The Eckert Number Phenomenon - An Experimental Investigation of the Heat Transfer from a Rotating Cylinder in a Cross-Flow

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ABSTRACT

The Eckert number phenomenon describes a reversal in heat transfer from a moving wall at an Eckert number Ec ≈ 1. In this report the Eckert number phenomenon is confirmed experimentally for the first time. For that purpose the heat transfer from a heated, vertically rotating cylinder in a cross-flow was investigated where the cylinder surface represents a heat rejecting, moving wall. In order to perform the experiments in a range where the predicted phenomenon occurs, rotational speeds of up to 30,000 rpm were necessary which required the development of a contact-free heating concept and a new optical measuring technique suitable for boundary layers. The results show, among other things, that the temperature difference between the wall and the surrounding fluid has a significant effect on the predicted reversal of heat transfer at the wall. The impact of rotation on flow patterns, boundary layer behavior and heat transfer could be clearly identified. It appears that the velocity-ratio Ω acts like an independent parameter, in that flow patterns correspond to this dimensionless number. Furthermore, it seems that rotation dominates over cross-flow, both fluid-dynamically and thermally above Ω = 2.

B INTRODUCTION

In the 1960s experiments were carried out to investigate the cooling of commutators of electric motors by fans. It was found that the heat transfer from a fast rotating cylinder is only determined by the rotational Reynolds number which relates in a dimensionless form to the peripheral velocity of the cylinder[1]. However, it was discovered that the heat transfer could not be increased any further above a rotational Reynolds number $Re_\Omega = 2.5 \times 10^6$.

This observation induced Geropp [2] to carry out a theoretical study of the correlations of the Nusselt, Reynolds and Eckert numbers. His main focus was thereby on the high rotational speed of an infinitesimally long cylinder in quiescent air where the heat is created by dissipative effects and therefore the Eckert number (defined as the ratio of kinetic energy at the wall to the specific enthalpy difference between wall and fluid) becomes an important factor for the heat transfer. Based on the boundary-layer equations, Geropp formed a theory which supports the observations in that the heat transfer stagnates at a particular rotational Reynolds number. Moreover, he predicted that the heat transfer even changes its direction at an Eckert number Ec ≈ 1 which means that the heat flux rate between a heated wall and the surrounding fluid is then directed from the fluid towards the heated wall. According to Geropp, this reversal will occur at a rotational Reynolds number $Re_\Omega \approx 6.9 \times 10^6$.

In this report the Eckert number phenomenon is confirmed experimentally for the first time. For that purpose the heat transfer from a heated, vertically rotating cylinder in a cross-flow was investigated where the cylinder surface represents a heat rejecting, moving wall. In order to perform the experiments in a range where the predicted phenomenon occurs, rotational speeds of up to 30,000 rpm were necessary. This imposed an enormous challenge upon both the experimental setup and the measurement instrumentation. A heating concept had to be developed which allowed an input of heating power independent of the speed and which therefore had to be contact-free. The heat transfer from the rotating cylinder was investigated by purely optical measuring techniques in order to avoid disturbance of the extremely sensitive boundary layer in which the source of the phenomenon was expected to be found.

While this paper can only give an overview of the work that had been carried out to confirm the Eckert number phenomenon a more detailed description of the theoretical background and the experimental strategy can be found in [3]. Furthermore, the results of the extensive optical measurements of the heat transfer and the flow in the vicinity of a rotating cylinder are described in more detail in [4] and can be found in full length in [5].

C EXPERIMENTAL APPARATUS

C.1 Basic Configuration Providing the three principal requirements of cross-flow, rotation and heating is achieved with the apparatus shown in Figure 1. The cross-flow is provided in a closed wind-tunnel in which a constant fluid temperature is maintained by a heat exchanger. Although the wind-tunnel is gas-proof up to a pressure of 2 bar, the experiments were only carried out in air at ambient pressure. In this set-up gas-velocities of 70 m/s can be achieved. The rotating cylinder consists of high-strength aluminum and is mounted at the top of a hollow shaft of an asynchronous motor which was originally designed for high-speed aluminum milling and allows rotational speeds up to 30,000 rpm. This motor is attached to the bottom of the wind-tunnel with the cylinder reaching vertically through the test-section. The heat is supplied by a high performance heating cartridge which is mounted on a tube in the inner of the rotating cylinder. The tube is held by a flange at the bottom of the electric motor and led upwards through a hollow shaft. Thus, the heat is transferred from the non-rotating cartridge to the rotating cylinder by radiation only and the system works reliably regardless of the speed.

