AUCKLAND UNIVERSITY OF TECHNOLOGY

MASTER'S THESIS

Metaphors we teach by

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Abstract

Many researchers have commented on the use of metaphor in education. This thesis considers the extent to which metaphor and metaphorical language is used in the case of undergraduate statistics education.

There are many aspects of an undergraduate statistics course that employ metaphor. I will consider a typical undergraduate statistics course in terms of the following components: educational policy, educational administration, spoken lectures, statistics textbooks, assessment, and course evaluation. Metaphor is present in all these aspects, and I discuss its importance in each aspect in turn.

Although much research into tertiary mathematics education has been undertaken, only a small proportion is devoted to statistics education; and it is not clear whether statistics should be viewed as a subset of mathematics for this purpose. I discuss the extent to which insights from the study of metaphor in the teaching of mathematics are applicable to the teaching of statistics.

Contents

Declaration of Authorship

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person (except where explicitly defined in the acknowledgements), nor material which, to a substantial extent has been submitted for the award of any degree or diploma of a university or other institution of higher learning.

Signed:

Date:

"Educators have been talking about foundations *for so long that it no longer seems metaphorical—but that is when metaphors become the most dangerous. All that "foundation" literally means in the context of instruction is something taught initially in order to facilitate future learning "*

Bereiter (2005), page 335

"We believe that educators need to organize curriculum, instruction, assessment and schools using metaphors that they choose intentionally and deliberately. Metaphors do shape educational practice. If we do not choose our metaphors, our metaphors will simply choose us "

Badley and Brummelen (2012), page 7

"Language, and metaphorical language in particular, constructs the worlds in which we live. Consequently, the metaphors we use to talk and to think about education have a profound effect on what it is like to be educated in the societies in which we use them. We have a long history of using metaphors in education, both as calls to arms and as supposed neutral descriptors. Both have ideological force, and both have the potential to steer us into particular ways of thinking about, and going about, the processes of education"

Paechter (2004), page 460

Sometimes, there is little awareness of the metaphors that guide our behaviours and shape our institutional structures. We may not even be aware of the negative consequences of the metaphors we live by

Marland (2005), page 37

1 Introduction and literature review

Metaphor is usually defined as an invitation for the reader or listener to consider one thing in terms of another (Steen, 1994), although its definition is far from clear; Knowles and Moon (2006) and others state that metaphor is generally easier to recognize than to define. Modern treatments go to some lengths to refute the notion that metaphor is confined to poetical use, pointing to the ubiquity of metaphor at all levels of formality (Deignan, 2005) in spoken (Cameron, 2003) and written (Charteris-Black, 2004) English. Textbooks usually distinguish metaphor from simile, in which X is held to be *like* Y; compare metaphor, in which X is said to *be* Y, although this characterization does not stand close scrutiny.

Here, I will investigate the use of metaphorical language in undergraduate statistics education. In this thesis, *metaphor* will be interpreted as meaning any non-literal language and will thus include idiosyncratic use such as word problems.

Metaphor is a ubiquitous phenomenon in language and this thesis examines metaphor as used in education, specifically undergraduate statistics education. Although research into tertiary mathematics education is reasonably well represented, only a small proportion is devoted to statistics education; and it is not clear whether statistics should be viewed as a subset of mathematics.

An example of metaphor in an educational context might be "this student is the *cream* of the cohort"[1]. Students are not dairy products and in this case we are invited to compare the student's elite academic performance (in relation to his peers) with the most desirable component of milk, viz. the cream. Observe that the comparison to cream stimulates only a small part of the semantic connotations of actual cream: specifically the desirable properties of tastiness and expensiveness. Cream itself has many undesirable features: it is unhealthy, it is sickly, and it is fattening but these properties are never used metaphorically.

A more pertinent example might be to describe a student as "top of the class". Paechter (2004) considers this metaphor to drive a particular view of assessment, specifically one based on comparisons between students rather than on whether each student has achieved the learning objectives of the unit of study at hand.

Skillful use of metaphor does not necessarily include any form of the verb *to be*, for example:

> An iron curtain has descended across the continent
> —W. Churchill (attributed; circa 1946)

In this famous quote, the verb is *to descend*: the reader (originally the listener) must infer that the dividing boundary developing between East and Western Europe is to be considered a physical object. Note too Churchill's mentioning iron, its utilitarian connotations underscoring the perceived economic poverty of communism when compared with the prosperous West. Curtains literally descending would be familiar to Churchill's theatre-going audience as marking the end of a performance, and it is reasonable to believe that Churchill was alluding to communism's extinguishing of democratic rights.

[1] Slanted type is used to denote metaphor, as in "all the world's a *stage*".

In this thesis, metaphor is discussed using standard terminology: the *topic* (sometimes *tenor*) is the concept being described, and the *vehicle* (sometimes *figure*) is the concept used to describe the topic. In Churchill's quote above, the topic would be the political divide between East and West Europe, and the vehicle would be a stage curtain made of iron.

It is clear that such metaphors can be informative about a speaker's thoughts; skillful orators can utilize metaphors effectively to encapsulate the mood of a nation, or indeed to provoke debate about a nation's educational system (Robinson, 2011). Theoretical linguists and sociologists have published a large amount of material dissecting and analyzing this form of language from many perspectives: workers have studied the effective use of metaphor (by politicians; see Perrez and Reuchamps (2015), for example), and the effect of rhetorical metaphor on listeners (Keating, 2015).

In this thesis, I investigate the usage of metaphor in education, and focus on one particular aspect of education that is familiar and important to me, that of undergraduate statistics. At the undergraduate level, "statistics" is both a practical and a mathematical discipline; it is practical in that many students will be expected to learn the skills required to extract useful information from data, but mathematical in the sense that many important statistical ideas can only be understood in a relatively sophisticated mathematical context.

1.1 Metaphors we live by

> Metaphors may create realities for us, especially social realities. A metaphor may thus be a guide for future action. Such actions will, of course, fit the metaphor. This will, in turn, reinforce the power of the metaphor to make experience coherent. In this sense, metaphors can be self-fulfilling prophecies
>
> Lakoff and Johnson (1980), page 156

The publication of *Metaphors we live by* (Lakoff & Johnson, 1980) ushered in the modern era of metaphorical thinking. In this short and accessible book, the authors argue that metaphor is in fact a cognitive phenomenon (in which one thing is considered in terms

of another), which happens to have a linguistic manifestation (for example, writing that "cancer is a *battle*".

As Lakoff and Johnson (1980) and others point out, metaphors are not confined to literary or rhetorical contexts, and are used frequently in everyday language. For example, one might say student attendance was *up*, indicating an increased in attendance. Lakoff and Johnson (1980) would classify this as an orientational metaphor: the word *up* denotes increased altitude in literal speech, but is used here as part of a systematic scheme whereby an orientational vehicle (up, down, in, out, etc) refers to a non-spatial topic (happy, sad, rich, hot, cold, etc). Such systematic schemes are traditionally denoted using small capitals, as in: THE MIND IS A CONTAINER; other apposite examples might include COMMUNICATION IS TRANSFER, or EDUCATION IS ACQUISITION. When sensitized to the issue, one tends to see conceptual metaphors everywhere. For example, we *pass* an exam (LIFE IS A JOURNEY or possibly CHALLENGES ARE OBSTACLES), and either *progress through* college (SITUATIONS ARE CONTAINERS), or *drop out* (DOWN IS BAD).

Such quotidian use of metaphor can easily pass unnoticed. However, Lakoff and Johnson (1980) argue persuasively that metaphors can and do inform our conceptual system and influence our actions; they make a strong case for a serious study of metaphor in a variety of contexts. Bereiter (2005), in particular, cautions against the MIND IS A CONTAINER metaphor, deriding it as "folk theory of mind", and goes on to argue that its uncritical adoption is damaging to education.

1.2 Cognitive metaphor theory

Cognitive metaphors such as THE MIND IS A CONTAINER are rarely used directly in speaking or writing but are influential in that they function at the level of thought, below language (Deignan, 2005); it is common to say that linguistic metaphors *realize* conceptual metaphors. Deignan goes on to give five tenets of cognitive metaphor theory:

1. Metaphors structure thinking
2. Metaphors structure knowledge
3. Metaphor is central to abstract language

4. Metaphor is grounded in physical experience

5. Metaphor is ideological.

These five tenets of metaphor theory may be used to structure thinking of corpus analysis when considering metaphors in education.

1.2.1 Metaphors structure thinking and knowledge

Corpus linguistics is the study of language as expressed in naturally-occurring text; the standard methodology (annotation-abstraction-analysis) is due to Wallis and Nelson (2001), although this is not well-suited to analysis of metaphor in education. Metaphorical analysis of corpora is discussed by Deignan (2005), in the context of understanding metaphor per se. This approach is not especially suitable for the investigation of a specific metaphor (or set of metaphors), as here, but the methodology has been adopted by Charteris-Black (2004) and others, to assess specific areas of language use. The author takes corpora from political rhetoric, financial reporting, and religious texts, and assesses metaphor use in a range of written corpora. But note that the texts all have one feature in common: they are written specifically to convince the reader to think in a particular way, to adopt a particular stance. The techniques used in such corpus analysis are suitable for political rhetoric or similar texts, but tend to focus on metaphor in general rather than specific metaphors such as the level metaphor or the foundation metaphor.

1.2.2 Metaphor is central to abstract language

In a very heavily-cited work, Reddy (1993) discusses one very important concept in education, that of communication. In essence, his thesis is that communication is frequently discussed using the *conduit* metaphor (Lakoff & Johnson, 1980); he later characterized this as a distillation of the conceptual metaphors IDEAS ARE OBJECTS, LINGUISTIC EXPRESSIONS ARE CONTAINERS, and COMMUNICATION IS SENDING. Reddy states that stories about communication (of which education figures prominently) are largely determined by semantic structures and it is clear that the primary structure he has in mind is (linguistic) metaphor.

1.2.3 Metaphors are grounded in physical experience

Many metaphors have their origins in physical space; Lakoff and Johnson (1980) term these *orientational* metaphors. Orientations may include up-down, in-out, central-peripheral, near-far, shallow-deep, and so on; the best examples include HAPPY IS UP; INTIMACY IS PROXIMITY and it is clear that the *level* and *foundation* metaphors are at least partially spatial. *Ascending the levels* of a course of study and indeed the foundation metaphor itself would be exemplars of SOPHISTICATED IS UP.

1.2.4 Metaphors are ideological

Although most authors discussing metaphor interpret the word "ideological" in terms of either corporate or national policy (Perrez & Reuchamps, 2015), it is worth remembering that ideology can apply to any system of ideas by any group or community; here the relevant community would be tertiary teachers and students. Consider the *level* metaphor as an example. Whether the implications of the metaphor constitute an ideology (or indeed whether considering current mathematical education's level-based philosophy from an ideological perspective is a coherent or desirable scheme) remains an open question.

1.3 Research methodologies

> Metaphor is both important and odd—its importance odd and its oddity important—Goodman (1979), page 125

A systematic citation analysis of Lakoff and Johnson (1980)'s seminal work revealed few studies of metaphorical language in specific contexts; the majority of those discussed the ramifications of metaphor in the medical profession. Montgomery (1991) discusses medicine-as-combat and Harrington (2012) presents cancer-as-war.

This type of research on metaphor in education is sparse, and what does exist is largely focused on its use by teachers at primary or secondary level (Munby, 1986; Cameron, 2003). Paechter (2004) is

one of the very few works discussing the level, foundation, or framework metaphor; perhaps this is because their use constitutes an "unquestioned norm", which renders investigation difficult.

