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An innovative pedagogical strategy for teaching and assessing critical thinking in mathematics

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ABSTRACT

This article describes and promotes an innovative pedagogical strategy for teaching and assessing critical thinking in mathematics. It can be applied at secondary and tertiary level. The idea is to use mathematics questions that are *deliberately* designed to mislead the solver and direct to an incorrect solution. We call such questions provocative questions. The intention is to encourage students to critically analyse a mathematical question first before applying certain techniques or software to solve it. That is to enhance a habit to question the question, pay attention to conditions and constraints, recognise mistakes and don't take anything for granted which is an essential part of a mathematical way of thinking. Several examples demonstrate the idea in the article. Attitudes of secondary school mathematics teachers and university lecturers towards the suggested pedagogical strategy are presented and compared. Implications for teaching practice are also discussed.

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KEYWORDS

Pedagogical innovation; critical thinking; mathematics assessment

1. Introduction and rationale

We begin with the premise that making mistakes is an inherent aspect of all human activity. While most errors occur unintentionally, some are deliberate – crafted to deceive, manipulate, exploit, mislead, or spread misinformation, as seen in deceptive advertising and fake news. The prevalence of conspiracy theories, information warfare, and deepfakes is increasing in our daily lives. The scientific community has expressed growing concern that we are living in an era dominated by misinformation, where 'our willingness to share content without thinking is exploited to spread disinformation' (Wardle, 2019, p. 15).

Regardless of whether a mistake is accidental or intentional, the ability to identify and respond to it effectively is a crucial skill and a fundamental component of critical thinking. Although definitions of critical thinking may vary, they frequently include concepts such as analysis, evaluation, examination, questioning, reasoning, identification of biases, and challenging assumptions. In response, many national curricula explicitly prioritise the development of students' critical and analytical thinking abilities. For instance, the New Zealand Curriculum highlights this focus by using the word 'critical' 60 times across

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its 67-page document (Ministry of Education, 2025). These competencies are also highly sought after in the workforce. According to the World Economic Forum's *Future of Jobs Report* (2020), analytical and critical thinking are among the top five most essential skills for the year 2025.

Critical thinking is valuable not only in academic settings but also in everyday life. It enables individuals to resist manipulation and avoid being deceived or misled. In his influential book *How to Lie with Statistics*, Huff (1954) provided numerous examples of how statistics can be misused in business to mislead the public. He observed, 'the crooks already know these tricks; honest men must learn them in self-defence' (Huff, 1954, p. 11).

Mathematical concepts – particularly percentages – are frequently misrepresented in media and political discourse. A common example is the misreporting of an increase from 10% to 20% as a 10% rise, when it is actually a 100% increase. Another instance is the statement by Vladimir Putin: 'The Belorussian economy equals 3% of the Russian, and the Russian, accordingly, 97% of the Belorussian' (Barbeau, 2013, p. 15).

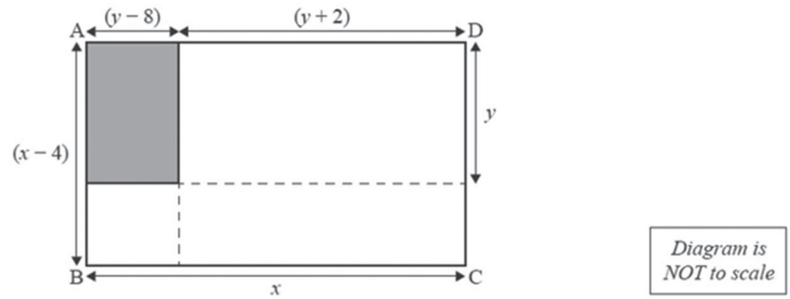
Unfortunately, some mathematical mistakes led to tragedies. For example, Parker (2019) presents many tragic real cases when mathematical mistakes caused the death of people. One such chilling story happened in 1986 when the space shuttle Challenger exploded because of the leak from the solid rocket boosters. A significant contributory factor was related to the simple geometric fact that the converse of the statement 'if we have a circle then the diameter is constant' is false, for example the Reuleaux triangle or the shape of the 50p UK coin. However, the NASA engineers used the wrong criterion of a constant diameter to identify a circular shape. That mathematical error contributed to the disaster as reported in Finding #5 of the Investigation Report: 'significant out-of-round conditions existed between the two segments' (Parker, 2019, pp. 75–77).

