Model Order Reduction for Control and Stability Analysis of Complex Dynamical Systems in a DC Microgrid

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***Abstract*—This paper proposes a method for controlling high-order complex systems using reduced-order models to reduce computational power and hardware requirements. The method is applied to a nonlinear sample system with six states, and different model order reduction (MOR) techniques are assessed. Linear quadratic regulators (LQRs) are then designed using the reduced-order models to control the system. The controllers are evaluated on the original system using MATLAB simulations. The results show that MOR techniques can provide satisfactory performance, and that the LQR controller designed using the reduced-order model can be implemented on the original system without changes.**

***Index Terms*—DC microgrid, model order reduction, Balanced truncation, Modal truncation, Hankel-norm approximation,Non-linear load, LQR controller**

I. INTRODUCTION

Reccently, the concept of microgrids has been proposed as a way to deploy distributed generators (DGs) and localize electric production. A microgrid combines several renewable energy sources, storage systems, and loads, and it offers several advantages, especially in terms of control [1, 2]. These complicated systems require a model to be developed in order to be controlled effectively. However, such systems have high-order state space models, resulting in high computational requirements. When microgrids are interconnected and combined to form multicluster microgrids, these computational requirements are even greater [3]. Moreover, it is difficult to establish every renewable energy resource's detailed state-space model with accuracy. For example, in the photovoltaic (PV) system as well as in the batteries, over ten orders can be determined, while a wind turbine model with different components may exceed over 20 orders [4, 5]. A microgrid system model may include hundreds of orders when it includes tie-lines, system networks, and load models. Consequently, model order reduction (MOR) techniques have become necessary to improve stability assessment, control design, or microgrid optimization [6].

In MOR techniques, the input-output behavior and essential characteristics of the original system are maintained while the system has been reduced in dimensionality and complexity. MOR can be accomplished using a variety of methods. Among MOR techniques, balanced truncation, balanced truncation of normalized coprime factors, modal truncation, balanced stochastic truncation, singular perturbation, and so on can be mentioned [7]. To reduce linear system order, each technique takes a different approach, with balanced truncation and balanced truncation of normalized coprime factors focusing on minimization of state count, whereas modal truncation concentrates on retaining only the dominant modes [8]. For different applications and scenarios, reduced-order models can be obtained using these methods for linearized or nonlinear models of microgrids [9].

Multi-input multi-output systems having error bounds are treated using the Gramian-based balanced truncation method in [10]. Similarly, a balanced truncation approach is also proposed in [11] for reducing the order of a large power system model. In [12], balanced residualization, balanced truncation, and balanced stochastic truncation MOR techniques have been used in a microgrid. A singular perturbation approach is employed in [13] to reduce a complicated system model that considers detailed electromagnetic and electromechanical transient modes in a microgrid cluster.

Controlling different states and parameters in microgrids is a challenging process that must be handled by appropriate controllers. In microgrids, there may be problems with conventional control methods, such as proportional integral (PI) controllers, in dealing with uncertainties and disturbances. In this regard, linear quadratic regulation (LQR) is one of the