Established research methodologies appear to be poorly suited to this work. Hermeneutic analysis would seem to be inappropriate on the grounds that there is no canonical text (Mantzavinos, 2005). It might be argued that the Education Act 1989 or the New Zealand Qualifications Framework constitute revelatory texts but a close word-by-word reading of these documents would seem to be unlikely to produce any insight into metaphor as used in the field.

Documentary analysis techniques (Fitzgerald, 2012) might be more promising although they are geared towards historical rather than social analysis and again there is no canonical document to analyze.

Harrington (2012) presents one of the very few discipline-specific surveys of metaphor usage in discourse, in this case medical science. She presents a careful and insightful study of metaphor use in (written) discourse about cancer—military and journey metaphors figure prominently—but her work does not appear to fall into any recognizable research methodology. She does analyze various documents which she believes to be representative or influential, but makes no attempt at a systematic survey or to trace any historical drift. Having said that, her work is extremely convincing, and very heavily cited, and provided inspiration for the present work.

In an educational context, Cameron (2003) considers metaphor in spoken English, using 10- and 11- year old students as informants. She employs concepts from applied linguistics to infer students' learning strategies, and to improve teaching methods. Her data comprised natural utterances (and a small amount of written material) and her conclusions centered around detailed analysis of carefully selected, and mostly very short, fragments.

These two approaches contrast sharply in their epistemological assumptions about the nature of knowledge: Harrington is clearly oriented toward propositional knowledge [of patients' and doctors' speech]; while Cameron is more focused on knowledge by acquaintance: she makes little attempt to generalize her findings beyond the confines of her classroom study.

Both works, however, share a common understanding of ontology: both writers maintain the existence of an entity, here the linguistic phenomenon of metaphor, with certain properties. The assumption is that it is possible to observe this entity, albeit imperfectly, and make inferences about its nature and properties. Admittedly, Cameron observes human behaviour through the lens of linguistic theory, while Harrington considers only published research, but both clearly have an entity in mind which they wish to learn about.

It appears that different individuals hold widely differing interpretations of the vehicles "level", "foundation" and "framework" when used in educational policy. For example, to me the level metaphor involves ascending a multistory building; but many of my colleagues view the level metaphor as actually constructing a large structure or edifice, something that was not part of my thinking.

One possibility might be to interview, say, ten practising mathematics or statistics lecturers in a semi-structured interview; this might produce interesting results. However, one potential pitfall might be the recruitment of informants, who would need to be chosen carefully: the interviewees would not be a random sample, but on the other hand statistical validity is not an issue in studies of this type (Ribbins, 2012). Also, merely stating the purpose of the interview might distort their perceptions; but an appropriately structured system of questioning might be able to mitigate this deficiency. Further work would be needed to assess whether this approach would be worthwhile.

Cornelissen (2012) points out that metaphor is a commonly considered aspect of organizational theory. Amernic, Russel, and Tourish (2007), for example, consider the metaphors in a series of letters written by a CEO to his shareholders; and Tourish and Hargie (2012) consider the metaphors used by disgraced bankers following the 2006 financial crisis. These authors study *root metaphors*—that is, metaphors which provide "rich summaries of the world and reveal dominant and powerful ways of seeing". Root metaphors differ from conceptual metaphors in that a root metaphor is generally used as a rhetorical device with the intent to frame public discussion. Tourish and Hargie (2012) do not classify root metaphor as deception as such, but rather as systematic distortion; compare conceptual metaphor, in which the emphasis lies more in its role as a cognitive mechanism.

Drawing on these ideas, this thesis presents a semi-systematic survey of language as used in the various phases of a standard undergraduate statistics course.

1.4 Metaphors in tertiary education

If Lakoff and Johnson are correct in their view of metaphor being a cognitive—rather than a linguistic—-phenomenon, then it is clear that metaphor will have an important part to play in education. This thesis considers metaphors as used in tertiary education, with a focus on undergraduate statistics.

The overwhelming majority of studies of metaphor in an educational context focus on its use by teachers, as opposed to students or administrators. Willox et al. (2010) observe that most literature focuses on metaphor as an "instructor driven pedagogical tool"); however, students too create and use metaphor in many educational contexts including learning activities such as lectures as well as in assessment.

In this thesis, I consider metaphor as used in written and spoken language by instructors and students with a focus is on the case of undergraduate statistics; most of my teaching is in this area. The thesis chapters cover the different aspects of an undergraduate statistics course; they are ordered roughly in chronological order as experienced by the lecturer. The aspects covered are as follows:

- Metaphor in educational planning (level/foundation metaphors and possibly the factory metaphor or the acquisition/participation metaphor) (chapter 2);
- Metaphor educational administration (chapter 3);
- Metaphor used in spoken or recorded lectures (chapter 4)
- Metaphor in statistics textbooks (chapter 5)
- Metaphor in statistics assessment (chapter 6)
- Metaphor in course evaluation (chapter 7).

1.5 Conclusions

Following Lakoff and Johnson's seminal publication, many scholars have written about the power of metaphor to shape and guide thinking. One way in which the effect of metaphor may be studied is via its linguistic manifestation, which is open to study in both corpora and spoken English. Metaphor is known to be influential in thinking about education (Sfard, 1998's acquisition metaphor; the factory metaphor), and also more directly in educational policy documents, and even more directly in classroom practice. There are a few fields, medical science in particular, where particular metaphors (martial metaphor for cancer) have been studied and shown to have powerful and sometimes harmful effects. Given the undeniable power of metaphor, one might expect that metaphorical language is a potentially important component of statistical education. The extent to which this is true in undergraduate statistics education is the topic of this thesis.

> We have no literal language for talking about what thoughts do. . . [there is] no possible way of literally saying what has to be said: so that if it is to be said at all, metaphor is essential
>
> Ortony (1975), page 49

> Your brain does not process information, retrieve knowledge or store memories
>
> Epstein (2016), front cover

> The "dead metaphor" account misses an important point: namely, that what is deeply entrenched, hardly noticed, and thus effortlessly used is most active in our thought. The metaphors listed above may be highly conventional and effortlessly used, but this does not mean that they have lost their vigor in thought and that they are dead. On the contrary, they are "alive" in the most important sense—they govern our thought: they are *metaphors we live by*. One example of this involves our comprehension of the mind as a machine.
>
> Kövecses (2010), page 12

Educators have been talking about foundations for so long that it no longer seems metaphorical—but that is when metaphors become the most dangerous. All that "foundation" literally means in the context of instruction is something taught initially in order to facilitate future learning. This may or may not have anything to do with foundational ideas of the discipline, but the metaphor disposes people to prejudge this issue.

Bereiter (2005), page 335

2

Metaphor in educational policy

2.1 Overview

In this chapter I investigate metaphor in educational planning and policy, with special reference to undergraduate statistics education. I discuss metaphor in general, and then show why metaphorical language is both important and informative in educational policy. I present a literature review of research that has been carried out in this area, and set out a proposal for further work.

Miller and Fredericks (1990) consider metaphor in the context of educational policy. They state that metaphors may serve as important clues in better understanding of the implicit ideological preferences of the policymakers themselves.

There are three metaphors that appear to be particularly pervasive in the structuring of undergraduate education: the *level* metaphor, the *foundation* metaphor, and the *framework* metaphor. These metaphors are commonly used in the context of undergraduate statistics education, and this chapter gives an overview of these and related metaphors.

2.2 The *level* metaphor

The Oxford English Dictionary (henceforth OED) gives a number of literal senses of the word "level", the most germane of which are "a horizontal plane" and "position on a real or imaginary scale". Much undergraduate mathematics is structured into "levels" (see, for example, the New Zealand Qualifications Framework) that broadly correspond to time spent in further education. Typically, a first year undergraduate course would be described as "level 5", a second year course as "level 6", and so on. However, it should be noted that the OED does not admit that levels are discrete, although other dictionaries include "a floor within a multi-storey building" which makes the discretization of the vehicle explicit.

Note that this metaphor is easily adapted to a constructivist framework: the students construct the multi-storey building as they ascend the levels. However, the metaphor fails in certain key respects. Firstly, the content of each level is generally held to be of equal, uniform, difficulty. This is questionable at best and, I would claim, demoralizing at worst. Secondly, higher levels are supposed to be of successively greater difficulty; this is unlikely to be true if "foundational" mathematics is studied. And thirdly, any cohort is imbued with some form of magical elevation from one level to the next at the beginning of a school year. Paechter (2004) points out that the *level* metaphor entails that every layer rests on the one before, observing that such metaphors are temporal rather than spatial.

Miller and Fredericks (1990) consider such metaphors in the context of national educational policies and point out that such metaphors reflect a desire for permanence, stability, and predictability.

In the context of statistics education, there are many concepts for which the discrete nature of levels is at odds with the wide range of sophisticated needed for their understanding. One example might be the concept of statistical independence. This is considered to be a "level 5" concept and indeed the basic definition is readily understandable: events A and B are independent if $P(A|B) = P(A)$. However, the concept of statistical independence is notoriously tricky, even for professional statisticians—with Dawid (1979) detailing a number of common fallacies surrounding the distinction between independence and conditional independence.

2.2.1 The International Standard Classification of Education

Explicit statements on the nature of educational levels appear to be rare[1]. However, one of the very few places where the level metaphor is discussed explicitly is the International Standard Classification of Education, ISCED (UNESCO, 2012). This is the "standard framework used to facilitate international comparisons of education systems". Item 47 is worth quoting in full:

> "The notion of "*levels*" of education is represented by an ordered set, grouping education *programmes* in relation to *gradations of learning experiences*, as well as the knowledge, skills and competencies which each programme is *designed* to *impart*. The ISCED level reflects the *degree* of complexity and specialization of the *content* of an education programme, from *foundational* to complex"—(UNESCO, 2012, item 47)

(here, salient metaphorical terms are indicated in italics). In these short quotes, ISCED uses the level, foundation, and framework policy in concert. All three are examples of the conceptual metaphor IDEAS ARE BUILDINGS; but note that the first and second are also examples of orientational metaphors, in this case UP IS GOOD; and the third is—arguably—an example of THE MIND IS A MACHINE; or just possibly THEORIES ARE OBJECTS. ISCED seems to be aware that the *level* metaphor is indeed only a metaphor. Item 48 reads:

> "Levels of education are therefore a construct based on the assumption that education programmes can be grouped into an ordered series of categories. These categories represent broad steps of educational progression in terms of the complexity of educational content. The more advanced the programme, the higher the level of education"—(UNESCO, 2012, item 47)

Documents such as the New Zealand Qualifications Framework employ the level metaphor extensively, presenting tables of properties of study from level 1 (certificate) through level 10 (PhD).

[1] The Bologna process (Keeling, 2006) discusses "cycles" corresponding broadly to BA, MA, and PhD degrees.

The other discrete orientational metaphor used in this context is that of a *taxonomy*, the most prominent examples of which are Bloom's (Anderson, Krathwohl, & Bloom, 2001) and SOLO (Briggs & Collis, 1982). However, there does not appear to be any connection between the taxonomy metaphor and the level metaphor as used in education.

2.3 The *foundation* metaphor

For *foundation*, the OED gives "the solid ground or base on which an edifice or other structure is erected" and the word is often used to refer to mathematics content that is more basic or fundamental than other material. In this context the phrase is an instantiation of the THEORIES ARE BUILDINGS conceptual metaphor.

