Mathematical Social Sciences 140 (2026) 102514

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

# Category-dependent preferences and stochastic choice ☆

Matthew Ryan

*School of Economics and Finance, Auckland University of Technology, New Zealand*

ARTICLE INFO

*Keywords:*
Stochastic choice
Random utility
Consideration set
Random attention

ABSTRACT

We introduce a generalisation of (Aguiar's 2017) *random categorisation rule (RCR)* that relaxes (Aguiar's 2017) *Acyclicity* axiom to an *Asymmetry* condition. Unlike other alternatives to the RCR, such as the models of Brady and Rehbeck (2016) and Cattaneo et al., (2020), our *generalised random categorisation rule (GRCR)* relaxes the linearity of preference, rather than varying the random consideration process. We show that the GRCR is also equivalent to allowing linear preferences to be *category-dependent*, with the requirement that the preferences associated with overlapping categories agree on the intersection. This allows the GRCR to capture intransitivities across categories, which may naturally arise when categorisation is used to frame choices. We provide a characterisation for the GRCR and compare it to the random utility model, as well as to the BR model, the RAM, and the model of Manzini and Mariotti (2014).

## 1. Introduction

Consumption items are elements of categories before they are elements of budget sets. Outside laboratory settings, consumers rarely "experience" budget sets.[1] Rather, consumers experience wants, needs and desires. It is therefore natural that they should organise the world into different categories that meet such needs, wants and desires. Consumers may also organise alternatives into categories in order simplify choice problems by facilitating comparisons. For example, they may focus on a category of goods which vary only along one particular dimension. Whatever the motivation, categorisation focusses attention on a salient subset of the budget set — the *consideration set*.

We imagine the following scenario. A consumer randomly selects a category on which to focus – she might, for example, randomly experience a desire for a particular category of good – and then determines which, if any, alternatives from that category are within her budget set. If none is available, she makes no acquisition; an action that is formally described as choosing the "default" option. Otherwise, she chooses her most preferred member of the category within her budget set according her given linear preference relation. Aguiar's (2017) *random categorisation rule (RCR)* models just such a scenario. Here, we introduce a *generalised random categorisation rule (GRCR)* that relaxes the requirement that preferences are transitive.

It is well known that decision-makers may be induced to violate transitivity by carefully curating the sequence of menus presented to them. The famous *preference reversal* experiments are striking examples (e.g., Lichtenstein and Slovic, 1973). The construction of the menus shapes the way that menu elements are compared, so that a given alternative may be evaluated differently in one menu context than another. Preferences are well-behaved within contexts, but may exhibit intransitivities across contexts. When decision-makers categorise-then-choose, they create their own internal contexts and similar phenomena may arise, as illustrated in Example 1 in Section 2. Such choice behaviour is incompatible with the Acyclicity property of the RCR (Aguiar, 2017, Condition 1). Our GRCR is obtained by relaxing Acyclicity to an Asymmetry property (Theorem 3). We show (Theorem 1) that this is equivalent to allowing a limited form of category-dependence of the linear preferences that guide choice: if two goods co-exist in two different categories, they must be ranked the same way by the linear preference orders associated with each of the two categories.

The motivation behind our generalisation of Aguiar's RCR is the idea that preferences are partially formed rather than entirely given. The mechanism of preference formation is *purposive comparison* – comparing alternatives for the purpose of making a choice. This is what enforces "rational" consistency on preferences. Only alternatives

---

☆ My thanks to the Editor and two anonymous referees for many insightful and constructive comments on previous drafts. Their suggestions have significantly improved the paper. I am also grateful to the School of Economics and Finance at Queen Mary University of London for its generous hospitality, and to Marco Mariotti for illuminating discussions during those visits. Earlier drafts circulated under the title "Generalised Random Categorisation Rules" . I have also benefitted from audience comments at the NZ Microeconomics Study Group (Auckland) and the BRIC conference (Aarhus).

*E-mail address:* mryan@aut.ac.nz.

[1] By "budget set" we simply mean the set of alternatives available for choice; this set need not be determined by a fixed budget and fixed prices for each good.

[2] For one attempt at a dynamic model in similar spirit, see Barokas (2021).

https://doi.org/10.1016/j.mathsocsci.2026.102514
Received 17 October 2024; Received in revised form 26 January 2026; Accepted 27 January 2026
Available online 5 February 2026

within the same category are ever directly compared for the purposes of making a choice. Moreover, alternatives that do not co-exist in any single category may be intrinsically more difficult to compare — apples versus oranges, rather than different varieties of apples. It is therefore natural that preference discipline may be weaker across categories than within. Of course, as a consumer gains more experience, *indirect* comparisons may gradually eliminate preference conflict *across* categories. Her behaviour will approximate more and more closely to an RCR. However, a fully dynamic model of preference formation is a task for future research.[2]

## 2. Random categorisation

Let $X$ be a finite set of alternatives and $2^X$ the power set of $X$. We omit braces from singleton sets whenever the intended meaning is clear.

A *choice set* will be an element of $2^X$. Let $a^* \notin X$ denote the default or outside option. Note that $a^*$ is excluded from $X$ by assumption: if the choice set is empty, then $a^*$ must be chosen. For each $A \subseteq X$ let $A^* = A \cup \{a^*\}$. A consumer facing choice set $A$ may choose any element of $A^*$; the latter is called the consumer's *budget set*.

A *random choice function (RCF)* is a mapping $p : X^* \times 2^X \to [0,1]$ that satisfies $p(x,A) = 0$ if $x \notin A^*$ and

$$\sum_{x\in A^*} p(x,A) = 1$$

for all $A \subseteq X$. If $E \subseteq X^*$ we write $p(E,A)$ for

$$\sum_{x\in E} p(x,A).$$

We interpret $p$ as the stochastic choice behaviour of some individual: $p(x,A)$ is the probability of choosing $x$ when facing choice set $A$.

The GRCR is a model for $p$. It has two components: a *preference relation* and an *attention index*.

A *preference relation* is a reflexive and antisymmetric binary relation. A preference relation is *linear* if it is complete and transitive. We define $\succ$ and $\sim$ from $\succsim$ in the usual way. Given a preference relation, $\succsim \subseteq X^2$, the weak and strict upper contour sets for $x \in X$ are

$$\succsim(x) \equiv \{y \in X \mid y \succsim x\}$$

and

$$\succ(x) \equiv \{y \in X \mid y \succ x\}$$

respectively. Note that $\succsim(x) = \succ(x) \cup \{x\}$.

An *attention index* (Aguiar et al., 2023) is a probability mass function (pmf) over $2^X$. If $S$ is a finite set, we use $\Delta(S)$ to denote the set of all pmfs on $S$. When $\pi \in \Delta(S)$ and $E \subseteq S$ we use $\mathrm{supp}(\pi)$ to denote the support of $m$ and we write $\pi(E)$ for

$$\sum_{s\in E} \pi(s).$$

If $m \in \Delta\left(2^X\right)$ is an attention index, we interpret $m(E)$ as the probability that the decision-maker only pays attention to alternatives in the "category" $E \subseteq X$, with $E = \emptyset$ being the null category: if attention is focussed on $E = \emptyset$ then $a^*$ is chosen.

We may now define Aguiar's (2017) random categorisation rule and our generalisation thereof.