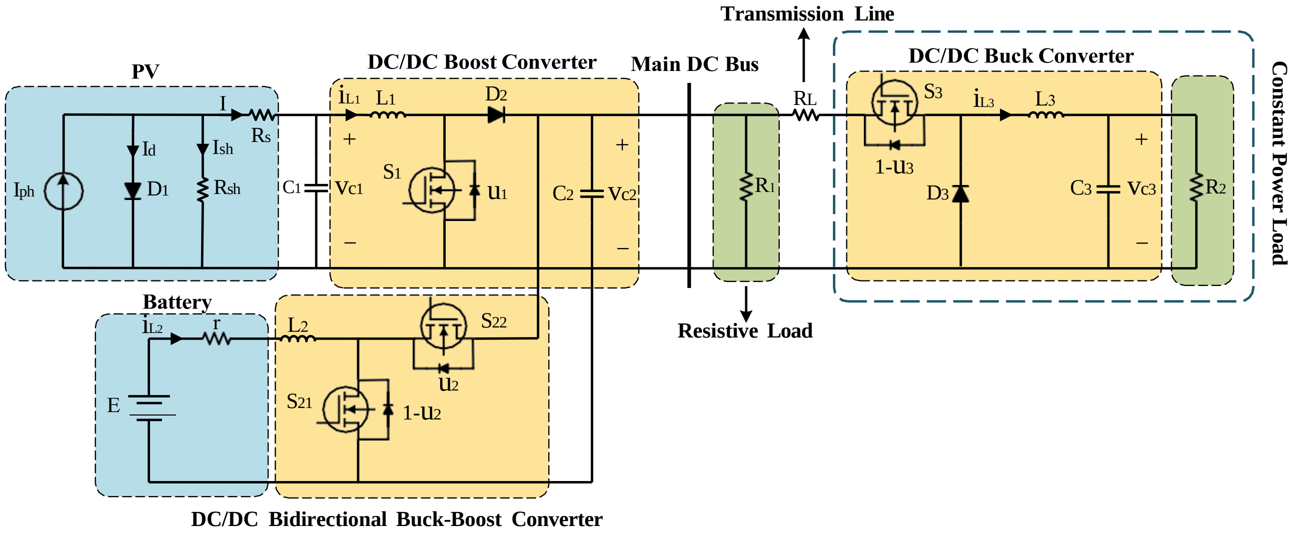

Fig. 1. The electrical circuit of the DC microgrid in the islanded mode.

control techniques that can be mentioned. In LQR, a cost function is minimized to optimize system performance. A more effective and efficient control system can be achieved this way. Furthermore, the cost function can be customized to achieve specific goals, for instance, minimizing power loss or maximizing renewable energy use, allowing for more versatility in design. In LQR, the goal is to identify the best control law to minimize the cost function [14]. To achieve this goal, different methods can be used, including analytical approaches, such as the Riccati equation, or learning algorithms, such as gradient descent.

By regulating the voltage, frequency, and power sharing among distributed generators and loads, LQR can improve the stability, efficiency, and power quality of microgrids. In [15], the design and implementation of LQR controllers for a hybrid energy system combining wind energy and ultracapacitor energy storage are presented. Moreover, a learning-based LQR is proposed in [16] to address the longstanding issue of model dependency regarding the control of vanadium redox flow batteries (VRBs). In addition, the decentralized output-feedback optimal LQR approach is implemented to develop a robust voltage and frequency controller designed to power an islanded microgrid that includes PV-battery systems in [17].

This study makes the following main contributions:

- The mathematical model of a microgrid is developed and linearized and then different MOR techniques, including the modal truncation method, balanced truncation method, and Hankel-norm approximation method are applied and compared to check how well the reduced models perform.
- The performance of LQR controllers is evaluated for the original and reduced-order models, making sure the controller developed for the reduced-order model performs satisfactorily for the original model as well.

Following is an overview of the paper. The model of the system is elaborated in section II. The basic concepts of developed LQR controller and MOR techniques are discussed in section III. In section IV, results and discussion are provided. Lastly, the paper ends with conclusion in section V.

II. MICROGRID MODEL

In Fig. 1, a typical PV/battery-based DC microgrid in islanded mode is illustrated. The configuration comprises a solar photovoltaic (PV) connected to a common DC link via a boost converter. Additionally,a battery storage system with a buck/boost converter is connected to the same DC link. The battery is represented by a constant voltage source (V_s) and internal resistor (r). Two loads exist-(R_1) and (R_2). Losses in the transmission line are represented by a line resistor (R_L), and switch duty cycles are denoted by u_i (i=1,2,3). The PV source, assumed to use a single-diode electrical circuit.

The PV generator employs a current source parallel (I_{ph}) to a diode and shunt resistance (R_{sh}), with a series resistor (R_s). By writing KCL and KVL in the microgrid, the following equations can be obtained:

$$\begin{aligned}
L_1\frac{di_{L_1}}{dt} &= v_{C_1} - v_{C_2}(1-u_1)\\
C_1\frac{dv_{C_1}}{dt} &= I_{ph} - I_o(\exp^{(v_{C_1}+R_s i_{L_1})a} - 1)\\
&\quad - (v_{C_1} + i_{L_1}R_s)/R_{sh} - i_{L_1}\\
L_2\frac{di_{L_2}}{dt} &= v_s - r i_{L_2} - v_{C_2}u_2\\
C_2\frac{dv_{C_2}}{dt} &= i_{L_1}(1-u_1) + i_{L_2}u_2 - v_{C_2}/R_1 - i_{L_3}(1-u_3)\\
L_3\frac{di_{L_3}}{dt} &= (v_{C_2} - i_{L_3}R_L(1-u_3))(1-u_3) - v_{C_3}\\
C_3\frac{dv_{C_3}}{dt} &= i_{L_3} - v_{C_3}/R_2
\end{aligned} \tag{1}$$