The term "foundation" in the context of statistics education has two meanings: firstly, it refers to statistical knowledge and techniques that are frequently assumed knowledge in more advanced courses; and secondly, it refers to the foundations of the discipline, usually meaning the relationship between statistical reasoning and the more fundamental science of probability.

Bereiter (2005) gives a disarming, yet devastating, observation on the first sense of *foundation* (itself rich in metaphor):

> "But the insidious effect of the foundation metaphor does not stop there. No builder would construct a foundation without having a pretty clear idea of the building to be erected upon it; only a subcontractor would do that. Beginning students, having no way to foresee the eventual structure of knowledge, are therefore cast into the role of subcontractors"—Bereiter (2005), page 336

The two operative conceptual metaphors, specifically UP IS GOOD and THEORIES ARE BUILDINGS, embody a spatial contradiction in the sense that *foundations* are the lowest level of a building, yet are often held to be of the *highest* importance. It is perhaps worth pointing out that, to the professional mathematician or physicist, those working in the foundations of the discipline enjoy the highest status in the profession: McCulloch and Crook (2013), pages 469–571 talk of the "high status" and "great prestige" of pure mathematics when

compared with applied; note that there is no equivalent of the *Apology* (Hardy, 1940) for applied mathematics (or indeed statistics).

2.3.1 Foundational statistics

The term "foundational statistics" usually refers to study of the relationship between probability and statistics. Probability—itself on a shaky and arguably meaningless logical footing[2]—has its roots in pure mathematics, and I consider metaphor usage in mathematics in Chapters 4 and 5.

The *foundation* metaphor might suggest that foundational statistics is somehow more fixed, more solid, or more well-established, than other branches of statistics. One could reasonably demand that "foundations" of any discipline be firm. How can a study of statistics be built on anything but the most sturdy of fundaments?

Even a cursory study of foundational issues in statistics reveals two surprising features: firstly, the large number of mutually exclusive and inconsistent statistical principles in common use (Edwards, 1984); and secondly, the deficiencies and unavoidable contradictions of inferential statistics as practised in the applied sciences (Wasserstein & Lazar, 2016). In the context of statistics and statistics education, Robins and Wasserman (2000) consider this and observe that there is no agreement on which principles are "right", nor on which should take precedence over others. Thus statistical education must be structured in such a way that these difficulties are obscured by pedagogical strategies.

2.4 The *framework* metaphor

For "framework", the OED gives "a structure made of parts joined to form a frame; esp. one designed to enclose or support; a frame or skeleton", with senses supporting this use in an industrial or horticultural context.

[2]The hugely influential treatise of de Finetti (1975) famously begins with the provocative statement that PROBABILITY DOES NOT EXIST (the intended sense was that probability has no objective meaning). Many subsequent authors, notably Nau (2001), quote this rather subversive assertion with approval, retaining the startling capitalization of the original.

In an educational policy, "framework" usually refers to an organized set of standards, aims, or learning objectives that loosely specify the type of material to be learned. This is frequently used to support the *level* and *foundation* metaphors. However, frameworks are generally held to be rigid and inflexible structures and the effect of the framework metaphor is not necessarily beneficial.

Paechter (2004) observes that the framework metaphor, along with its close relation the *scaffolding* metaphor, is an exemplar of a wider class of structural spatial metaphors. Paechter goes on to speculate that these metaphors are unusual in that they explicitly privilege space over time.

2.5 Metaphors in educational policy

The value of metaphor has been clear to practising educators for a very long time; Cameron (2003) observes that metaphors are peculiarly susceptible to being misinterpreted in a classroom context but emphasizes the fact that education simply cannot function without them.

In a wide-ranging review, Botha (2009) considers the epistemic and ideological freight carried by metaphor in an educational context; yet she omits entirely any mention of metaphor in educational policy. One of the very few scholarly writings to consider metaphor's role in educational policy per se is that of Bessant (2002). Bessant considers the political rhetoric surrounding an influential period of educational reform in Australia, focusing on the use of metaphorical language. Like Charteris-Black (2004) and Deignan (2005), Bessant considers metaphor as a persuasive device, but emphasizes metaphor's ability to inform our thinking without us being aware of its influence.

There are two further metaphors that appear in connection with education: the *factory* metaphor, and the *acquisition metaphor* of Sfard (1998). I discuss each in turn below.

The factory metaphor

By far the best-known educational metaphor is the factory metaphor. Claxton (2013), following Toffler (1990), draws several paragraphs of

analogies between modern schools and production lines: cohorts become batches; (educational) standards and indeed examination grading become quality control; and so on. Mass production of literate, honest, punctual and dutiful workers was conceived of in exactly the same way as mass production of anything else. Although Claxton did not actually use the term "conceptual metaphor" here, he emphasized elsewhere the power of metaphor to guide and sculpt thinking.

Participation vs acquisition

No study of metaphor in educational theory would be complete without mentioning the work of Sfard (1998), who points out that "human learning [has always been] conceived of as an acquisition of something". She goes on to develop this acquisition metaphor, observing its ubiquity in educational discourse, and its implicit comparison of education with accumulation of material wealth. Sfard then posits a new *participation* metaphor, on the grounds that education is something one *does*, rather than *gets*.

This dichotomy has proved fruitful and persistent: Wegner and Nückles (2015), for example, point out that the very existence of tuition fees and credit points highlights the acquisition metaphor and actively discourages the participation metaphor. They go on to observe that the acquisition metaphor is predicated on the assumption that knowledge can be seen as an entity; the appropriate conceptual metaphors are IDEAS ARE OBJECTS and THE MIND IS A CONTAINER.

Note too that the acquisition metaphor is neutral with respect to constructivism: learners may either receive knowledge entities or actively construct them.

Expressions like "knowledge *transfer*", "intellectual *property*" or "*grasping* ideas" show how deeply engrained this metaphor is in western language. The acquisition metaphor includes both transmissive views (the assumption that knowledge can be passed by transmission from one person to the other) and constructivist views (knowledge is constructed individually by each person), because both conceptualize knowledge as an entity.

Sfard (1998) considers these issues and, using the central thesis of Lakoff and Johnson (1980), applies them in the context of educational policy. She leaves us in no doubt that metaphors are important and

influential: "Different metaphors lead to different ways of thinking and to different activities" (Sfard, 1998, page 5).

2.6 Conclusions

Metaphor thus appears to be threaded through educational policy, and exerts a powerful yet hidden effect on our thought. Our unthinking use of the *level* metaphor, for example, normalizes the notion that mathematics and indeed statistics has discrete units of ascending difficulty.

> The educational discourse in England has thus become so dominated by metaphors of height-privileged hierarchical space that almost everything now operates in relation to it, including teachers' and students' views of themselves and each other…spatial metaphors of schooling can capture us, almost unthinking, in particular, pernicious, discourses
>
> Paechter (2004), page 458

The framework [for higher education qualifications] should be regarded as a framework, not a straitjacket

Quality Assurance Agency for Higher Education, 2008, page 3

[W]hen there are explicit culturally sanctioned warnings not to do something, you can be sure that people are doing it. Otherwise there would be no point to the warnings.

Lakoff and Núñez (2000), page 164

3

Metaphor in educational administration

3.1 Overview

In education a *course descriptor* is a terse, self-contained specification for a unit of study; a *syllabus* lists the specific course requirements a student must complete. The term *curriculum* is usually reserved for the entirety of student experience while attending the institution.

In this chapter I will consider metaphor in course descriptors, using first year statistics course from AUT and the University of Cambridge as examples.

3.1.1 The Metaphor Identification Procedure

In this thesis, I will analyse language (text and speech) using the metaphor identification protocol (MIP) of Pragglejaz (2007). This is a formal procedure in which metaphor may be identified; "Pragglejaz" is the name of a group of scholars.

Slightly paraphrased, the MIP is as follows:

1. Establish a general understanding of the text's meaning

2. Determine appropriate lexical units for analysis

3. For each lexical unit, establish whether the contextual meaning is the same as the *basic meaning*, which is effectively the meaning of the lexical unit when used in isolation

4. If the basic meaning differs from the contextual meaning, mark the unit as metaphorical.

The term "metaphor" is not used in its literary or poetic sense, and the procedure identifies many metaphors that would not be recognised as such by ordinary listeners or readers without prompting. Also, the MIP does not address the issue of *intent:* the speaker or writer's intentions are not part of the procedure. This procedure, while somewhat subjective, has been used with some success on a variety of spoken and written sources.

3.2 Introduction

The syllabus and course descriptor are an ideal place to investigate metaphor usage in education: they are terse, short documents, with an intended audience comprising both educators and students. Course descriptors are official documents, representing a "legal contract of sorts between academy and student" (Luke, Woods, & Weir, 2013)

O'Brien, Millis, and Cohen (2009) consider syllabus from a learning perspective, with a strong emphasis on students' taking responsibility for their own learning. They observe that a syllabus can serve a wide variety of functions that will support, engage, and challenge students; and it can establish an early point of contact and connection between student and instructor.

Rubin (1985), however, takes a rather pessimistic view of the institutional course syllabus, mentioning "badly thought-out efforts". She classifies syllabuses into *listers* which emphasize reading lists and topics covered (rather than selection principles); and *scolders* which give little more than lengthy sets of instructions detailing what will happen if a student commits any of a list of minor misdemeanours such as handing work in after the deadline.

In the field of undergraduate statistics education, many syllabuses appear to be somewhat unhelpful. Take The University of Sydney's

STAT5002, for example. This promises to introduce students to "basic statistical concepts" and develop "computer-oriented estimation procedures".

Observe that in the absence of any contextual information, these phrases are highly ambiguous. "Basic statistical concepts", for example, might be the use of histograms, or Fisher sufficiency; "computer-oriented estimation procedures" might be using `t.test()` in an R session, or writing recursive Archimedean copulæ. It is only when accounting for the fact that STAT5002 is a first-year service course that one would be able to say what sense these phrases are being used; but if this is the case, what is the point of a syllabus at all?

3.3 Auckland University of Technology

Auckland University of Technology (AUT) is the newest of New Zealand's universities, having attained university status in 2000. AUT has the lowest QS ranking of the six and its promotional literature emphasizes the practical nature of the study and the employability of its graduates.

AUT's STAT500 paper is a first-year mandatory course in introductory statistics, intended to ensure that second-year science subjects have appropriate statistical underpinning. The course descriptor has several sections, the most salient of which is the informal course description:

> An introduction to applied statistics. Provides techniques for describing and summarizing a data set. Delivers an understanding of how to use a sample to infer the properties of the population it was taken from. Students in this course also learn how to use statistical software to undertake descriptive analysis as well as perform statistical inference.

The metaphors in this course description may be identified by following the metaphor identification protocol (MIP) of Pragglejaz (2007). There are several groups of words that are used possibly metaphorically:

introduction The OED actually gives this sense of the word ("bringing into use or practice") as the first sense. Conclusion: not metaphorical (or, at best, lexicalized metaphor)

applied the OED gives "put to practical use"; not metaphorical

provides; delivers Clearly metaphorical. The notion that a course *provides* anything is a clear example of the conduit metaphor, COMMUNICATION IS TRANSFER (Reddy, 1993); this is supported by *delivers* as used in the next sentence. Together these suggest that the student is a passive recipient of information, specifically information which is delivered by the institution. The form of words specifically excludes the constructivist approach.

Authors such as Krippendorff (1993) consider the transmission metaphor of lecturing, stating that students becoming "passive and uninformed receivers or consumers" is a side-effect of the conduit metaphor. Yoon, Kensington-Miller, Sneddon, and Bartholomew (2011) concur, associating the transmission model with a social norm of student passivity in the context of undergraduate mathematics lectures.

use; infer; population metaphorical but part of the disciplinary jargon; see Chapter 7

in (this course) The use of the preposition "in" suggests that the cognitive metaphor COURSES ARE CONTAINERS is being used here.