**Definition 1** (*Aguiar, 2017*). If $p$ is an RCF, then $p$ can be represented by a **random categorisation rule (RCR)** iff there exist an attention index, $m$, and a linear preference relation, $\succsim \subseteq X^2$, such that

$$p(x,A) = \sum_{E:(E\cap A)\cap\succsim(x)=\{x\}} m(E) \tag{1}$$

for any $A \subseteq X$ and any $x \in A$, and

$$p\left(a^*,A\right) = \sum_{E:E\cap A=\emptyset} m(E) \tag{2}$$

for any $A \subseteq X$. In this case, we say that $(m,\succsim)$ is an RCR for $p$.

**Definition 2.** If $p$ is an RCF, then $p$ can be represented by a **generalised random categorisation rule (GRCR)** iff there exist an attention index, $m$, and a preference relation, $\succsim \subseteq X^2$, such that Eqs. (1) and (2) hold for all $A \subseteq X$ and every $x \in X$. In this case, we say that $(m,\succsim)$ is a GRCR for $p$.

The GRCR relaxes the linearity restriction on the preference relation in an RCR. Importantly, the GRCR is observationally equivalent to a model with category-dependent linear preferences.[3]

**Theorem 1.** *Let $p$ be an RCF. If $(m,\succsim)$ is a GRCR for $p$ then there exists a linear preference relation $\succsim_E \subseteq X^2$ for each $E \in 2^X$ such that: (i) if $E,F \in \mathrm{supp}(m)$ then $\succsim_E$ and $\succsim_F$ agree on $E\cap F$; and (ii) for all $A \subseteq X$ and every $x \in X$,*

$$p(x,A) = \sum_{E:(E\cap A)\cap\succsim_E(x)=\{x\}} m(E) \tag{3}$$

*and*

$$p\left(a^*,A\right) = \sum_{E:E\cap A=\emptyset} m(E) \tag{4}$$

*Conversely, if there exists an $m \in \Delta\left(2^X\right)$ and a linear preference relation $\succsim_E \subseteq X^2$ for each $E \subseteq X$ such that (i) and (ii) hold, then $p$ has a GRCR.*

The following example uses Theorem 1 to confirm that the GRCR is a non-vacuous generalisation of the RCR. We provide possible interpretations after presenting the example.

**Example 1.** Let $X = \{a,b,c\}$, $E_1 = \{a,b\}$, $E_2 = \{b,c\}$ and $E_3 = \{a,c\}$. Let $m \in \Delta\left(2^X\right)$ be the attention index satisfying

$$m\left(E_i\right) = \frac{1}{3}$$

for each $i \in \{1,2,3\}$. Let $\left\{\succsim_{E_i}\right\}_{i=1}^{3}$ be linear preference relations on $X$ satisfying $b \succ_{E_1} a$, $c \succ_{E_2} b$ and $a \succ_{E_3} c$. Defining $p$ using Eqs. (3) and (4) gives an RCF. By Theorem 1 it has a GRCR representation.[4] We will show that it has no RCR representation. Suppose, to the contrary, that $(\hat{m},\succsim)$ is an RCR that rationalises $p$. Since

$$p(a,\{a,b\}) = \frac{1}{3} < \frac{2}{3} = p(a,\{a\}) = \sum_{A:a\in A} \hat{m}(A)$$

it follows that $a$ is not always chosen when it is considered alongside $b$, so we must have $b \succ a$. Likewise, we deduce $c \succ b$ from $p(b,\{b,c\}) < p(b,\{b\})$ and $a \succ c$ from $p(c,\{a,c\}) < p(c,\{c\})$. This contradicts the transitivity of $\succ$.

The generic scenario in Example 1 may arise naturally in many contexts.

- Consider goods defined by three measurable characteristics. In particular, let $X \subseteq \mathbb{R}^3_+ \setminus \{0\}$ and $a^* = 0$. Suppose a particular consumer always focusses attention on some characteristic (i.e., on the alternatives which are strictly positive in that dimension — the salient category), with each characteristic being equally likely to attract their attention. When focussed on a given category the consumer ranks alternatives by the quantity of the relevant characteristic. Now suppose $X = \{a,b,c\}$, where $a = \left(a_1,0,a_3\right)$, $b = \left(b_1,b_2,0\right)$ and $c = \left(0,c_2,c_3\right)$ with $0 < a_1 < b_1$, $0 < b_2 < c_2$ and $0 < c_3 < a_3$. Let $E_i$ be the category associated with characteristic $i$ and define $\succsim_{E_i}$ to be the linear order on $X$ satisfying $x \succsim_{E_i} y$ iff $x_i \geq y_i$. This gives the situation in Example 1.[5]

---

[3] Proofs of all results may be found in the appendix.

[4] It is straightforward to check that $(m,\succsim)$ provides such a representation, where

$$\succsim = \{(a,a),(b,b),(c,c),(b,a),(c,b),(a,c)\}.$$

[5] My thanks to an anonymous referee for suggesting this interpretation of Example 1.

- One may generalise the previous interpretation of Example 1 by considering choice behaviour that exhibits *similarity effects*. Categories may reflect a subjective classification of alternatives based on some perceived similarity relationship. This categorisation is used to simplify complex choices by focussing on subsets of alternatives for which comparison is more straightforward, and over which preferences are linear. Such similarity effects have been used to explain intransitivities in binary choices between lotteries (as well as other anomalies, such as the Allais paradox), due to changes in the similarity frame between choice sets.[6] In experimental settings, choices can be kept simple – typically binary – so full consideration is naturally presumed, and the choice sets are carefully constructed to control similarity dimensions. In real-life decision-making, with large and complex choice sets, it is more likely that there will be less-than-full consideration, with endogenous selection of a similarity category on which to focus. The GRCR captures precisely such a process of choice.

It is important to note that the sort of transitivity that is violated in Example 1 is different from, though similar in spirit to, the usual notions of stochastic transitivity. It is based on a distinct, but equally compelling, revealed preference relation. As is well-known, the RCR can already accommodate violations of weak stochastic transitivity (Aguiar, 2017, p.49).[7]

Theorem 1 conveys another important lesson. When we relax the transitivity requirement on preferences in an RCR, we can no longer pair an arbitrary attention index with an arbitrary preference relation and expect to construct a valid RCF via Eqs. (1) and (2).

**Example 2.** Suppose $m \in \Delta\left(2^X\right)$ assigns equal probability to each subset of $X$ and $\succsim = \{(x, x) \,|\, x \in X\}$. Then Eq. (1) implies

$$p(a, A) \;=\; \sum_{B \subseteq X : a \in B} m(B) \;=\; \frac{1}{2}$$

for any $A \subseteq X$ and any $a \in A$. Hence $p$ cannot be an RCF when $|X| > 2$, since $p(X, X) > 1$.

Conditions Eqs. (1) and (2) impose implicit restrictions on $\succsim$. The proof of Theorem 1 unpacks these restrictions.[8] They require preferences to be linear on every category in the support of $m$. This allows $\succsim$ to be replaced by a family of **category-dependent** and **linear** preference relations that satisfy condition (i). In this reformulation of the GRCR model, Eq. (1) is transformed into Eq. (3). Theorem 1 provides a useful recipe for constructing GRCRs that respect the coherence constraints imposed by the properties of a random choice function; a recipe that we exploited to construct Example 1 above.