In these quations, $a = 1/(nV_t)$, n represents the number of cells connected in series in a PV module, V_t denotes the junction's thermal voltage, I_o indicates the dark saturation current of the solar PV, i_{L_i} and v_{C_i} (i=1,2,3) denote the instantaneous inductor currents and capacitor voltages, respectively.

$$x^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6] = [i_{L_1}\ v_{C_1}\ i_{L_2}\ v_{C_2}\ i_{L_3}\ v_{C_3}] \tag{2}$$

By writing dynamical equation of the system (eqution 1) in the canonical state-space form, expressed as $\dot{x}(t) = f(x(t), u(t))$. It is noteworthy that the state variables are voltage and current associated with capacitors and inductors:

Hence, the state-space model is reformulated into:

$$f(x,u) = \begin{bmatrix} \frac{1}{L_1}(x_2 - x_4(1-u_1)) \\ \frac{1}{C_1}(I_{ph} - I_o(\exp^{(x_2+R_s x_1)a} - 1) - (x_2 + x_1 R_s)/R_{sh} - x_1) \\ \frac{1}{L_2}(v_s - r x_3 - x_4 u_2) \\ \frac{1}{C_2}(x_1(1-u_1) + x_3 u_2 - x_4/R_1 - x_5(1-u_3)) \\ \frac{1}{L_3}((x_4 - x_5 R_L(1-u_3))(1-u_3) - x_6)) \\ \frac{1}{C_3}(x_5 - x_6/R_2) \end{bmatrix} \tag{3}$$

An equilibrium point is reached when the state trajectory of the DC microgrid system converges. In simpler terms, stability is ensured if the real parts of all eigenvalues in the linearized system are negative near the equilibrium point.

III. MODEL ORDER REDUCTION TECHNIQUES

In MOR, the input-output behavior of a higher-order system is approximated using a lower-order system. A linear time-invariant system can be expressed by the state-space model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{4}$$

In this model, $x(t) \in R^n$ represent the state vector, $y(t) \in R^p$ represents the output vector, and $u(t) \in R^m$ represents the input vector, and $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $D \in R^{p\times m}$ correspond to the system matrices. The goal of MOR involves trying to find a reduced-order system with a state vector $\hat{x}(t) \in R^r$, where $r < n$:

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}u(t) \\ \hat{y}(t) &= \hat{C}\hat{x}(t) + \hat{D}u(t) \end{aligned} \tag{5}$$

In this model, $\hat{A} \in R^{r\times r}$, $\hat{B} \in R^{r\times m}$, $\hat{C} \in R^{p\times r}$, $\hat{D} \in R^{p\times m}$ represent the reduced-order system matrices. There are several methods for reducing model order, each of which has benefits and drawbacks. In this paper, modal truncation, balanced truncation, and Hankel-norm approximation are implemented.

A. Modal Truncation

Modal method relies on the system matrix eigenvalues and eigenvectors. Identifying the dominant modes of a system involves finding their eigenvalues, which are the ones that have the greatest magnitude or influence. Once the eigenvectors are known, the system is projected on the modal vectors that span the subspace where the eigenvectors span [18]. The modal vectors corresponding to the dominant r modes are represented in the matrix V_r. The matrices for the reduced system can be expressed by:

$$\hat{A} = V_r^T A V_r,\ \hat{B} = V_r^T B,\ \hat{C} = CV_r,\ \hat{D} = D \tag{6}$$

B. Balanced truncation

The balanced truncation method aims to balance the reachability and observability of Gramians. Gramians matrices measure the amount of energy required to achieve a state from the input, and the amount of energy observable from the output. By using this method, the system is converted into a balanced realization with equal and diagonal Gramians, and then the smallest Gramian eigenvalues are truncated [19]. Assume that Q and P are the observability and reachability Gramians of the system, fulfilling the following equation:

$$A^T Q + C^T C + QA = 0,\ AP + BB^T + PA^T = 0 \tag{7}$$

Consider T is the transformation matrix that makes the system balanced in a way that $T^{-1}AT,\ T^{-1}B,\ CT,\ D$ represent the system matrices of the balanced realization. Matrices of reduced-order system are:

$$\hat{A} = T_r^{-1} A T_r,\quad \hat{B} = T_r^{-1} B,\quad \hat{C} = CT_r,\quad \hat{D} = D \tag{8}$$