Mathematics is not exempt from mistakes. Mistakes can be found in mathematics textbooks and dictionaries published by reputable publishers, in leading international research journals on mathematics education, and even in national school examinations. Several real examples of mathematical mistakes in educational contexts are presented in Klymchuk and Sangwin (2021). The example below highlights the most recent mistake from a New Zealand national mathematics examination (Figure 1).

This apparently routine question contains a contradiction. Using basic algebra one can easily find that $y = 9\,\text{cm}$, $x = 12\,\text{cm}$ and the area of the rectangle ABCD is $96\,\text{cm}^2$. This was the model 'correct' answer, and many students solved the question that way. However, on a closer look one can see that this rectangle does not exist (with the given dimensions the side (x–4) is 8 cm and it is bigger than y which is 9 cm). The journalists reported that the students who noticed the contradiction were 'baffled', 'shocked' and 'confused' by the question (New Zealand Herald, 2019).

To foster critical thinking in the mathematics classroom, we suggest incorporating so-called provocative or 'impossible' questions into both teaching and assessment. These questions resemble standard, routine problems but they are *intentionally* designed to include a subtle flaw or misleading element. The goal is to better prepare students for real-life situations, where not all problems follow familiar patterns. Many students – especially those studying mathematics as a service subject – primarily engage with routine exercises that involve applying formulas, rules, or theorems where all conditions are met. However, in real-world contexts, these conditions may not always be met, and misapplying a rule or procedure without careful analysis can result in significant errors.

The shape below is divided into rectangles. All measurements are in cm.



The shaded rectangle has an area of 9 cm².

Find the area of the rectangle ABCD.

Figure 1. The 2019 algebra level 2 examination question.

As a teaching strategy, provocative mathematics questions highlight the importance of remaining alert and critically evaluating each problem. They encourage students to 'question the question' – a fundamental habit in inquiry-based learning and a key component of mathematical thinking.

2. Design and examples of provocative questions

There are several ways to design provocative questions. The most common is asking someone to do an impossible task, e.g.

- prove something that is *not provable*
- show the existence of something that *doesn't exist*
- apply of a rule or formula or law that is *inapplicable* due to their conditions/constraints
- show that something is *impossible* whereas it is *possible*
- give a multi-choice question where *all* suggested answers are wrong.

Below are several examples of provocative/impossible questions.

Question 1. Prove the identity $\sin x = \sqrt{(1 - \cos^2 x)}$.

The 'identity' is not true. Squaring both sides does not prove it because this operation is irreversible. It is not an identity but an equation with infinitely many solutions $x \in [2\pi n, \pi(2n+1)]$. Here the deceptive formulation leads to a false assumption that the given formula is a provable identity.

Question 2. Explain why the function $g = \log_5(5 - x)$ is monotone increasing and the function $h = \log_{0.5}(5 - x)$ is monotone decreasing.

One might mistakenly 'explain' that using the fact that the log function $y = \log_a x$ is monotone increasing for $a > 1$ and decreasing for $0 < a < 1$. In fact, the function $g = \log_5(5 - x)$ is monotone decreasing and the function $h = \log_{0.5}(5 - x)$ is monotone increasing on their domain which is easy to show using the definition.

Question 3. Find the derivative of the function $y = \ln(\ln(\sin x))$.

Although it looks like a routine question on differentiation techniques using the Chain Rule, the rule is not applicable as the function has an empty domain and therefore the derivative does not exist.

Question 4. Sketch a graph of a function that is differentiable on the interval (a,b) and discontinuous at least at one point on (a,b).

Any sketch would be incorrect as the task is impossible: a function differentiable on interval (a,b) is continuous on it.

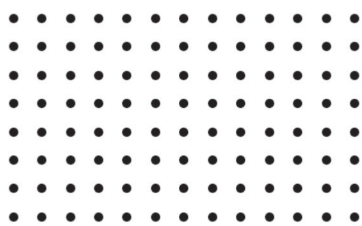
Question 5. Find the following definite integral using the Newton-Leibniz formula: $\int_{-1}^{1} \frac{dx}{x}$.

This question looks like a routine question on integration, but this is not a definite integral since the integrand function is not continuous on the interval $[-1, 1]$. Therefore, the Newton-Leibniz formula is not applicable.