The fact that $m$ and $\succsim$ cannot be chosen independently complicates the task of further generalising the model to allow preferences, as well as the consideration set, to be random (cf., Aguiar et al., 2023). The collection of viable support sets for random preferences depends on $m$ in a complex fashion. Similarly, if we randomly choose the category-dependent linear preferences in the Theorem 1 version of the model, these must be correlated across categories to ensure (i). We have not been able to characterise this further generalisation of the RCR.

## 3. Characterising the GRCR

The following is necessary for existence of a GRCR representation for $p$.

**Axiom 1** (*Asymmetry*). *For any $a, b \in X$, if $p(b, A \cup a) \neq p(b, A)$ for some $A$ containing $b$, then $p(a, B \cup b) = p(a, B)$ for any $B$ containing $a$.*

This is a strengthening of the *i-Asymmetry* axiom of Manzini and Mariotti (2014). Axiom 1 asserts the asymmetry of the following binary relation on $X$ (cf., Aguiar, 2017, Definition 2): $a \rhd b$ iff $p(b, A \cup a) \neq p(b, A)$ for some $A$ containing $b$. Note that $\rhd$ is irreflexive. It will be useful to define $\trianglerighteq$ to be the minimal reflexive extension of $\rhd$. That is: $a \trianglerighteq b$ iff $a = b$ or $a \rhd b$.[9] In particular, Axiom 1 implies that $\trianglerighteq$ is a preference relation.

Aguiar (2017) shows that acyclicity of $\rhd$ is necessary for $p$ to have an RCR representation. Axiom 1 imposes a weaker restriction on $\rhd$, in the same way that WARP is weaker than SARP. Asymmetry is, correspondingly, more accessible and compelling to our intuition than Acyclicity.

Axiom 1 characterises a model that relaxes the positivity constraint on the attention index in a GRCR but is otherwise identical.[10]

**Theorem 2.** *Let $p$ be an RCF. Then $p$ satisfies Asymmetry iff there exists a preference relation $\succsim$ and a mapping $m : 2^X \to \mathbb{R}$ with $\sum_{B \subseteq X} m(B) = 1$, such that Eqs. (1) and (2) hold for any $A \subseteq X$ and any $x \in A$.*

The possibility of negative values of $m$ makes the interpretation of this model problematic — though signed measures have appeared elsewhere in representations of preference: see Brandenburger et al. (2024). Nevertheless, it does have the merit of being fully characterised by the intuitive Axiom 1, and is also very flexible. For example, it can accommodate violations of regularity[11]; though not, unfortunately, of a sort that might provide a natural explanation of the attraction effect (Huber et al., 1982).[12]

Aguiar (2017) shows that $p(a^*, \cdot)$ uniquely determines $m$ via Eq. (2), and uses this fact to impose constraints on $p(a^*, \cdot)$ that ensure the

---

[6] The literature is voluminous. Rubinstein (1988) is a classic theoretical treatment. Tversky et al. (1988) explain intransitivities observed in preference reversal experiments using similarity-based arguments. Leland (1994) posits similarity-based rationalisations of other instances of intransitivity, with supporting experimental evidence.

[7] Weak stochastic transitivity (WST) is defined for choice environments without a default. The most appropriate adaptation to choice sets with default is not entirely obvious. Manzini and Mariotti (2014) translate it as follows: $p(a, \{a, c\}) \geq \frac{1}{2}$ whenever $p(a, \{a, b\}) \geq \frac{1}{2}$ and $p(b, \{b, c\}) \geq \frac{1}{2}$. They construct an example to show that their model, which is more restrictive than the RCR, may violate this property. The other way to generalise WST is to require $p(a, \{a, c\}) \geq p(c, \{a, c\})$ whenever $p(a, \{a, b\}) \geq p(b, \{a, b\})$ and $p(b, \{b, c\}) \geq p(c, \{b, c\})$. It is straightforward to show that the RCR may also violate this property.

[8] It a stochastic cousin of Lemma 2 in Dutta and Horan (2015). If $m$ is an attention index with $\mathrm{supp}(\pi) = \{E\}$, then a GRCR $(m, \succsim)$ generates a degenerate RCF (i.e., a choice function) which may also be rationalised by the *rational shortlist method (RSM)* of Manzini and Mariotti (2007): let $P_1$ be the asymmetric order defined by $x P_1 y$ iff $x \in E$ and $y \notin E$, and let $P_2 = \succ$. Dutta and Horan (2015, Lemma 2) give a sufficient condition for the RSM to yield a valid choice function. In the present case, this condition is satisfied if $P_2$ is linear on $E$. My thanks to the Co-Editor for alerting me to this connection.

[9] Note that $\trianglerighteq$ need not coincide with $\{(a, b) \in X \times X \mid (b, a) \notin \rhd\}$.

[10] My thanks to an anonymous referee for suggesting this result, and for the reference to Brandenburger et al. (2024).

[11] Let $X = \{a, b\}$ and let $\succsim$ be the linear preference relation on $X$ satisfying $a \succ b$. Suppose $m : 2^X \to \mathbb{R}$ satisfies $\sum_{B \subseteq X} m(B) = 1$ with $m(E) < 0$ iff $E = \{a\}$ and $|m(\{a\})| \leq \min\left\{m\left(\emptyset\right), m(X)\right\}$. Then it is easily checked that Eqs. (1) and (2) determine a well-defined RCF, with $\succsim = \trianglerighteq$ and $p(a^*, \{b\}) < p(a^*, X)$.

[12] Suppose $X = \{a, b, c\}$, with $c$ the decoy and $c$ dominated by $b$ but not by $a$. Then it is natural to assume that preferences satisfy $c \notin \succsim(b)$. In this case, for any $E \subseteq X$:

$$(E \cap \{a, b\}) \cap \succsim(b) = \{b\}$$

iff

$$(E \cap \{a, b, c\}) \cap \succsim(b) = \{b\}\,.$$

It follows from Eq. (1) that $p(b, \{a, b\}) = p(b, \{a, b, c\})$ for any $m : 2^X \to \mathbb{R}$.

positivity of $m$. In particular, defining $f(A) = p(a^*, X \setminus A)$ for each $A \subseteq X$, we observe from Eq. (2) that

$$f(A) = \sum_{B \subseteq A} m(B),$$

so $m$ is the (unique) Möbius inverse of $f$ (Shafer, 1976, Lemma 2.3):

$$m(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} p\left(a^*, X \setminus B\right) \tag{5}$$

Since $f(X) = p\left(a^*, \emptyset\right) = 1$, it follows that $m \in \Delta\left(2^X\right)$ provided

$$\sum_{B \subseteq A} (-1)^{|A \setminus B|} p\left(a^*, X \setminus B\right) \geq 0 \tag{6}$$

for all $A \subseteq X$.

