Vibration of lightweight floor/ceiling systems

Hyuck Chung

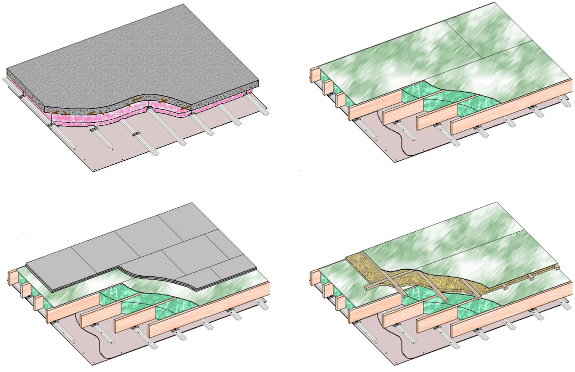
Department of Physics, University of Otago
New Zealand

Dunedin, July, 2009

Outline

- Project by wood product manufacturers: plaster board, LVL, aerated concrete, etc.
- Fast construction
- Better use of timber than wood-chip or pulp

- Design a lightweight floor/ceiling system with good low-frequncy sound insulation
- Predict vibration level of various designs
- Predict vibration of wider frequency ranges



Lightweight floor/ceiling system can be better than concrete slab.

Timber-based high rise building

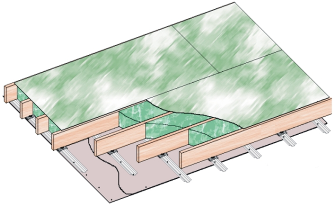
7 story apartment building in Välle Broar project in Växjö, Sweden.

Modelling procedure

Many components interacting with each other.

Energy contribution from components: Boards and joists flexing and moving, and junctions stretching and contracting, e.g. spring = $\frac{1}{2}\tau w^2$

Use variational principle



- Floor: plywood, particle board
- Joists: solid timber, LVL, I-beams
- Ceiling panels: plasterboard

Modelling technique

Lagrangian of the whole structure is minimized using the Fourier basis

Potential and kinetic energy of plate and joists

$$\frac{1}{2}\int_0^A\int_0^B \left\{D\left[\left(\nabla^2 w\right)^2 + 2\left(1-\nu\right)\left(w_{xx}w_{yy} - w_{xy}^2\right)\right] - m\omega^2 w^2 - fw\right\}\, dxdy$$

$$\frac{1}{2}\sum_{\text{joists}}\int_0^A \left\{EIw_{xx}^2 - m_j\omega^2 w^2 - f_i w\right\}\, dx$$

Helmholtz equation for acoustic pressure in cavity

$$\nabla^2_{x,y,z}p\left(x,y,z\right) + \frac{\omega^2}{c^2}p\left(x,y,z\right) = 0$$

$$w_i\left(x,y\right)=\sum_{m,n=1}^{N}c_{mn}^{i}\phi_m\left(x\right)\psi_n\left(y\right):\ \text{2D plates}$$

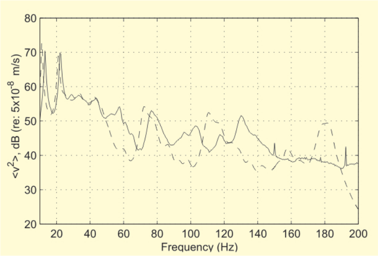
$$w_1\left(x,j\right)=\sum_{m=1}^{N}c_{mj}^{1}\phi_m\left(x\right):\ \text{1D joists}$$

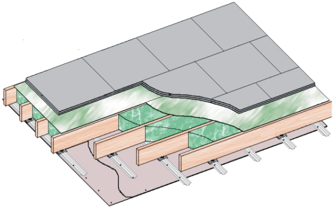
For a simply supported structure,

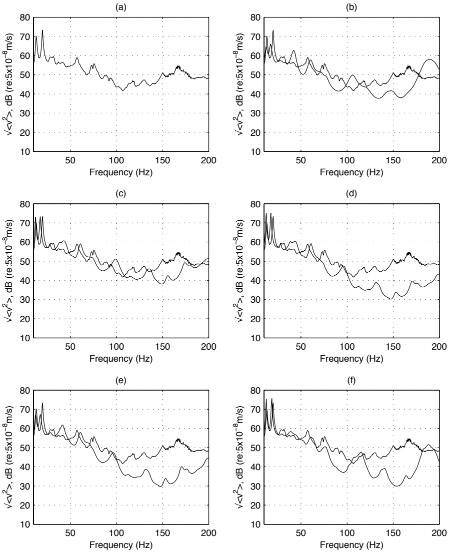
$$\phi_m\left(x\right)\propto\sin k_m x,\quad \psi_n\left(y\right)\propto\sin\kappa_n y$$

and $k_m=\pi m/A$, $\kappa_n=\pi n/B$ for $m,n=1,2,...,N$.