Question 6. Show that the equation $(2x+7)^2 = (2x-1)^2$ doesn't have solutions.

One can mistakenly 'forget' about signs when taking square roots and cancel $2x$ on both sides. In fact, the equation does have the solution $x = -1.5$.

Question 7. 'Below is part of an infinite integer lattice. A lattice triangle is a triangle where the coordinates of all vertices are integers. What is the size of the smallest equilateral lattice triangle?



The trick here is that it is impossible to draw an equilateral triangle (a triangle that has three distinct finite vertices) on the lattice. Proving that no triangles exist is an interesting task in its own right' (Badger et al., 2012, pp. 6–7).

Apart from impossible questions, there is a cunning way to create a provocative question. A question can be possible but be formulated in a way that the attention of a solver diverts into a wrong direction of thinking. Different distractors can be used for that, for example: additional non-essential information, text formatting/layout, a wording style to formulate a question. It demonstrated below by the following famous conjunction fallacy created by psychologists Tversky and Kahneman (1983).

Question 8. 'Linda Problem. Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which alternative is more probable?

(a) Linda is a bank teller.
(b) Linda is a bank teller and is active in the feminist movement.' (p. 297).

In the Linda problem, background information about Linda leads participants toward incorrect reasoning. This effect is caused by the inclusion of non-essential, stereotypical details that make the less probable option seem more believable, thereby interfering with logical judgment. Despite the clear principle that a conjunction of two events is less likely than a single event, 'about 85% to 90% of undergraduates at several major [American] universities chose the second option, contrary to logic' (Kahneman, 2011, p. 158).

Kahneman (2011) also found that this reasoning error persisted among highly educated individuals: '85% of doctoral students in the decision-science program at the Stanford Graduate School of Business, all of whom had taken several advanced courses in probability and statistics, also incorrectly ranked 'feminist bank teller' as more likely than 'bank teller'' (p. 159). He further noted that when a similar question was asked without the stereotypical narrative, participants were less likely to make the same logical mistake. For example:

Which alternative is more probable?

(a) Jane is a teacher.
(b) Jane is a teacher and walks to work.

Kahneman (2011) concluded that 'the most coherent stories are not necessarily the most probable, but they are plausible, and the notions of coherence, plausibility, and probability are easily confused by the unwary' (p. 159).

3. Practitioners' attitudes towards the usage of provocative questions

Several studies have explored the attitudes of school mathematics teachers and university lecturers toward the use of provocative questions in teaching and assessment. The initial investigation involved four groups of school mathematics teachers from Germany, Hong Kong, New Zealand, and Ukraine (Klymchuk, 2015), and was later replicated in the fifth group in Australia in (Brown, 2018). Across all five groups, there were 127 participants in total. Approximately 60% indicated they would modify their teaching practices by incorporating 'trick' questions, encouraging students to 'question the question', and prompting careful analysis of conditions and constraints before applying formulas or theorems. In contrast, the remaining 40% stated they would not adopt provocative questions, citing their exclusion from formal assessments as the primary reason.

Another study by Klymchuk (2022) involved 82 school mathematics teachers who were introduced to the proposed pedagogical strategy at conference workshops. An overwhelming majority (96%) expressed their willingness to incorporate provocative questions into their mathematics teaching, indicating a strong appreciation for the educational benefits of such questions. Most of the teachers' feedback emphasised the promotion of critical thinking skills. Additional comments highlighted advantages such as fostering a deeper understanding of mathematical concepts, enhancing creativity, and increasing student engagement and interest in mathematics. However, 37% of participants stated they would not use provocative questions in assessment contexts. The two main concerns cited were the potential for increased student stress during examinations and the belief that students were insufficiently prepared for such questions.

A recent pilot study involving eight experienced university mathematics lecturers was conducted by Klymchuk (2024). Of the participants, six lecturers (75%) indicated they would incorporate provocative questions into their teaching. However, five lecturers (62%) stated they would not include such questions in assessments. Their primary concerns centred on the potential for reduced pass rates, which could attract negative feedback from university management, as well as possible complaints from students who might perceive the questions as 'unfair' or 'abnormal.'

In all studies above a mixed methods approach was used and the data were analysed through coding and thematic analysis.