C.2 Measurement Instrumentation In order to examine the influence of rotation on the flow conditions in the surroundings of the cylinder the knowledge of the characteristic flow parameters is of great importance. Gas velocities are therefore measured with a two-dimensional Laser-Doppler-velocimeter (LDV) which is a highly complex optical measuring technique but offers the advantage of accurate measurements without disturbing the sensitive flow. In order to determine the temperature gradients (being the driving force for heat transfer) an optical measuring technique was developed which is based on the deflection of a light beam in a temperature field according to Schmidt´s analysis [6]. Furthermore, the real-time observation of temperature fields in selected areas should allow insights into the actual processes around the cylinder. With a Michelson interferometer a third optical method is employed with which the fluctuations of isothermals can be recorded on video. Finally, the Nusselt numbers which represent the heat transfer and thus any reversal thereof are determined by an energy balance on a segment of the heated cylinder.

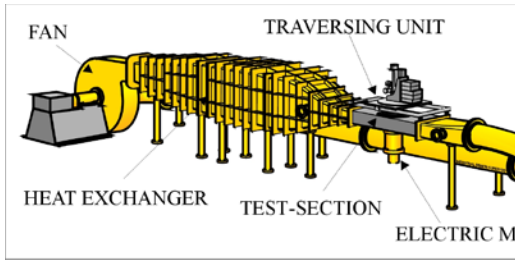


Figure 1. Experimental set-up of the test-section.

C.2 .1 Laser-Doppler-Velocimeter A two-dimensional Laser-Doppler-Velocimeter (LDV) is employed to measure local gas velocities in the vicinity of the rotating cylinder. The principle of this measuring technique is based on the light-scattering of particles which are added to the flow, passing through an interference pattern created by two monochromatic light-beams. This complex optical measuring technique offers the advantage of quick measurements at high local resolution without disturbing the flow, as would be the case if hot wire probes, for example, were used. Also, data can be taken almost without delay which allows dynamic measurements. Furthermore, in using two wavelengths of the laser, as in this case, two velocity components can be measured simultaneously at one position. Apart from that, this measuring technique is independent of the pressure, temperature or density of the fluid.

C.3 .2 Light-beam Deflection Temperature gradients can hardly be measured in a direct way let alone contact-free. For this purpose an optical measuring technique was developed which allows the determination of local temperature gradients at the wall of the rotating cylinder without disturbing the sensitive boundary layer. The principle is based on E. Schmidt´s analysis [6] describing the path of parallel light near heated walls. According to this theory, light is being deflected in a density field towards higher density. With neglect of pressure differences the deflection of a light beam can be correlated with temperature gradients. Figure 2 depicts the experimental set-up.

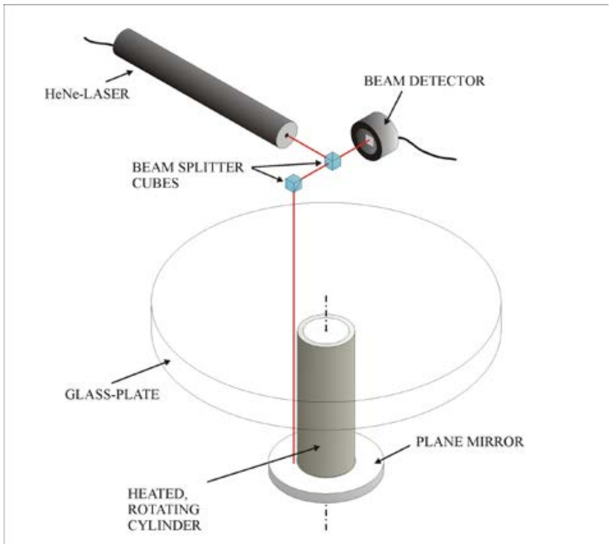


Figure 2. The optical set-up of the light-deflection technique.