**Axiom 2** (*WDMP*). *Condition Eq. (6) is satisfied for all $A \subseteq X$.*

Our Axiom 2 is equivalent to Aguiar's *Weakly Decreasing Marginal Propensity (of Choice)*, or *WDMP*, condition, but expressed in the form of a standard Block-Marschak inequality. Aguiar's WDMP condition requires *C-total monotonicity* (Aguiar, 2017, Definition 10) of the function $\varphi : 2^X \rightarrow [0,1]$ defined by $\varphi(A) = 1 - p(a^*, A)$. To see that this is equivalent to our Axiom 2, we refer the reader to the discussion on p.48 of Aguiar (2017) and Section 7.2.3 of Grabisch (2016).[13]

We have therefore established the following theorem.

**Theorem 3.** *Let $p$ be an RCF. Then $p$ satisfies Axioms 1-2 iff it has a GRCR representation.*

While the attention index in a GRCR is unique, preferences are not uniquely identified: any preference relation containing $\trianglerighteq$ will suffice.

**Proposition 1.** *Let $p$ be an RCF with a GRCR representation, $(m, \succsim)$. Then*

$$\trianglerighteq = \bigcup_{E \in \mathrm{supp}(m)} \left(\succsim \cap E^2\right).$$

*Moreover, $\left(m', \succsim'\right)$ is a GRCR for $p$ iff $m' = m$ and $\trianglerighteq \subseteq \succsim'$.*

It is therefore without loss of generality to require the preference relation in a GRCR to be complete (i.e., to require $\succ$ to be a *tournament*). Proposition 1 also implies that if $p$ possesses a GRCR representation with attention index, $m$, then $(m, \trianglerighteq)$ is a GRCR for $p$. This may be regarded as the *canonical* GRCR for $p$. It is fully identified.

## 4. Comparison with other models

If $\mathcal{R}^*$ denotes the set of linear preference relations on $X^*$, then a random utility model (RUM) an element of $\Delta(\mathcal{R}^*)$. Aguiar (2017, Lemma 1) shows that every RCF with an RCR representation also has a RUM. It is not hard to see why. When every menu contains a common default option, random preference can replicate random categorisation by randomly demoting alternatives below $a^*$ in the ranking. Rather than choosing category $E$ with probability $m(E) > 0$, we choose (with the same probability) a linear preference relation on $X^*$ that ranks everything in $E$ above $a^*$, everything in $X \setminus E$ below $a^*$, and matches the RCR preference relation on $E$. This logic carries over directly to the GRCR, since $\succsim$ must be linear on every $E$ in the support of $m$ (see Lemma A.1 in Appendix A), so we omit the proof of the following:

**Proposition 2.** *Any RCF with a GRCR representation also has a RUM.*

If $p$ has a RUM, then $p$ will necessarily satisfy WDMP, which is one of the necessary Block-Marschak inequalities (McFadden, 2005, Theorem 3.3).[14] Suppose $\pi \in \Delta(\mathcal{R}^*)$ is a RUM for $p$. Then $p$ has a GRCR representation iff it satisfies Asymmetry. The Asymmetry Axiom is equivalent to the following: for any $a, b \in X$ with $a \neq b$, either

$$p(a, A \cup \{a\}) = p(a, A \cup \{a, b\}) \tag{7}$$

for all $A \subseteq X \diagdown \{a, b\}$, or

$$p(b, A \cup \{b\}) = p(b, A \cup \{a, b\}) \tag{8}$$

for all $A \subseteq X \diagdown \{a, b\}$. It is straightforward to show that Eq. (7) holds iff $\pi\left(\{\succsim^* | b \succ^* a \succ^* a^*\}\right) = 0$ and Eq. (8) holds iff $\pi\left(\{\succsim^* | a \succ^* b \succ^* a^*\}\right) = 0$. Therefore, $p$ satisfies Asymmetry iff

$$\pi\left(\left\{\succsim^* | a \succ^* b \succ^* a^*\right\}\right) \pi\left(\left\{\succsim^* | b \succ^* a \succ^* a^*\right\}\right) = 0 \tag{9}$$

for any $a, b \in X$. Condition Eq. (9) determines the set of RUMs for which there is an observationally equivalent GRCR. It requires that any distinct $a, b \in X$ be ranked the same way by all supported linear preference relations that rank both $a$ and $b$ above the default. This is clearly a significant restriction.

It follows from Proposition 2 that any $p$ with a GRCR representation must satisfy *regularity*: $p(x, A) \geq p(x, B)$ when $x \in A \subseteq B$. It is therefore natural to wonder whether we could replace Axiom 1 in Theorem 3 with asymmetry of the following alternative binary relation: $a \blacktriangleright b$ iff $p(b, A \cup a) < p(b, A)$ for some $A$ containing $b$. In fact, we cannot.

**Example 3.** Let $X = \{a, b, c\}$ and consider the RCF described in the table below. The value of $p(x, A)$ is the entry in row $x$ and column $A$. The probabilities of choosing $a^*$ are the column residuals, together with $p\left(a^*, \emptyset\right) = 1$.

| | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{a,b\}$ | $\{b,c\}$ | $\{a,c\}$ | $X$ |
|---|---|---|---|---|---|---|---|
| $a$ | 1/2 | 0 | 0 | 7/12 | 0 | 7/12 | 7/12 |
| $b$ | 0 | 1/2 | 0 | 1/6 | 1/2 | 0 | 1/6 |
| $c$ | 0 | 0 | 1/2 | 0 | 1/4 | 1/6 | 1/8 |

Noting that $p(a^*, A) = 2^{-|A|}$ for any $A \subseteq X$, it is clear that

$$p\left(a^*, X \setminus A\right) = \sum_{B \subseteq A} m(B)$$

when $m(E) = \frac{1}{8}$ for each $E \subseteq X$. Hence, $p$ satisfies WDMP. Moreover:

$$\blacktriangleright = \{(a,b), (b,c), (a,c)\}.$$

See this by checking each row of the table: row "$a$" contributes no elements of $\blacktriangleright$; row "$b$" contributes $(a, b)$; row "$c$" contributes $(b, c)$ and $(a, c)$. Thus, $\blacktriangleright$ is asymmetric. However, $p$ cannot have a GRCR representation since regularity is violated: for example, $p(a, \{a\}) < p(a, \{a, b\})$.

It is also useful to compare the GRCR to other popular random consideration models. Besides the RCR, the representations of Brady and Rehbeck (2016;[BR]) and Manzini and Mariotti (2014;[MM]) are also well known. The next proposition is summarised in Fig. 1.

**Proposition 3.** *Let $p$ be an RCF. Then the following are equivalent:*

(i) *$p$ has an MM representation.*
(ii) *$p$ has both RCR and BR representations.*
(iii) *$p$ has both GRCR and BR representations.*

---

[13] Aguiar's terminology is somewhat non-standard. What he calls the C-total monotonicity property is better known in the literature as the $\infty$ *-alternating* (or $\cup$-alternating) property. The implied property of the *dual function* $\overline{\varphi} : 2^X \rightarrow [0,1]$, defined by $\overline{\varphi}(A) = 1 - \varphi\left(\overline{A}\right)$, is conventionally called "total monotonicity" (equivalently, "$\infty$ -monotonicity" or "$\cap$ -monotonicity").

[14] McFadden's theorem generalises Falmagne (1978) to situations where choice probabilities are defined for a collection of budget sets that forms a net. The collection $\{A^* | A \subseteq X\}$ is a net.

