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METHOD OF CALCULATING THE RESISTANCE FORCE UPON AN IMPACT ON A COMPOSITE TARGET

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Abstract: A kinematically possible velocity field allowing calculation of all the necessary integrals in quadratures and obtaining an analytical solution for the resistance force induced by impactor penetration into the target is constructed. The Saint-Venant model of a rigid-plastic body and the theorem on the upper bound of the limit load are used in solving the problem. The essence of the method applied is using the equilibrium equation in the form of the Lagrange equation. The kinematically possible velocity field allows obtaining an upper bound of the limit load, i.e., estimating the resistance force to impactor penetration.

Keywords: kinematically possible velocity field, impactor, rigid-plastic body, upper bound of the limit load, composite target.

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Problems of penetration of an impactor into a target in an exact formulation are very complex, so it is necessary to apply modern computational tools, as well as a detailed analysis of numerical schemes used to obtain their solutions. Along with exact solutions of the problem, it is reasonable to find approximate solutions, which make it possible to obtain approximate estimates of unknown quantities easily and quickly. It should be noted that approximate solutions are not a substitute for numerical solutions, and they can only be considered as the first approximation to the solution.

Apparently, Babakov and Karimov [1] and Fomin et al. [2] were the first to propose using the upper bound method for solving problems of impactor penetration into the target. This method was developed in [3, 4].

There are various methods of the analytical study of the penetration process. The most important of these methods involves the use of simple mathematical models of a solid body, reliably describing the phenomenon under study and allowing us to obtain the solution in an analytical form. In this paper, we consider the range of low velocities of penetration, in which the strength properties of the material should be taken into account. A significant difference of strengths of the impactor and the target (e.g., iron–soil and iron–concrete) allows us to select a class of problems in which the impactor deformation can be neglected, as the impactor is assumed to be absolutely rigid.

As a model of the target, we used a model of an ideal incompressible rigid-plastic material, which allows us to describe large plastic strains. The legitimacy of this model in analyzing the process of strains is justified by the following arguments. In the case of large deformations, their elastic components, can be neglected because of their smallness, and the medium can be considered as rigid-plastic. In a solid, the energy coming from outside is spent on changing its volume, shape, and kinetic energy. The relatively shallow penetration of the impactor and the fact that the target has free surfaces (front or back) do not allow development of large volume strains. The hypothesis of the material incompressibility only takes into account the fact that the work spent on uniform compression of the material is substantially smaller than that spent on the shear, that is, on changing the shape.

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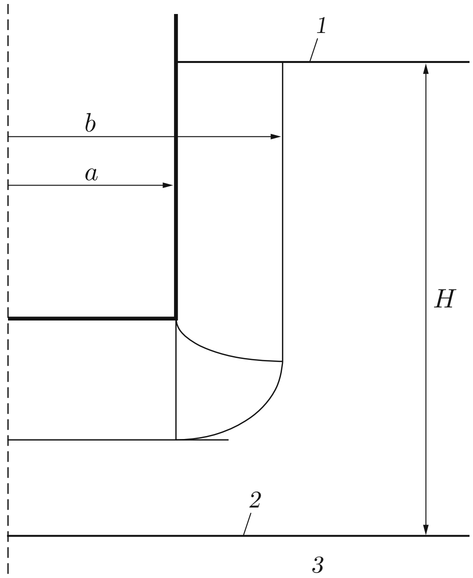

Fig. 1.

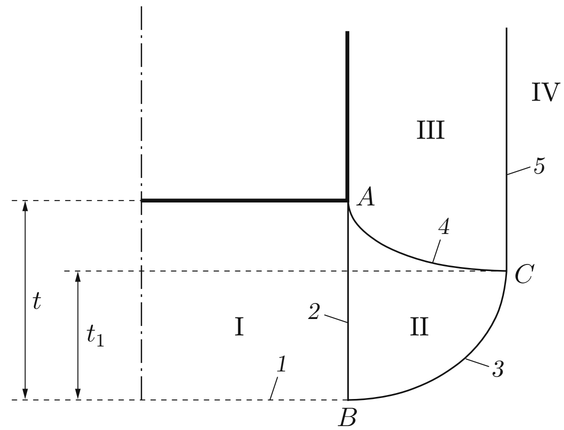

Fig. 2.