C. Hankel-norm approximation

In the Hankel-norm approximation method, anti-causal inputs are converted into causal outputs via the linear transformation of the Hankel operator. An input-output model is described by a Hankel operator, whose singular values are known as Hankel singular values. In the Hankel method, we try to minimize the Hankel seminorm of the error system $G - \hat{G}$, where G and $\hat{G}$ represent the original and reduced-order transfer functions, respectively. Assume that H represents the Hankel operator for the system, which has the definition as follows:

$$Hf(t) = u(t)y(t),\ y(t) = F^{-1}\{G(j\omega)F\{f(t)\}\} \tag{9}$$

In this equation, $G(s)$ represent the system's transfer function, $u(t)$ represents a step function, and F and F^{-1} represent the Fourier and inverse Fourier operators, respectively. Consider H_r as the set of all Hankel operators by order r, and $\hat{H}$ is the optimal Hankel-Norm approximation of H, fulfilling

$$\|\hat{H} - H\|_H = min_{\tilde{H}\in H_r}\|\tilde{H} - H\| \tag{10}$$

where, $\|H\|_H = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\|G(i\omega)\|_F^2\, d\omega}$ is the Hankel seminorm and $\|G(j\omega)\|_F$ represents the Frobenius norm of the frequency response matrix. Afterward, reduced system matrices are derived from $\hat{H}$ realization, which can be determined iteratively [20].

IV. LQR CONTROLLER

LQR is one of the robust control strategies that considers system states and control inputs to make optimal decisions. Consider a state-space model of a system as presented in (4). Using the linear state feedback, the closed loop poles can be allocated to an arbitrary position once the complete state x is available for feedback. Therefore, the control law is [21].

$$u(t) = -K_{lqr}x(t) \tag{11}$$

where K_{lqr} represents the feedback gain matrix, leading to the following closed-loop system equation:

$$\dot{x}(t) = [A - B\quad K_{lqr}]x \tag{12}$$

The optimal signal u in LQR is derived by minimizing the cost function J as follows:

$$J = \int (x^T(t)Qx(t) + u^T(t)Ru(t))dt \tag{13}$$

In this cost function, Q and R represent the weighting matrices, indicating how the error and energy consumption are weighted. In order to determine the best control action, we can use the Riccati equation, which is as follows:

$$A^T P + Q + PA - PBR^{-1}B^T P = 0 \tag{14}$$

Assuming the matrices R and Q are chosen, solving the Riccati equation yields the matrix P, which then can be used to find K_{lqr}:

$$K_{lqr} = R^{-1}B^T P \tag{15}$$

V. SIMULATION AND RESULTS

The nonlinear system depicted in Fig. 1 has been implemented in MATLAB. The steady-state values of the system variables $[x_1, x_2, x_3, x_4, x_5, x_6]$ are $[8.379, 36.48, -3.073, 121.6, 2.225, 20.03]$. Then with the help of Jacobian matrix linearization technique, and these steady states, system has been linearized.

$$A = \begin{bmatrix} 0 & 200 & 0 & -200 & 0 & 0 \\ -5000 & -1723 & 0 & 0 & 0 & 0 \\ 0 & 0 & -200 & 0 & 0 & 0 \\ 500 & 0 & 0 & -3.472 & -500 & 0 \\ 0 & 0 & 0 & 200 & -2000 & -200 \\ 0 & 0 & 0 & 0 & 3333 & -370.4 \end{bmatrix} \tag{16}$$

$$B = \begin{bmatrix} 24320 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -24320 & 0 \\ -4190 & -1536 & 1112 \\ 0 & 0 & -2281 \\ 0 & 0 & 0 \end{bmatrix} \tag{17}$$

The matrix in (16) and (17) represent the linearized model of the system around an operating point. This system order is reduced to four by employing modal and balanced truncation, as well as the Hankel-Norm approximation. Subsequently, an LQR controller is implemented on each reduced system, and the output results are mapped back to the original six-order system. According to Fig. 2, the pole diagrams for the original system and the three reduced systems are shown.

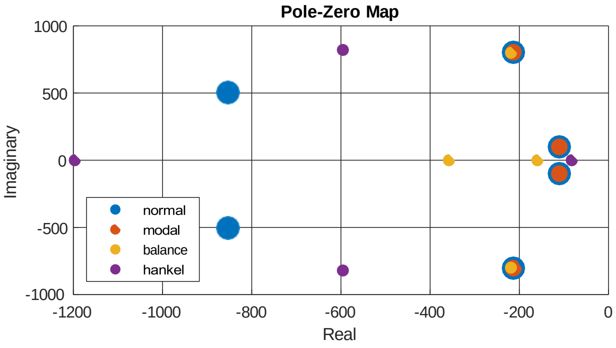

Fig. 2. Pole Diagram of different models of the system.