Deterministic model

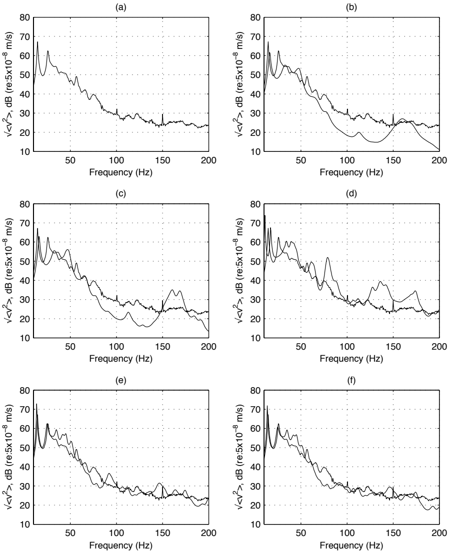






Refine the model by changing parameters:

Elasticity of rubber connectors, sound absorption by glassfibre wool, e.g. constant or frequency dependent



Refine the model by changing parameters:

Damping by sand and sawdust mix.

$$\text{Upper layer stiffness} = D\left[1 + \mathrm{i}\,\frac{0.4f}{100}\right]$$

Formulation with irregularities

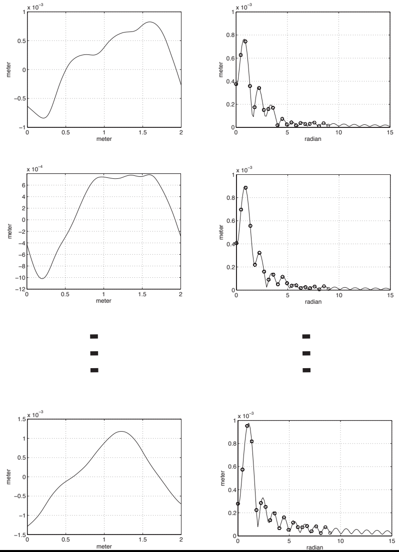
Contact condition between floor and joists

$$\mathbf{c}_1 = [L + L_\theta]\,\mathbf{c}_0.$$

L_θ : deviation from the straight line.

$$\mathcal{L} = \frac{1}{2}\mathbf{c}^{\mathsf{T}} \begin{bmatrix} M^1 + J^{11} & J^{12} & 0 & \cdots & J^{1L} \\ J^{21} & M^2 + J^{22} & J^{23} & & \\ 0 & J^{32} & M^3 + J^{33} & & \\ \vdots & & & \ddots & \\ J^{L1} & & & & M^L + J^{LL} \end{bmatrix} \mathbf{c} + \mathbf{f}^{\mathsf{T}}\mathbf{c}$$

Twisting joists



Sampling of the Fourier transform of the shape

$$[L_\theta] \sim \left[\hat{\theta} * \{\delta\left(k_{m-m'}\right) - \delta\left(k_{m+m'}\right)\}\right]$$
$$= \hat{\theta}\left(k_{m-m'}\right) - \hat{\theta}\left(k_{m+m'}\right)$$

$\hat{\theta}$ = Fourier transform of θ

Other irregularities

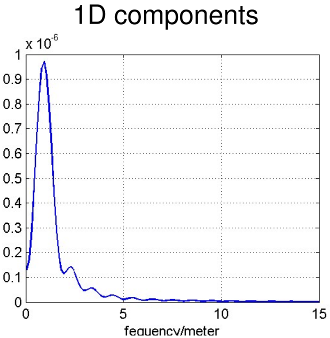
Stiffness of joist: $E + E_\delta(x)$, the matrix for strain energy

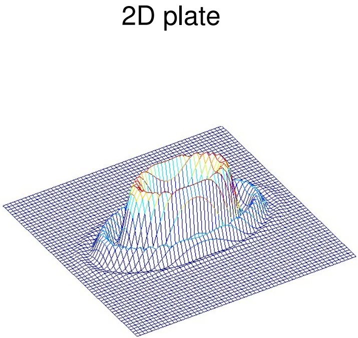
$$\begin{bmatrix} \ddots & 0 & 0 \\ 0 & EIk_m^4 & 0 \\ 0 & 0 & \ddots \end{bmatrix} + \left[Ik_m^2 k_{m'}^2 \hat{E}_\delta \left(k_{m\pm m'} \right) \right]$$

Stiffness of floor: $D + D_\epsilon(x, y)$, the matrix for strain energy

$$\begin{bmatrix} \ddots & 0 & 0 \\ 0 & D\left(k_m^2 + \kappa_n^2\right)^2 & 0 \\ 0 & 0 & \ddots \end{bmatrix} + \left[f\left(k_m.\kappa_n\right) \hat{D}_\epsilon \left(\pm k_{m\pm m'}, \pm \kappa_{n\pm n'} \right) \right]$$

Power Spectra





Our modelling procedure

PSD of component → Realizations of components → Solutions

True stochastic modelling

Probability density of components → Probability density of solutions

Currently there are no such models:

- Finite element method is good at continuum mechanics, but not at composites
- Statistical energy analysis is useful in high-frequency range

Other models

The whole structure is modelled as a single plate

$$\mathrm{ISPL} = V - f_{\mathrm{t}} - 6 \quad \mathrm{dB}$$

V is maximum velocity of the floor, and f_{t} is transfer function

Energy balance: Joists $\leftrightarrow$ Floor, Cavity $\leftrightarrow$ Floor, Joists $\leftrightarrow$ Ceiling, Floor $\leftrightarrow$ Ceiling, etc.

There are various ways to relate components, which are either numerical simulation or experiment based

Summary

- Simple procedure to include irregularities
- Irregular features given by power spectral density
- Hope for better estimation in mid-frequency?