It is clear that this particular course descriptor is rich in metaphor, which reveals underlying assumptions and attitudes to education.

3.4 The University of Cambridge

The University of Cambridge is consistently ranked as one of the world's best universities and some measure of its status is indicated by the fact that its alumni include 95 Nobel laureates. Undergraduate mathematics at Cambridge includes a substantial amount of compulsory statistics education. The course descriptor for the first year statistics course reads, in part:

> The course introduces the basic ideas of probability and should be accessible to students who have no previous experience of probability or statistics. While developing the underlying theory, the course should strengthen students' general mathematical background and manipulative skills by its use of the axiomatic approach. There are links with other courses [a list is given]. Students should be left with a sense of the power of mathematics in relation to a variety of application areas. After a discussion of basic concepts [list] the course studies [list] . . . Through its treatment of discrete and continuous random variables, the course lays the foundation for the later study of statistical inference.

This text is rich in metaphor. In the list below, I discuss the metaphorical language used, with the exception of the *agency* metaphor, which is discussed at the end.

The course *introduces* (agency metaphor); also an example of IDEAS ARE OBJECTS. Note the active voice here, contrasted with the passive voice ("the course introduces") used by AUT.

the *basic* ideas of probability the OED gives "fundamental, essential". Note the absence of the word "foundation" here: the authors presumably wished to avoid confusion with foundational statistics, a separate academic discipline (not one typically taught at undergraduate level).

and should be *accessible* clearly an orientational metaphor. The basic meaning of "accessible" given by the OED is "readily approached or reached"; however, the contextual meaning is that the course can be understood by a student with no specialized knowledge.

to students who *have* no previous experience ABILITIES ARE ENTITIES; KNOWLEDGE IS A POSSESSION. The word *have* is being used metaphorically: familiarity with mathematics is expressed using the language of ownership.

of probability or statistics Both words used metonymically for "formal courses of study covering probability or statistics".

while *developing* The OED gives "to bring out what is implicitly contained". But note the inherent ambiguity in the contextual meaning: is it the theory that is being developed, or the

student's understanding of it? If the former, this is metaphorical usage: there is no ready sense in which underlying theory is implicitly contained in anything. If the latter, the word is being used literally (although then one would arguably be using the conceptual metaphor THE MIND IS A CONTAINER).

the *underlying* theory Underlying: "lying under or beneath". Possibly an example of SOPHISTICATED IS UP or just possibly THEORIES ARE BUILDINGS

the course should (agency metaphor)

strengthen THEORIES ARE CONSTRUCTED OBJECTS; in particular, Kövecses (2010) offers ABSTRACT STABILITY IS PHYSICAL STRENGTH.

students' The apostrophe would indicate KNOWLEDGE IS A POSSESSION again. Observe its placing: the possessor is plural, suggesting that the students as a group (in contrast to individual students) somehow possess the knowledge in question.

mathematical *background* An orientational metaphor; the course is being metaphorically identified with an object, to which previous mathematical learning is cast as a "background". Lakoff and Johnson (1980) observe that the use of such terms imputes two features on the course: firstly, the course is rendered as an object; and secondly, it imposes an anthropomorphic orientation on it. In this case the course *per se* is not only personified but given an orientation facing the speaker (student?).

and *manipulative* skills IDEAS ARE OBJECTS. The OED gives "reposition or reshape [a physical object] manually" as the literal sense—and offers "handle" as a synonym—before moving on to metaphorical senses in which mental or logical operations are emphasised.

In mathematical education, one so often encounters the word "manipulate" (or various declensions thereof) that it is easy to forget that the word is being used metaphorically. Kövecses (2010) lists CONTROL IS HOLDING SOMETHING IN THE HAND. Note that the word "skill" is being used *literally* here. Skill is not

necessarily adroit physical manipulation of objects—one can be a skilled orator or investment banker (or indeed educator).

by its use of (agency metaphor)

the axiomatic *approach*. Clearly an orientational metaphor, but does not appear to be systematic.

There are *links* with other courses... COURSES ARE OBJECTS, or possibly THEORIES ARE OBJECTS. In this case *link* is understood in its sense of a component of a chain; a rare example of a non-hierarchical vehicle used to describe relationships between mathematical ideas.

Students should be *left with* THE MIND IS A CONTAINER. This phrase suggests very explicitly that the student is a passive receptacle of information. Also A COURSE IS A JOURNEY: in this case the student is visualized as being stationary while the course moves past him or her; hardly conducive to a constructionist attitude.

a *sense* of A lexicalized metaphor. Here, *sense* is being used in sense 17 of the OED: "Mental apprehension, appreciation", or possibly sense 13: "more or less vague perception or impression".

the *power* of mathematics In modern usage, largely lexicalized. Possibly IDEAS ARE OBJECTS or THEORIES ARE MACHINES. The OED explicitly lists mental strength and effectiveness in the primary sense for "power" but the metaphorical interpretation is one of personification.

in relation to a variety of *application* Lexicalized metaphor. The sense intended is that the ideas presented in course are useful in other (perhaps non-mathematical) disciplines such as medicine or physics.

areas. clearly metaphorical: THEORIES ARE CONTAINERS.

After a *discussion* Not quite metaphorical but not literal either. This is arguably a personification metaphor (the course does

not *discuss* anything); Knowles and Moon (2006) would classify it as a nominalization metaphor, observing that nominalization generally tends to de-emphasize human agency, while Miller and Fredericks (1990) observe that personification metaphors contribute towards a sense of real individuals with real concerns.

of *basic* concepts as above

the course studies… (agency metaphor)

Through its treatment Personification again, this time in the passive voice.

of discrete and continuous *random variables* The phrase "random variables" is another lexicalized metaphor, occurring frequently in learning resources and research literature alike. It is discussed at length in chapters 3 and 4.

The course lays the foundation (agency metaphor); also THEORIES ARE BUILDINGS, as discussed in 5.

for the later study of statistical inference Note the implicit assumption the the course in question is merely a pre-requisite for more advanced study.

One notable conceptual metaphor in the above course description is the *agency* metaphor: the course itself is ascribed agency. Tourish and Hargie (2012) consider agency metaphors as language that portrays events as volitional actions that reflect internal mental states, and this seems appropriate in the present context. Lakoff and Johnson (1980) might categorize this such agency metaphors as CONTROLLED FOR CONTROLLER, whereas Knowles and Moon (2006) suggest that ascribing agency directly to a controlled item is a *personification* metaphor.

3.5 Graduate outcomes

State support for tertiary education is often justified by the belief that graduates have superior cognitive abilities to non-graduates, and such superiority will result in a more effective workforce.

Such beliefs are often embodied in terms of institutionalized *graduate outcomes*, brief idealized statements of the qualities that successful graduates are expected to possess.

Graduate outcomes, in general, make heavy use of ABILITIES ARE POSSESSIONS, also reinforcing the EDUCATION IS ACQUISITION metaphor of Sfard (1998). Australia's Higher Education Council, for example, explicitly state that graduate outcomes are "the personal attributes and values which should be *acquired* by all graduates…" (my emphasis). Such wording echoes course descriptors (chapter 2) in their promotion of the acquisition metaphor at the expense of the participation metaphor.

3.6 Conclusions

Metaphor occurs frequently in educational administration, with metaphorical language being a pervasive component of the course descriptors chosen for detailed analysis. Pre-eminent among the metaphors used in course descriptors is the agency metaphor: a course is held to possess agency and as such is personified. Such metaphors help to create an impression of "a sense of real individuals with real concerns".

Metaphor occurs in other parts of educational planning, notably institutional *graduate outcomes*. The metaphorical language appears to promote the acquisition metaphor at the expense of the participation metaphor.

> Metaphors, and their frequency of use, may serve as important clues not only in better understanding the stated intent of the policy but also the implicit ideological preferences of the policymakers themselves. In our view, the use of metaphorical expressions in major policy statements reflects a largely unconscious process whereby implicit beliefs, attitudes, and ideological presuppositions concerning the desirability or utility of a course of action are made explicit
>
> Miller and Fredericks (1990), page 68

Metaphor is used repeatedly [in undergraduate lectures]... but there are few elaborated or developed metaphors; those there are tend to be short, unconnected with later metaphors and used primarily to serve local, rather than global purposes.

Low, Littlemore, and Koester (2008), page 428

4

Metaphor in spoken undergraduate statistics lectures

4.1 Chapter overview

In spoken academic discourse, deliberate metaphor seems to be a powerful tool. Published research on spoken metaphor use in education appears to be focused on its use by teachers at primary or secondary level (Munby, 1986; Cameron, 2003), with an emphasis on science education. Low et al. (2008) is one of the very few publications to consider metaphor use in university lectures, although attention is confined to humanities subjects.

As far as academic discourse is concerned, it is widely accepted that metaphor is a "basic epistemological, discourse-organizational, and pedagogical device" (Beger, 2015). As such, one might expect metaphor to be part of spoken education at an undergraduate level. This chapter will consider the extent to which metaphor is part of the most widely recognized aspect of spoken language in undergraduate statistics education: the lecture.

4.2 Introduction

The traditional spoken lecture is a pedagogical genre that has been much maligned as a learning tool (Friesen, 2011); authors such as King (1993) decry lectures as an antiquated "sage on the stage" and urge their replacement with a constructivist "guide on the side".

One of the most cogent and fierce critics of the traditional lecture is Laurillard (1993): "Lectures are profoundly defective, inefficient, and outmoded". They are, she asserts, "a very unreliable way of transferring the lecturer's knowledge to the students". Perhaps this is true, but observe the casual use of COMMUNICATION IS TRANSFER metaphor and the MINDS ARE CONTAINERS; also note the implicit use of Sfard's EDUCATION IS ACQUISITION metaphor.

However, lectures also have their champions: Burgan (2006), for example, lauds the "public display of daring and dazzling intellectual expertise" that only a live lecture can provide. Students too defend lectures, specifically citing the "efficiency" of lecturing, but note here too the unquestioned use of the transmission metaphor: LEARNING IS ACQUISITION.

Yoon et al. (2011) report that students overwhelmingly defend the transmission mode of lecturing, while simultaneously acknowledging that lectures did little to contribute towards understanding. Students, in interviews, emphasized the efficiency of this mode of teaching and noted the practical necessity for the lecturer to get through the allotted lecture content.

Nevertheless, lectures are an important feature of undergraduate education, with the traditional lecture comprising just over half a student's contact hours in a typical statistics course. Yoon et al. (2011) attribute the intransigence of lectures to a combination of academic inertia and students' familiarity with the format.

What role does *metaphor* play in this problematic learning environment, peculiar as it is to higher education? In this section, I will consider spoken lectures and their use of metaphor from a pedagogical perspective.

4.3 Initialization: call for quiet

As a lecture is a performance, there are certain normative standards that are necessary for the process to function as intended. One of these is the maintenance of silence among the audience so the lecturer can hold the floor (student questions are dealt with later in this chapter).

Many lecturers signal the start of the lecture with a stylized speech act[1] ranging from a simple "good morning" to more sophisticated rituals which may include non-verbal components such as dimming the lights.

My own lectures have a mixture of these two things: the system clock is visible to the students on the display screen and when the second hand passes the precise start time I say "right, let's go". The utterance is metaphorical; nothing is "going". In this case the first person plural is inclusive (compare chapter 5, in which "we" is used idiosyncratically). It is interesting to observe that the subjunctive mood is used: the intention is clearly one of inclusion.

