Received: 21 February 2023 | Accepted: 26 July 2023

DOI: 10.1002/jeab.876

RESEARCH ARTICLE

Journal of the Experimental Analysis of Behavior

SEAB SOCIETY FOR THE EXPERIMENTAL ANALYSIS OF BEHAVIOR

Divided stimulus control depends on differential and nondifferential reinforcement: Testing a quantitative model

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Editor-in-Chief: Mark Galizio
Handling Editor: Brent Alsop

Abstract

We investigated the effects of differential and nondifferential reinforcers on divided control by compound-stimulus dimensions. Six pigeons responded in a delayed matching-to-sample procedure in which a blue or yellow sample stimulus flashed on/off at a fast or slow rate, and subjects reported its color or alternation frequency. The dimension to report was unsignaled (Phase 1) or signaled (Phase 2). Correct responses were reinforced with a probability of .70, and the probability of reinforcers for errors varied across conditions. Comparison choice depended on reinforcer ratios for correct and incorrect responding; as the frequency of error reinforcers according to a dimension increased, control (measured by log *d*) by that dimension decreased and control by the other dimension increased. Davison and Nevin's (1999) model described data when the dimension to report was unsignaled, whereas model fits were poorer when it was signaled, perhaps due to carryover between conditions. We are the first to test this quantitative model of divided control with reinforcers for errors and when the dimension to report is signaled; hence, further research is needed to establish the model's generality. We question whether divided stimulus control is dimensional and suggest it may instead reflect joint control by compound stimuli and reinforcer ratios.

KEYWORDS
conditional discrimination, divided control, pigeon, reinforcement for errors, reinforcer rate

In conditional-discrimination tasks, responses are differentially reinforced depending on a sample stimulus. For example, a left-key response may be reinforced following a blue stimulus, whereas a right-key response may be reinforced following a yellow stimulus (*matching to sample*). The proportion of correct responses provides an indication of conditional control by the sample. This procedure may be used to examine control by relatively simple stimuli, such as two colors, or the division of control between the dimensions of compound stimuli. In the latter case, subjects are presented with a compound stimulus composed of two or more dimensions (e.g., a vertical or horizontal line superimposed on a green or blue background) and report the identity of one of its dimensions (e.g., color) in a subsequent choice phase. Typically, above-chance accuracy is observed on both dimensions, suggesting that stimulus control is shared between the dimensions (e.g., Brown & Morrison, 1990; Davison, 2018; Maki & Leith, 1973; Maki & Leuin, 1972; Podlesnik et al., 2012; Shahan & Podlesnik, 2006, 2007; see Zentall, 2005, for a brief review).

Shahan and Podlesnik (2006) arranged a delayed matching-to-sample (DMTS) task with compound sample stimuli composed of a color and a line orientation and varied the probability of reinforcer deliveries for correct responses according to each stimulus dimension. As the probability of reinforcers on one dimension increased, so did accuracy on that dimension, whereas accuracy on the other dimension decreased. In a similar experiment, Davison and Elliffe (2010) arranged a symbolic DMTS task in which the sample stimulus alternated between red and green every 0.1 s or 0.5 s (fast–slow dimension) for 2 s or 10 s (long–short dimension). Left-key responses were reinforced following fast or long stimuli and right-key responses after slow or short stimuli (the dimension to report in a trial was selected randomly and was not signaled to the pigeons). Like Shahan and Podlesnik, Davison and Elliffe found that as the probability of reinforcer deliveries for correct responses according to one dimension increased, so did the frequency of correct responses according to that dimension, whereas correct responses according to the other dimension decreased. Thus, these experiments demonstrate that divided control between compound-stimulus dimensions depends on relative reinforcer rates (see also Davison, 2018; Gomes-Ng et al., 2019a, 2019b; Podlesnik et al., 2012; Shahan & Podlesnik, 2007, 2008).

Davison and Elliffe (2010) likened the contingencies in their experiment to those in a "reinforcement-for-errors" procedure, in which some reinforcers are delivered after *incorrect* responses. The typical result in the reinforcement-for-errors procedure is a decrease in accuracy and to a greater extent when the probability of reinforcers for errors exceeds the probability of reinforcers for correct responses (Davison & McCarthy, 1980; Nevin et al., 1975, 1982). According to Davison and Elliffe, in the DMTS task used to study divided stimulus control, some reinforcers following correct responses according to one dimension may also be reinforcers following *incorrect* responses according to the other dimension. To illustrate, consider trials in which both elements of the sample matched the same key in Davison and Elliffe's procedure (i.e., fast and long, which both matched left, or slow and short, which both matched right). In these trials, a correct reinforced response according to one dimension was also a correct reinforced response according to the other dimension. Now consider trials in which the elements matched different keys (fast and short, or slow and long stimuli); in these trials, a correct reinforced response according to one dimension was also an *incorrect* reinforced response according to the other dimension. Thus, increasing the frequency of reinforcers for correct responses according to one dimension also increased the frequency of reinforcers for errors according to the other dimension. Davison and Elliffe suggested that these rates of differential and nondifferential reinforcers determine the extent of control by each dimension (see also Davison, 2018).

To test their suggestion, Davison and Elliffe (2010) fit Davison and Nevin's (1999) model of conditional discrimination to their data. Davison and Nevin's model can be summarized by a two-by-two signal-detection matrix, depicted in Figure 1, in which B_1 responses are correct following stimulus S_1 and B_2 responses are correct following S_2. R_{11} and R_{22} represent the reinforcers obtained for these correct responses. According to Davison and Nevin, such reinforcers may generalize to surrounding cells in the matrix. As a result, some reinforcers obtained following correct responses (B_{11} and B_{22}) may be *discriminated* as following incorrect responses (B_{21} and B_{12}). Such generalization between stimuli (that is, from S_1 to S_2 and vice versa) is quantified by the parameter d_{sb}, which ranges from 1 (no discrimination, maximum generalization) to ∞ (perfect discrimination, no generalization). Likewise, generalization between responses (from B_1 to B_2 and vice versa) is quantified by d_{br}, which also ranges from 1 to ∞. Manipulations that degrade stimulus or contingency discriminability, such as reducing the physical disparity between stimuli or inserting a delay between responses and reinforcers, decrease d_{sb} and d_{br}, respectively (Davison & Nevin, 1999; see also e.g., Davison & Jenkins, 1985; Everly, 2016; Godfrey & Davison, 1998). This model is easily extended to the reinforcement-for-errors procedure, in which B_{21} and B_{12} responses are reinforced, and such reinforcers (R_{21} and R_{12}) generalize via the same mechanisms.

	$\boldsymbol{B_1}$	$\boldsymbol{B_2}$
$\boldsymbol{S_1}$	B_{11}, R_{11}	B_{12}
$\boldsymbol{S_2}$	B_{21}	B_{22}, R_{22}

FIGURE 1 A signal-detection matrix. The contingencies for a conditional-discrimination task in which responses (B_x) are reinforced differentially after different stimuli (S_x). B_{xy} represents responses to Alternative y following Stimulus x, and R_{xy} represents reinforcers obtained for such responses.

Davison and Elliffe (2010) showed that Davison and Nevin's (1999) model described the relation between relative reinforcers and divided stimulus control well, lending support to their suggestion that divided stimulus control depends on differential and nondifferential reinforcer rates. The present study tested further Davison and Elliffe's suggestion, by filling two important gaps in the literature. First, no studies have explicitly examined the effects of reinforcers for errors on divided stimulus control; in Davison and Elliffe, reinforcers for errors were a by-product of the arranged procedure. Thus, in the present experiment, we arranged reinforcers following incorrect responses according to each dimension and varied the probability of reinforcers for errors across conditions. This allowed us to directly test the effects of reinforcers for errors on divided stimulus control, and the ability of Davison and Nevin's model to account for any such effects.

Second, a key difference between Davison and Elliffe's (2010) procedure and other studies' procedures (Davison, 2018; Podlesnik et al., 2012; Shahan & Podlesnik, 2006, 2007) is that the dimension to report was unsignaled in the former but signaled in the latter. In other words, Davison and Elliffe's pigeons reported whether the sample was fast or slow, or long or short, but which question was being asked was not signaled; this is why reinforcers for correct responses according to one dimension were also error reinforcers according to the other dimension. In contrast, in other studies, the dimension to report was signaled with different stimuli.

Davison (2018) described application of the model to cases where the dimension to report is signaled, but this has not yet been tested empirically. Therefore, we fit Davison and Nevin's model to data from conditions in which the dimension to report was unsignaled (Phase 1) or signaled (Phase 2).

The present experiment was a systematic replication of Davison and Elliffe (2010), with reinforcer contingencies analogous to Davison and McCarthy's (1980) reinforcement-for-errors procedure. Pigeons responded in a symbolic DMTS task in which the sample stimulus was a compound composed of a color (blue or yellow) and an alternation frequency (fast or slow). Although these dimensions may differ in their discriminability—color is discriminable immediately, whereas alternation frequency is only discriminable across time—we presented the sample stimulus for at least 5 s and arranged maximally different alternation frequencies with the goal to enhance the discriminability of the alternation-frequency dimension. We have previously used these two stimulus dimensions in other studies of divided stimulus control (Cowie et al., 2020; Gomes-Ng et al., 2019a, 2019b), and previous research—including Davison and Elliffe (2010)—shows that pigeons can discriminate well between frequencies of keylight-color alternation (e.g., Cowie et al., 2020; Gomes-Ng, Elliffe, et al., 2018; Gomes-Ng et al., 2019a, 2019b; Krägeloh & Davison, 2003; Roberts, 1997; Roberts et al., 2000; Roberts & Mitchell, 1994).

After presentation of the sample stimulus, the pigeons chose between left and right comparison keys. As in Davison and Elliffe (2010), left responses were defined as correct following one color or alternation frequency and right responses were defined as correct following the other color or alternation frequency. As in Davison and McCarthy (1980), correct responses were reinforced with a probability of .70 and error responses were reinforced with different probabilities across conditions. The probabilities of reinforcers for errors according to the color and alternation-frequency dimensions were complementary and varied from 0 to 1. In Phase 1, the dimension to report (color or alternation frequency) was unsignaled; in Phase 2, it was signaled by different colored stimuli.

METHOD

Subjects

Six pigeons, numbered 71 to 76, served as subjects. The pigeons were maintained at 85% ± 15 g of free-feeding body weight by supplementary feeding of mixed grain, when necessary, at around 10 a.m. each morning. Water and grit were freely available. Pigeons were housed in a colony room with a shifted light–dark cycle (lights on at 12 a.m., off at 4 p.m.). All pigeons had previous experimental experience, most recently with the peak procedure (Gomes-Ng et al., 2019b).