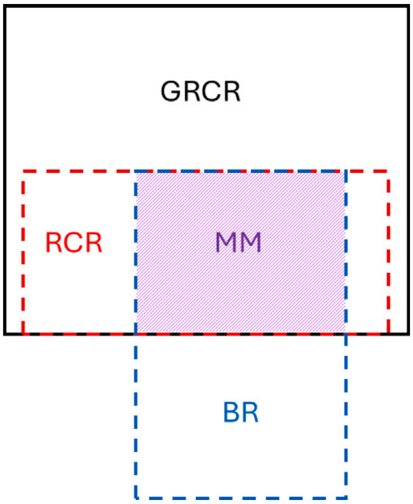

**Fig. 1.** Relationships implied by Proposition 3.

The equivalence of (i) and (ii) is an important and well-known result in the literature: see Kovach and Suleymanov (2023) and its precursor, Suleymanov (2018).[15] However, we include a new proof (see Appendix D), which may be of independent interest: in particular, it shows that any RCF which has both RCR and BR representations must satisfy the MIDO axiom of Brady and Rehbeck (2016). The equivalence of (i) and (iii) is the novel claim in Proposition 3. One implication of Proposition 3 is that neither the RCR nor the BR representation can accommodate the behaviour in Example 1: see Fig. 1.

The BR representation permits violatations of Asymmetry,[16] and therefore so does any model that generalises the BR representation, such as the *random attention model (RAM)* of Cattaneo et al. (2020).[17] Note, however, that the RAM is compatible with Example 1.[18] In short, while the RUM, RAM and GRCR can all capture the behaviour in Example 1, of these, only the GRCR satisfies Asymmetry. Since the Asymmetry property embodies a very natural and appealing consistency constraint on stochastic choice behaviour, it seems useful to explore models that respect this constraint.

## 5. Concluding remarks

In an RCR, choice is guided by a rational (linear) preference relation applied to a random consideration set — the intersection of the choice set with a randomly selected category. We have shown that allowing preference to be *category-dependent*, but with agreement on category intersections, is equivalent to relaxing the *linearity* requirement of the RCR preference relation. Our *generalised* RCR may be characterised by weakening Aguiar's (2017) Acyclicity restriction on his revealed preference relation to an Asymmetry restriction.

The ill-behavedness of the preference relation in a GRCR does not undermine the possibility of rational choice, since preferences need only be well-behaved on sets of options which are *considered together*. How much preference discipline this imposes is endogenous, depending on the individual's consideration process. If one assumes that preferences are disciplined by the experience of making choices, then it is natural to suppose that preferences will exhibit more consistency on sets of alternatives which are considered together. This is precisely what the GRCR model requires. Of course, the ill-behavedness of preferences may, in principle, result in choice cycles over a *sequence* of choice problems (recall Example 1), thereby prompting some further process of preference revision — but this dynamic process is outside the current model.

An auxiliary contribution of the present paper is to provide a new proof of the fact that Manzini and Mariotti's (2014) model characterises the intersection of the RCR and BR classes. The MM model also, it turns out, characterises the intersection of the GRCR and BR classes (Fig. 1).

Finally, let us comment briefly on the problem of testing the GRCR model. Extant testing methodologies (of which we are aware) cannot distinguish the GRCR from the RCR. The methodology of Aguiar et al. (2023) requires a strictly positive probability of full consideration for any choice set. In the GRCR context, this necessitates $m(X) > 0$, which implies that $\succsim$ must be linear (Proposition 1). The design of a test that can differentiate between the RCR and GRCR models is therefore an important open problem.

We might alternatively test the compatibility of choice probabilities with a *mixture* of GRCRs, since most datasets aggregate across heterogeneous individuals. However, an RCF is generated by a mixture of GRCRs if and only if it is generated by a mixture of RCRs: the two mixture models are empirically equivalent. This fact is established in Appendix E, which also describes how to test the compatibility of an RCF with a mixture of (G)RCRs.

## Appendix A. Proof of Theorem 1

We first prove:

**Lemma A.1.** *Let $p$ be an RCF and let $m \in \Delta\left(2^X\right)$. There exist linear preference relations $\left\{\succsim_E\right\}_{E \subseteq X}$ on $X$ satisfying (i) and (ii) iff there exists a preference relation, $\succsim$, such that $(m, \succsim)$ is a GRCR for $p$ and the restriction of $\succsim$ to $E$ is linear for each $E \subseteq X$ with $m(E) > 0$.*

**Proof.** The "if" part is straightforward: for each $E \subseteq X$ with $m(E) > 0$, define $\succsim_E$ to be any linear extension (to $X$) of the restriction of $\succsim$ to $E$, while if $m(E) = 0$, define $\succsim_E$ to be any linear preference relation on $X$. For the "only if" part, first observe that Eq. (2) follows from Eq. (4). Now let

$$F = \left\{ x \in X \;\middle|\; \sum_{E : x \in E} m(E) = 0 \right\}.$$

Thus, $F$ contains the alternatives excluded from all categories in the support of $m$. Define the binary relation, $\succsim \subseteq X^2$, as follows:

$$\succsim \cap F^2 = \{(x, x) \mid x \in F\}$$

and for all $E \subseteq X$ with $m(E) > 0$, $\succsim \cap E^2 = \succsim_E \cap E^2$. This construction is well-defined, since $\succsim_E$ and $\succsim_F$ agree on $E \cap F$ whenever $m(E)\, m(F) > 0$; it is also obvious that $\succsim$ is a preference relation and that $\succsim$ is linear on $E$ whenever $m(E) > 0$. Since $\succsim$ coincides with $\succsim_E$ on $E$ when $m(E) > 0$, condition Eq. (1) is satisfied.

This completes the proof of Lemma A.1. ■

The second claim in Theorem 1 follows directly from Lemma A.1. To complete the proof of Theorem 1 it suffices to show that if $(m, \succsim)$ is a GRCR for $p$ then $\succsim$ is linear on each $E \subseteq X$ for which $m(E) > 0$.

---

[15] It also appears in Strzalecki (2025).

[16] Example 1 in Brady and Rehbeck (2016) shows that the BR representation may violate *i-Asymmetry* (Manzini and Mariotti, 2014), and therefore also our (stronger) Asymmetry axiom.

[17] Strictly speaking, the RAM is defined for choice environments without a default, but is easily adapted to our set-up. Kovach and Suleymanov (2023) adapt the RAM to environments with a default, but in their model preferences are not required to rank the outside option last.

[18] Suppose $c \succ b \succ a$ and define the consideration mapping $\mu$ (where $\mu(B, A)$ is the probability of considering $B \subseteq A$ from menu $A$) as follows: $\mu\left(\emptyset, A\right) = 0$ if $|A| > 1$, $\mu\left(\emptyset, \{x\}\right) = \frac{1}{3}$ for all $x \in X$, $\mu(\{a, c\}, \{a, c\}) = \mu(\{c\}, \{a, c\}) = \frac{1}{6}$, $\mu(\{a\}, \{a, c\}) = \frac{2}{3}$, $\mu(\{a, b\}, X) = \mu(\{b, c\}, X) = \mu(\{a, c\}, X) = \mu(\{c\}, X) = 0$, and $\mu(B, A) = \frac{1}{3}$ otherwise.