A light-beam from a small 5 mW helium-neon-laser is deflected by two beam splitter cubes so that it vertically enters the test-section through the optical glass-plate. The beam touches the cylinder wall, hits a mirror at the bottom of the test-section and is reflected back on itself. While passing through the beam splitter cubes again one half of the remaining beam coming from the test-section hits a beam-position detector which measures the x,y-position of the centre of area with an accuracy of ± 50 µm. Laser, beam splitter cubes and detector, all mounted on a small plate, can be attached to the LDV-unit. Thus, the highly precise traversing unit can be used and the laser-beam can be positioned in polar-coordinates. A detailed explanation and the experimental verification of the interpretation of the results obtained by this method are described in [4] and [5].

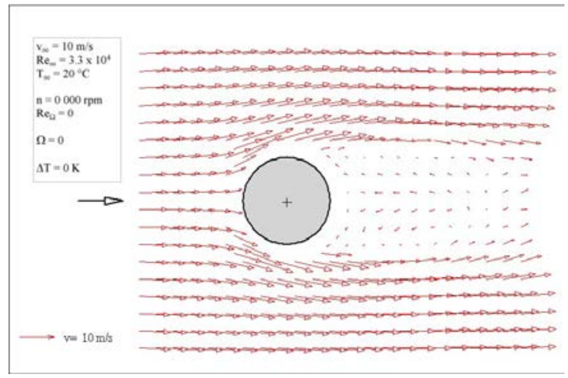
C.3 .3 Michelson-Interferometer In order to visualize heat transfer processes and the impact of rotation in the vicinity of the cylinder, an interferometer is installed. The principle of this measuring technique is based on the superposition of two monochromatic and coherent light-beams which create a two-dimensional pattern of dark and light fringes. Assuming that the gas obeys the laws of an ideal gas and that pressure differences change the density of the gas only marginally compared to temperature differences, interference fringes can be regarded as isothermals. The original intention to use the interferometer as an instrument for the qualitative determination of isotherms had soon to be rejected. This is because the produced pictures do not allow the identification of isotherms due to the extreme sensitivity of the optical set-up. (By means of a hot-wire-probe the temperature difference between two interference fringes was determined to be approximately 0.05 K.) In other words, the density of the recorded fringes is beyond the optical resolution of the system. It is unavoidable to expose the interferometer to vibrations stemming from the wind-tunnel and the electric motor as there is no possibility of separating the mechanical and the optical systems. Experiments to try to eliminate the fluctuations of the interference pattern by using extremely short exposure times of only a few µs did not achieve the desired outcome. For all these reasons the use of the interferometer is reduced to low gas velocities and small temperature differences, whereas the value of this measuring instrument mainly lay in the visualization of the ongoing fluid-dynamic and thermal processes.

C.3 .4 Heat Transfer Measurements Since it was the goal to confirm the Eckert number phenomenon in practice, i.e. the decrease or even reversal of the heat transfer, the experiments had to be carried out at Eckert numbers around unity. In order to reach this value, it was of critical importance to know that, apart from the circumferential velocity of the cylinder, which was high but limited, an additional parameter was available in the form of the temperature difference between the cylinder wall and the fluid. For experimental reasons it was advantageous to choose this temperature difference to be as large as possible, which consequently meant operating at the maximum rotational speed of the experimental apparatus. With a maximum rotational speed of 30,000 rpm the temperature difference between cylinder wall and surrounding fluid should not exceed 5 K according to Geropp's theory in order to show the dependence of the Nusselt numbers on the rotational Reynolds number. The disadvantage of this procedure was that the absolute error of the temperature measurements was larger; however, the advantage was that the temperature dependency of fluid properties could be neglected at these small temperature differences.