Apparatus

Pigeons were housed individually in home cages (375 mm high × 375 mm deep × 370 mm wide), which also served as experimental chambers. Each cage contained two wooden perches, spaced 135 mm apart and parallel to the back wall. The back perch was 110 mm away from the back wall. An operant panel was mounted on the back wall 115 mm above the cage floor. The operant panel contained five circular 20-mm diameter response keys arranged in three rows. The top and bottom rows each contained two keys spaced 113 mm apart from center to center, and the middle row contained one key centered horizontally between the keys in the top and bottom rows. The vertical distance between the bottom of the keys in one row and the top of the keys in the row below was 25 mm. Only the keys in the middle and bottom rows were used, excluding Pigeon 73, for whom the top-right key was used instead of the center (middle) key due to an equipment issue. The keys could be illuminated blue, white, or yellow, and responses exceeding 0.1 N to illuminated keys were recorded. Centered on the back wall and 53 mm below the operant panel was a magazine aperture measuring 40 mm by 40 mm. A hopper, filled with wheat, was located behind the magazine aperture. During a reinforcer delivery, the hopper was raised for 2 s, the magazine aperture was lit, and the keylights were darkened. In an adjacent room, a computer running MED-PC IV software ran the experiment and recorded all experimental events.

Procedure

Experimental sessions began at 1 a.m. daily, and the pigeons were run successively and in numerical order. Each session lasted for 65 min or 160 trials, whichever occurred first.

Phase 1: Unsignaled conditions

A symbolic 0-s delayed matching-to-sample (DMTS) procedure was arranged in which the sample stimulus was a flashing blue or yellow keylight that alternated with a white keylight every 0.1 s or 0.9 s. Thus, the sample stimulus was a compound comprising two stimulus dimensions, a color (blue or yellow) and an alternation frequency (fast or slow). There were four sample stimuli in total: blue-fast, blue-slow, yellow-fast, and yellow-slow. In each trial, the pigeons reported the identity of either the color or alternation frequency of the sample stimulus by choosing between left and right comparison keys. In Phase 1, the dimension to report was unsignaled. Thus, the comparison keys were both lit white in all trials.

At the beginning of each trial, the experiment program randomly selected a sample stimulus and a dimension to report, with the following constraints: In each session, there was a maximum of 40 trials per sample stimulus, and of those 40 trials, 20 were trials in which the dimension to report was color and the other 20 were trials in which the dimension to report was alternation frequency. These constraints ensured that the pigeons had equal experience with all sample stimuli and dimensions to report.

After selecting a sample stimulus and dimension to report, the trial began with the presentation of the sample on the center key (or top-right key, for Pigeon 73) for 5 s, after which a single peck to the sample resulted in its offset and the onset of the comparison stimuli. The comparison stimuli were the bottom-left and bottom-right keys, both lit white. The correct comparison depended on the dimension to report in the current trial. If color was selected as the dimension to report, then the comparison assigned to match symbolically the color of the sample stimulus was the correct response. Likewise, if alternation frequency was selected as the dimension to report, then the comparison assigned to match the alternation frequency of the sample was correct. The stimuli assigned to match each comparison key were counterbalanced across pigeons (Table 1). For example, for Pigeon 71, choice of the left key was correct after a blue or fast sample stimulus and choice of the right key was correct after a yellow or slow sample stimulus. A peck to either comparison key resulted in the offset of the comparison stimuli and either a 2-s reinforcer delivery or equivalent-length blackout depending on whether a reinforcer was arranged. Thereafter, there was a 3-s intertrial interval during which the keys remained darkened.

Correct and incorrect responses were reinforced probabilistically, depending on the dimension to report and the condition. In some conditions, the probability of reinforcers following incorrect responses was higher than the probability of reinforcers following correct responses. Although this effectively changes the contingencies (in the sense that choosing the incorrect response maximizes obtained reinforcement), for brevity, we will continue to refer to "correct" responses as those that were assigned to match the color or alternation frequency of the sample (as in Table 1), and "incorrect" responses as the other response.

Table 2 shows the sequence of conditions and the arranged and mean obtained probabilities of reinforcer deliveries in each condition. In all conditions and for both stimulus dimensions, correct responses were reinforced with a probability of .70. In Conditions 1 and 10, no reinforcers were delivered following incorrect responses. In Conditions 2 to 9, incorrect responses according to the color and alternation-frequency dimensions were reinforced with complementary probabilities ranging from 0 to 1 (Table 2). Obtained reinforcers closely approximated arranged probabilities. Each condition lasted for 50 sessions.

Phase 2: Signaled conditions

In Phase 1 conditions, the comparison keys were always lit white regardless of the dimension to report. In Phase 2 conditions (Conditions 11 to 19; Table 2), we differentially signaled the dimension to report using different colored comparison keys. Thus, the procedure used in Phase 2 was the same in Phase 1, except that the bottom-left and bottom-right keys were both lit red for one dimension and were lit green for the other dimension. The colors corresponding to each dimension were counterbalanced across pigeons (Table 1). Conditions lasted for 30 sessions in Phase 2.

Data analysis

Visual inspection across sessions indicated that accuracy was generally stable for most pigeons by the last 15 sessions of each condition. The only exception was Pigeon 71 in Condition 16, for whom an equipment issue disrupted responding in the last few sessions (see Appendix Figure A4). Nevertheless, as this was the only exception, we analyzed data from the last 15 sessions of each condition.

TABLE 1 Counterbalancing information

	Stimuli assigned to match comparison keys in Phases 1 and 2		Color of comparison keys for each stimulus dimension in Phase 2	
Pigeon	**Matching left**	**Matching right**	**Color**	**Alternation freq.**
71	Blue, Fast	Yellow, Slow	Red	Green
72	Blue, Slow	Yellow, Fast	Green	Red
73	Blue, Fast	Yellow, Slow	Green	Red
74	Yellow, Slow	Blue, Fast	Red	Green
75	Yellow, Fast	Blue, Slow	Red	Green
76	Yellow, Slow	Blue, Fast	Green	Red

Note. Left columns show the color and alternation-frequency elements assigned to match the left and right keys for individual pigeons in Phases 1 and 2. In Phase 1, the dimension to report was unsignaled, whereas it was signaled by the color of the comparison keys in Phase 2. The right columns show the color of the comparison keys corresponding to each stimulus dimension in Phase 2.

TABLE 2 Arranged and obtained probability of reinforcer deliveries following error responses according to the color and alternation-frequency (freq) dimensions in each condition

		Arranged *p*(food)		Mean (*SD*) obtained *p*(food)			
Phase	Condition	Error$_{color}$	Error$_{freq}$	Correct$_{color}$	Correct$_{freq}$	Error$_{color}$	Error$_{freq}$
1	1*	0	0	.70 (.01)	.70 (.02)	0	0
1	2	.50	.50	.70 (.02)	.69 (.01)	.51 (.01)	.50 (.02)
1	3	.90	.10	.70 (.03)	.71 (.01)	.90 (.01)	.10 (.02)
1	4	.25	.75	.70 (.02)	.71 (.01)	.24 (.01)	.75 (.02)
1	5	.10	.90	.69 (.02)	.69 (.01)	.10 (.01)	.91 (.02)
1	6	.75	.25	.70 (.01)	.69 (.01)	.75 (.03)	.26 (.03)
1	7	0	1	.70 (.01)	.71 (.01)	0	1
1	8	.50	.50	.69 (.02)	.70 (.02)	.49 (.03)	.51 (.03)
1	9	1	0	.69 (.03)	.69 (.02)	1	0
1	10*	0	0	.70 (.01)	.71 (.01)	0	0
2	11*	0	0	.71 (.02)	.71 (.01)	0	0
2	12	.50	.50	.69 (.02)	.70 (.01)	.49 (.05)	.51 (.02)
2	13	.10	.90	.70 (.02)	.70 (.02)	.09 (.02)	.91 (.01)
2	14	.75	.25	.70 (.02)	.69 (.02)	.76 (.02)	.26 (.03)
2	15	.90	.10	.70 (.01)	.70 (.02)	.90 (.02)	.10 (.04)
2	16	.25	.75	.70 (.01)	.71 (.03)	.25 (.02)	.76 (.02)
2	17	0	1	.70 (.02)	.70 (.01)	0	1
2	18	.50	.50	.70 (.01)	.68 (.03)	.50 (.04)	.49 (.01)
2	19	1	0	.71 (.01)	.70 (.01)	1	0

Note. In all conditions, correct responses were reinforced with a probability of .70. Asterisks indicate conditions without reinforcers for errors. In Phase 1, the dimension to report was unsignaled, whereas in Phase 2, the dimension to report was signaled.

Appendix A shows acquisition data for each condition (Figures A1, A2, and A3), and data (numbers of correct and incorrect responses and obtained reinforcers) used for steady-state analyses are provided in the supplementary material.

To assess control by the color and alternation-frequency dimensions, we calculated a bias-free measure of discrimination, log d_x, for each stimulus dimension x (Davison & Elliffe, 2010; see also Davison & Nevin, 1999):

$$\log d_x = 0.5\log\left(\frac{B_{corr\,|\,S1}}{B_{err\,|\,S1}}\cdot\frac{B_{corr\,|\,S2}}{B_{err\,|\,S2}}\right), \qquad (1)$$

where x is a placeholder representing the stimulus dimension, and $S1$ and $S2$ represent the two color elements (blue and yellow, respectively) or two alternation-frequency elements (fast and slow). Thus, $B_{corr\,|\,Sy}$ and $B_{err\,|\,Sy}$ represent the number of correct and incorrect responses, respectively, according to element Sy. For example, for log d_{color}, $B_{corr\,|\,S1}$ represents the number of responses to the comparison that matched the blue element in blue-fast and blue-slow trials, and $B_{err\,|\,S1}$ represents the number of comparison responses that did not match the blue element in those same trials (see Table 1). Because some response counts were zero, we added 0.5 to all counts (Hautus, 1995). A log d_x value of zero indicates no discriminative control by that stimulus dimension, and more positive values indicate stronger control. We also calculated a related measure, log b_x, which quantifies bias:

$$\log b_x = 0.5\log\left(\frac{B_{left\,|\,S1}}{B_{right\,|\,S1}}\cdot\frac{B_{left\,|\,S2}}{B_{right\,|\,S2}}\right), \qquad (2)$$

where the parameters are as in Equation 1, except that left and right response counts (rather than correct and incorrect responses) are used. A log b_x value of zero indicates no bias, positive values indicate bias toward the left key, and negative values indicate bias toward the right key.

