Tertiary Student Experiences Connecting Context and Mathematics While Mathematically Modelling

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Abstract

Connecting mathematics and contexts is fundamental when mathematically modelling. However, connecting mathematics and contexts appears unfamiliar for mathematics students. To inform teaching practices, this study looked at: What are the student experiences for connecting mathematics and contexts while participating in mathematical modelling? Tertiary student mathematical modelling experiences with modelling activities from three different New Zealand university modelling courses were looked at. Data was collected through semi-structured interviews with the students. Reflective thematic analysis was used to identify themes in students' experiences. The results showed connecting mathematics and contexts was: new for students; can make mathematics more relatable; a different experience for different students depending on their backgrounds.

30.1 Introduction

Connecting mathematics and contexts is a fundamental requirement for mathematical modelling. In addition, there are claims that connecting mathematics to contexts deepens students mathematical understanding (Ainley et al., 2006; Blomhøj, 2019; Winkel, 2016), with contexts acting as an anchor for the mathematics (Blomhøj, 2019; Freudenthal, 1973). However, students do not naturally see the links between contexts and mathematics (Klymchuk et al., 2010; Soon et al., 2011). To gain insight into ways of supporting students to make connections between contexts and modelling a qualitative study was undertaken to investigate the research question, "What are the student experiences for connecting mathematics and contexts while participating in mathematical modelling?"

30.2 Literature Review

Research shows that when students are mathematically modelling, they have different experiences with connecting mathematics and contexts. These experiences can depend on whether a full or partial modelling experience is undertaken; a student is already familiar with the contexts; diagrams are used to explore the context; or the student has previous experience with applied mathematics. Taking a holistic approach, that is using the full modelling process, can help students realise the connections between the context and the mathematics (Lesh & Harel, 2003). Lesh and Harel (2003) believe that it is the iterative nature of the modelling process that helps students to realise these connections. Not surprisingly, when students are familiar with the context being modelled, they recognise important aspects of the situation (Kadijevich, 2009) and thus make connections between the important aspects of the context and appropriate mathematics. In addition, Cole et al. (2010) and Oke and Bajpai (1982) found using diagrams helps students to identify the essential aspects of the situation. Student's previous experience with physics, an applied mathematics subject, may have an influence on students' ability to make connections between mathematics and contexts. Oke and Bajpai (1982) believed that students gifted in physics were potentially able to identify key variables of a situation to be modelled more easily; however, this was not always what they observed.

Students do not generally link mathematics with contexts (Klymchuk et al., 2010; Soon et al., 2011). Nguyen and Rebello (2011) and Bollen et al. (2015) found that when students were taught calculus in a physics course, they did not use the taught calculus to solve physics problems. It is thought this happens because students are used to being taught calculus in their mathematics classes in isolation and not in conjunction with applied situations. There are several studies showing that students do not tend to link between physical processes, physics and mathematics (Chaachoua & Saglam, 2006; Domínguez et al., 2015; Nguyen & Rebello, 2011; Rowland & Jovanoski, 2004; Rowland, 2006). It is seen that most students have difficulty connecting mathematics with contexts when students are taught traditionally, that is when mathematical modelling is not one of the main teaching strategies (Boaler, 2001; Crouch & Haines, 2004; Lesh & Harel, 2003).

In the early Realistic Mathematics Education (RME) writings, Gravemeijer and Doorman (1999) stated that contexts needed to be experientially real for students. Similarly, Van den Heuvel-Panhuizen (2003), states contexts in RME are not limited to everyday life or the real world but can also include formal mathematics and fictional situations.

In order to help students realise the connections between mathematics and contexts, it is recommended to use modelling as the main teaching strategy for mathematics (Boaler, 2001; Crouch & Haines, 2004; Lesh & Harel, 2003), and include word problems and applications into general Mathematics and Calculus courses (Quimson, 2021). Both Armstrong and Bajpai (1988) and Quimson (2021) believe students need practical experience to be able to connect mathematical concepts to reality.

30.3 Background of Study

To further advance knowledge for supporting students to make connections between mathematics and real world or fictional contexts, a study involving students from three tertiary mathematical modelling courses across New Zealand was undertaken.