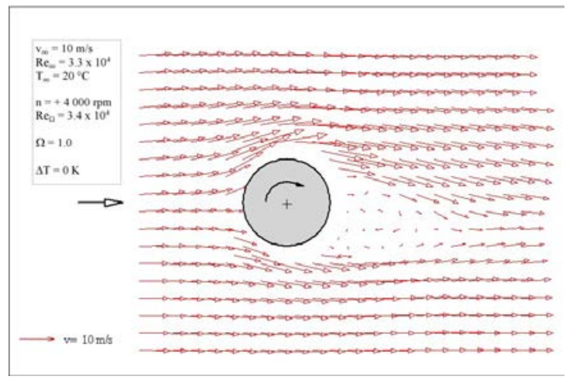
For the determination of the Nusselt number three variables have to be measured with the heat flow, the wall temperature and the temperature of the surrounding fluid. Although the temperatures can be measured with infrared thermometers, the heat flow can be determined only indirectly. This is because the waste heat of the electric motor plays a non-negligible role in the experiments. The axial conductive heat flow along the shaft on which the cylinder is mounted increases with rising speed and contributes additionally to the heating of the cylinder. Since only a few Watts are necessary for the desired small temperature differences, the contribution of this waste heat has to be taken into account. However, the problem is that this amount of heat cannot be measured or isolated with comparative reference measurements since an additional speed-dependent effect is involved, namely the “self-heating” caused by dissipation in the boundary layer. The solution to this problem resulted from the observed axial temperature difference at the cylinder wall. With increasing rotational speed, a gradient in the wall temperature can be observed from the bottom to the top. With known temperatures and a known electrical heat input from the heating cartridge an energy balance can be performed on a segment of the cylinder which yields the rejected heat from the cylinder wall. The strategy for the heat transfer experiments was such that a constant heating rate was set until temperature equilibrium was reached. This solved both the controlling problem due to thermal inertia of the heating system, and also the case of a constant wall heat flux seemed to be closer to reality than a constant wall temperature.

D. EXPERIMENTAL RESULTS

D.1 Fluid Velocities The purpose of the gas velocity measurements was to show the influence of rotation and heat transfer on the flow in the vicinity of the cylinder. Approximately 400 data points were taken on a grid with side lengths of 3 and 4 times the cylinder diameter. The chosen rotational speeds correspond to even numbers of the velocity ratio Ω. The temperature difference ΔT between the cylinder wall and the surrounding flow was 50 K, whereas the fluid temperature was maintained at 20 °C. The experiments were carried out mainly at sub-critical flow speeds, i.e. at Reynolds numbers $Re_\infty = 3.3 \times 10^4$, which related to a main-flow velocity of 10 m/s corresponding to the cylinder diameter. Figure 3 illustrates the flow around the cylinder in the unheated case, in sub-critical flow and velocity-ratios Ω = 0,1 and 2. The arrows thereby represent the time-averaged, local velocity vector, according to magnitude and direction. Figure 3a shows the case of the non-rotating cylinder. The flow is attached symmetrically to the cylinder and separates at an angle of about 90° measured from the front-stagnation point. The wake is characterized by turbulence and re-circulation and stretches up to three times the cylinder diameter downstream.



a)



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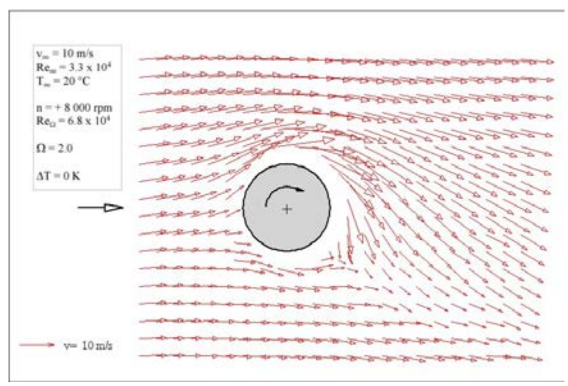

c)

Figure 3. LDV measurements of flow velocities around a rotating cylinder at sub-critical flow and velocity ratios Ω = 0, 1 and 2.