In Phase 1 conditions, the dimension to report was unsignaled because the comparison keys were always lit white (hereafter, *unsignaled* conditions). Thus, for calculations of log d_x and log b_x (Equations 1 and 2), data from trials in which the color (for log d_{color}) or alternation frequency (for log d_{freq}) was the same were aggregated regardless of the programmed dimension to report. For example, for log d_{color}, data from all trials in which the color element was blue (i.e., *blue*-fast and *blue*-slow) were aggregated. In Phase 2 conditions, the dimension to report was signaled with different colored comparison keys (hereafter, *signaled* conditions); thus, data were separated according to the dimension to report, and log d_x values were calculated using only data from trials in which the dimension to report was x.

RESULTS

Replicability and carryover effects

Figures 2 and 3 show log d_x values (Equation 1), plotted as a function of the probability of reinforcer deliveries for incorrect responses according to the color dimension, in Phases 1 (unsignaled) and 2 (signaled), respectively. Although there were individual-pigeon differences in the extent of change in log d_x values between conditions (e.g., changes in log d_x were smaller for Pigeons 73 and 74 than for Pigeons 71, 72, 75, and 76), log d_x followed a similar general pattern across conditions for all pigeons—log d_{color} increased and log d_{freq} decreased as the probability of reinforcers for incorrect responses according to the color dimension increased. Thus, the effects of reinforcers for errors were replicable across subjects.

Even though responding was stable in the last 15 sessions of each condition, some carryover effects between conditions were also apparent. This is particularly evident in the replication conditions (Conditions 8, 10, and 18, which replicated Conditions 2, 1, and 12, respectively; see Table 2); although the same reinforcer contingencies operated in each pair of replication conditions, log d_x values differed. Figure 4, which shows mean log d_x values in condition order, shows the carryover between conditions more clearly: log d_x in one condition changed relative to the preceding condition, and particularly when the preceding condition arranged a more extreme probability

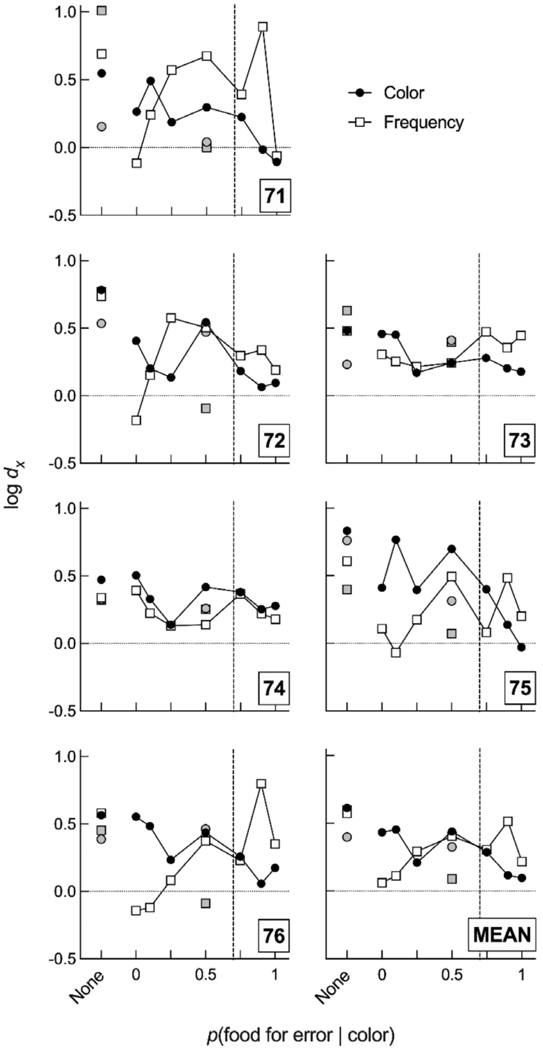

FIGURE 2 Discriminative control (log d_x) in Phase 1 (unsignaled) conditions. Shaded gray symbols indicate replication conditions. "None" indicates conditions in which no reinforcers for errors were arranged.

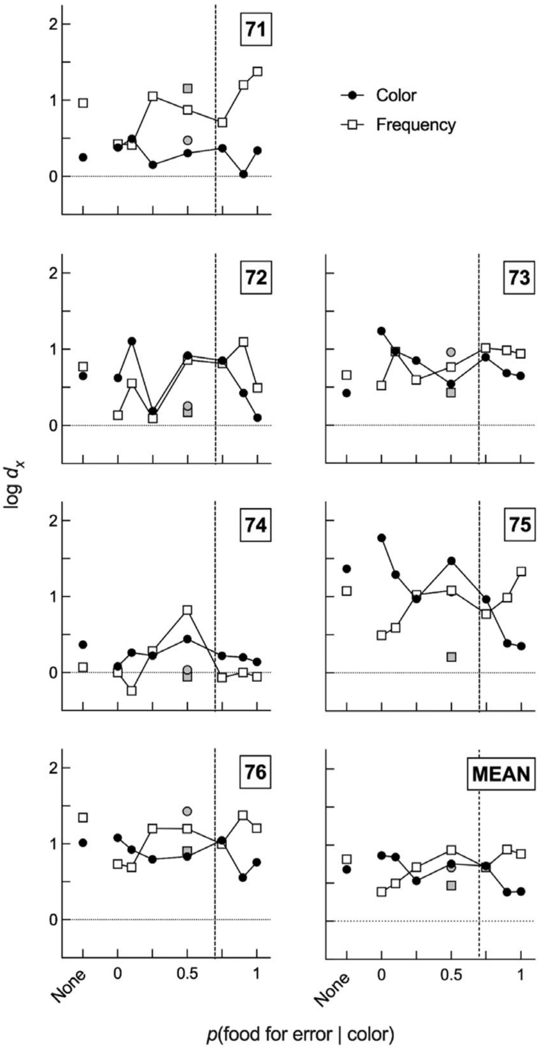

FIGURE 3 Discriminative control (log d_x) in Phase 2 (signaled) conditions. Shaded gray symbols indicate replication conditions. "None" indicates conditions in which no reinforcers for errors were arranged.

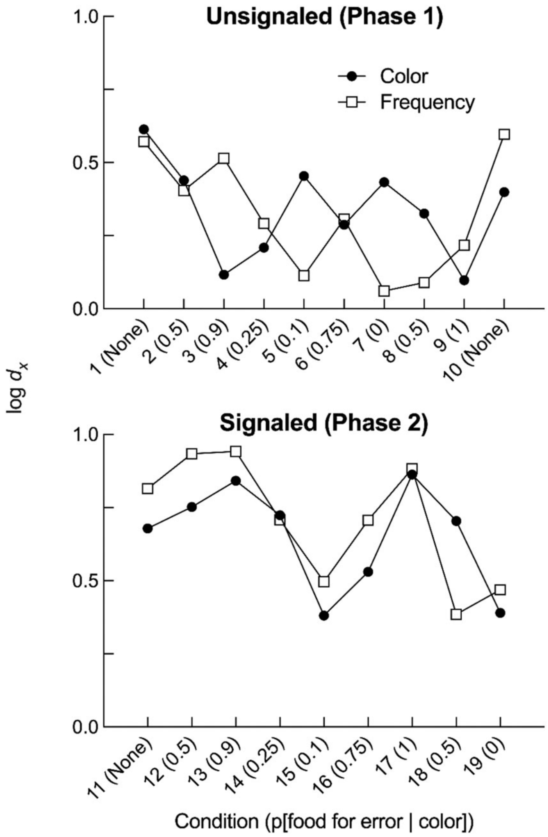

FIGURE 4 Mean log d_x in order of conditions. “None” indicates conditions in which no reinforcers for errors were arranged.

of reinforcers for errors (i.e., closer to 0 or 1; see Conditions 8 and 18, which occurred after the probability of reinforcers for errors according to the color dimension was 0 or 1, respectively).

Such carryover (Figure 4) is probably related to the dynamic relation between responses and reinforcers. In the present procedure, subjects must obtain reinforcers for correct and incorrect responses, and this must happen often enough to allow for accurate discrimination of reinforcer probabilities. In other words, discrimination of the probability of reinforcers for errors required subjects to emit incorrect responses. To the extent that behavior from one condition carried over to the next condition, such discrimination may have been less accurate, especially if subjects made few incorrect responses in the previous condition. Additionally, obtained reinforcers may have served to maintain behavior from the previous condition.

Effects of error reinforcers on discriminative control

Despite carryover between conditions, log d_x was still controlled by the arranged contingencies; in both unsignaled and signaled conditions, shifts in log d_x were consistent with the probabilities of reinforcers for errors in each condition (Figures 2 to 4). Log d_x was overall higher in signaled conditions than in unsignaled conditions, suggesting stronger control by the stimulus dimensions when the dimension to report was signaled than when it was unsignaled. Additionally, for individual pigeons, one stimulus dimension tended to exert stronger control than the other dimension, as evidenced by overall higher log d_x values for that dimension. For example, the alternation-frequency dimension exerted stronger control for Pigeon 71, whereas the color dimension exerted stronger control for Pigeon 75. This is unsurprising, as previous studies have shown similarly asymmetric control by different dimensions (e.g., Davison, 2018; Davison & Elliffe, 2010; Shahan & Podlesnik, 2006).

In general, when no error reinforcers were delivered, log d_x values were positive, indicating some degree of control by each dimension (Figures 2 to 4). Introducing reinforcers for errors generally decreased log d_x values, and as the probability of reinforcers for incorrect responses according to the color dimension increased, log d_{color} decreased and log d_{freq} increased, although not always monotonically. One-tailed nonparametric trend tests (Elliffe & Elliffe, 2019; Kendall, 1955) confirmed these patterns in log d_x; trend tests were significant for unsignaled conditions ($\Sigma S = -60$ and 54 for log d_{color} and log d_{freq}, respectively, both $p < .001$) and signaled conditions ($\Sigma S = -53$ and 52 for log d_{color} and log d_{freq}, respectively, both $p < .001$). Thus, discriminative control by a dimension depended on the relative probability of reinforcers for errors according to that dimension. Interestingly, log d_x values rarely fell below zero, even in conditions with higher probabilities of reinforcers for errors, indicating that correct responding according to a dimension was maintained despite reinforcers following incorrect responses.