It was the first encounter with mathematical modelling in tertiary studies for students in all three courses, with only one student having encountered modelling briefly at secondary school. A different modelling activity that involved student group work was used for each course. The lecturer was available for guidance during course two's modelling activity, but not in course one and three. Different contexts were used for the modelling activities ranging from real contexts to fictional contexts. Table 30.1 shows the type of modelling activity, examples of context, focus on the modelling cycle, and level of student responsibility for the modelling process for each course. For further details of the modelling activities and their contexts used for each course see Spooner (2024).

Table 30.1 Characteristics of course modelling activities. (Based on Table 30.1; Spooner, 2024)

Full size table

30.4 Methodology and Method

Interpretative Description (Thorne, 2008) was used to investigate the research question, "What are the student experiences for connecting mathematics and contexts while participating in mathematical modelling?" The objective of the study was to identify ways lecturers can support students to make connections between mathematics and contexts. This aligns with an Interpretive Description methodology, which aims to use participants' experiences within their natural context to contribute to advance existing and developing knowledge to improve practice (Hunt, 2009; Thorne, 2008). By using practice-based questions, Interpretive Description generates findings with practical implications.

Three tertiary mathematical modelling courses were identified using a combination of purposive and judgement sampling (Sharma, 2017). The focus of the selected courses was on the process and techniques of mathematical modelling and provided students with their first opportunity to study modelling at the corresponding institution. By inviting students from all three courses to participate, the study aimed to capture a broad range of experiences to provide valuable insights into connecting mathematics and contexts through mathematical modelling, thus establishing a level of rigour with the findings. Students were invited to participate at the end of a course tutorial session and interviewed after the modelling activity. The indicative qualifications students were taking included Engineering, Mathematics, and Natural and Environmental Sciences. Twenty individual semi-structured interviews were conducted with students with a range of mathematical abilities.

Themes across the students' experiences for connecting mathematics and contexts were identified using reflexive thematic analysis (Braun et al., 2019; Terry et al., 2017). Reflexive thematic analysis, which uses inductive coding and acknowledges the researcher as a key component of the data analysis, is a flexible, iterative method. These attributes are considered essential and strongly advised characteristics of an Interpretive Description methodology (Hunt, 2009; Thorne, 2008). "The general phases of reflexive thematic analysis are: familiarisation with the data; systematic generation of codes for the data set; constructing themes; reviewing themes; defining themes; producing a findings report" (Spooner, 2024, p. 368).

30.5 Findings

Relating and applying mathematics to reality appeared to be a fairly new concept for all students. How students responded to this new practice differed across the courses and the participants. The need to fully engage in the modelling process appeared to influence students' experience. It was observed that students can model if the context of the problem is relatable, not necessarily real or authentic. See Table 30.1 for types of contexts used. Even though the experience was new for students, relating mathematics with contexts has the potential to make mathematics easier to understand for students.

30.5.1 New and Unfamiliar

Relating and applying mathematics to contexts was outside of students' normal range of mathematical activity. For example, Maria (course 2) stated that "when you're in school, no one understands how to apply math. You don't actually get to see how it's applied". Similarly, Lucas (course 3) discussed "at school you only do theoretical maths for calculus and algebra and do not show how math's is applied except in the odd extension questions. Do not get to experience real-life applications of the hard maths subjects" (Interview summary, Lucas). Rhys (course 2) illustrated that connecting mathematics with contexts was something that also did not regularly occur during tertiary mathematics courses saying:

Mathematical modelling is obviously ah a lot different to just studying kind of abstract algebra. It's very applied, about applying things back to the real world and applying things from the real world to mathematics. Those steps are very important and with other mathematics papers you kind of just focus on the mathematics. (Rhys, course 2).

These student comments illustrate that connecting maths and the real world was not generally a standard practice and was not experienced by these students at school or university.

30.5.2 Range of Experiences

Students across the courses had varying degrees of experience with relating mathematics to context. Students who had previous experience with physics tended to be more comfortable with applying mathematics to the real world. As discussed by Freddy (course 1) and Rhys (course 2), having a physics background made students aware that relationships exist between mathematics and the physical world "giving you a place to start". In contrast, students with no background in physics recognised that their peers drawing on prior experience applying mathematics to concrete examples, and further considered physics important for successful modelling.