The typical twin-vortex structure can be recognized in this area. With the onset of rotation (Fig. 3b) the wake is asymmetrically deformed in the rotational direction, with one separation point moving downstream and the other moving upstream. The twin-vortex structure is increasingly destroyed. At a velocity-ratio of Ω = 2 (Fig. 3c) the turbulent wake is already significantly shifted. A further shift, however, cannot be observed at even higher rotational speeds (not shown here). This supports the observation by other authors that both separation points coincide at a velocity-ratio $\Omega = 2$ and that their position remains unchanged with increased speed. Thus, above $\Omega = 2$, a similar flow situation exists near the cylinder wall as in the case of a rotating cylinder without cross-flow. A significant change happens to the wake of the cylinder, however, when rotation is present (for lack of space further diagrams are omitted here but can be found in [3], [4] and [5]). The complete wake is highly turbulent in the case of the non-rotating cylinder, and this is especially evident at the border to the main flow where flow separation takes place. A reason for this may be found in the higher shear-forces in this particular flow area. However, as soon as rotation comes into play a significant reduction of turbulence can be observed. An increase of the flow velocity up to the critical range has no effect on the flow pattern as further experiments showed. A comparison of the obtained results with the pictures in Fig. 3 does not indicate any obvious differences. Furthermore, heat rejection has no significant influence on the flow characteristic in the surroundings of the cylinder either as repeated experiments with a heated cylinder demonstrated. No significant differences could be found between the heated and the corresponding unheated cases.

D.2 Temperature Gradients Only a few examples shall be given here from the large number of experiments. Figure 4 shows a series of measurements at sub-critical flow at a Reynolds number $Re_\infty = 3.3 \times 10^4$, a temperature difference of 50 K and the rotational speed as a parameter. It can be seen that the downstream separation point (represented by the maximum at about $\varphi \approx 90°$) is shifted in rotational direction with increased speed. At about $\Omega = 2$ this point starts to disappear and the curve becomes flatter. The opposite separation point, however, is hardly shifted. Even at $\Omega = 3$ a maximum of light-deflection can still be observed, but not as marked as at lower speeds. Generally speaking, rotation seems to have an equalizing effect on the shape of the graph as maxima and minima become less marked. Local, and for the heat transfer, characteristic points transit to a more and more uniform course.

D.3 Temperature Fields The interferometer served the purpose of the qualitative assessment of the thermal processes at the rotating cylinder with dark and light lines representing near-isothermals. Figure 5 serves as a single example out of a large number of pictures cut from video recordings. Only the downstream half of the cylinder with its separation points and the turbulent wake is shown since the high line-density in the upstream half (caused by the high temperature gradients there) was too high for the optical resolution. Two quarter circles of similar experiments but reversed speed have been assembled to form one picture. Flow- and rotational direction are marked by arrows. Since the vibrations caused by the fan and the electric motor had a bad influence on the quality of the produced interferograms, the experiments were carried out at low flow speeds of 1 to 2 m/s and rotational speeds of 800 to 1600 rpm. Even so, the expected processes are clearly evident.

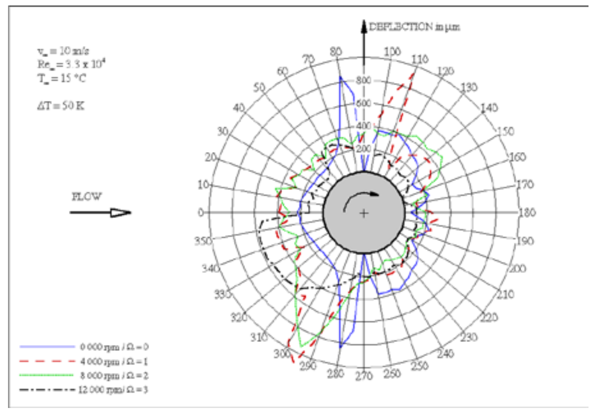

Figure 4. Temperature gradients obtained by the light-deflection technique in the boundary layer of the heated, rotating cylinder at various velocity ratios.