The attitudes of school mathematics teachers and university lecturers across the above studies showed similar trends. Both groups generally responded positively to the idea of using provocative questions to enhance students' critical thinking skills. However, they expressed reservations about including these questions in assessments – though for different reasons. School teachers were primarily concerned about the potential stress and lack of preparedness among students, while university lecturers were more apprehensive about the possible negative impact on their professional standing. Additionally, only a few school teachers mentioned the broader applicability of critical thinking skills beyond the classroom, often viewing their role as preparing students for the next academic level. In contrast, half of the university lecturers explicitly connected critical thinking to students' future careers, employability, and civic engagement.

4. Implications for teaching practice

Erroneous statements are sometimes used in assessments, but they are normally labelled as incorrect, with students being asked to correct them. For instance, a question may prompt students to critique a misleading graph, refute a false statement using a counterexample, identify an error in a sophism, or verify the accuracy of a given claim with justification. These types of questions are effective in assessing students' conceptual understanding and critical thinking. However, such tasks often include cues or hints that alert students to the presence of an error, prompting a different cognitive response than when solving a standard procedural problem. In contrast, real-life situations rarely offer such explicit warnings. Therefore, provocative questions go beyond these guided error-identification tasks by presenting misleading or flawed scenarios without any indication that something is amiss, thereby more closely simulating real-world scenarios.

Traditional mathematics assessments generally include three main categories of questions: procedural, conceptual, and applications. In most of these, the conditions required for the correct application of theorems, formulas, or rules are fully satisfied. As a result, students may become accustomed to applying procedures without critically evaluating whether all conditions are met. In real-life contexts, however, such assumptions can lead to significant mistakes. To address this issue, we propose incorporating provocative questions as a fourth category alongside the conventional trio. These questions are specifically designed to challenge students' assumptions, encouraging them to verify conditions and constraints and think critically before proceeding.

To mitigate potential student anxiety and lack of preparedness for provocative questions, we recommend implementing them within a supportive classroom environment and providing ample opportunities for practice. Their introduction should be gradual – initially as optional or extracurricular tasks, followed by integration into the formal curriculum, then into formative assessments, and eventually into summative assessments. An instructional note might be added to tests or examinations to prepare students: 'Some of the questions in this assessment are standard routine questions, while others are intentionally designed to mislead you. You should distinguish between them and respond accordingly.' Once students gain sufficient experience with such problems, they may come to view them as routine. As Hughes et al. (2006) noted, 'routineness has to do with what the solver is used to' (p. 91).

This pedagogical strategy is not limited to mathematics and can be effectively applied in other disciplines at both school and university levels. In physics, for example, a deceptively simple question might be: Give an example of the most well-known quadratic equation in the theory of relativity. A common (but incorrect) response is $E = mc^2$. This is not a quadratic equation, since c is a constant, making the equation linear. Another misleading physics question could be: In an ideal state, prove that when two balls are dropped from the same height, the heavier one will hit the ground first. Any attempted proof is incorrect, as in a vacuum (ideal state), both objects fall at the same rate, regardless of mass. In chemistry, a misleading question might be: Why can't a thermometer show −1°C in ocean water? The expected (incorrect) answer is that water freezes at 0°C, so at −1°C, it would be ice. The correct explanation is that ocean water contains salt and freezes at around −2°C, so −1°C is a perfectly reasonable temperature reading. In engineering, students might be asked to design a bridge using given parameters that make the task impossible – without being told so – or to create a faulty electrical circuit diagram, again without being informed of the flaw.

5. Conclusion and further studies

Whether we like it or not, misleading statements and incorrect recommendations pervade many aspects of modern life – including the media, politics, business, and the economy. The issue of fake news has become even more pressing in the digital age, particularly with the rise of artificial intelligence and conspiracy theories. Deepfakes and other technological manipulations have a profound impact on individuals and organisations, fuelling misinformation campaigns and information warfare between opposing nations. To describe this growing phenomenon, scientists, journalists, and political leaders have adopted strong terms such as information disorder, weaponization of information, computational propaganda, troll armies, and information arms race. The acronym VUCA – volatility, uncertainty, complexity, and ambiguity – is increasingly used by leaders in politics, business, and science to characterise the rapidly evolving world we inhabit.