It is difficult to study this aspect of language use. Lecturers rarely allow "outsiders" to attend their lectures, having "entrenched norms" of autonomy and privacy (Evans, 2012); and when they do, this is likely to change the atmosphere in the lecture hall.

4.4 Lecture content

Metaphor use in spoken undergraduate lectures has been studied by Low et al. (2008), who observed that metaphorical language was used in large quantities in social science lectures. They found that metaphors occurred in *clusters*: conceptually coherent segments of speech, rich in metaphor. Low et al. hypothesized that metaphor clusters marked the boundary between two distinct themes in the lecturer's narrative, signalling a major turning point. Metaphor also appeared when the lecturer was placed under pressure to think quickly.

[1] A *speech act* is an utterance considered as an action; the canonical example is "I pronounce you man and wife" and "I name this ship…". In this case, the start signal is simultaneously a call for quiet, a statement that the lecture has started, and the actual beginning of the lecture. Searle (1969) would classify this utterance as an *assertive*, a *directive*, a *commissive*, and a *declaration*.

The extent to which these findings from teaching in the social sciences apply to statistics lectures is investigated in this chapter.

One might expect that, given the pervasiveness of metaphor in lectures, that figures of speech such as simile would also be common. However, Low, Todd, Deignan, and Cameron (2010) found a "virtual absence" of simile in a large corpora of academic English, which included undergraduate lectures.

In the following, I will discuss a number of metaphors used in my own videotaped lectures, using the protocol developed by Bergsten (2007) for undergraduate pure mathematics courses. Bergsten split lectures into fragments of a few words and analysed the fragments individually, focusing on the relation between the spoken and written content and observing other features of the lecture such as student questions and the lecturer's gestures.

The source material used here is taken verbatim from a lecture in which I introduced the Poisson approximation to the binomial distribution. This particular lecture was chosen because the limiting process discussed is an exemplar of the basic metaphor of infinity (Lakoff & Núñez, 2000). The lectures were recorded two years ago. The sentence fragments are those containing metaphorical language, as determined by the Pragglejaz metaphor identification protocol; the intervening utterances contained no metaphor.

***We* have been talking quite a lot about the Binomial distribution...**
An example of a metaphorical *we*. The audience is almost totally silent; the *we* is actually *I*. Pimm (1984) discusses the use of the first person plural in this context[2], pointing out that the "educational we" often effectively excludes the speaker. Pimm expresses bafflement as to exactly which community *we* indicates, and conjectures that it induces (either deliberately or inadvertently) "passive acquiescence" in the student[3].

[the Poisson] is one of a *family* of distributions A lexicalized metaphor, one that is standard terminology in statistics. In undergraduate statistics, "family" is usually reserved to describe a set of

[2]Pimm also writes about this issue in educational contexts using written English; I draw on his work in Chapters 5 and 6.

[3]This interpretation has been cited in a small number (A *Web of Science* cited reference search gives 10 citations at the time of writing) of published sources. They uniformly refer to Pimm (1984) only in passing; and none of them offers any conflicting viewpoints.

distributions indexed by one or more (possibly real) parameters. The literal meaning of "family" is sociological: a group consisting of one or two parents and one or more dependent children living together. However, one striking misalignment of this metaphor is that the vehicle is a set (of humans)that is not only *discrete* and *finite*, but also has a very small membership, typically in the range 2-5. Note also the culturally specific nature of this metaphor. Contrast the topic, which is not only continuous but generally infinite. There are other differences: familiarity and ready identification are salient features of the vehicle, yet in the topic, complicated and unreliable mathematical inference is needed. Such differences are the essence of pedagogical metaphor, as the topic is rendered comprehensible due to familiarity with the vehicle.

Bernoulli trials with a probability of *success*... Standard statistical terminology is to refer to the support of any random variable with two outcomes as "success" and "failure". However, there is no value judgment inherent in these words and one finds (in studies of family sex balancing, for example), that a birth being male is a "success" and female a "failure". Mathematically, the two terms are equivalent as they are symmetric with respect to $p \longleftrightarrow 1 - p$.

The terminology is arguably metaphorical: the topic (support of the random variable) is described using the vehicle of an abstract experiment which may succeed or fail. This abstractness is, I would argue, a virtue on the grounds that one *wants* to extirpate any traces of value judgments from the narrative.

simply because this $1 - \mathrm{p}$ here *turns into* a q there The context was that the binomial probability mass function $\binom{n}{r}p^r(1-p)^{n-r}$ was rewritten as $\binom{n}{r}p^rq^{n-r}$, with q being substituted for $1 - p$. This is another example of agency metaphor, but one of a peculiar kind: the equation is somehow imbued with the ability to transform itself from one form to another.

a much more *symmetric* way of writing it Here the binomial probability mass function was written in the form $\binom{a+b}{a\,b}p^aq^b$, with a being the number of successes and b the number of failures

(the intent was to introduce the Dirichlet distribution). The words "symmetry" and "symmetric" are problematic for mathematicians; the words originally referred to spatial harmony and, for most people—including the lecture audience, I would suggest—symmetry is an inherently geometric concept. Here the contextual meaning is that of algebraic symmetry between parts of an equation, a concept likely to be new to much of the audience.

because *we have* asymmetry between a and b The context was referring to $\binom{a+b}{a\,b}p^a(1-p)^b$, clearly placing a and b on a different footing. This is not really an example of Pimm's "we": in this case the asymmetry was undesirable.

it's a much more pleasing way of *handling* this There are two metaphors here. Firstly, the use of "pleasing": note the passive voice. Pleasing to whom? The intended sense is that the form of the equation is intrinsically appealing, independently of any particular viewer. This is not an uncommon viewpoint (Rota, 1997). The intended sense is that the community of practice into which the students are being acculturated is one which collectively finds that particular expression pleasing.

The other metaphor is the use of the word "handling". The intended sense is that the equation under consideration is a physical object, and expressing the equation in different mathematical ways corresponds to physical manipulation of the object. This might suggest a new conceptual metaphor: EQUATIONS ARE OBJECTS.

and I'll *return to* this formulation later This is an example of AN ARGUMENT IS A JOURNEY, here underscoring the difficulty of the material.

One disadvantage of the binomial distribution... Arguably a peculiar metaphor, perhaps EQUATIONS ARE TOOLS. The intended sense was that the distribution included analytically intractable terms such as the factorial function, which made the equation hard to deal with.

...is that it has this factorial function *in it* AN EQUATION IS A CONTAINER

The first thing that should *come into your head* is to verify [that formula]... This is an example of IDEAS ARE OBJECTS AND THE MIND A CONTAINER; Bereiter would dismiss this as "folk theory of mind". However, the form of words used is interesting because there is no indication of the *conduit* metaphor (Reddy, 1993): there is no suggestion that the concept of verification originated from the lecturer. The clear import is that the (habit of) verification should spontaneously arise, unbidden, in the student's mind: this would be the *agency* metaphor.

Am I doing what I think I'm doing?...can *I* check it, can *I* verify it... (the context was an exhortation to the audience to check their work continually, to identify and rectify algebraic and conceptual errors). This is an interesting use of the metaphorical "I". This is arguably an inversion of Pimm's educational "we": here, "I" is clearly intended to indicate what the class should be doing.

Fauvel (1988) would observe that such rhetorical devices are Cartesian rather than Euclidean: the audience is being personified directly and quotes attributed directly to them.

however this is quite a difficult and unwieldy process EQUATIONS ARE TOOLS, in this case undesirable qualities of algebraic manipulation.

I will *give you* this formula in a different form This is one of a large number of "give" metaphors used in this series of statistics lectures (e.g., "I can *give* you an exact answer to that"). Such phrases are direct examples of the *conduit* metaphor of Reddy (1993). However, note the simultaneous use of the *acquisition* metaphor of Sfard (1998). In this case, the contextual meaning is a promise to re-write the formula; but the basic meaning is both acquisitive and transferative.

The factorial function isn't easy *to deal with* (also, later, "the standard deviation is harder to deal with than the mean"). Arguably a personification metaphor: the factorial function is given agency. Low et al. (2008) assert that personification is by far the most common metaphor in humanities lectures and it is certainly common in these statistics lectures.

I'll *cover* the first two or three members of the series In this context, *covering* is a very commonly used metaphor. Many authors (Biggs and Tang (2011) and Vella (2007), for example) criticize the very notion of "covering" a topic, on the grounds that it obscures any learning objectives; Paechter (2004) observes that it is a spatial metaphor.

Note also that the context also carries the implication that this material will be assessed at some point, as the concept being covered appears on the learning objectives, which are explicitly assessed.

the probability of success is p=0.5, so it is *a fair coin*... A coin toss is a prototypical example of a Bernoulli trial: it is indisputably random, the probability of success (heads) is known precisely, and successive tosses are demonstrably independent. But to say that a Bernoulli trial *is* a fair coin is clearly metaphorical. The coin metaphor is standard terminology in statistics.

I can hear my mathematical colleagues *howling in outrage* This was in the context of a somewhat low-status technique involving numerical approximation; also, later, "these disadvantages wouldn't *cut much mustard* with a mathematician".

The thrust of these comments is that there are differing schools of thought in mathematics and the approach taken in the lecture sacrifices exactness (which is highly prized in some disciplines) for computational convenience (which is highly prized in this particular course).

These metaphors are used in rapid succession, and qualify as part of a metaphor *cluster* in the sense of Low et al. (2008). However, this cluster did not mark a "major turning point" in the lecture, unlike the clusters identified by Low et al. (2008).

last time we had the binomial distribution $\mathrm{Bin}(n, p)$ ...I'm going to *make* n *get larger* in a particular way This qualifies as a metaphor, in this case the basic meaning of "make" is "force" but the topic is an equation, in this case a probability mass function.

Last time we discussed n *getting larger*, with p fixed In terms of the Basic Metaphor of Infinity (BMI) of Lakoff and Núñez (2000)

(see chapter 5), this would be "potential infinity". The mathematical statement is here the well-known Gaussian approximation to the Binomial, another limiting distribution this time arising from the central limit theorem.

I asserted that the limit (in scare quotes, I'm not defining formal limits) as n *approaches infinity*, of the distribution of r, the number of successes, is normal or Gaussian with mean np and standard deviation $\sqrt{npq}$. It is difficult to interpret this utterance in terms of the BMI, yet metaphor is definitely used. The phrase "approaches infinity" is, although standard terminology, metaphoric: nothing actually *approaches* anything; and in any event, "approaches" implies "getting closer (to something)", which is emphatically not occurring.

Now n is getting larger and p is fixed... and *it looks like this* (draws a Gaussian on the whiteboard) I would suggest that this is a metonym: a random variable is identified with its probability density function. Also note that the random variable is imbued with a visual appearance.

I'm going to think about the two parameters, n and p, and I'm going to consider a sequence in which n gets bigger, and simultaneously p gets smaller in such a way that np stays fixed. The very essence of the (non-metaphorical) Cauchy sequence (Hardy, 1952): this shows that metaphor is *not* necessary for everyday mathematical teaching.

4.4.1 Summary

The sentence fragments quoted above illustrate the frequency and ubiquity of metaphor in statistics lectures. Conceptual metaphors were frequent, specifically EQUATIONS ARE TOOLS which was used several times.

Metaphor, at least in the source material above, is a key pedagogical tool in the sense that many of the utterances could not easily be rephrased nonmetaphorically. None of the metaphors (with the possible exception of the highly idiomatic *cut much mustard*) are natural part of language and would not draw attention to themselves.