Fig. 5 shows the case of a velocity ratio $\Omega = 2$. It can be seen that the downstream separation point has shifted in the direction of rotation and is about to dissolve. Also the turbulent wake has moved in this direction. From qualitative assessment and comparison of further interferograms it can be concluded that the velocity-ratio Ω behaves like an independent parameter if the flow velocity is varied and the rotational speed is kept at a constant Ω. With reduced flow velocity and rotational speed reduced proportionately in order to keep Ω constant the same characteristics of the flow pattern persist. With further increased speed of the cylinder up to $\Omega = 3$ the downstream separation point completely vanishes. Even more compelling than the shown interferometric photographs are moving pictures recorded by a video camera during the experiments. Those real-time pictures appear richer in contrast as the human eye can identify dynamic changes better than can be detected from still pictures. Thus, the video recordings deliver a very vivid impression of the oscillating processes in the turbulent wake of the cylinder.

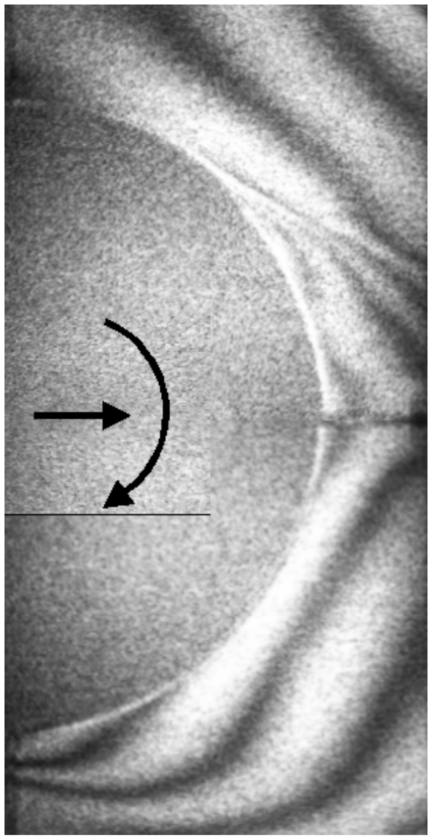

Figure 5. Interferogram for Re_∞ = 0.7 x 10^4, ΔT = 25 K and Ω = 2.

D.4 Nusselt Numbers Based on the energy balance described above the calculated Nusselt numbers derived from the three temperature differences are compared with the theoretical ones in Figure 6. By comparing the experimental results with the theory the following statements can be made:

- The overall trend of the measured values is similar to the theoretically predicted ones, which confirms the existence of the Eckert number-phenomenon with its reversal in heat transfer.
- The measured values are almost twice as high as the theoretical ones.

In order to explain this discrepancy it is useful to look at the heat transfer measurements of the non-rotating cylinder as a reference to exclude any systematic errors in the experiments. To do so, the measured Nusselt number of Nu = 125 at a Reynolds number Re_∞ = 1.6 x 10^4 has to be corrected to the theoretical case of an infinitely long, freely rotating cylinder in an ideal cross-flow in order to be comparable at all. In applying correlations that can be found in literature in order to take effects into account such as free-stream turbulence, aspect ratio, blockage ratio etc which enhance the heat transfer compared to the ideal case, the experimental Nusselt number is corrected down to about 60% of its value (see [3] for more details). Due to the lack of experimental data on above mentioned influences in the situation of a rotating cylinder one can only speculate that those heat transfer enhancing effects also apply here.

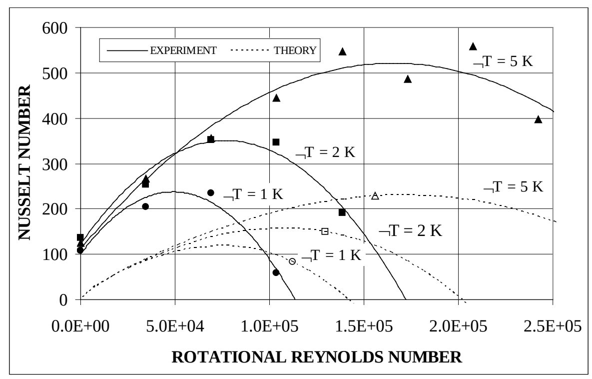

Figure 6. The measured Nusselt numbers with their trendlines (solid) at a cross-flow Reynolds number Re_∞ = 1.6 x 10^4 and the theoretical Nusselt numbers according to Geropp (dotted)

It should also be noted that a maximum heat transfer for each temperature difference can be obtained from Geropp's equations. It appears that the maximum heat transfer converges towards Ec $\approx$ 0.3 with increasing temperature difference (see [3]).