In contrast to discriminative control (log d_x; Figures 2 to 4), the probability of reinforcers for errors should have had little effect on bias (log b_x) because we did not manipulate the relative probabilities of left and right reinforcers (and thus expected no systematic change in bias for the left or right comparison key). Indeed, log b_x values changed unsystematically in unsignaled conditions, and nonparametric trend tests on log b_x were not significant (see Appendix Figure B1; $\Sigma S = 12$, $p = .251$ for log b_{color}; $\Sigma S = 10$, $p = .292$ for log b_{freq}). In signaled conditions, there was no consistent trend across pigeons in log b_{color} ($\Sigma S = 12$, $p = .351$), whereas log b_{freq} tended to decrease as the probability of reinforcers for errors increased ($\Sigma S = -48$, $p = .002$; see Appendix Figure B2). Nevertheless, as this was the only significant trend, there appeared to be no overall consistent effect of reinforcer probabilities on log b_x. This is unsurprising, as we did not manipulate the relative probabilities of left and right reinforcers.

Thus, the main effect of varying the probability of reinforcers for errors was to change discriminative control by the color and alternation-frequency dimensions (log d_x; Figures 2 to 4): As the probability of reinforcer deliveries for errors according to a stimulus dimension increased (and, complementarily, the probability of reinforcer deliveries for errors according to the other dimension decreased), control by that dimension decreased and control by the other dimension increased. This was the case both when the dimension to report was unsignaled (Phase 1) and when it was signaled by different colored comparison keys (Phase 2).

Obtained frequencies of reinforcers

In the present procedure, reinforcers for correct and incorrect responses were arranged independently; this meant that such reinforcers were only obtained if the pigeons emitted correct or incorrect responses, respectively. As a result of this dynamic relation between response and reinforcers, a higher arranged or obtained *probability* of reinforcers for errors did not necessarily mean a higher obtained *frequency* of reinforcers for errors. That is, if the pigeons emitted few incorrect (or correct) responses in a condition, then the frequency of obtained reinforcers for errors (or for correct responses) would have been low. Furthermore, this means that the frequency of obtained reinforcers following correct responses may have been higher than the frequency of obtained reinforcers following incorrect responses, even in conditions in which the probability of reinforcers for errors was high. This may explain why log d_x values rarely fell below zero (Figures 2 to 4), even when the probability of reinforcers for errors according to one dimension exceeded the probability of reinforcers for correct responses.

Figure 5 shows the mean number of obtained reinforcers following correct and incorrect responses in unsignaled and signaled conditions. Comparing unsignaled and signaled conditions in which the probability of reinforcers for errors was equal (see Table 2), the frequency of reinforcers for correct responses was higher and the frequency of reinforcers for incorrect responses was lower in signaled conditions. Although these differences did not reach statistical significance in most conditions (binomial sign tests: $p > .05$), they were evident for most (four or more) pigeons in most conditions. Thus, in general, the pigeons had greater experience with the reinforcement-for-errors contingencies (due to emitting a larger number of incorrect responses and thus obtaining more reinforcers following incorrect responses) in unsignaled conditions.

Additionally, the frequencies of reinforcers following correct responses were overall larger than the frequencies of reinforcers following incorrect responses. Figures 6 and 7 show this more clearly, by plotting log correct/error reinforcer ratios for each dimension in unsignaled and signaled conditions. Because no reinforcers for errors were delivered in some conditions, we added 0.5 to all reinforcer counts. Log reinforcer ratios were almost always positive, indicating that the pigeons indeed obtained more reinforcers for correct than incorrect responses in most conditions. Overall, discriminative control by a dimension clearly depended on the ratio of reinforcers for correct and incorrect responses; log d_x values were higher when more reinforcers were obtained for correct than incorrect responses. Sensitivity to log correct/error reinforcer ratios (measured by the slope of the best-fitting line through each set of data) varied between pigeons and, in some cases, between dimensions as well (e.g., Pigeon 71, Phase 2; Figure 7).

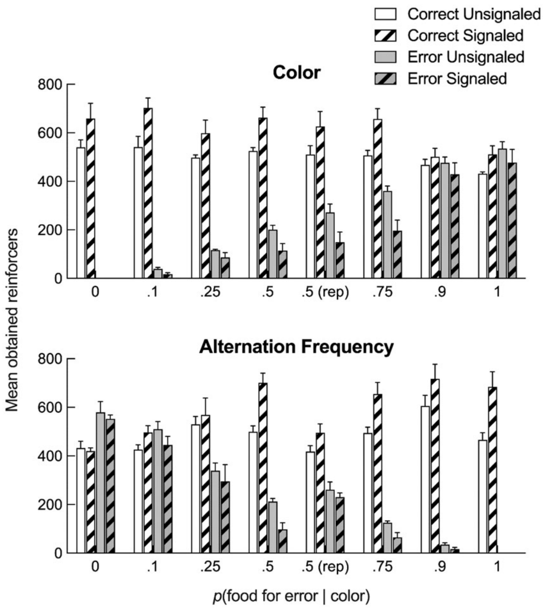

FIGURE 5 Mean obtained reinforcer frequencies after correct and incorrect responses in unsignaled and signaled conditions. White bars show mean obtained reinforcers after correct responses, and gray bars show mean obtained reinforcers after incorrect responses. Solid bars show data from unsignaled conditions, and patterned bars show data from signaled conditions.

Modeling the effects of differential and nondifferential reinforcers

To model the effects of reinforcers for correct and incorrect responses on divided stimulus control, we fit Davison and Nevin's (1999) model of conditional-discrimination performance to log d_x values. The basic logic of the model is that reinforcers obtained for response B_{xy} after stimulus S_y (see Figure 1) may generalize to other stimuli and to other responses. These two modes of generalization are described by the parameters d_{sb} and d_{br}, respectively. In the present procedure, responses to the left and right comparison keys were occasionally reinforced depending on either the color or alternation frequency of the sample stimulus. Thus, there were two discriminations, one for each stimulus dimension. Because discriminability

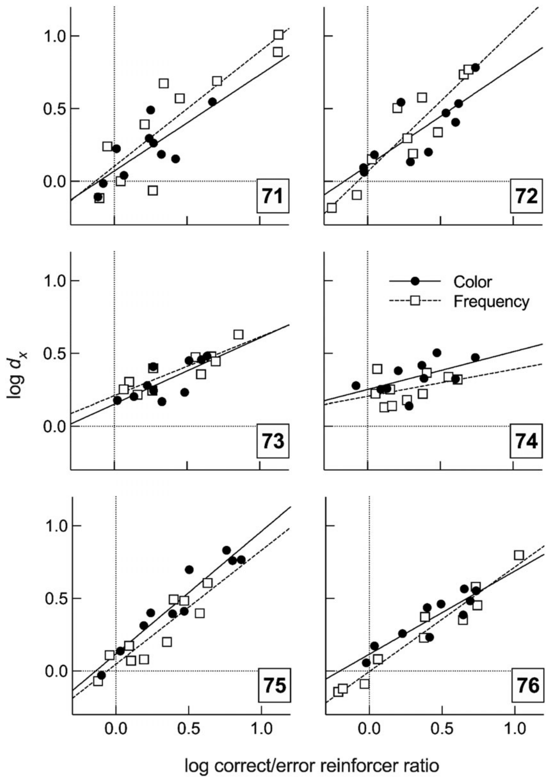

FIGURE 6 Log d_x as a function of log correct/error reinforcer ratios in Phase 1 (unsignaled) conditions

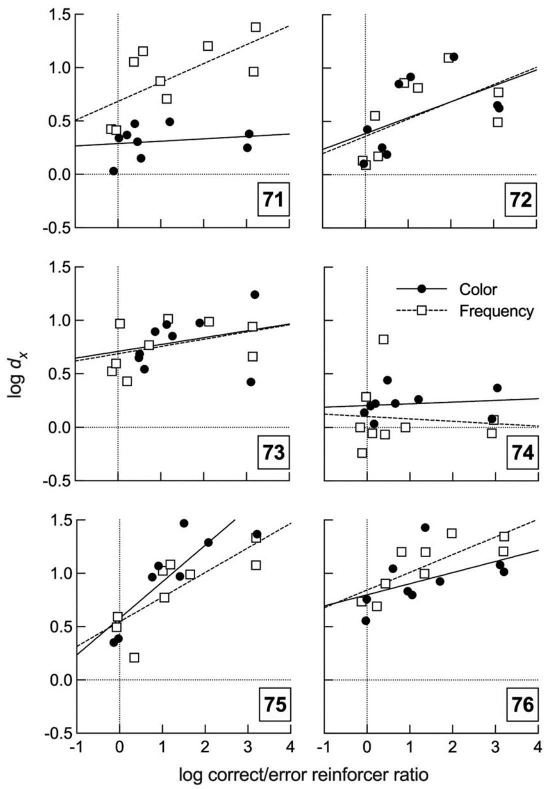

FIGURE 7 Log d_x as a function of log correct/error reinforcer ratios in Phase 2 (signaled) conditions

may differ for different stimulus dimensions, our model included two d_{sb} parameters, one for the color discrimination ($d_{sb\text{-}color}$) and one for the alternation-frequency discrimination ($d_{sb\text{-}freq}$; see Davison & Elliffe, 2010). In contrast, we included only one d_{br} parameter, because the comparison stimuli were always the left and right keys (and there was no a priori reason to assume differences in left versus right discrimination; see Davison & Elliffe, 2010).

As a result of reinforcer generalization, some reinforcers obtained for response B_{xy} after stimulus S_y will be effectively “lost” to other stimuli and responses. Additionally, some reinforcers will be effectively “gained” from other stimuli and responses (Davison, 2018). Thus, the equation for the model is as follows:

$$\log d_x = 0.5\log\left(\frac{R_{11}-\frac{R_{11}}{d_{sbx}}-\frac{R_{11}}{d_{br}}-\frac{R_{11}}{d_{sbx}d_{br}}+\frac{R_{12}}{d_{br}}+\frac{R_{21}}{d_{sbx}}+\frac{R_{22}}{d_{sbx}d_{br}}}{R_{12}-\frac{R_{12}}{d_{sbx}}-\frac{R_{12}}{d_{br}}-\frac{R_{12}}{d_{sbx}d_{br}}+\frac{R_{11}}{d_{br}}+\frac{R_{22}}{d_{sbx}}+\frac{R_{21}}{d_{sbx}d_{br}}}\cdot\frac{R_{22}-\frac{R_{22}}{d_{sbx}}-\frac{R_{22}}{d_{br}}-\frac{R_{22}}{d_{sbx}d_{br}}+\frac{R_{21}}{d_{br}}+\frac{R_{12}}{d_{sbx}}+\frac{R_{11}}{d_{sbx}d_{br}}}{R_{21}-\frac{R_{21}}{d_{sbx}}-\frac{R_{21}}{d_{br}}-\frac{R_{21}}{d_{sbx}d_{br}}+\frac{R_{22}}{d_{br}}+\frac{R_{11}}{d_{sbx}}+\frac{R_{12}}{d_{sbx}d_{br}}}\right), \tag{3}$$

where x is a placeholder for the stimulus dimension and R_{xy} corresponds to reinforcers obtained following B_{xy} responses in the matrix in Figure 1. Equation 3 is the model for a situation in which the dimension to report is unsignaled (as in Phase 1 conditions and in Davison & Elliffe, 2010).