The LQR controller is applied to these models based on the cost function coefficients of $Q = [1, 0, 0, 0; 0, 1, 0, 0; 0, 0, 1, 0; 0, 0, 0, 1]$ and $r = [20, 0, 0; 0, 150, 0; 0, 0, 60]$ for the reduced-order models, and a 6x6 identity matrix for Q of the original system. The transformed state values are illustrated in Fig. 3. As shown, the balanced truncation method exhibits a faster response, while the Hankel-Norm approximation method provides a smoother response. Then transformed-state values are then mapped back to the original coordinate system as depicted in Fig. 4.

The performance of the different reduction methods is compared to the original system with the LQR controller, and the results are presented in Fig. 4. The findings demonstrate that controlling a complex system with its reduced-order model is feasible and can be generalized to higher-order systems.

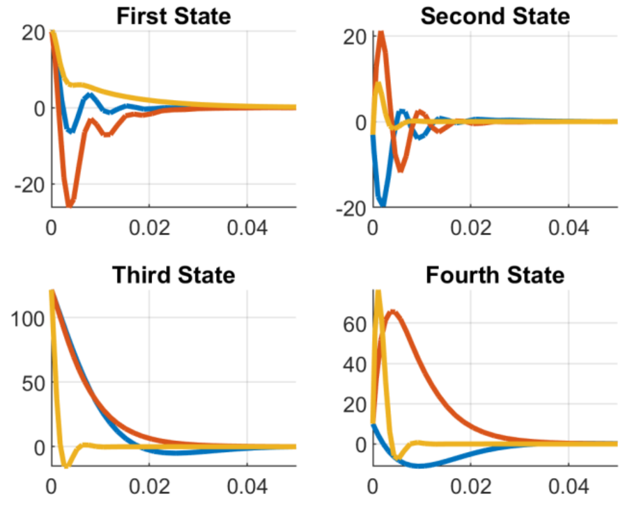

Fig. 3. LQR Controller responses: — is the modal truncation response, — is the balanced truncation response, — is Hankel-Norm approximation response.

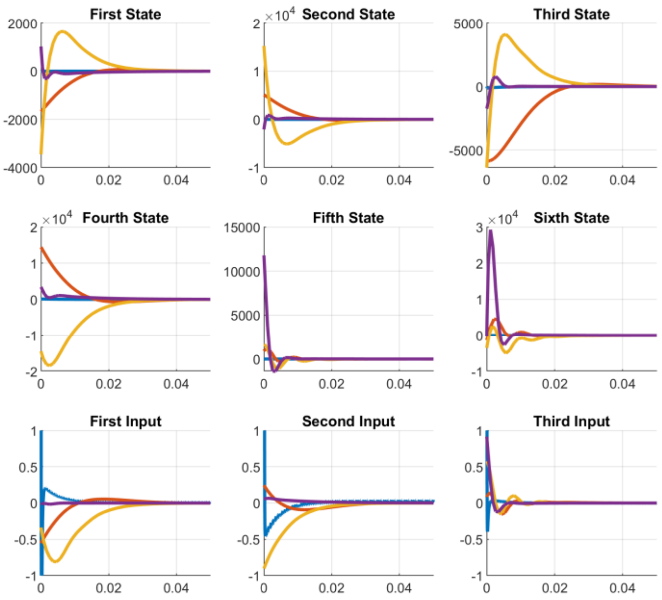

Fig. 4. LQR Controller responses: — is the original system response, — is the modal truncation response, — is the balanced truncation response, — is Hankel-Norm approximation response.

Moreover, the Hankel-Norm approximation method yields the closest response to the controller on the original system, making it a promising candidate for reducing the degree of the system's model.

VI. CONCLUSION

In this paper, the idea of controlling a complex nonlinear system was investigated based on a simpler system, provided by popular model order reduction (MOR) techniques, including modal truncation, balanced truncation and Hankel-norm approximation. The proposed method was applied to a complex nonlinear system with six states. Using the developed MOR methods, the system order was reduced to four. In order to control the system, linear quadratic regulator (LQR) controllers were designed based on reduced-order models. The LQR controller's performance on the original and reduced-order models was evaluated with MATLAB. The results showed that the Hankel-Norm approximation technique outperformed the other MOR methods regarding accuracy and robustness, while the LQR controller provided stable outputs for different states of the system.

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