Among the students that had little or no experience, there were conflicting experiences. Some students, for example Natalia (course 2) and Ngaire (course 2), struggled with relating mathematics to context. Natalia found the experience completely foreign as she had been taught mathematics separately from context and English was her second language. As she described: "This paper is quite hard for language. It is quite hard for me ... it's not just the mathematics. It's the whole things" (Natalia, course 2). Ngaire (course 2) found thinking about context with mathematics added to her confusion and she became overwhelmed with "too many things to think about", finding it hard to simplify the situation hindering her progress with the modelling. Natalia and Ngaire's experiences suggest extra support and guidance is needed to make connections between mathematics and contexts. However, Steph (course 1) had a different experience. She expressed appreciating relating mathematics to context saying it really "brought mathematics alive" for her and made it easier to understand the mathematics. Steph's experience suggests that relating context potentially facilities understanding mathematics.

30.5.3 Full or Partial Immersion in the Modelling Process?

The need to fully engage in the modelling process, as required by the level of responsibility that students held for making decisions within the process, appears to influence students' experience. Course one students did not discuss issues relating mathematics to context, suggesting they had accepted this as part of the modelling process. However, exploration and full engagement with the context was needed in order to recognise key elements that could be important for their future model. Sophie (course 1) mentioned, "I think though it didn't give us the right answer ... it gave us more details about the question", suggesting exploration of the context enabled her group, as it did others, to identify important components of the situation in proposed approaches for the model development. When referring to an approach proposed within her group Sophie (course 1) went on to say "at least it made us feel like we knew what he is talking about you know ..." showing how exploring the context enabled the group to understand, and potentially critique, what the group member was suggesting.

In contrast, course two students did discuss issues relating to context, suggesting that integrating the context involved some level of complexity for them. As mentioned, Natalia (course 2) and Ngaire (course 2) struggled with adopting this practice. Rhys (course 2) discussed how he perceived his previous experience helped in this area. Course two students were provided the context and key elements reducing the need to explore the context. However, this partial experience did not minimise their need to understand the context. It seems that the total immersion experience of course one students made it easier for them to accept the need to relate maths to context, where as providing the key elements of the model to course two students did not necessarily help students with their process. By doing this, course two students could have been denied the opportunity to engage with the context to such a degree that it would inform the development of the modification of their model. In comparison, engagement with the context by course one students meant they could appreciate contributions from their fellow group members. This created a situation for recognising proposed viable models for further development. These experiences illustrate that strategies for engaging with the context need to be provided for students or situations created where they can appreciate the need to explore the situation enough to be able to recognise key elements of the situation.

30.5.4 Relatable Contexts

Course three students were able to successfully connect maths to popular fictional contexts and produce models. Hannah (course 3) commented:

We got to apply modelling to, I wouldn't say a real situation, a situation and actually have a go at trying to model something. It made me feel smart. Ours was insanely complicated because we tried to incorporate all the variables, but we did alright. We ended up writing all the differential equations out and everything. (Hannah, course 3).

Hannah's group's experience was similar to other course 3 student groups. These experiences show, given the right conditions, the context of the situation needs to be relatable, not necessarily real or authentic, in order to have a successful modelling experience.

30.5.5 Understanding Mathematics

For some students using mathematics with contexts made the mathematics easier to comprehend. As mentioned, Steph (course 1) discussed how having a context to apply the mathematics to, brought the mathematics alive for her, made it easier to understand the mathematics and potentially enabled her to see how she might possibly use it. This may explain why she had struggled with this paper previously when it had not been so context based. Steph (course 1) stated:

If I didn't do summer school and I just did 211 then we're just doing like differentiation and like, you know they're just equations but putting it into context, like oh, so this is how you model like a car moving up and down. Its way more interesting doing it that way. (Steph, course 1).

The context of "the real-life examples" gave Steph something to relate the mathematics to, thus reducing the abstract thinking required. Steph's different experiences with both the semester and summer school version of the paper illustrates the potential of using contexts with mathematics. Steph's experiences suggest she obtained a depth and breadth of knowledge about the mathematics that may not have been accessible to her otherwise.

30.6 Discussion and Teaching Implications

Relating mathematics to fictional or real world contexts is an essential component of modelling. The findings show relating mathematics to contexts is a key new behaviour and way of thinking for students. Prior exposure to using mathematics in context was found to help students see the connections with mathematics in reality. This supports literature that suggests experience is needed to be able to connect mathematical concepts with reality (Armstrong & Bajpai 1988; Quimson, 2021) and thus help future modelling experiences (Cole et al., 2010; Soon et al., 2011). To normalise the use of mathematics with contexts, in conjunction with lecturers role modelling moving between contexts and mathematics and vice versa in lectures, lecturers need to allow students to “have a go” at moving backwards and forwards between familiar contexts and possible mathematical connections themselves. The more students are provided with these opportunities, the more likely it will be that using mathematics with contexts is seen as a normal mathematics practice. As students start to see these connections and see the applicability of mathematics, this may reduce the view that mathematics is hard and inaccessible (Spooner, 2022). This in turn would aid the development of “seeing the world through the eyes of mathematics” which then will further assist the development of mathematical modellers.