**Lemma A.2.** *Let $p$ be an RCF and suppose $(m, \succsim)$ is a GRCR for $p$. Then $m$ assigns zero probability to any $B \subseteq X$ such that (i) there exist $a, b \in B$ with $a \notin \succsim (b)$ and $b \notin \succsim (a)$, or (ii) $B$ contains a cycle with respect to $\succ$.*

**Proof.** Suppose $B$ satisfies (i) and $m(B) > 0$. Let $A = \{a, b\}$. It follows that $\{a, b\} \cap \succsim (a) = \{a\}$ and hence

$$\begin{aligned} p(a, A) &= \sum_{C : C \cap \{a,b\} \cap \succsim(a) = \{a\}} m(C) \\ &= \sum_{C : a \in C} m(C) \\ &= \sum_{E : E \cap \{a,b\} = \{a\}} m(E) + \sum_{F : F \cap \{a,b\} = \{a,b\}} m(F). \end{aligned}$$

Similarly,

$$p(b, A) = \sum_{E : E \cap \{a,b\} = \{b\}} m(E) + \sum_{F : F \cap \{a,b\} = \{a,b\}} m(F).$$

Moreover,

$$p\left(a^*, A\right) = \sum_{C : C \cap \{a,b\} = \emptyset} m(C)$$

so

$$p\left(A^*, A\right) = 1 + \sum_{F : F \cap A = \{a,b\}} m(F) \geq 1 + m(B) > 1.$$

This is the desired contradiction.

Next, suppose $B$ satisfies (ii) and $m(B) > 0$. By what we have already established, we may assume that $m(E) = 0$ for any $E$ with $a, b \in E$ such that $a \neq b$, $a \notin \succ (b)$ and $b \notin \succ (a)$. Let $A = \left\{a_0, a_1, \dots, a_n\right\} \subseteq B$ with $a_0 = a_n$ and $a_i \succ a_{i+1}$ for each $i \in \{0, 1, \dots, n-1\}$. Since $\succ$ is asymmetric, $n \geq 3$. It follows that $m(B)$ does not contribute to $p(x, A)$ for any $x \in A^*$. To avoid the conclusion that $p(A^*, A) < 1$ there must be some $E$ with $m(E) > 0$ that contributes to the probability of choosing *more than one* element of $A^*$. This requires that there exist $a_j, a_k \in A$ with $a_j \neq a_k$,

$$E \cap A \cap \succsim \left(a_j\right) = \left\{a_j\right\}$$

and

$$E \cap A \cap \succsim \left(a_k\right) = \left\{a_k\right\}.$$

But this implies $\left\{a_j, a_k\right\} \subseteq E \cap A$, and hence $a_j \notin \succsim \left(a_k\right)$ and $a_k \notin \succsim \left(a_j\right)$. Once again, we have a contradiction. ■

Lemma A.2 means that $\succ$ is connected and acyclic, hence transitive, on any $E$ with $m(E) > 0$. Since $\succsim$ is a preference relation it follows that $\succsim$ is linear on any such $E$. This completes the proof of Theorem 1.

## Appendix B. Proof of Theorem 2

Our proof closely follows the arguments in Aguiar's (2017) proof of his Theorem 1, as readers who are familiar with that paper will easily recognise. It will be useful to define the following (revealed) upper contour sets for each $a \in X$:

$$\triangleright (a) \equiv \{b \in X | b \triangleright a\}$$

$$\trianglerighteq (a) \equiv \{b \in X | b \trianglerighteq a\} = \{a\} \cup \triangleright (a).$$

The following important observation is noted by Aguiar (2017), p.51):

**Lemma B.1.** *Let $p$ be an RCF. Then*

$$p(a, A) = p(a, \trianglerighteq (a) \cap A)$$

*for every $A \subseteq X$ and every $a \in A$.*

**Proof.** If $b \in A$ and $b \notin \trianglerighteq (a)$ then $b \neq a$ and $p(a, B \cup b) = p(a, B)$ for all $B$ containing $a$. We may therefore successively remove each such $b$ from $A$ without affecting the probability that $a$ is chosen. ■

Next, we have the key implication of Axiom 1:

**Lemma B.2** (*cf.*, *Aguiar, 2017, Lemma 2*). *Let $p$ be an RCF. If $p$ satisfies Asymmetry, then*

$$p\left(\left\{a^*, a\right\}, A \cap \trianglerighteq (a)\right) = p\left(a^*, A \cap \triangleright (a)\right)$$

*for every $A \subseteq X$ and every $a \in A$.*

**Proof.** Suppose $a \in A \subseteq X$. For any $b \in \triangleright (a) \cap A$ the asymmetry of $\triangleright$ implies $p(b, \trianglerighteq (a) \cap A) = p(b, \triangleright (a) \cap A)$. Therefore:

$$\begin{aligned} p\left(\left\{a^*, a\right\}, A \cap \trianglerighteq (a)\right) &= 1 - \sum_{b \in \triangleright (a) \cap A} p(b, \trianglerighteq (a) \cap A) \\ &= 1 - \sum_{b \in \triangleright (a) \cap A} p(b, \triangleright (a) \cap A) \\ &= p\left(a^*, A \cap \triangleright (a)\right) \end{aligned}$$

which is the desired result. ■

Finally, we will need the following:

**Lemma B.3.** *Let $p$ be an RCF. Suppose there exists a preference relation $\succsim \subseteq X^2$, and a mapping $m : 2^X \to \mathbb{R}$ satisfying $\sum_{B \subseteq X} m(B) = 1$, such that Eq. (1) holds for any $A \subseteq X$ and any $x \in A$. Then $\triangleright \subseteq \succ$.*

**Proof.** Suppose $a, b \in X$ with $a \triangleright b$ and $a \notin \succ (b)$. Since $a \triangleright b$, we have $p(b, A) \neq p(b, A \cup a)$ for some $A \subseteq X$ with $b \in A$. In particular, $a \notin A$. Since $a \notin \succ (b)$ we deduce that

$$E \cap (A \cap \succ (b)) = E \cap ((A \cup a) \cap \succ (b))$$

for any $E \subseteq X$, and therefore $p(b, A) = p(b, A \cup a)$. This is the desired contradiction. ■

We now prove the "if" part of Theorem 2. Suppose $a, b \in X$ with $a \triangleright b$. Hence $a \neq b$ and (Lemma B.3) $a \succ b$. By the asymmetry of $\succ$ we therefore have $b \notin \succ (a)$, from which it follows that

$$C \cap \succsim (a) = (C \cup b) \cap \succsim (a)$$

for any $C \subseteq X$ with $a \in C$. Hence $p(a, C) = p(a, C \cup b)$ for any $C \subseteq X$ with $a \in C$, so $(b, a) \notin \triangleright$. Hence $\triangleright$ is asymmetric.