Such observations are consistent with those of Geary (2011): metaphors are indeed "hiding in plain sight". Their very unremarkability, even invisibility, combined with their frequency and power, suggests that we take metaphor very seriously in education.

4.5 Students' use of language in lectures

The lecturer is not the only source of language in lectures: students also, on occasion, ask questions. Marbach-Ad and Sokolove (2000) consider students' asking of classroom questions as they progress from elementary level to college, and conclude that students learn not to ask questions in class. At undergraduate level, the lecture environment has implicit social norms which generally enforce a passive role during lectures (Yoon et al., 2011).

Questions are generally infrequent, with Pearson and West (1991) reporting an average of only three questions per hour, the majority of which were on-task but restricted to procedural clarification, such as due dates on assignments or venues for lectures. The only questions in the chosen lectures on the Poisson distribution were clarificatory. Student questions do not appear to be sufficiently frequent to make any claims about students' use of metaphor in this context, either from the literature or my own observations in my lectures.

4.6 Conclusions

Metaphor is certainly a component of spoken undergraduate statistics lectures, but with only two exceptions ("howling in outrage" and the idiomatic "cut the mustard"), none of them call attention to the language used, nor are likely to be perceived as metaphorical by the audience.

A number of metaphors in the corpus analysed appear to relate to "domestic" aspects of the lecture such as promises to discuss certain content, or standard pedagogical constructs such as COMMUNICATION IS TRANSFER (the most apposite example being "I will *give you* this formula…")

A certain amount of metaphor is unavoidable in any comprehensible mathematical or statistical discourse. However, much of mathematics is arguably metaphorical, as argued by Lakoff and Núñez

(2000). Specifically, the *basic metaphor of infinity* (BMI) occurred several times in the corpus under study. A more detailed discussion of the BMI is given in Chapter 5.

How much can we infer about the basic cognitive mechanisms used in mathematics from what we find in texts and curricula? A study of navigation based on the standard manuals would tell us very little indeed about how the task was actually accomplished on the bridge of a large ship.

Madden (2001), page 1185

5

Metaphor in statistics textbooks

5.1 Overview

This chapter gives a discussion of metaphor and metaphorical language as used in statistics textbooks. The majority of the relevant literature covers mathematics in general; few articles consider metaphor in statistics education. In this chapter I use literature that analyses metaphor in mathematics textbooks and consider the extent to which the findings are applicable to statistics education.

The chapter is split into two main parts. The first part will consider metaphor in mathematics generally, specifically as interpreted in the controversial *Where mathematics comes from* (Lakoff & Núñez, 2000), henceforth WMCF. I will consider this book from the perspective of statistics education.

The second part of the chapter concentrates on one often-overlooked aspect of language frequently used in mathematics textbooks: the mathematician's *we*. This usage is considered to be metaphorical because the referents are not a well-defined group. I will consider the educational implications of this language use.

5.2 Introduction

A *textbook* is a standard work for the study of a particular subject, here statistics; attention will be confined to those used for undergraduate study. Mathematics textbooks are a valuable and often-consulted resource for university students (Weinberg, Wiesner, Benesh, & Boester, 2012). One might expect metaphorical language to be widely and effectively deployed.

Three textbooks were chosen for detailed study:

- Crawley (2015)
- Feller (1968)
- Casella and Berger (2001)

These books span a range of sophistication—Crawley [1] is practitioner-oriented, heavy on computational examples and light on mathematical detail. Casella and Berger is classified as high-end undergraduate or mid-range postgraduate study material; while Feller is a classic work, emphasizing rigour. These are books that I use in my own teaching and broadly correspond to first, second and third year courses.

The Pragglejaz (2007) protocol, applied to these three texts, revealed that metaphor (in the sense of Lakoff and Johnson (1980)) was rare to nonexistent. This is perhaps not surprising in such a mathematical context where accuracy is more highly valued than clarity or even educational value.

There was, however, one non-literal usage of language that occurred frequently throughout all three books: the mathematician's *we*, which is discussed in section 5.5.

5.3 Random variables and metaphor

The notion of *random variable* is a central concept in statistics. The formal definition of a random variable is as follows.

> Suppose we have a probability space $(\Omega, \mathcal{F}, P)$. Then if E is some set and $X : \Omega \longrightarrow E$ is measurable function from Ω to X, we say that X is a *random variable*.

[1] It is standard practice to refer to textbooks by the author(s) name.

Note the abstract and unhelpful nature of such a rigorous definition; mathematically, the difficulty lies in ensuring consistent behaviour when E is uncountably infinite (one prominent example would be $\mathbb{R}$, the real numbers; one would hope that such definitions do not let one down in such a practically important case).

Crawley is one of many introductions to inferential statistics that discusses random variables from a more practical perspective. While declining to offer a formal definition, several examples are given, the canonical one being

$$X = \begin{cases} 0 & \text{if coin lands tails} \\ 1 & \text{otherwise} \end{cases}$$

Crawley goes on to state that, together with the specification that $p(X = 0) = p(X = 1) = 1/2$ fully characterizes X.

Undergraduate statistics textbooks typically offer a level of rigour between these two extremes. To what extent do linguistic or cognitive metaphors enter in to such discussions? Lakoff and Núñez (2000) would suggest that metaphor plays a large part in all of mathematics, and indeed claim that *all* mathematical reasoning is inherently metaphorical.

Of all the metaphorical mathematics presented in WMCF, by far the most relevant is the *basic metaphor of infinity*, discussed in the next section.

5.4 The basic metaphor of infinity

Metaphorical language and reasoning is common in mathematics and mathematics education (Pimm, 1981). However, the study of metaphor in mathematics was kick-started by publication of the controversial *Where mathematics comes from* (Lakoff & Núñez, 2000) which set out the authors' contention that *all* mathematical reasoning is metaphorical. The authors also make a case for mathematics *per se* being a human construct.

Statistics, like many branches of mathematics, often uses the concept of *infinity*. Here I draw on the ideas of Lakoff and Núñez (2000),

in a controversial work often referred to as "WMCF" (being the initials of the book title, *Where Mathematics Comes From*). WMCF suggests that all mathematical thought is metaphorical and the authors make a case for even such fundamental branches of mathematics as axiomatic set theory being metaphorical: for example, the authors point out that the conceptual metaphor SETS ARE CONTAINERS AND ELEMENTS OBJECTS IN THEM is purely metaphorical, yet almost universally used when thinking about set membership. Notions such as the number line are also held to be metaphorical: WMCF points out that natural numbers are not points on a line; counting (enumeration) is not temporal progression along a marked rod; sets are not containers with elements objects inside them.

WMCF makes a case, echoing that of Lakoff and Johnson (1980), for many if not all such metaphors to be rooted in sensory-motor experience. Here the most germane is the *basic metaphor of infinity* (BMI) in which processes that go on indefinitely are conceptualized as having an end and an ultimate result. Motivating examples are discussed, including the one-point compactification of the plane, limits of sequences, and mathematical induction.

It should be pointed out that WMCF has come under severe criticism and indeed the ideas have been met with little interest among mathematicians. The authors do not put forward any empirical support whatsoever (Madden, 2001) for their assertions about the ways metaphorical reasoning is used when mathematics is carried out. Schiralli and Sinclair (2003), for example, considers the book to make "fundamental oversimplifications" and observe that the authors use the word *metaphor* to serve so many purposes that "the notion of metaphor itself begins to lose its meaning".

The book received at best mixed reviews from both mathematicians and cognitive scientists. Neither of the authors is a mathematician (and certainly no non-elementary mathematics is presented in the book). The authors present "misconceptions of mathematics [that] are prevalent among non-mathematicians" (Henderson, 2002). Indeed, many reviewers point to the "rather frequent" mathematical errors (Gold, 2001); at many points in the book, metaphorical reasoning is invoked to explain mathematical cognitive phenomena, yet a slightly more sophisticated analysis would show an appropriate mathematical framework. Nevertheless, as the authors point out,

cognitive mathematics is a sorely neglected field of study; and the book provides a coherent account of cognition's role in mathematics.

Other aspects of the book are unsatisfactory. Goldin (2001), for example, considers the book to be "fundamentally flawed" on the grounds that it was poorly sourced in both cognitive science and philosophy of mathematical thought. When mathematicians review the book, they observe that WMCF includes "numerous errors of mathematical fact" (Henderson, 2002) and also conflates at least three distinct mathematical activities: learning, using and research.

The only evidence that the authors adduce for their assertion that metaphor underlies all mathematical thinking is textual. This, if nothing else, suggests that it is at least plausible that textual analysis of the type given in chapter 2 of the current thesis is a respectable source of information in its own right.

My own reading of WMCF suggests that the authors appear to be ignorant of mathematical techniques that render much of their metaphorical interpretation unnecessary. For example, in the context of elementary group theory, the authors give an extended discussion of what I would call C_3, the cyclic group of three elements. They insist that the different examples of this group (plane rotations by $2n\pi/3$, arithmetic modulo 3, etc) are "metaphorically linked", and give extensive tables; yet they appear to be ignorant of the notion of isomorphism, a formal and non-metaphorical concept that would render their analysis superfluous.

Considering the BMI, the authors again appear not to have understood (and certainly have not mentioned) the concept of *Cauchy sequence*, which again would render much of their discussion superfluous. A sequence $x_1, x_2, \ldots$ is Cauchy if, for any $\epsilon > 0$, one can find an integer n_0 such that $n, m \geqslant n_0$ implies $|x_n - x_m| < \epsilon$. It is easy to show that a Cauchy sequence approaches a limit as $n \longrightarrow \infty$.

Cauchy's startling and elegant definition neatly sidesteps any confusion between "actual infinity" and "realized infinity" as the limit itself is not mentioned; observe that no metaphor is needed. For this reason, Cauchy sequences are fundamental to the understanding of many diverse mathematical concepts such as compactness in Hilbert spaces and completeness of p-adic numbers.

From an undergraduate statistics education perspective, the BMI

is used when considering convergence of random variables. The example I will use is the same topic as discussed in the spoken lectures discussed in Chapter 5: the elementary observation that the Poisson distribution is a limiting case of the binomial. The formal statement I am expressing is as follows:

Theorem. if $X_n \sim \mathrm{Bin}\,(n, r/n)$ assuming $0 \leqslant r \leqslant n$, then

1. $\lim_{n \longrightarrow \infty} X_n = X$ exists, and
2. $X \sim \mathrm{Poisson}\,(r)$; that is $\Pr(X = n) = \frac{e^{-r} r^n}{n!}$.

This fact is neither formally stated, nor any proof given; but the underlying idea is both simple and important for statistics at this level. Observe that the concept of Cauchy sequence is applicable to probability mass functions just as well for real numbers[2].

In this context, WMCF (Where Mathematics Comes From, Lakoff and Núñez (2000)) asserts directly that the BMI is unavoidable in mathematical language, yet the concept of Cauchy sequence neatly avoids any need for arguably metaphoric concepts of "limit" and "the infinite". The excerpts shown above demonstrate that careful use of Cauchy sequences can illustrate the concepts of infinite limits—certainly in the case of Borel probability measures—without any potentially confusing metaphorical language; and that such methods are available in a written or spoken context.

5.5 Rhetorical metaphor in textbooks

Mathematical textbooks, including statistics textbooks, frequently use rhetorical devices as part of their communication strategy (Kane, 1970). One such rhetorical device is the use of *we* which is metaphorical in the sense that the writer is not using literal language: the reader is identified with a poorly-defined "community of practice", of which the writer of the textbook is one example. Observe that the author of any textbook would serve as a pre-eminent member of such a community.