Fig. 1. Diagram of the problem: (*1*) free surface; (*2*) layer boundary; (*3*) half-space.

Fig. 2. Diagram of the plastic flow of the material: regions of the plastic flow are denoted by I–IV and surfaces of discontinuity of tangential velocities are denoted by *1*–*5*.

Certainly, a constrained deformation of the same material requires taking into account its volume compression, and the hypothesis of incompressibility becomes inacceptable.

Determination of limit loads, i. e., loads at which the process of deformation begins, is quite a complex problem even when a simple model of an ideal incompressible rigid-plastic material is used. However, two limit theorems on the upper and lower estimates of the limit loads allow estimating the limit loads. There is a fairly simple procedure of constructing the upper bound on the basis of the equilibrium equation in the form of the Lagrange equation in the Saint-Venant–Mises model (the Prandtl–Reuss model, which neglects elastic strains) [5]. Thus, the problem is reduced to specification of a velocity field that more accurately describes the kinematics of the process. The construction of the adequate velocity field should involve using the results of field experiments and laboratory modeling (see, e.g., [6]).

To construct the lower bound of the limit load is much more difficult because it is necessary to solve a system of three equations in partial derivatives (Cauchy equations) satisfying all the boundary conditions. At the same time, the stresses in plasticity zones should not satisfy the condition of plasticity. It should be noted that, with the use of the upper bound of the limit load, we can determine the range of initial velocities, in which a certain depth of impactor penetration is provided (the estimated depth of penetration is measured from the bottom, i. e., determined with a margin).

Let us consider a smooth, absolutely rigid impactor of radius a, which impinges onto a composite target with an initial velocity V_0 (layer of thickness H in a half-space), which results in formation of a flow region with an external radius equal to b (Fig. 1). The channel size is determined in the solution process. Figure 2 shows a diagram of a plastic flow of the material (regions I–IV). In regions I and II, the particles of the target are directed toward the free surface. In region III, the flow is vertical. In region IV, the material is fixed ("dead" zone).

In accordance with [1, 2], we propose the following analytical description of the velocity field. Region I is a plastically deformable disc. In the cylindrical coordinate system (the origin is at the bottom of the flow channel), the velocity component v_z satisfies the boundary conditions at the end of the impactor and the bottom of the flow channel, and the component v_r is determined from the condition of incompressibility (V_0 is assumed to be equal to unity):

$$v_r^{(\mathrm{I})} = \frac{r}{2t}, \qquad v_z^{(\mathrm{I})} = \frac{z}{t}$$

(t is the distance between the end of the impactor and the bottom of the channel). Region III is the flow region of the vertical flow:

$$v_r^{(\mathrm{III})} = 0, \qquad v_z^{(\mathrm{III})} = V_1 = \frac{1}{\delta^2 - 1}$$

($\delta = b/a$). Region II is the region of plastic deformation in which the velocity components are chosen in the following form:

$$v_r^{(\mathrm{II})} = \frac{a^2}{2t}\,\frac{1}{r}, \qquad v_z^{(\mathrm{II})} = \frac{t_1}{t}\,\frac{1}{\delta^2 - 1}.$$

Region II is separated by the surface AB from region I, by the surface AC from region III, and by the surface BC from region IV. These are the surfaces of discontinuity of tangential velocities. Note that the equations of the surfaces AB and AC are unknown and will be determined in the solution process. In region IV, $\boldsymbol{v}_4 = 0$.

It is easy to show that the proposed velocity fields satisfy the equation of incompressibility

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0$$

and all boundary conditions.