To extend the model (Equation 3) to situations in which the dimension to report is signaled (as in Phase 2 conditions), Davison (2018) suggested two additional parameters that quantify generalization between the stimulus dimensions ($d_{COLOR\text{-}FREQ}$) and between the comparison stimuli associated with those dimensions (red or green keys in this experiment; $d_{RED\text{-}GREEN}$). The model and associated equations for Phase 2 (signaled) conditions are not presented here due to their complexity; a full description is provided in Appendix C. We also fit a model without these additional parameters to Phase 2 data (hereafter, "standard" model). The model fits were done simultaneously for the color and alternation-frequency dimensions using Microsoft Excel Solver.

Table 3 shows log parameter estimates from fits of the Davison–Nevin model to individual-pigeon data from unsignaled and signaled conditions. In unsignaled conditions, the model fits were generally good, with values for variance accounted for (VACs) above .85 for four of the six pigeons. Estimates of log d_{sb} and log d_{br} varied between pigeons. For individual pigeons, stimulus discriminability was generally higher for one dimension (alternation frequency for Pigeons 71 to 73, color for Pigeons 74 to 76), as evidenced by larger log d_{sb} values. Additionally, estimates of log d_{br} tended to be larger than estimates of log d_{sb}, suggesting that the extent of reinforcer generalization between left and right comparison responses was smaller than generalization between the elements within a dimension.

Similar patterns were evident in signaled conditions, with higher stimulus discriminability for one dimension and larger estimates of log d_{br} (Table 3). However, compared with unsignaled conditions, fits of the model were much poorer in signaled conditions, with VACs ranging from 0 to .83 (Table 3). Generally, the pigeons with higher VACs were those with larger changes in log d_x between conditions (see Figure 3), suggesting that the model provided a better fit for the pigeons that better discriminated (as evidenced by changes in log d_x) changes in the probability of reinforcers for errors.

TABLE 3 Log parameter estimates and the proportion of data variance accounted for (VAC) from fits of the Davison–Nevin model

	Pigeon					
	71	72	73	74	75	76
Phase 1 (Unsignaled conditions)						
log $d_{sb\text{-}color}$	1.02	0.97	0.85	0.86	2.14	0.96
log $d_{sb\text{-}freq}$	1.46	1.29	0.95	0.77	1.26	0.91
log d_{br}	2.20	8.81	8.83	9.13	1.39	9.20
VAC	.91	.87	.58	.14	.93	.91
Phase 2 (Signaled conditions; standard model, no extra parameters)						
log $d_{sb\text{-}color}$	0.59	0.92	1.05	0.52	1.55	1.25
log $d_{sb\text{-}freq}$	1.32	0.93	1.06	0.60	1.27	1.43
log d_{br}	3.91	6.24	10.97	1.60	3.53	5.98
VAC	.76	.59	0	.18	.83	.34
Phase 2 (Signaled conditions; full model)						
log $d_{sb\text{-}color}$	0.59	6.88	1.05	0.52	1.55	1.25
log $d_{sb\text{-}freq}$	1.32	2.79	1.06	0.60	1.27	1.43
log d_{br}	3.91	7.00	12.52	1.60	6.98	11.79
log $d_{COLOR\text{-}FREQ}$	4.55	0.35	7.66	2.60	1.55	7.80
log $d_{RED\text{-}GREEN}$	4.48	0.35	7.60	2.65	1.63	7.88
VAC	.76	.64	0	.18	.83	.34

There was no change in log d_{sb} or log d_{br} values for five pigeons and no increase in VACs for any pigeons[1] when two additional parameters quantifying generalization between the stimulus dimensions (log $d_{COLOR\text{-}FREQ}$) and between the comparison stimuli associated with those dimensions (log $d_{RED\text{-}GREEN}$) were included in the full model (Table 3; see Equation 2 in Appendix C for the full model). This was because estimates of these additional parameters were large. Larger values reflect less generalization and thus little to no contribution of reinforcers generalized from one stimulus dimension (or comparison stimulus) to another. As a result, the effects of these parameters are effectively null (i.e., no reinforcers generalized). Thus, overall, our findings provide no support for Davison's (2018) extended version of the Davison–Nevin model and only limited support for the standard model.

DISCUSSION

The present experiment tested Davison and Elliffe's (2010) suggestion that divided stimulus control is governed by relative rates of differential and nondifferential reinforcers. To that end, pigeons reported the color or alternation-frequency element of a compound sample stimulus, and the probability of reinforcers for errors according to each dimension varied across conditions (Table 2). Overall, divided stimulus control depended on relative frequencies of reinforcers for correct and incorrect responses; as the frequency of reinforcers for errors according to one dimension increased, control by that dimension tended to decrease while control by the other dimension tended to increase (Figures 2 to 7). These effects of differential and nondifferential reinforcers were generally well described by Davison and Nevin's (1999) model when the dimension to report was unsignaled (Phase 1, unsignaled conditions), whereas model fits were poorer when the dimension to report was signaled (Phase 2, signaled conditions; Table 3).

The present findings replicate and extend previous research demonstrating that relative reinforcer rates determine divided stimulus control (Davison, 2018; Davison & Elliffe, 2010; Podlesnik et al., 2012; Shahan & Podlesnik, 2006, 2007). However, a notable difference between our results and those of past research is that changes in log d_x across conditions were less systematic in the present experiment. Here, changes in the probability of error reinforcers were not always accompanied by proportional or monotonic changes in log d_x

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[1] Thus, using the Akaike (1973) information criterion to compare the full and standard models would be redundant.

(Figures 2 and 3). In contrast, in previous studies—in which only reinforcers for correct responses were delivered—increasing the rate of differential reinforcers according to a stimulus dimension produced increases in log d_x for that dimension and concomitant decreases in log d_x for the other dimension (see e.g., Figure 2 in Davison & Elliffe, 2010; see also Davison, 2018; Shahan & Podlesnik, 2006, 2007).

Why might explicitly arranged reinforcers for errors have less systematic effects on divided stimulus control than differential reinforcers? One possibility is that without reinforcers for errors, rates of differential and nondifferential reinforcers are *inversely* related, whereas this is not the case when reinforcers for errors are arranged. To elaborate, in the divided-stimulus-control procedure, the compound-stimulus dimensions match the *same* comparison in half of trials and *different* comparisons in the other half (hereafter "compatible" and "incompatible" trials, respectively). When reinforcers for errors are arranged, as in the present experiment, they occur in both compatible and incompatible trials, and thus error reinforcers in compatible trials follow responses that are incorrect according to *both* stimulus dimensions. In contrast, when only correct responses are reinforced, as in previous studies, reinforcers following incorrect responses only occur in incompatible trials and thus are incorrect according to only one dimension.

Put differently, in a procedure with reinforcers for errors, some reinforcers are nondifferential with respect to both dimensions, whereas all reinforcers are differential according to at least one dimension in procedures without reinforcers for errors. As a result, changing the probability of reinforcers for errors has different effects than only changing the probability of reinforcers for correct responses. Figure 8 illustrates this, by plotting the log ratio of reinforcers following correct and incorrect responses for one stimulus dimension against the log correct/error reinforcer ratio for the other dimension in Davison and Elliffe's (2010) and the present experiments. As is evident from Figure 8, the log correct/error reinforcer ratios in Davison and Elliffe's experiment were reciprocal, whereas no such reciprocal relation was evident in the present experiment. Thus, it is perhaps unsurprising that changes in log d_x in the present experiment were less systematic than in previous studies that did not arrange reinforcers for errors.

Additionally, previous research using the reinforcement-for-errors procedure suggests that changes in discriminative control may not be proportional to reinforcer rates when errors are reinforced with *asymmetrical* probabilities in different trials (Nevin et al., 1982). Here, the probability of reinforcers for errors differed between stimulus dimensions (Table 2). Additionally, the obtained frequency of reinforcers for errors probably differed between compatible and incompatible trials. This was because in compatible trials, reinforcers for errors were only delivered if subjects made an incorrect response. In comparison, as described above, in incompatible trials, reinforcers for errors according to one dimension were confounded with reinforcers for correct responses according to the other dimension. As a result, *all* obtained reinforcers in incompatible trials were essentially reinforcers for errors according to one dimension, whereas this was not the case in compatible trials. Therefore, the unsystematic changes in log d_x may also be partly related to these asymmetrical probabilities of reinforcers for errors.

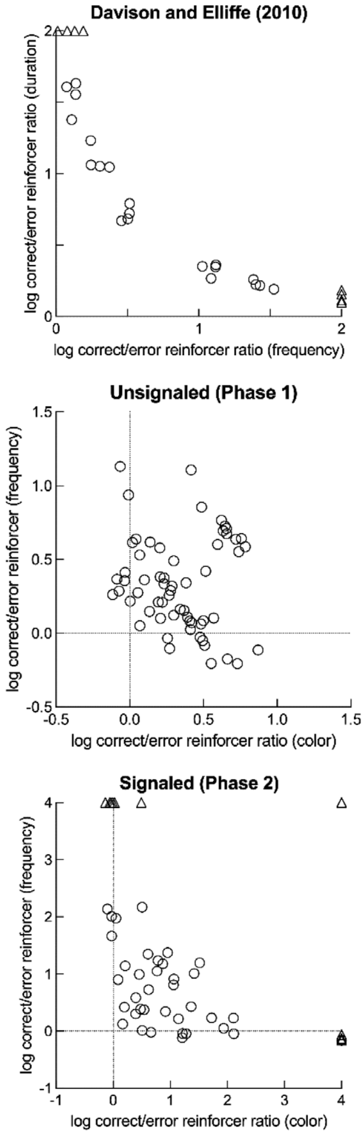

FIGURE 8 Relation between log obtained correct/error reinforcer ratios in Davison and Elliffe (2010) and in the present experiment. Each data point represents a pair of log reinforcer ratios for one subject in one condition. Triangles plotted at +2 or +4 represent data from conditions in which no correct or no error reinforcers were arranged for a dimension.