[2]The "distance" between two probability mass functions is simply the supremum of the differences between their cumulative distribution functions.

Pimm (1984) asserts that textbooks' use of *we* attempts to "enrol the tacit acquiescence of the reader", and serves as an imposition which fails to take into account the wishes or interests of participating individuals.

This peculiar use of *we* among mathematicians is not limited to textbooks; it is a ubiquitous construction in research articles (Kuo, 1999). Consider, for example, the first article in the most recent edition at time of writing in *The Journal of Topology*[3], (Lange, 2016). This is a typical article in the field, the abstract of which starts "We characterize finite groups G generated by orthogonal…"; the *we* must be inclusive because the article is single-author. Kuo (1999) considers that the almost complete absence of first-person singular pronouns (I/me) to be evidence of effort to reduce personal attributions, and this tendency is presumably operating in textbooks too.

5.5.1 Inclusivity in mathematics

The mathematicians' *we* is thus an attempt to draw the reader in to the community of practice. In this context, Fauvel (1988) considers the issue of inclusivity in mathematics, drawing a distinction between the *Euclidean* and *Cartesian* styles of rhetoric. Fauvel characterizes the Euclidean style as follows:

> There is no sign he notices the existence of readers at all. Rather, he seems engaged in laying down inexorable eternal truths. The reader is never addressed.

and compares with the Cartesian approach:

> The mathematics described is clearly created, not unveiled, in rhetoric which veers from grabbing the reader by the lapels to treating you with utter disdain…

The three textbooks use the mathematicians' *we*, and are thus more Cartesian than Euclidean in outlook (at least, if the inclusive sense is understood).

[3]The discipline of topology is a theoretical branch of pure mathematics, notable for its extreme abstraction

5.5.2 Grammatical inclusivity

It is interesting to note that English does not distinguish between inclusive and exclusive *we*[4], so there is no grammatical way to detect whether the reader is included in the writer's utterances. Such considerations can be important in political speech (Chen, 2006). Languages such as Te Reo Māori do maintain a distinction between inclusive and exclusive forms (the words are tātau and mātau respectively), so perhaps Māori textbooks would afford some insight here.

5.6 Conclusions

Metaphor is an unavoidable component of exposition used in statistics textbooks. Three statistics textbooks, used in my own teaching, were chosen for detailed analysis of metaphorical language using the Pragglejaz procedure.

Metaphor was rare in all three textbooks, with the exception of the basic metaphor of infinity, and the mathematicians' "we", which is not specific to statistics.

[4]Inclusive *we* specifically includes the addressee while the exclusive form does not

For a very long time, word problems have played their role as an unproblematic and transparent bridge between the world of mathematics and the real world.

Verschaffel, Depaepe, and Van Dooren (2014), page 644

6 Metaphor in assessment

6.1 Overview

In this chapter, I consider metaphor in the assessment phase of an undergraduate statistics course. Metaphor occurs in both the assessment cue and the student's response and I consider both separately below.

Metaphor is common in statistics examination questions, specifically occurring when (proper) placeholder nouns are used in word problems.

I also briefly discuss students' use of metaphor when being assessed, considering both controlled assessment (examination) and uncontrolled (portfolio) assessment items.

6.2 Essay-type questions

Essay-type questions *per se* are rare in undergraduate statistics, with one exception: a relatively open-ended cue to analyze a dataset using the methods taught in the course, and communicate any findings. Horowitz (1986) considers the use of language in such assessment cues, with a typical exam prompt being along the lines of

> *Using the Yanomamo as an example briefly explain Marvin Harris's theory of primitive warfare*

Horowitz went on to identify a number of "micro-functions" of such cues which characterize acceptable responses. He ordered these micro-functions along an axis from content-oriented ("identify the topic") to form-oriented ("specify the length of the essay"). From the perspective of undergraduate statistics education, the most germane micro-function was his number 5: *specify the writer's persona*. For a typical written assignment, Horowitz (1989) points out that students must pretend that the marker is not familiar with the issues discussed.

However, in the context of undergraduate statistics, a typical assignment might be to analyze a specific dataset using algebraic and visual methods. In this situation, a student need only make the realistic assumption that the marker has not actually carried out such analysis.

Is *metaphor* part of this aspect of language use? I would argue that typical undergraduate statistics assessments do use metaphoric language, in several senses. Firstly, the students generally treat the dataset provided with the assessment as just one representative of an ensemble of possible datasets, all of which would elicit identical statistical analyses: they would perform the same steps if the data were perturbed slightly. The data is thus meronymically defined. Secondly, the student understands that the analysis is not actually important and the premises of the cue are merely a plausible fiction which may or may not exist.

6.3 Word problems

A *word problem* is a verbal description of a problem situation wherein one or more questions are raised, and the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement (Verschaffel et al., 2014). Word problems are "considered to be an important part of mathematics education" (Reusser & Stebler, 1997).

Gerofsky (1996) states that the overwhelming majority of word problems have three components: firstly, a backstory which establishes the characters and possibly the location of the putative story;

an information component, in which the information needed to solve the problem is given; and a question.

However, word problems are a problematic assessment item in terms of educational value (Gerofsky, 1996), transferability (Reusser & Stebler, 1997), and low achievement rate (Cummins, Kintsch, & Weimer, 1988). Given these concerns, it is somewhat surprising that M. Johnson (1976), in a 166-page book devoted purely to the solution of word problems, offers not the slightest motivation for their study.

In the context of statistics education, Quilici and Mayer (1996) show that word problems encourage students to attend to surface elements of the question (such as inclusion of words such as "compare" which induce the use of a two-sample t-test) at the expense of underlying structural features.

In the following sections I consider the extent to which metaphor occurs in this assessment trope.

6.3.1 Truth value and word problems

Metaphor analysis of word problems is not straightforward because a word problem is semantically ambiguous. One concept useful in the analysis of word problems is "truth value" as originally defined by Frege in 1891: the truth value is the attribute assigned to a proposition in respect of its truth or falsehood. Frege considered the relation between propositions and truth from a philosophical perspective; but the relevance of truth value to language encountered in fiction remains "problematic" (Lamarque & Olsen, 1994).

In the context of education, Gerofsky (1996) considers the semantics of word problems and observes that their truth value has no clear status. He observes that word problems may be rephrased without altering their truth value and offers:

> Every year (but it has never happened), Stella (there is no Stella) rents a craft table at a local fun fair (which does not exist). She has a deal for anyone who buys more than one sweater (we know this to be false…there are no people, or sweaters, or prices)

As Gerofsky (1996) points out, word problems are a peculiar trope in which one is instructed to pretend that the background story is true,

under (implicit) direction from the writer of the problem; but simultaneously the competent reader considers the background story to be irrelevant. Reusser and Stebler (1997) contrast the chimerical text of the backstory with the implicit web of mathematical relations in the problem itself; they conclude that these two "interwoven semiotic worlds" are poorly aligned and largely irreconcilable.

The entire backstory may thus be considered to be meronymic in the sense that the one provided is but one representative of many possible, functionally identical, backstories.

Boaler (2000) describes one enthusiastic student who, meeting word problems for the first time, was dismayed to find that bringing her competent, adult-level situational knowledge to bear on the problems was counterproductive to academic success. Boaler went on to question the (practical) competence of the question-setter, although she does not query the ontological status—or educational value—of the word problem.

6.4 Students' use of metaphor in assessment

> Essay examination writing is, indeed, a "display" for the purposes of evaluation, a time to show that one has studied hard, not that one is especially clever or possessed of broad general knowledge
>
> Horowitz (1986)

The nature of the language used by students in the assessment phase of a statistics course is problematic: Horowitz (1986) argues that a student's response to any assessment cue is a perlocutionary act, specifically one that persuades the reader to proffer an acceptable grade in a course.

This viewpoint makes the analysis of metaphor difficult, as the purpose of the writing is not clear. On the one hand, the student is expected to create an exposition to convince the reader of the truth of a proposition, but on the other, the reader is already convinced.

Read, Francis, and Robson (2001) consider the ways in which students develop a "voice" and points out that students must master the complex culture of academic language in order to succeed. These authors point out that the conflict between the desire to score a high

grade is counter to the desire for a student to have their own voice: high-GPA students were reported to have sought out their tutors' viewpoints in order to write from their perspective. Read et al. (2001) go on to pose the rhetorical question: is such writing the students' voice or that of their tutors?

In pure mathematical disciplines, essay-type questions are rare but do exist (M. L. Johnson, 1983). However, by far the most common type of mathematical examination cue requires the student to prove a (given) statement. The linguistic status of a student's proof (under examination conditions) is again problematic. The definition of "proof" is a logically watertight demonstration of the truth of a statement: all the student has to do is to reproduce an existing proof.

However, analogously to an undergraduate essay, an examination proof is intended to have a perlocutionary effect, specifically of inducing the reader or marker to give an acceptable grade to the student. A poor student will (attempt to) reproduce an existing proof, with no understanding. This, however, is a very difficult task as without the cognitive scaffolding that understanding provides, a proof typically has no discernible structure and is very difficult to memorize. The natural way for a more able student to proceed is to convince the marker that the writer has actually grokked (sic) the proof and can convey this to the reader. In this sense, the mathematics examination is a peculiar form of performance assessment in which one has to convince the examiner that you have indeed had the flash of insight which mathematicians call "proof".

Under these circumstances, can the student be said to employ metaphor? Mi-Kyung and Oh Nam (2004) argue that students constantly employ mathematical metaphor in class and, as such, identifies the machine metaphor and the fictive motion metaphor as dominant metaphors in the case of partial differential equations.

However, in the context of undergraduate statistics, a typical assignment might be to analyze a specific dataset using algebraic and visual methods. In this situation, a student need only make the realistic assumption that the marker has not actually carried out such analysis (Horowitz, 1986).

6.4.1 The British Academic Written English corpus

Assessed student writing is difficult to study owing to a scarcity of suitable corpora (Nesi, Sharpling, & Ganobcsik-Williams, 2004); and those that exist are typically focused on English as a second language.

One of the few systematic corpora of assessment is the British Academic Written English corpus, the BAWE (Heuboeck, Holmes, & Nesi, 2016). The BAWE is a collection of student writing from undergraduate to taught Masters level, restricted to assignments consistent with an upper second or first class honours degree (Nesi & Gardner, 2012). The BAWE is unique in the wide range of disciplines represented.

The BAWE includes samples of statistics assessment. It is clear from context (the anonymization protocol redacted the cue) that the student was a first year undergraduate required to assess the relative merits of two measures of central tendency: the mode and the median. I will analyse these two pieces of work for metaphor using the Pragglejaz (2007) protocol.

The first piece of student work was responding to a cue asking to characterize the median as a measure of central tendency:

> "Median is useful in this case because it tells us that half the sample has more money than this with them and half has less. It is not influenced by outliers as the mean is. For example if we had a very high value such as £200 this would increase the mean greatly so that it is no longer as representative of the sample as the median"—Anonymous student, quoted in Heuboeck et al. (2016)

It is possible to analyze this assessed writing using the Pragglejaz (2007) metaphor identification protocol;

[The] median *is useful* in this case An example of the conceptual metaphor EQUATIONS ARE TOOLS. In this case, the metaphor is likely to originate in the course itself.

because *it tells* us that A peculiar agency metaphor; the estimator is personified, and in addition given a "voice".

half the sample has more…and half less. No metaphor: this is a literal definition of the median.