Next, we prove the "only if" part. Defining $f(A) \equiv p(a^*, X \setminus A)$, let $m : 2^X \to \mathbb{R}$ be the Möbius inverse (Shafer, 1976, Lemma 2.3) of $f$. Therefore

$$m(A) = \sum_{B : B \subseteq A} (-1)^{|A \setminus B|} f(B) \qquad (10)$$

and

$$f(A) = \sum_{B \subseteq A} m(B)$$

for any $A \subseteq X$. Hence, Eq. (2) holds for any $A \subseteq X$. Since $f(X) = 1$, it follows that $\sum_{B \subseteq X} m(B) = 1$. Using Lemmas B.1 and B.2, we now have, for any $A \subseteq X$ and any $a \in A$:

$$\begin{aligned} p(a, A) &= p(a, \trianglerighteq (a) \cap A) \\ &= p\left(a^*, \triangleright (a) \cap A\right) - p\left(a^*, \trianglerighteq (a) \cap A\right) \\ &= \left[1 - p\left(a^*, \trianglerighteq (a) \cap A\right)\right] - \left[1 - p\left(a^*, \triangleright (a) \cap A\right)\right] \\ &= \left[\sum_{B : B \cap A \cap \trianglerighteq (a) \neq \emptyset} m(B)\right] - \left[\sum_{B : B \cap A \cap \triangleright (a) \neq \emptyset} m(B)\right] \\ &= \sum_{B : B \cap A \cap \trianglerighteq (a) = \{a\}} m(B) \end{aligned}$$

Since $\trianglerighteq$ is a preference relation, this completes the proof of Theorem 2.

## Appendix C. Proof of Proposition 1

Suppose $a, b \in X$ with $b \succ a$. Then $p(b, A \cup a) = p(b, A)$ for any $A \subseteq X$ with $b \in A$. Hence, if $a \rhd b$ then $(b, a) \notin \succ$. But if $a \rhd b$ then there must exist some $E \in \mathrm{supp}(m)$ with $\{a, b\} \subseteq E$. It follows from Lemma A.1 that $\succsim$ is linear on $E$ and therefore $a \succ b$. Thus, if $a \rhd b$ then $a \succ b$ and there exists some $E \in \mathrm{supp}(m)$ with $\{a, b\} \subseteq E$. Conversely, if $a \succ b$ and there exists some $E \in \mathrm{supp}(m)$ with $\{a, b\} \subseteq E$, then $p(b, \{a, b\}) < p(b, \{b\})$ and therefore $a \rhd b$. This proves the first claim. It follows that $\unrhd \subseteq \succsim$.

Consider the second claim. Given the first claim, and the fact that the attention index in a GRCR is unique, it suffices to show that $\left(m, \succsim'\right)$ is a GRCR for $p$ when $\unrhd \subseteq \succsim'$. From the first claim, together with Lemma A.1, we deduce that $\unrhd$ is linear on any $E \in \mathrm{supp}(m)$. It follows that, in any GRCR, only those elements of the preference relation that are also in $\unrhd$ matter for choice probabilities. Any additional elements rank alternatives that do not co-exist in any category. We may therefore add or remove such elements to obtain an observationally equivalent GRCR. The second claim now follows.

## Appendix D. Proof of Proposition 3

It is obvious that (ii) implies (iii). That (i) implies (ii) follows from Proposition 1 in Kovach and Suleymanov (2023). We complete the proof by showing that (ii) implies (i) and (iii) implies (ii).

We start by showing that (iii) implies (ii). Suppose $p$ has a GRCR and a BR model.[19] Since $p$ has a GRCR, $\rhd$ is asymmetric (Theorem 3). Recall that $p$ has a BR representation if there is a linear preference relation, $\succsim \subseteq X^2$, and $\hat{m} \in \Delta\left(2^X\right)$ with $\hat{m}(E) > 0$ for all $E \subseteq X$, such that

$$p(x, A) = \sum_{B \subseteq A : B \cap \succsim(x) = \{x\}} \frac{\hat{m}(B)}{\sum_{E \subseteq A} \hat{m}(A)}$$

and

$$p\left(a^*, A\right) = \frac{\hat{m}\left(\emptyset\right)}{\sum_{E \subseteq A} \hat{m}(A)}$$

for all $A \subseteq X$ and all $x \in A$. We will show that $\succsim = \unrhd$. If $a \succ b$ then

$$p(b, \{a, b\}) = \frac{\hat{m}(\{b\})}{\sum_{E \subseteq \{a,b\}} \hat{m}(E)} < \frac{\hat{m}(\{b\})}{\sum_{E \subseteq \{b\}} \hat{m}(E)} = p(b, \{b\})$$

since $m$ has full support. Hence $\succ \subseteq \rhd$. It follows that $\unrhd$ is complete and antisymmetric, and contains the linear order, $\succsim$. We now easily deduce that $\succsim = \unrhd$: if $a \rhd b$ we must have $a \succ b$ since otherwise we obtain $b \rhd a$ from $\succ \subseteq \rhd$, which contradicts the asymmetry of $\rhd$. Hence, the canonical GRCR for $p$ is an RCR.

Finally, we must show that (ii) implies (i). Let us first observe that if $p$ has a BR model then $p$ satisfies *positivity*: $p(x, A) > 0$ for all $A \subseteq X$ and all $x \in A^*$. We also recall the MIDO axiom of Brady and Rehbeck (2016), which requires[20]:

$$\frac{p(a^*, A \setminus \{b\})}{p(a^*, A)} = \frac{p(a^*, B \setminus \{b\})}{p(a^*, B)} \tag{11}$$

for any $\{A, B\} \subseteq 2^X$ and any $b \in A \cap B$.

**Lemma D.1.** *Let $p$ be an RCF which has BR and RCR representations. Then $p$ satisfies the MIDO axiom.*[21]

**Proof.** Let $\gamma(x) = p(x, \{x\})$ for each $x \in X$. Positivity of $p$ ensures that

$$\gamma(x) = 1 - p\left(a^*, \{x\}\right) \in (0, 1).$$

We will show that

$$p\left(a^*, E\right) = \prod_{x \in E} (1 - \gamma(x)) \tag{12}$$

for any $E \subseteq X$, from which Eq. (11) easily follows. We argue by induction on $|E|$. The case $|E| = 1$ is immediate. Fix some integer $k > 1$ and suppose Eq. (12) holds for any $E$ with $|E| < k$. Let $E \subseteq X$ with $|A| = k$. It follows, as shown above, that the BR and RCR representations for $p$ have a common linear order: when $\unrhd$ is complete it is the unique linear order in any RCR representation for $p$. Let $\succsim$ denote this common linear order and let $x \in E$ be $\succsim$-maximal in $E$. Then Lemma 3.1 in Brady and Rehbeck (2016) implies that

$$p\left(a^*, E\right) = p\left(E^* \setminus \{x\}, E\right) p\left(a^*, E \setminus \{x\}\right).$$

By the inductive hypothesis we have:

$$\begin{aligned} p\left(a^*, E\right) &= p\left(E^* \setminus \{x\}, E\right) \prod_{y \in E \setminus \{x\}} (1 - \gamma(y)) \\ &= (1 - p(x, E)) \prod_{y \in E \setminus \{x\}} (1 - \gamma(y)). \end{aligned}$$

Since $p$ has an RCR representation and $x$ is $\succsim$-maximal in $E$, it follows that $p(x, E) = p(x, \{x\})$ so we have:

$$p\left(a^*, E\right) = (1 - p(x, \{x\})) \prod_{y \in E \setminus \{x\}} (1 - \gamma(y)) = \prod_{y \in E} (1 - \gamma(y)).$$

This completes the proof of the lemma. ■

Theorems 3.1 and 3.3 in Brady and Rehbeck (2016) imply that any RCR that has a BR representation and satisfies the MIDO axiom also has an MM representation. This establishes that (ii) implies (i) and completes the proof.