The equilibrium equation in the form of the Lagrange equation has the form

$$\int\limits_S \sigma_{ni} v_i \, dS = \tau_s \int\limits_V H \, dV + \sum_{k=1}^{N} \tau_s \int\limits_{L_k} |[v_\tau]| \, dL_k,$$

where $H = \sqrt{2\varepsilon_{ij}\varepsilon_{ij}}$, $\varepsilon_{ij} = (1/2)(\partial v_i/\partial x_j + \partial v_j/\partial x_i)$, v_i are the components of the velocity vector, L_k are the surfaces of velocity discontinuity, $[v_\tau]$ is the jump of the tangential component of velocity, and τ_s is the plasticity limit.

Using the expressions for the components of the strain rate tensor

$$\varepsilon_r^{(\mathrm{I})} = \frac{1}{2t}, \qquad \varepsilon_\varphi^{(\mathrm{I})} = \frac{1}{2t}, \qquad \varepsilon_z^{(\mathrm{I})} = -\frac{1}{t},$$

$$\varepsilon_r^{(\mathrm{II})} = -\frac{a^2}{2tr^2}, \qquad \varepsilon_\varphi^{(\mathrm{II})} = \frac{a^2}{2tr^2}, \qquad \varepsilon_z^{(\mathrm{II})} = 0,$$

$$\varepsilon_{ij}^{(\mathrm{III})} = 0,$$

we obtain

$$H^{\mathrm{I}} = \frac{\sqrt{3}}{t}, \qquad H^{\mathrm{II}} = \frac{a^2}{tr^2}, \qquad H^{\mathrm{III}} = 0.$$

Depending on the impactor penetration depth, the boundary between the layer and the half-space can be located as follows: below the flow channel (case 1); intersecting the surface BC (case 2); intersecting the surface AC (case 3); located above the impactor bottom (case 4).

Next, we consider case 3 characterized by the following inequalities:

$$t - t_1 > H - h \qquad (H > h)$$

(h is the distance from the free surface to the impactor bottom; t_1 is the distance from the point C to the channel bottom).

Equations of the surfaces AC and BC, respectively, can be obtained from the continuity conditions for normal velocities across these surfaces [1]:

$$z = \frac{t - t_1}{a^2(\delta^2 - 1)}\,(a^2\delta^2 - r^2) + t_1, \qquad z = \frac{t_1}{\delta^2 - 1}\Big(\frac{r^2}{a^2} - 1\Big).$$

The coordinate of the point O (the intersection point of the layer boundary and the half-space with the surface AC) $r = R_{01}$ is determined as follows:

$$R_{01} = a\sqrt{\frac{(H - h)(\delta^2 - 1) + t - t_1}{t - t_1}}.$$

In region I, the plastic strain work A^{I} is expressed in the form of the integral

$$A^{\mathrm{I}} = \int_0^{2\pi} d\varphi \int_0^a \Big(\tau_{s1} \int_{t+h-H}^{t} + \tau_{s2} \int_0^{t+h-H} \Big) H_1 r\, dr\, dz = \frac{\sqrt{3}\,\pi a^2}{t}\,[(H-h)(\tau_{s1}-\tau_{s2}) + t\tau_{s2}],$$

in region II, the plastic strain work A^{II} is expressed as the sum of three integrals A_{21}, A_{22}, and A_{23}:

$$A_{21} = \tau_{s1} \int_0^{2\pi} d\varphi \int_a^{R_{01}} \int_{t+h-H}^{z_{C_4}} H_2 r\, dr\, dz$$

$$= \tau_{s1}\,\frac{\pi a^2}{t}\Big[\frac{t-t_1}{\delta^2-1}\Big(1-\frac{R_{01}^2}{a^2}\Big) + \Big(\frac{t-t_1}{\delta^2-1} + H - h\Big)\ln\frac{R_{01}^2}{a^2}\Big],$$