It is not *influenced by outliers* This is metaphorical: what the student is trying to say (successfully) is that the *value* of the median does not change as a result of a marginal change to an outlier. However, she is here using a metonym: the topic is the measure of central tendency known as the median, but the vehicle is the value of the median.

as the mean is. non-metaphorical.

For example if *we* had Another example of Pimm's "we"

a *very high* value such as £200 Orientational metaphor: INCREASE IS UP.

this would increase the mean greatly no metaphor

so that it is no longer as representative of the sample as the median no metaphor

The second piece of assessed work is from the same student considering the mode of a dataset.

> "The mode is of a varying degree of usefulness—it tells us the most common value but here this is very low and not representative of the sample values as a whole. If we look at various major peaks in the data set then this is a fairly useful tool. For example, here the data is bimodal—there is a split between people carrying a relatively large amount of money and a small amount."

The mode is of a varying degree of *usefulness*— EQUATIONS ARE TOOLS

***it tells us* the most common value** Clear personification metaphor.

but here this is very low Orientational metaphor (LESS IS DOWN)

and not representative of the sample values as a whole. Non-metaphoric.

If *we* look at Another example of Pimm's "we"

various major peaks *in* the data set Possible conceptual metaphor DATA IS A CONTAINER AND FEATURES OF THE DATA OBJECTS IN IT. This is a reasonably common metaphor.

then this is a fairly useful tool. Lakoff and Johnson (1980) state that explicit use of conceptual metaphors is rare but here is an example of EQUATIONS ARE TOOLS being used explicitly.

For example, here the data is bimodal— Note that DATA IS A CONTAINER AND FEATURES OF THE DATA OBJECTS IN IT is not being used here: the bimodality is attributed directly to the dataset rather than asserting that the feature is contained inside it.

there is a split between people carrying a relatively large amount of money and a small amount. Metaphoric use of *split*. Here, "split" is being used in the sense of chasm or divide: the data-points (here people) are placed on a number line (itself a metaphor for the real numbers) and the student is observing that there is a region along this line that is sparsely populated.

In this case, there is one metaphor that is conspicuous by its absence. The data comprised a finite number of observations, each one being the amount of money carried by a *specific* person in the sample. The student is not metonymically referring to a person by the amount of money they were carrying: the student refers directly to the split between the people.

These pieces of assessed writing use metaphor in a routine and unremarkable manner. The metaphors used appear to be of the same general type as used by educators in typical course resources.

6.4.2 Measures of central tendency in undergraduate statistics

Calculating measures of central tendency is a distressingly common trope in undergraduate statistics education to the extent that one sees derogatory descriptions of "mean-median-mode" education. It is an easy matter to assess whether students can evaluate the measures by requiring students to calculate the three measures for different datasets.

From the perspective of a practising statistician, the mean and mode are simply measures of central tendency of a sample[1] and do not have "usefulness". The concept of one measure being more "representative" than another is meaningless: the measures summarize a sample in different ways and illustrate different aspects of the sample. The situation is analogous to an engineer asking whether the radius is more "representative" of the size of a given a circle than the diameter. So in this case the conceptual metaphor EQUATIONS ARE TOOLS is leading to flawed thinking.

But it should be noted that my comments above—correct as they are from a theoretical statisticians' perspective—would probably not attract a passing grade at undergraduate level: as I argue in section 2.3.1, deep thinking about any aspect of inferential statistics inevitably interferes with successful assessment

6.5 Conclusions

In assessed work, students do employ metaphor when enrolled in tertiary statistics courses; such prompts might appear in a first year undergraduate descriptive statistics paper. However, ethical considerations mean that there is very little assessed work that may be analysed for metaphorical language: fully informed consent is difficult to obtain.

The British Academic Written English corpus is one of the very few corpuses of assessed undergraduate student writing available, and this contains two pieces of assessed work which were part of a statistics course. The students used the conceptual metaphor EQUATIONS ARE TOOLS in much the same way as textbooks do.

[1] In the assessed writing above, the student was discussing the sample mean and sample mode. The sample mean is the arithmetic mean of one's observations and is useful because it is an estimate of the population mean (if defined). The mode is an altogether more problematic measure, having different definitions for continuous and discrete distributions.

7

Metaphor in course evaluation

7.1 Overview

Many institutions mandate polling of students' opinions on the quality of instruction. Such polling is almost universally administered by questionnaires distributed to students at or close to the end of a course (Wachtel, 1998). Reasons given for the collection of student evaluation include instructional improvement, and as a personnel or management tool (student evaluation is an input to promotion and tenure decisions).

One aspect of student evaluation that is often overlooked is student *comments*: written responses to open-ended questions (Stewart, 2015). In this chapter, I discuss students' spontaneous[1] use of metaphor in the free-form comments section of course evaluation.

[1] A few authors (Kemp, 1999; Starr-Glass, 2005) report on the results of explicitly asking students to produce metaphors for the course (the cue is along the lines of "For us, this programme was like…because…".

7.2 Introduction

The overwhelming majority of research into student evaluation focuses on *ratings* or *scores*, that is, quantitative assessments attributed to various aspects of a course (Stewart, 2015). A typical rating might be a point on a five- or seven- point Likert scale, given in response to a cue such as "The course was interesting and stimulating".

There are two situations in which metaphorical language may be used in course evaluation: the cues used in the questionnaire design; and the comments made by students in response to open-ended questions.

The cues used in questionnaires do not appear to have been studied in detail (Aleamoni & Hexner, 1980) but if AUT evaluation are typical, the salient component of the process is requesting students to give a Likert score to various prompts. The Likert cues are generally literal—and non-metaphorical—noun phrases: "appropriate workload"; "overall experience"; "good teaching".

Cues for comment were, in contrast, phrased as questions: "What were the best aspects of this paper?"; "What aspects of this paper are most in need of improvement?" and these were again devoid of anything but the most lexicalised metaphor.

7.3 Students' comments in course evaluation

The only aspect of student evaluation in which metaphor may be used by students is the in the *comments* section: written responses to open-ended questions (Stewart, 2015). Despite the extensive research literature on student *ratings*, comparatively little is known about the quality of data obtained from students' written comments, their content, and the relationship between them and other variables.

Student comments are time-consuming to interpret, as standard descriptive and inferential techniques cannot be directly applied to free-form text.

One over-riding difficulty in the study of students' written comments is ethical: evaluation usually operates under strict confidentiality guarantees. Stewart (2015), for example, considers only an aggregated corpus of comments, being prevented from analysing individual responses by ethical concerns.

The few studies which have been reported in the literature tend to focus on whether the comments are positive or negative. Alhija and Fresko (2009), considering the frequency with which students' feedback includes comments, reports that the response rate lies between 10% and 70% and indicates that such a wide range reveals the "relatively small body of research" in this area (page 38).

Braskamp, Ory, and Piper (1981) is one of the few reports on comments *per se*; these authors report close agreement between written comments and objective responses. Nevertheless, comments can be informative in ways not possible for ratings. Tucker (2014), considering students' evaluation comments, provides a comprehensive review of students' use of language in this aspect of a course, and observes that student comments can provide "valuable insights" into the student experience. To what extent does metaphor play a part in this aspect of education?

Alhija and Fresko (2009) consider open-form requests for comment and report a range of student responses. The students appear to use metaphor very sparingly, with the majority being lexicalized ("time was wasted"; TIME IS A RESOURCE) or standard educational metaphors ("we never got to the end", A COURSE IS A JOURNEY). Such metaphors are strikingly similar to those used in documents such as course descriptors (discussed in section 3.1).

7.4 Conclusions

Course evaluation typically includes a free-form comments section in which students are invited to give feedback on a course. This section is the only section in which metaphor occurs.

It is difficult to study students' use of metaphor in this context and only a limited amount of research has been carried out to date. Students appear to use metaphor sparingly, and those that are used are similar to the metaphors used in institutional documents such as course descriptors.

8 Conclusions

In this thesis, I have looked across the AUT educational system within which an undergraduate statistics course was couched, and made a study of metaphorical language as used in the various aspects of such a course. The aspects range from course descriptors, through spoken lectures, to assessment cues and responses. For each aspect, I have identified metaphorical language using the Pragglejaz procedure, and analyed using methods drawn from similar studies in the fields of medicine, law, and business. Strengths of this methodology include wide coverage over various aspects of a course, and leverage of similar metaphorical studies in other disciplines. Weaknesses include lack of systematic and statistically representative corpus of data, although it is unclear how this could be achieved.

Conceptual metaphor, whether viewed as a cognitive or a linguistic phenomenon, appears in each of these aspects of a statistics course, to a greater or lesser extent. Metaphor is such a ubiquitous phenomenon that it is easy to overlook, but it is undoubtedly a component of many aspects of statistical education.

Following Lakoff and Johnson's seminal publication, metaphor is frequently viewed as a cognitive phenomenon which may be structured in terms of conceptual metaphors such as ARGUMENT IS WAR.

One way in which the metaphor may be studied is via its linguistic manifestation in both corpora and spoken English, and both are carried out in this thesis. Metaphor is shown to be influential in educational thought—notable examples including the acquisition metaphor of Sfard (1998).

Taking the *level* metaphor as another example, this metaphorical system structures thinking about education and encourages particular perceptions about mathematics—specifically, that mathematical knowledge is organised into a discrete set of standalone building blocks possessing a natural sequence. The result of this perception may well facilitate the organization of education (into annual cohorts), but the pedagogical effect of this is not clear.

It is not clear whether statistics education should be viewed as a subset of mathematics education, or a separate field. Statistics courses do share many points of similarity with mathematics courses such as hierarchical discipline structure, heavy dependence on abstraction, and an emphasis on numerical and algebraic techniques. The two subjects are also similar in their use of standard mathematical metaphors such as the BASIC METAPHOR OF INFINITY but this does not have the central role that it has in (for example) analysis or topology.

Metaphor is a component of educational administration. In particular, metaphorical language is a commonly occurring component of course descriptors. Pre-eminent among the metaphors used in this context is the *agency* metaphor: a statistics course is implicitly imbued with the ability to act independently and intelligently. Such language encourages the perception of the course content itself as active and personified. Such metaphors are often used deliberately to counteract the somewhat impersonal nature of many course descriptors. Metaphor occurs in other parts of educational planning too, notably institutional *graduate outcomes*. The metaphorical language appears to promote the acquisition metaphor at the expense of the participation metaphor.

Metaphor is a component of spoken lectures, but only rarely would the metaphors used be recognised as such by the audience. Also, in spoken lectures the metaphors seldom draw attention to the language used.

Metaphor is an unavoidable component of statistics textbooks. In

the three textbooks chosen for detailed study, metaphor was rare, with the exception of the basic metaphor of infinity, and the mathematicians' "we", which used throughout mathematics and is not specific to statistics.

It is not straightforward to analyse assessed work for metaphor (ethical considerations make informed consent difficult to obtain). However, a small number of assessed pieces of work is available, and it appears that tertiary statistics students do employ metaphor when assessed. The British Academic Written English corpus is one of the very few corpuses of assessed undergraduate student writing available, and this contains two pieces of assessed work which were part of a statistics course. The students used the conceptual metaphor EQUATIONS ARE TOOLS in much the same way as textbooks do.

Course evaluation is another component of statistics education in which metaphor is potentially used. Typically, course evaluation forms include a free-form comments section in which students are invited to give feedback on a course. It is again difficult to study students' use of metaphor in this context and only a limited amount of research has been carried out to date. Students appear to use metaphor sparingly, and those that are used are similar to the metaphors used in institutional documents such as course descriptors.

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