## Appendix E. Testing

Let $\mathcal{R}$ denote the set of all linear preference relations on $X$. Using Theorem 1, the "parameters" of a GRCR may be represented as vectors in $\Delta\left(\mathcal{R} \times 2^X\right)$. We write $\sigma(\succsim, E)$ for the $(\succsim, E)$-component of this vector. Let us associate each $(\succsim, E) \in \mathcal{R} \times 2^X$ with a linear preference relation on $X^*$ that ranks elements of $E$ the same way as $\succsim$, ranks all elements of $E$ above $a^*$ and ranks $a^*$ above all elements of $X \setminus E$, and define $p_{(\succsim, E)}$ to be the (deterministic) RCF generated by a decision-maker who maximises this preference relation over the entire menu. If $p$ has a (G)RCR representation, then it may be expressed in the form:

$$p(x, A) = \sum_{(\succsim, E) \in \mathcal{R} \times 2^X} \sigma(\succsim, E)\, p_{(\succsim, E)}(x, A) \tag{13}$$

for all $A \subseteq X$ and all $x \in A^*$. We have an RCR representation if there exists some $\succsim \in \mathcal{R}$ such that $\sigma\left(\succsim', E\right) = 0$ whenever $\succsim' \neq \succsim$. We have a GRCR representation if the following two restrictions are satisfied: first, for every $E \subseteq X$, we have $\sigma(\succsim, E)\, \sigma\left(\succsim', E\right) = 0$ whenever $\succsim \neq \succsim'$; and second, the restriction implied by condition (i) in Theorem 1 is satisfied. Note that each vertex of $\Delta\left(\mathcal{R} \times 2^X\right)$ corresponds to an RCR with $m$ having singleton support. Therefore, any RCF that can be represented by a mixture of GRCRs can also be represented by a mixture of RCRs. The mixtures of these two models are empirically indistinguishable: their empirical content is described by the set of RCFs that satisfy Eq. (13) for some $\sigma \in \Delta\left(\mathcal{R} \times 2^X\right)$.

The foregoing observations suggest a methodology for testing this common set of empirical restrictions. Let us first observe that satisfaction of condition Eq. (13) ensures that the components of $\sigma$ sum to

---

[19] Brady and Rehbeck (2016) refer to their model as a *random conditional choice set rule (RCCSR)*.

[20] The MIDO axiom comprises part of the *i-Independence* condition in Manzini and Mariotti (2014) – the part that restricts the probability of choosing the default.

[21] Note that any RCF with a BR model satisfies $p(a^*, E) \in (0, 1)$ for any non-empty $E \subseteq X$, so *Eq.* (11) is always well-defined when $p$ possesses such a model.

1.[22] Hence, there exists a vector $\sigma \in \Delta\left(\mathcal{R} \times 2^X\right)$ satisfying the linear system Eq. (13) iff there exists a vector $\sigma \in \mathbb{R}_+^{\mathcal{R}\times 2^X}$ satisfying this linear system. This is analogous to the problem of testing the *random attention and utility model (RAUM)* of Kashaev and Aguiar (2022).[23] The latter authors note that the testing methodology of Deb et al. (2023) is adequate to this task.

## Data availability

No data was used for the research described in the article.

## References

Aguiar, V.H., 2017. Random categorization and bounded rationality. Econom. Lett. 159, 46–52.

Aguiar, V.H., Boccardi, M.J., Kashaev, N., Kim, J., 2023. Random utility and limited consideration. Quant. Econ. 14, 71–116.

Barokas, G., 2021. Dynamic choice under familiarity-based attention. Soc. Choice Welf. 57, 703–720.

Brady, R.L., Rehbeck, J., 2016. Menu-dependent stochastic feasibility. Econometrica 84 (3), 1203–1223.

Brandenburger, A., Ghirardato, P., Pennesi, D., Stanca, L.M., 2024. Event valence and subjective probability. In: Carlo Alberto Notebooks, No. 717.

Cattaneo, M.D., Ma, X., Masatlioglu, Y., Suleymanov, E., 2020. A random attention model. J. Political Econ. 128 (7), 2796–2836.

Deb, R., Kitamura, Y., Quah, J.K., Stoye, J., 2023. Revealed price preference: theory and empirical analysis. Rev. Econ. Stud. 90 (2), 707–743.

Dutta, R., Horan, S., 2015. Inferring rationales from choice: Identification for rational shortlist methods. Am. Econ. J.: Microeconomics 7 (4), 179–201.

Falmagne, J.C., 1978. A representation theorem for finite random scale systems. J. Math. Psych. 18 (1), 52–72.

Grabisch, M., 2016. Set Functions, Games and Capacities in Decision Making. Springer International Publishing.

Huber, J., Payne, J.W., Puto, C., 1982. Adding asymmetrically dominated alternatives: violations of regularity and the similarity hypothesis. J. Consum. Res. 9 (1), 90–98.

Kashaev, N., Aguiar, V.H., 2022. A random attention and utility model. J. Econom. Theory 204, 105487.

Kovach, M., Suleymanov, E., 2023. Reference dependence and random attention. J. Econ. Behav. Organ. 215, 421–441.

Leland, J.W., 1994. Generalized similarity judgments: An alternative explanation for choice anomalies. J. Risk Uncertain. 9, 151–172.

Lichtenstein, S., Slovic, P., 1973. Response-induced reversals of preference in gambling: An extended replication in Las Vegas. J. Exp. Psychol. 101 (1), 16–20.

Manzini, P., Mariotti, M., 2007. Sequentially rationalizable choice. Am. Econ. Rev. 97 (5), 1824–1839.

Manzini, P., Mariotti, M., 2014. Stochastic choice and consideration sets. Econometrica 82 (3), 1153–1176.

McFadden, D.L., 2005. Revealed stochastic preference: a synthesis. Econom. Theory 26, 245–264.

Rubinstein, A., 1988. Similarity and decision-making under risk (is there a utility theory resolution to the Allais paradox?). J. Econom. Theory 46 (1), 145–153.

Shafer, G., 1976. A Mathematical Theory of Evidence. Princeton University Press.

Strzalecki, T., 2025. Stochastic Choice Theory. Cambridge University Press..

Suleymanov, E., 2018. Stochastic attention and search. Working Paper.

Turansick, C., 2025. An alternative approach for nonparametric analysis of random utility models. J. Econom. Theory 226, 105998.

Tversky, A., Sattath, S., Slovic, P., 1988. Contingent weighting in judgment and choice. Psychol Rev 95 (3), 371–384.

---

[22] This observation is essentially that of Turansick (2025), and may be seen as follows:

$$1 = \sum_{x\in A^*} p(x, A) = \sum_{(E,\succsim)\in\mathcal{R}\times 2^X} \sigma(E,\succsim) \sum_{x\in A^*} p_{(E,\succsim)}(x, A)$$
$$= \sum_{(E,\succsim)\in\mathcal{R}\times 2^X} \sigma(E,\succsim).$$

[23] Indeed, it is easy to show that any mixture of (G)RCRs may be expressed as a RAUM that satisfies the *stability* and *set monotonicity* conditions of Kashaev and Aguiar (2022). However, the latter class of models is strictly larger.