E. A SCENARIO FOR THE ECKERT NUMBER PHENOMENON

Concluding from Geropp's considerations and the supporting experiments the following scenario can be established for the heat transfer of a cylinder with constant temperature difference (T_W -T_∞) and increasing rotational speed: Initially, rotation has a positive effect on the heat transfer. However, with increasing circumferential speed the sheer stress due to viscosity of the fluid creates more and more dissipation. The maximum heat transfer occurs when the latter effect is still small while the former continues to increase. With a further increase of the rotational speed, however, dissipation plays the dominating role. In this regime the location where most of the dissipation occurs can be regarded as a local heat source. Since dissipation takes place mainly where the greatest velocity gradients are, this location is not situated at the wall where the fluid adheres, but in the boundary layer. In other words, the boundary layer encloses a virtual, concentric area around the cylinder which assumes the "adiabatic wall temperature" T_{ad}. The consequence for the heat transfer is that the cylinder does not "see" the temperature of the surroundings, that is T_∞, but the thin annular area around the cylinder with the "adiabatic wall temperature" T_{ad}. With this temperature increasing, the effective temperature difference becomes smaller and so does the heat transfer. Once the "adiabatic wall temperature" reaches the value of the cylinder wall (at an Eckert number Ec ≈ 1), the temperature gradient reverses and the cylinder is heated by the locally-created entropy production rate caused by friction within the boundary layer. At the same time, a temperature difference (T_{ad} -T_∞) exists for the surrounding fluid. Therefore a portion of the dissipation being produced within the boundary layer is also transferred to the fluid. As a result of these considerations it becomes clear why Eckert numbers greater than unity have never been reached in experiments. If one wants to keep the chosen temperature difference (T_W -T_∞) at a constant level, one has to start rejecting the entropy-rate transferred to the cylinder from Ec ≈ 1 on, which is impossible without an active cooling system.

F. CONCLUSIONS

1. LDV-field-measurements of fluid velocities indicate a significant influence of rotation on the flow past a rotating cylinder up to a distance of several cylinder diameters. The shift of the flow separation points due to rotation, the turbulent wake of the cylinder, and the existence of vortices in the wake can be clearly identified. However, a heated wall with a temperature difference of 50 K does not seem to change the flow pattern. The degree of turbulence of the flow in the wake of the cylinder is significantly reduced by rotation.
2. A contact-free optical measurement technique was developed and verified to measure local temperature gradients in the boundary layer of a heated, rotating cylinder. Temperature gradient measurements by the light-deflection technique in the vicinity of the cylinder also detected similar thermal patterns, as flow patterns could be detected by LDV-measurements, such as flow separation and changes in the boundary layer thickness.
3. Thermal fluctuations and the impact of rotation on heat transfer could be visualized by a Michelson-interferometer. It seems that the velocity-ratio Ω behaves like an independent parameter, as flow and thermal patterns seem unchanged at various cross-flow Reynolds numbers but constant velocity ratios. The interferograms also confirm the results of the light-deflection technique.
4. With the temperature difference $\Delta T = (T_W$ - $T_\infty)$ an additional parameter is hidden in the Eckert number. This means that a general boundary Reynolds number as suggested by Geropp does not exist, however, there is an individual function of the Nusselt number for each temperature difference.
5. The “self-heating” is a process of its own and is only a function of the rotational Reynolds number. In other words, the creation of dissipation is independent of the temperature difference between cylinder wall and fluid, but the interaction of $\Delta T = (T_W - T_\infty)$ and the “adiabatic wall temperature difference” $\Delta T_{ad} = (T_{ad} - T_W)$ determines the heat transfer. Thus, a boundary Reynolds number as suggested by Geropp does not exist.
6. Experiments qualitatively confirm Geropp´s theory and the existence of the Eckert number phenomenon although the measured values are nearly twice as high as the theoretical ones. However, looking at the non-rotating case as a reference this discrepancy can be explained by well known effects like the degree of turbulence, blockage and aspect ratio which each increases the heat transfer.

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