own institutions and just think one more time about the issue of the transition period from the school to university mathematics.

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