When the dimension to report is signaled, as in Phase 2, the response (correct or incorrect) and stimulus dimension producing a reinforcer are signaled by the comparison stimuli, whereas this is not the case when the dimension to report is unsignaled (as in Phase 1).

Control by arranged contingencies is stronger when the stimuli signaling or the response alternatives associated with those contingencies are more discriminable (see Cowie & Davison, 2016; Davison & Nevin, 1999, for reviews). For example, choice is more sensitive to changes in reinforcer ratios when the stimuli associated with each alternative are more disparate (e.g., Alsop & Davison, 1991, 1992; Davison & Jenkins, 1985; Godfrey & Davison, 1998; Miller et al., 1980), and likewise conditional-discrimination performance is better when correct and incorrect responses are more disparate (e.g., Davison & Cowie, 2022; Eckerman, 1970; Godfrey & Davison, 1998; Jones, 2003). Based on this research, discrimination of—and thus control by—reinforcer contingencies should be stronger when the dimension to report was signaled in Phase 2 than when it was unsignaled in Phase 1. At first glance, our findings appear inconsistent with this expectation; in contrast, changing the probability of reinforcers for errors had *less* systematic effects on log d_x and pigeons were *less* sensitive to reinforcer ratios in signaled conditions than in unsignaled conditions (compare Figures 2 and 3 and Figures 6 and 7).

In addition to the discriminative stimuli signaling contingencies, sensitivity to changes in reinforcer ratios critically depends on subjects' experience with those changes; in order to discriminate changes in reinforcer contingencies for a behavior, subjects must emit and obtain reinforcers following that behavior, and this must occur often enough such that a change in contingency can be accurately detected (e.g., Cowie et al., 2016, 2020; Mazur, 1997; Gomes-Ng, Landon, et al., 2018). Thus, in the present experiment, detecting changes in the probability of reinforcers for errors from condition to condition required that the pigeons emit incorrect responses and obtain reinforcers for doing so. Our pigeons tended to emit more correct than incorrect responses, so the frequency of obtained reinforcers for correct responses tended to exceed the frequency of reinforcers for errors (Figures 5 to 7). This was especially true in signaled conditions. Therefore, the pigeons probably had less contact with the contingencies associated with incorrect responding in signaled conditions, resulting in weaker discrimination of and control by the probability of reinforcers for errors in those conditions. Thus, the dynamic relation between behavior and consequences may explain why the probability of reinforcers for errors exerted less control over log d_x in signaled conditions. This explanation may also account for carryover between conditions (Figure 4): to the extent that behavior in one condition carried over, this may have reduced the pigeons' contact with, and thus weakened control by, the arranged contingencies in subsequent conditions.

To model quantitatively divided stimulus control, we fit Davison and Nevin's (1999; see also Davison, 2018; Davison & Elliffe, 2010) model to the data (Table 3). The model described data from unsignaled conditions adequately, although fits were slightly poorer than in Davison and Elliffe (2010). Because the dimension to report was signaled in Phase 2, the model for Phase 2 data contained additional parameters quantifying generalization between stimulus dimensions (log $d_{COLOR\text{-}FREQ}$) and between the comparison stimuli signaling those dimensions (log $d_{RED\text{-}GREEN}$; see Davison, 2018 and Appendix C). Compared with unsignaled conditions, fits of the model to data from signaled conditions were poorer, and there was no evidence to support the inclusion of the two additional parameters. In fact, estimates of these parameters were large, implying that there was little to no reinforcer generalization between stimulus dimensions and between the comparison stimuli signaling those dimensions. Thus, overall, these findings demonstrate that Davison and Nevin's model describes divided stimulus control when the dimension to report is unsignaled (Phase 1), whereas they provide little support for the same model, or for Davison's extended model, when the dimension to report is signaled (Phase 2).

The current study is the first to test a quantitative model of divided stimulus control with reinforcers for errors and when the dimension to report is signaled. Thus, it is presently unclear why the model fits for unsignaled conditions were worse than Davison and Elliffe's (2010) and why fits to most pigeons' data from signaled conditions were poor. One possibility is that carryover between conditions (Figure 4) meant that the contingencies in a condition exerted weaker-than-expected control over behavior. Indeed, in signaled conditions, the model fits appeared better when changes in log d_x between conditions were larger (e.g., for Pigeons 71 and 75), implying that the model fit the data better when behavior was more strongly controlled by the reinforcer contingencies. To be sure, future studies could increase the likelihood of subjects encountering, and thus discriminating, changes in contingencies—perhaps by manipulating rates of differential and nondifferential reinforcers or by arranging shorter conditions, as rapidly changing contingencies may promote faster learning and reduce carryover (see Cowie & Davison, 2016, for a brief discussion).

Our findings call into question whether a quantitative model of divided stimulus control requires parameters quantifying generalization between stimulus dimensions and between the comparison stimuli signaling those dimensions. At least here, such a model did no better at describing data in signaled conditions than a model without such parameters (Table 3). This may have been because our stimulus dimensions (color vs. alternation frequency) and comparison stimuli (red vs. green) were quite discriminable such that little generalization between them occurred. In a future study, varying the disparity between stimulus dimensions and/or comparison stimuli to see how this is reflected in the parameter estimates would provide a good test of the full model as well as further insight into the conditions under which the full or standard model can describe divided stimulus control.

In closing, what do our findings suggest about the mechanisms underlying divided stimulus control? Certainly, they confirm that divided stimulus control depends on differential and nondifferential reinforcement. They also raise an important theoretical question: To what extent should we conceptualize divided stimulus control as *dimensional*? Conceptualizing divided stimulus control as dimensional assumes that subjects "separate" a compound stimulus into its dimensions and individual dimensions exert control in proportion to relative reinforcer rates (i.e., *elemental* processing; Blough, 1975; Rescorla & Wagner, 1972). Although previous work describes divided control by dimensions (e.g., Davison & Elliffe, 2010; Shahan & Podlesnik, 2006, 2007) and quantitative models of divided stimulus control assume dimensional control, comparison choice after a compound stimulus may instead be controlled by the reinforcer ratio signaled *by that stimulus as a whole*, so subjects may match comparison choice to that reinforcer ratio (see Gomes-Ng et al., 2019b, for similar discussion). Divided stimulus control, then, may perhaps be understood as joint control over choice by complex multidimensional stimuli and reinforcer ratios.

ACKNOWLEDGMENTS

The authors thank the members of the Behaviour Research Group at The University of Auckland for their help in running the experiment and Lydia Beetham who took excellent care of the pigeons. Open access publishing facilitated by Auckland University of Technology, as part of the Wiley - Auckland University of Technology agreement via the Council of Australian University Librarians.

CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

ETHICS APPROVAL

This experiment was approved by the University of Auckland Animal Ethics Committee (Ref. 2657).

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SUPPORTING INFORMATION

Additional supporting information can be found online in the Supporting Information section at the end of this article.

How to cite this article: Gomes-Ng, S., Cowie, S., & Elliffe, D. (2023). Divided stimulus control depends on differential and nondifferential reinforcement: Testing a quantitative model. *Journal of the Experimental Analysis of Behavior*, *120*(3), 344–362. https://doi.org/10.1002/jeab.876

APPENDIX A

A.1 Acquisition data

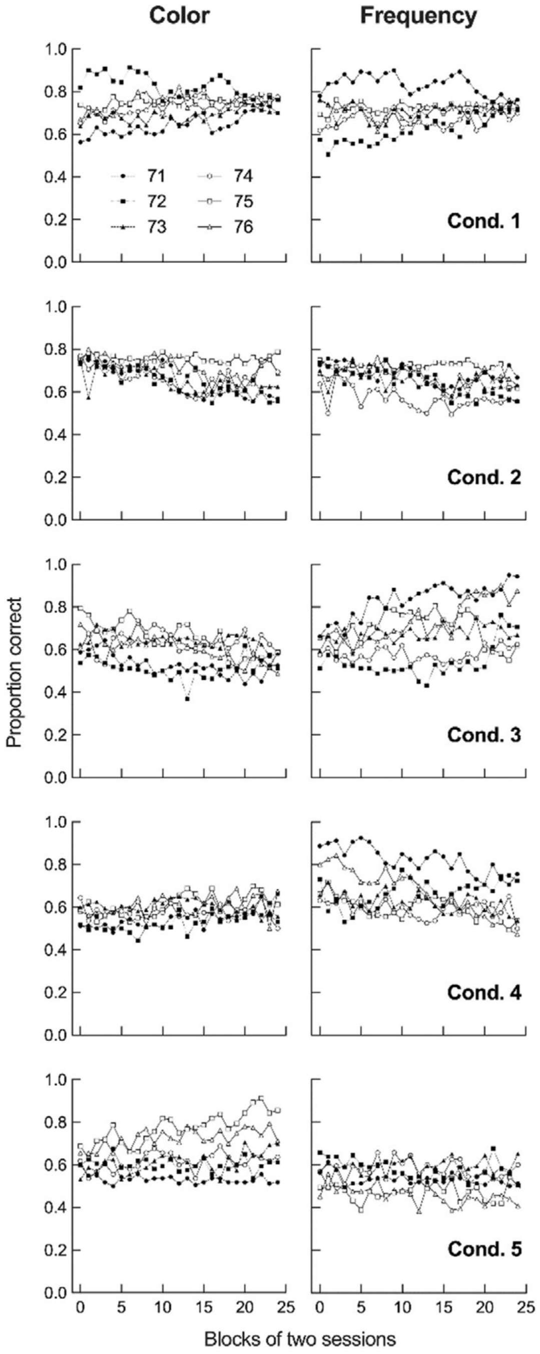

FIGURE A1 Proportion correct across blocks of two sessions in Conditions 1 to 5 (unsignaled conditions). Left columns show accuracy when the dimension to report was color, right columns show accuracy when the dimension to report was alternation frequency.