We hypothesise that the critical thinking skills fostered by provocative questions in mathematics classrooms may help students build resilience against misinformation and reduce their vulnerability to misleading claims. These skills have the potential to transfer beyond the classroom into broader aspects of everyday life, contributing to the development of well-informed citizens and thoughtful decision-makers. Researchers in mathematics education – particularly at the tertiary level – have emphasised the importance of focusing on the transferability of such skills. For example, Laursen and Rasmussen (2019) advocate for 'explicit attention to skills that are transferable to other disciplines and to work settings – student competencies such as communication to experts and non-experts, writing, working in teams, critical thinking, metacognition, and thinking ethically' (p.

139). The benefits of strong critical thinking extend to recognising errors and misinformation, identifying inconsistencies, ruling out implausible scenarios, conducting sceptical and unbiased evaluations, and making sound, evidence-based decisions. Investigating the transferability of critical thinking skills from a mathematics classroom into workplace contexts would require a series of longitudinal studies.

Author contributions

CRediT: **Sergiy Klymchuk:** Conceptualization, Project administration, Writing – original draft, Writing – review & editing; **Chris Sangwin:** Writing – review & editing

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Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this article.

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References

Badger, M., Sangwin, C. J., Ventura-Medina, E., & Thomas, C. R. (2012). *A guide to puzzle-based learning in STEM subjects*. University of Birmingham.

Barbeau, E. J. (2013). *More fallacies, flaws & flimflam*. Mathematical Association of America.

Brown, P. (2018). Conspiracy in senior school mathematics. In J. Hunter, P. Perger, & L. Darragh (Eds.), *Proceedings of the 41st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 186–193). MERGA. https://merga.net.au/publications/annual-conference-proceedings/2018-conference-proceedings/

Huff, D. (1954). *How to lie with statistics*. Norton.

Hughes, R., Monaghan, J., Shingadia, E., & Vaughan, S. (2006). Revisiting routine questions. *Teaching Mathematics and its Applications: An International Journal of the IMA*, *25*(2), 90–96. https://doi.org/10.1093/teamat/hri011

Kahneman, D. (2011). *Thinking, fast and slow*. Farrar, Straus and Giroux.

Klymchuk, S. (2015). Provocative mathematics questions: Drawing attention to a lack of attention. *Teaching Mathematics and Its Applications*, *34*(2), 63–70. https://doi.org/10.1093/teamat/hru022

Klymchuk, S. (2022). An innovative way of teaching and assessing critical thinking in mathematics. In J. Morska & A. Rogerson (Eds.), *Proceedings of the 16th International Conference of The Mathematics Education for the Future Project* (pp. 267–272). WTM-Verlag. https://doi.org/10.37626/GA9783959872188.0.049.

Klymchuk, S. (2024). A novel approach to teaching and assessing students' critical thinking in university mathematics. In J. Morska & A. Rogerson (Eds.), *Proceedings of the Third International Symposium "New Ways of Teaching and Learning"* (pp. 167–172). WTM-Verlag. https://doi.org/10.37626/GA9783959872881.0.31.

Klymchuk, S., & Sangwin, C. (2021). Provocations in mathematics: Teachers' attitudes. *Journal of Higher Education Theory and Practice*, *21*(10), 254–263. https://doi.org/10.33423/jhetp.v21i10.4639

Laursen, S., & Rasmussen, C. (2019). I on the prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *5*(1), 129–146. https://doi.org/10.1007/s40753-019-00085-6

Ministry of Education. (2025). *The New Zealand curriculum*. Learning Media for the Ministry of Education.

New Zealand Herald. (2019). NCEA maths exam mistake: National Party calls for independent review. https://www.nzherald.co.nz/nz/ncea-maths-exam-mistake-national-party-calls-for-independent-review/VUPTN5L2OHOD2U5FPCQ3AUWZ4U/?c_id=1&objectid=12287883&ref=clavis

Parker, M. (2019). *Humble Pi: A comedy of maths errors*. Penguin.

Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. *Psychological Review*, *90*(4), 293–315. https://doi.org/10.1037/0033-295X.90.4.293

Wardle, C. (2019). Misinformation has created a new world disorder. *Scientific American*, *321*(3), 88–93. https://www.nzherald.co.nz/nz/ncea-maths-exam-mistake-national-party-calls-for-independent-review/VUPTN5L2OHOD2U5FPCQ3AUWZ4U/?c_id=1&objectid=12287883&ref=clavis

World Economic Forum. (2020). *The future of jobs report 2020*. World Economic Forum. https://www.weforum.org/publications/the-future-of-jobs-report-2020/