$$A_{22} = \tau_{s2} \int_0^{2\pi} d\varphi \int_a^{R_{01}} \int_{z_{C_3}}^{t+h-H} \frac{a^2}{tr^2}\, r\, dr\, dz$$

$$= \tau_{s2}\,\frac{\pi a^2}{t}\Big[\frac{t_1}{\delta^2-1}\Big(1-\frac{R_{01}^2}{a^2}\Big) + \Big(\frac{t_1}{\delta^2-1} + t + h - H\Big)\ln\frac{R_{01}^2}{a^2}\Big],$$

$$A_{23} = \tau_{s2} \int_0^{2\pi} d\varphi \int_{R_{01}}^{a\delta} \int_{z_{C_3}}^{z_{C_4}} \frac{a^2}{tr^2}\, r\, dr\, dz = \tau_{s2}\,\frac{\pi a^2}{t}\Big[\frac{t}{\delta^2-1}\Big(\frac{R_{01}^2}{a^2} - \delta^2\Big) + \frac{t\delta^2}{\delta^2-1}\ \ln\frac{R_{01}^2}{a^2}\Big]$$

(z_{C_3} and z_{C_4} are the z coordinates on discontinuity surfaces *3* and *4*, respectively).

We only need to calculate the work of plastic strain on the surfaces of velocity discontinuity:

$$B_1 = \tau_{s2}\,\frac{\pi a^3}{3t},$$

$$B_2 = \int_0^{2\pi} d\varphi \Big(\tau_{s1} \int_{t+h-H}^{t} + \tau_{s2} \int_0^{t+h-H} \Big)\Big(\frac{z}{t} + \frac{t}{t_1(\delta^2-1)}\Big) a\, d\varphi\, dz$$

$$= \frac{2\pi a}{t}\Big(\frac{tt_1\tau_{s1} + t_1(t+h-H)(\tau_{s2}-\tau_{s1})}{\delta^2-1} + \frac{1}{2}\,(\tau_{s1}t^2 + (t+h-H)^2(\tau_{s2}-\tau_{s1}))\Big),$$

$$B_3 = \tau_{s2}\,\frac{\pi a}{t}\Big(a^2(\delta-1) + \frac{4t_1(\delta^3-1)}{\delta(\delta^2-1)^2}\Big),$$

$$B_4 = \int_0^{2\pi} d\varphi \Big(\tau_{s1} \int_a^{R_{01}} + \tau_{s2} \int_{R_{01}}^{a\delta} \Big)\Big(\frac{2(t-t_1)^2}{ta^2(\delta^2-1)^2} + \frac{a^2}{2t}\Big) dr$$

$$= \frac{\pi a}{t}\Big\{\frac{4(t-t_1)^2}{3(\delta^2-1)^2}\Big[\tau_{s1}\Big(\frac{R_{01}^3}{a^3} - 1\Big) + \tau_{s2}\Big(\delta^3 - \frac{R_{01}^3}{a^3}\Big)\Big]$$

$$+ a^2\Big[\tau_{s1}\Big(\frac{R_{01}}{a} - 1\Big) + \tau_{s2}\Big(\delta - \frac{R_{01}}{a}\Big)\Big]\Big\},$$

$$B_5 = \frac{2\pi a\delta}{\delta^2-1}\,[\tau_{s1}H + \tau_{s2}(t - t_1 + h - H)].$$

Here, B_1 is the integral along the channel bottom the channel, B_2, B_3, and B_4 are the integrals along the surfaces AB, BC, and AC, respectively, and B_5 is the integral along the cylindrical surface of the channel.

If $\tau_{s1} = \tau_{s2}$ (homogeneous material), the formulas given above are simplified and coincident with the known formulas [1].

To complete the solution, it is necessary to substitute all the calculated integrals into the Lagrange equation and find the minimum of the function. As a result, the flow channel size and the resistance force to impactor penetration are found.

Cases 1, 2, and 4 are treated similarly. All the integrals are calculated analytically. Being too cumbersome, the results are not given here

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