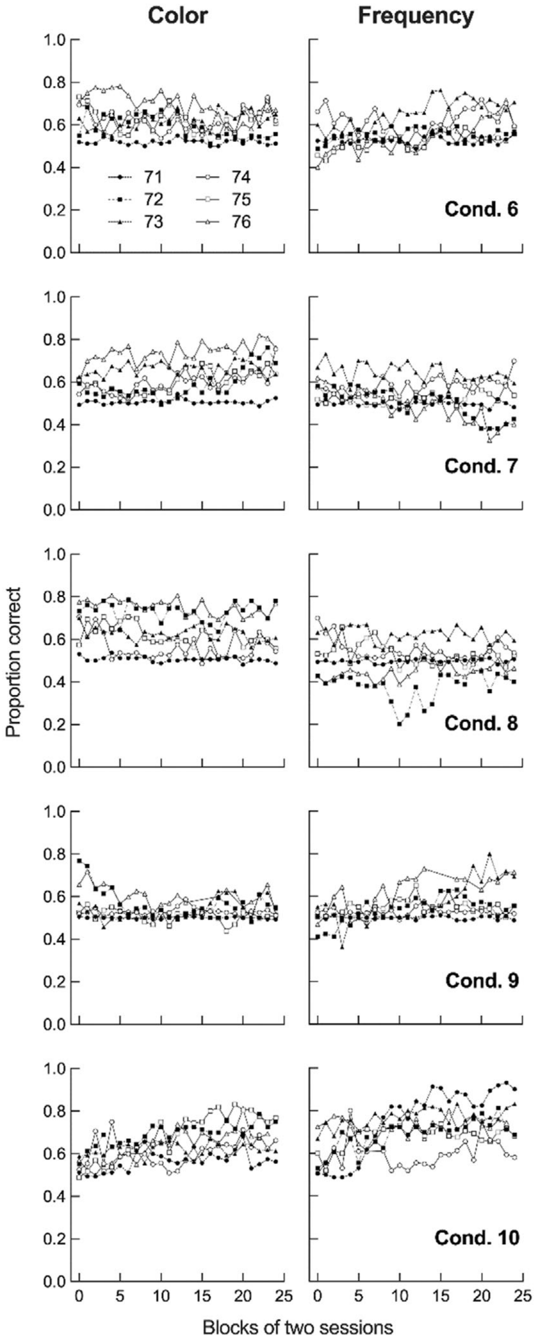

FIGURE A2 Proportion correct across blocks of two sessions in Conditions 6 to 10 (unsignaled conditions). Left columns show accuracy when the dimension to report was color, and right columns show accuracy when the dimension to report was alternation frequency.

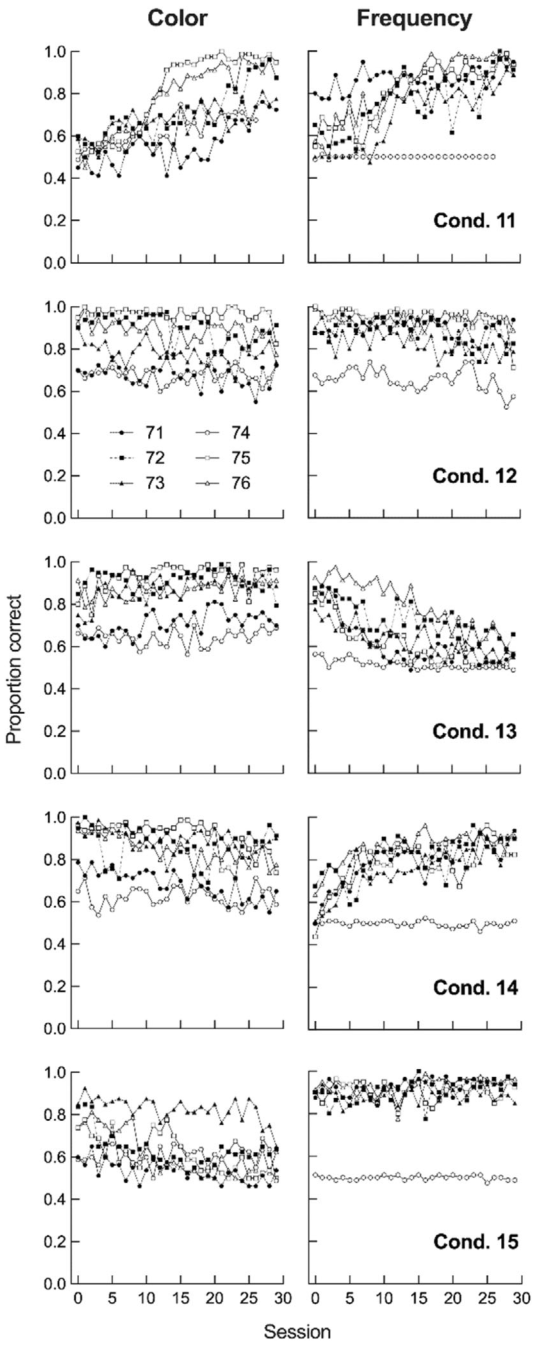

FIGURE A3 Proportion correct across sessions in Conditions 11 to 15 (signaled conditions). Left columns show accuracy when the dimension to report was color, and right columns show accuracy when the dimension to report was alternation frequency.

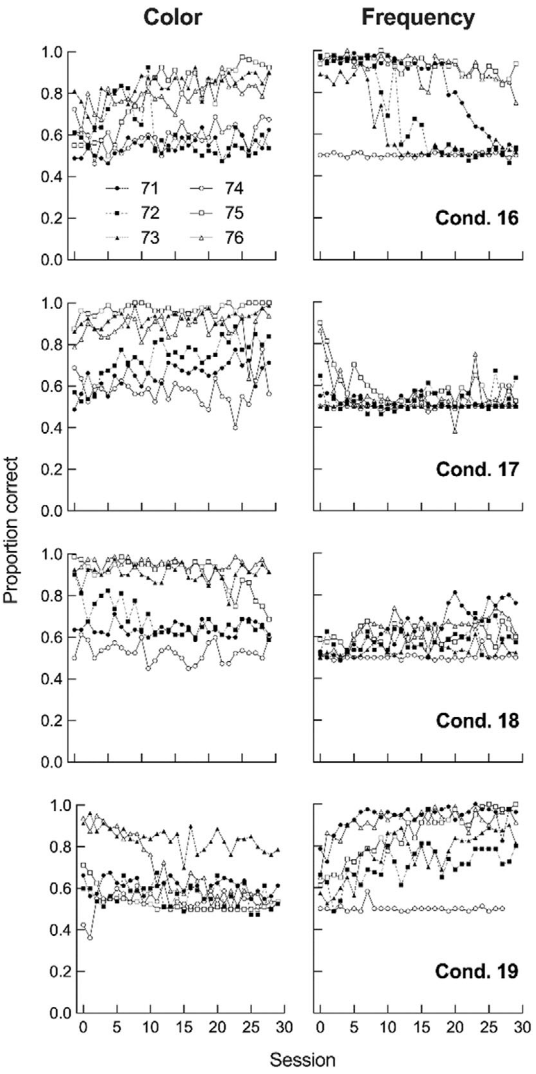

FIGURE A4 Proportion correct across sessions in Conditions 16 to 19 (signaled conditions). Left columns show accuracy when the dimension to report was color, and right columns show accuracy when the dimension to report was alternation frequency. Pigeon 71's responding was disrupted toward the end of Condition 16 due to an equipment issue.

APPENDIX B

B.1 Log b_x across conditions in Phases 1 and 2

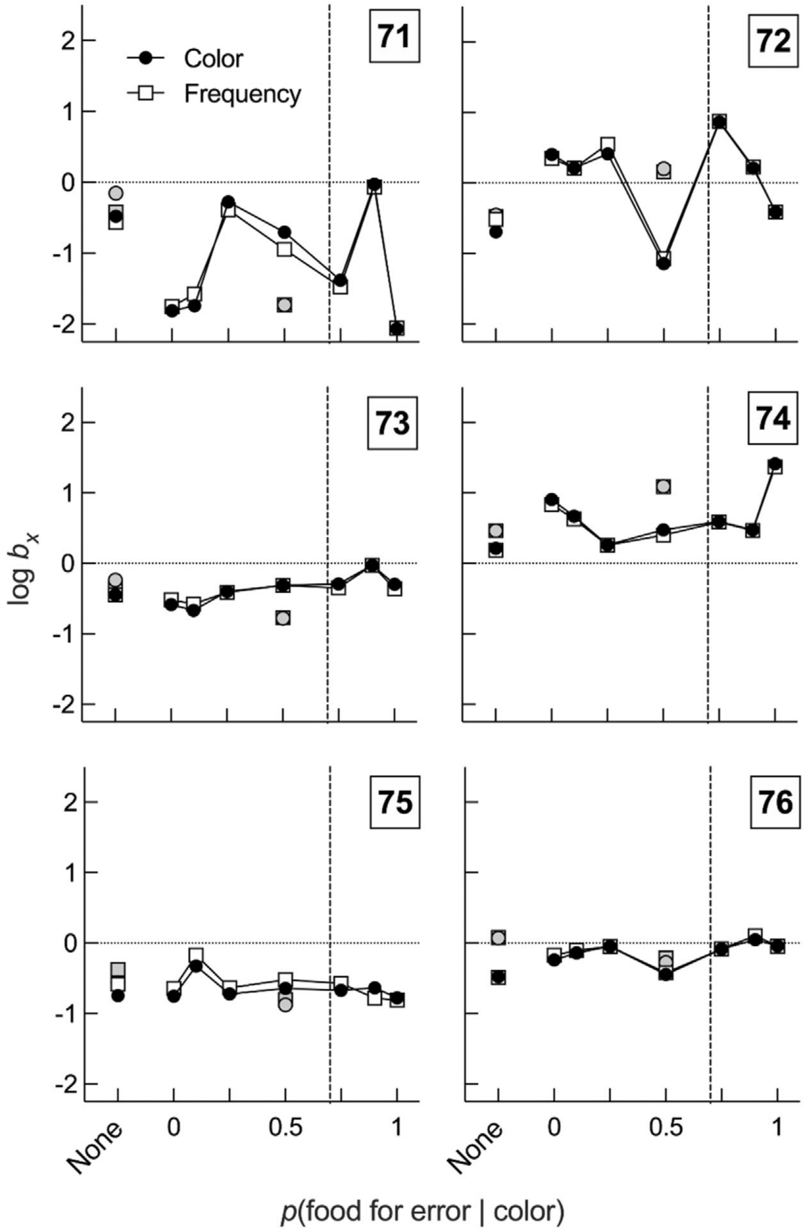

FIGURE B1 Bias in Phase 1 (unsignaled) conditions. Bias was calculated using Equation 2. Shaded gray symbols indicate replication conditions.