The findings showed students’ experiences with physics prepared them needed to see connections between mathematics and contexts within a mathematical modelling framework. However, some studies (Chaachoua & Saglam, 2006; Dominquez et al., 2015; Nguyen & Rebello, 2011; Rowland & Jovanoski, 2004; Rowland, 2006) have indicated that students tend not to naturally make links between physics, mathematics and physical processes. However, while these studies focused on connecting mathematics with physics and physical process, they did not examine the connections between using mathematics within a modelling context and how experience with physics can influence this. In this study, the contexts used across the courses ranged from physical processes, population growth, to spread of infectious diseases, and did not necessarily utilise physics principles. It is more likely that students’ previous experience with physics, an applied subject, had made them aware of connections between physical processes and mathematics and the thinking needed to make these connections. Further research is needed into the nuances of how previous physics experience can influence the process of making connections between mathematics and contexts within a modelling situation.

The findings also showed that students who had a total immersion experience with mathematical modelling more readily embraced relating mathematics and context. This experience provided the space to truly engage with the context and mathematics as opposed to avoiding it, with the goal of developing a model motivating the students to embrace these connections. The iterative nature of the modelling process helped students attain these connections (Lesh & Harel, 2003). In contrast some students who had a partial immersion experience struggled more with relating mathematics with contexts. This result extends on Boaler, (2001), Crouch and Haines (2004), and Lesh and Harel's (2003) recommendations suggesting it is the aspects of the total immersion experience that help students realise these connections. The findings also indicated specific activities or guidance that focused on relating mathematics to context would be beneficial for students who only experiencing parts of the modelling process. These activities would in turn be beneficial to all students, regardless of a partial or total immersion modelling experience. Further research is needed to establish the key characteristics of the activities and guidance.

Contexts further do not need to be authentic, only that the student can relate to the context (Gravemeijer & Doorman, 1999; Van den Heuvel-Panhuizen, 2003). The findings of this study showed that the use of popular contexts, as in the zombies used for course three, did make the context relatable for students and thus gave some "roots" for the modelling to occur. This result supports Gravemeijer and Doorman (1999) and Van den Heuvel-Panhuizen (2003) whose findings indicate that the contexts used need to be relatable, and not just "toy" problems.

Steph (course 1) was an example of when mathematics is applied in contexts, student's internalisation of mathematical procedures and understanding of the mathematical concepts meaning can deepen (Ainley et al., 2006; Blomhøj, 2019; Winkel, 2016)**.** While relating mathematics and context may not reduce cognitive load for students, the context can give access to productive cognitive conflict that may not have been accessible otherwise. Although Steph's experience initially suggests making mathematics relatable has the potential to reduce cognitive conflict, it may be more likely that the use of contexts gives students something to anchor the mathematics to. This enables the mathematics to become relatable, opening up the possibilities of students examining their previous understandings, challenging them, adding to them and thus allowing further understanding (Blomhøj, 2019).

Teaching implications for connecting context and mathematics include: using authentic or relatable contexts and relating these to mathematics during lectures, both in mathematical modelling lectures and in general mathematics courses; using authentic contexts and relating these to mathematics earlier in mathematics education both at secondary and primary school; allowing time for students to collaboratively explore connections between mathematics and the real world; and using simple contextual situations when first starting relating contexts and mathematics.

30.7 Conclusion

Student experiences relating mathematics and contexts suggest that lecturers and teachers need to be consciously including the application of mathematics to contexts in our mathematics and mathematical modelling courses. This inclusion ranges from using examples involving real world contexts through to translating word problems to mathematics, to start to normalise this way of using mathematics. It would be expected that a modelling course, by its nature, involves applying mathematics to contexts. What is clear from the findings is that more opportunities are needed for students to apply mathematics to contexts outside of the modelling course. This means for teaching and lecturing, if mathematical modelling is valued as a discipline within mathematics, that teaching and learning activities showing connections with mathematics and real world or fictional contexts are needed. This in turn will make mathematics more relatable to students, normalise using mathematics with context and address the common student question of "when will I need this?"

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