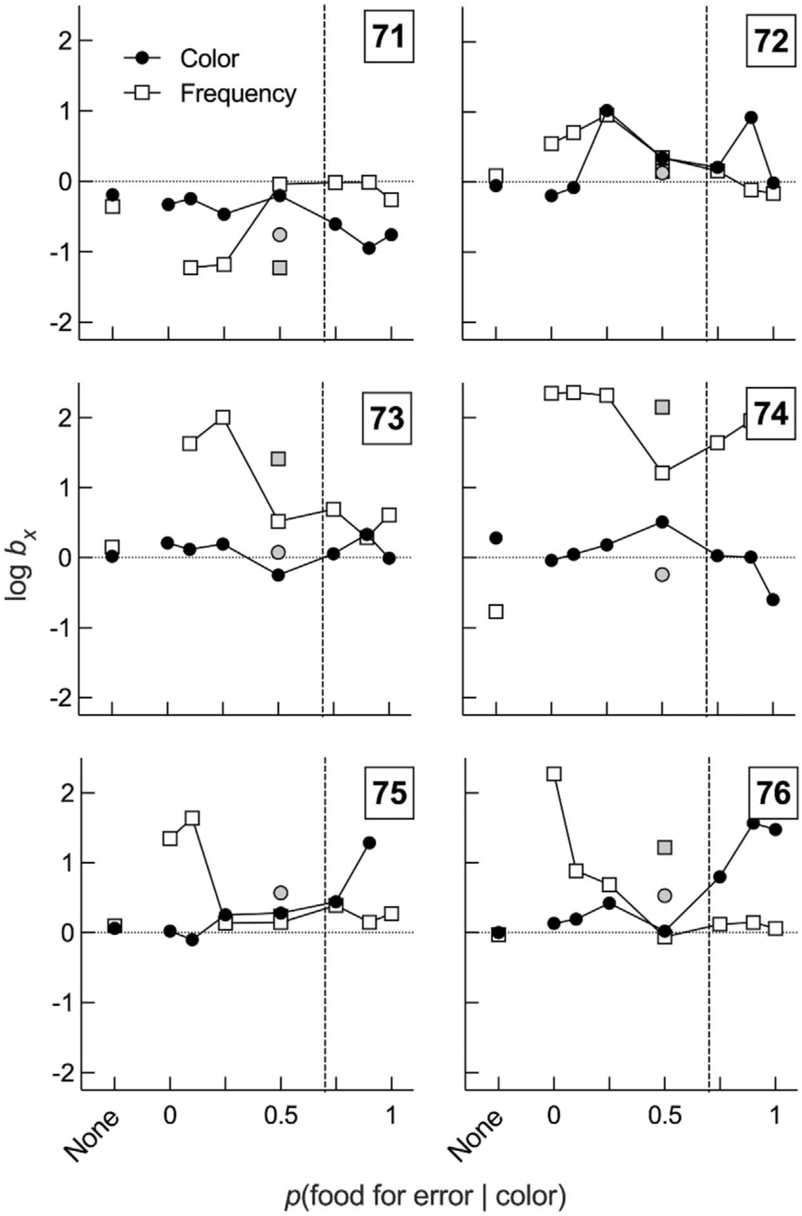

FIGURE B2 Bias in Phase 2 (dignaled) conditions. Bias was calculated using Equation 2. Shaded gray symbols indicate replication conditions.

APPENDIX C

C.1 Application of Davison and Nevin's (1999) model to Phase 2 (signaled conditions) data

This appendix describes application of Davison and Nevin's (1999) model to data from Phase 2, in which the dimension to report was signaled by different colored comparison keys (see also Davison, 2018). Figure C1 shows an example of a matrix of possible events in Phase 2 of the present experiment. The two discriminations are in separate submatrices because different colored comparisons were used. In this example, the comparison keys were lit red for a color discrimination and green for an alternation-frequency discrimination. Left responses were defined as correct after blue or fast stimuli and right responses after yellow or slow stimuli. The reinforcers for correct responses are shown in the relevant cells of the matrix. Reinforcers for errors are not shown, but these would fill the blank cells.

Davison (2018) suggested that reinforcers generalize between the cells *within* each submatrix (solid arrows in Figure C1), and also *between* the two submatrices (dashed arrow in Figure C1). Thus, the general logic for calculating the effective reinforcer count (i.e., reinforcers after generalization) in a particular cell is as follows:

> *Effective Reinforcers = Obtained Reinforcers – (Obtained Reinforcers lost via generalization to other cells in the submatrix) – (Obtained Reinforcers lost via generalization to the other discrimination) + (Obtained Reinforcers gained via generalization from other cells in the*

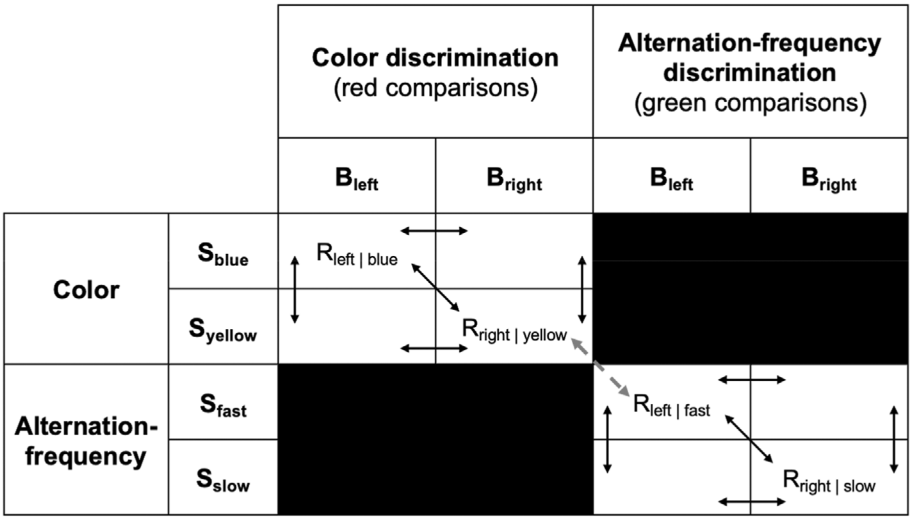

FIGURE C1 Signal-detection matrix depicting an example of the Phase 2 contingencies in the present experiment. There are two submatrices, one for the color discrimination and one for the alternation-frequency discrimination. Solid arrows represent generalization within the cells of a submatrix. The dashed arrow represents generalization between submatrices.

submatrix) + (Obtained Reinforcers gained via generalization from the other discrimination).

This results in a total of five parameters: (1) $d_{sb\text{-}color}$, which quantifies discriminability and thus reinforcer generalization, within the color submatrix; (2) $d_{sb\text{-}freq}$, which quantifies generalization within the alternation-frequency submatrix; (3) d_{br}, which quantifies generalization between the left and right keys; (4) $d_{COLOR\text{-}FREQ}$, which quantifies discriminability of one dimension from another (and thus generalization between the stimulus dimensions); and (4) $d_{RED\text{-}GREEN}$, which quantifies discriminability of the comparison stimuli (and thus generalization between the red and green comparisons).

Focusing first on generalization *within* the color submatrix, using the general formula above, the following equation shows the effective reinforcer count for correct responses after a blue stimulus:

$$R'_{left\,|\,blue} = R_{left\,|\,blue} - \frac{R_{left\,|\,blue}}{d_{sb-color}} - \frac{R_{left\,|\,blue}}{d_{br}} - \frac{R_{left\,|\,blue}}{d_{sb-color}d_{br}} + \frac{R_{right\,|\,blue}}{d_{br}} + \frac{R_{left\,|\,yellow}}{d_{sb-color}} + \frac{R_{right\,|\,yellow}}{d_{sb-color}d_{br}} \tag{1a}$$

The subtracted terms represent reinforcers *lost* via generalization to other cells in the submatrix, and the added terms represent reinforcers *gained* via generalization from the other cells in the submatrix.

Now, focusing on generalization *between* the submatrices, we must add a term to Equation 1a to account for reinforcers generalized from the alternation-frequency submatrix. Davison (2018) suggested that all reinforcers obtained in a submatrix generalize *equally* to all four cells of the other submatrix. Thus, the following should be added to Equation 1a:

$$+\left(\frac{R_{left\,|\,fast}+R_{right\,|\,fast}+R_{left\,|\,slow}+R_{right\,|\,slow}}{d_{COLOR-FREQ}d_{RED-GREEN}}\right)\Big/_{4} \tag{1b}$$

In Equation 1b, all obtained reinforcers in the four cells of the alternation-frequency matrix are added together, and these generalize equally to the four cells of the color matrix (thus the division by 4) via the parameters $d_{COLOR\text{-}FREQ}$ and $d_{RED\text{-}GREEN}$. Adding Equations 1a and 1b results in the following equation:

$$R'_{left\,|\,blue} = R_{left\,|\,blue} - \frac{R_{left\,|\,blue}}{d_{sb-color}} - \frac{R_{left\,|\,blue}}{d_{br}} - \frac{R_{left\,|\,blue}}{d_{sb-color}d_{br}} + \frac{R_{right\,|\,blue}}{d_{br}} + \frac{R_{left\,|\,yellow}}{d_{sb-color}} + \frac{R_{right\,|\,yellow}}{d_{sb-color}d_{br}} + \left(\frac{R_{left\,|\,fast}+R_{right\,|\,fast}+R_{left\,|\,slow}+R_{right\,|\,slow}}{d_{COLOR-FREQ}d_{RED-GREEN}}\right)\Big/_{4}. \tag{2}$$

Equation 2 represents the effective reinforcers for left responses after blue stimuli. Similar logic can be applied to calculate the effective reinforcers in all other cells in each submatrix.

The predicted log d_x values are therefore as follows:

$$\log d'_x = 0.5\log\left(\frac{R'_{correct\,|\,S1}}{R'_{error\,|\,S1}} \cdot \frac{R'_{correct\,|\,S2}}{R'_{error\,|\,S2}}\right), \tag{3}$$

where $R'_{x|Sy}$ represents the effective reinforcers for a correct or incorrect response (x) after stimulus Sy. We fit the Davison–Nevin model to obtained log d_x values in Phase 2 using Microsoft Excel Solver to obtain estimates of $d_{sb\text{-}color}$, $d_{sb\text{-}freq}$, d_{br}, $d_{COLOR\text{-}FREQ}$, and $d_{RED\text{-}GREEN}$.