ARTICLE TEMPLATE

Modelling and Forecasting COVID-19 Stock Returns using Asymmetric GARCH-ICAPM with Mixture and Heavy-Tailed Distributions

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ABSTRACT
COVID-19 pandemic is an extreme event that created turmoil in stock markets around the world. This unexpected circumstance poses a critical question of whether the prevailing models can help predict the plummets of indices, hence the returns. In this study, we aim to analyze and forecast the daily stock returns using various generalized autoregressive conditional heteroscedastic (GARCH) models with intertemporal capital asset pricing structure and innovation following (1) a mixture of generalized Pareto and Gaussian distributions and (2) generalized error distribution that can capture extreme events. We also employ the parallel griddy Gibbs (GG) sampling, which is a Markov chain Monte Carlo method, to facilitate parameter estimation. Our simulation study shows that the GG estimation method outperforms the benchmark quasi-maximum likelihood estimation method. We then proceed to the empirical study of seven stock markets where the results from the in-sample period before the COVID-19 pandemic justify the use of the proposed GARCH models. The out-of-sample forecasts during the early COVID-19 period also show satisfactory results.

KEYWORDS
COVID-19 pandemic, Bayesian Markov chain Monte Carlo, Asymmetric GARCH, ICAPM, Mixture distribution, Generalized Pareto distribution

JEL CLASSIFICATION C11; C22; C53; G12

1. Introduction

It is widely known that stock markets are prone to shocks and the COVID-19 pandemic is the most recent shock that generates the instantaneous risk that most markets faced but to different degrees. Given the prevalent risk, having a suitable model to analyze stock returns is a challenging task. The trade-off between risk and expected return of a financial asset is a fundamental problem in financial econometrics as its functional form is still debated in the literature. One of the leading models is the intertemporal capital asset pricing model (ICAPM), which is originally introduced by Merton [36].

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The ICAPM is a linear function between the expected return and the variance of the market portfolio's return. Due to its simplicity, this model has been widely used and remains crucial in financial econometrics. For instance, Bali & Engle [4] study ICAPM with dynamic conditional correlations and determined daily intertemporal relation between the expected return and risk of 30 stocks in the Dow Jones Industrial Average. Later, Engle et al. [19] introduce the generalized autoregressive conditional heteroscedasticity (GARCH)-in-mean model and specified the linear function for the risk-return relationship in the bond market. The model is called GARCH with linear-in-variance risk premium or GARCH-ICAPM or GARCH-M. However, it does not provide a robust solution for the estimation.

Due to its conditional volatility component, since ARCH was invented by Engle [17] and extended by Bollerslev [9], the (G)ARCH family becomes a crucial model in financial time series analysis and forecasting. The GARCH model has evolved due to its properties that can easily fit with various probability distributions of financial returns. Many extensions of the GARCH family have been proposed to capture the important financial market characteristics that are volatility clustering, non-normal, fat-tailed, and asymmetrically distributed properties; see, among others, Engle [18] for a survey.

Within the GARCH family, the exponential-type GARCH family is the popular model. Geweke [25] and Milhøj [37] recommend the LogGARCH while Nelson [39] suggests the exponential GARCH (EGARCH). Zakoian [52] proposes the threshold GARCH (TGARCH), whereas Bildirici & Ersin [7] study the GARCH family models including GARCH, EGARCH, GJR-GARCH, TGARCH, nonlinear-GARCH, simple-asymmetric-GARCH, power-GARCH, asymmetric-power-GARCH, and nonlinear-power-GARCH. The GARCH models are also extended, for example, to the artificial neural network, called the ANN-GARCH-type model. Besides, GJR-GARCH, introduced by Glosten et al. [26], is a popular class under the exponential-type GARCH model.

GARCH(1,1) appears to be the most popular specification from its simplicity with decent forecasting performance, as the autocorrelation of the conditional volatility and the innovation does not usually have a long-lasting effect. As mentioned earlier, various distributions have been applied to the innovation of the GARCH family. One class of distributions, that might fit well with the bullish and bearish natures of stock markets, is a mixture of distributions. Given the COVID-19 pandemic, which is a rare event that produces extreme negative returns, having the innovation that follows the mixture of distributions with an extreme distribution, e.g., generalized Pareto distribution (GPD), might improve the GARCH performance in the analysis and forecasting under the severe shocks. We also compare the performance of the GPD innovation with that of the generalized error distribution (GED), which is a heavy-tailed distribution.

In this study, we have three major contributions. First, we propose a hybrid of the exponential-type GARCH and the ICAPM models that statistically fit better to the real data and provide insightful information on financial returns. Precisely, our proposed model reveals the empirical risk-free rate and risk premium of the financial market while capturing important volatility clustering characteristics of the returns. Further, we show that this combined model is easier to handle the coefficient terms of ICAPM than those of the standard GARCH models. It is also a constraint-free model that can facilitate the estimation for inference. See, among others, Francq et al. [22], Francq et al. [23], and Hafner & Kyriakopoulou [28] for the stochastic properties of the exponential-type GARCH model, which is compatible with the ICAPM model. Second, we apply a mixture of distributions that include a GPD to the exponential-type GARCH innovation. The mixture with the GPD allows us to analyze the returns in a pre-specified quantile level so that the model can capture the tail behaviour or extreme events, i.e., the lower and upper tails in our case. See Sahamkhadam et al. [45] and Caporale & Zekokh [10], among others, for more details on the related mixture distributions. Third, we implement the proposed models using the Bayesian griddy Gibbs (GG) method with a parallel computing technique that results in better accuracy performance than that of traditional QMLE.

Recently, a popular estimation method for the exponential-type GARCH model is a quasi-maximum likelihood (QMLE) approach, see Francq et al. [22] and Hafner & Kyriakopoulou [28]. However, in general, the QMLE estimator is known for its dependence on initial conditions and is less statistically efficient for this type of model. Hence, we use the Bayesian Markov chain Monte Carlo (MCMC) that provides robustness and superiority of forecasting performance over that of the QMLE. Note, however, that the MCMC method requires further mathematical complexity and more computational time. See Chang et al. [11], Kolm et al. [32], Pareek & Thakkar [41], and Patton [42], among others, for the relevant survey of estimation method using the QMLE and the MCMC. To facilitate the estimation and reduce the computational time for the MCMC, many algorithms have been proposed. Andrieu et al. [1] and Creal & Tsay [14] use particle Gibbs sampling algorithm while Wichitaksorn & Choy [51] and Ausín & Galeano [3] use the GG sampling. Multiple-try Metropolis forward filtering backward sampling (FFBS) approach and the multiple-trial Metropolis independent sampler FFBS approach are applied in Billio et al. [8]. See also Ardia et al. [2] and Li & Kang [34] for the Metropolis-Hastings algorithm and its variations.

Hence, in this study, we choose the MCMC approach to estimate our proposed exponential-type GARCH-ICAPM-Mixture model. To mitigate the adverse effect of longer computational time, we incorporate the parallel computing technique, into the GG algorithm. This allows us to compute a massive amount of data at the same time. See Benczúr [5], Fujiwara et al. [24], Lam et al. [33], Lopatka & Czyzewski [35], Prudencio & Cheung [43], Sanchez-Vazquez et al. [46] and Wichitaksorn & Choy [51], among others, for more details on the parallel computing technique. Based on the simulation study results, our parallel-GG algorithm outperforms the QMLE. The empirical study using the data during the early period of the COVID-19 pandemic also shows the favorable forecasting performance of our proposed models and method.

The rest of the paper is organized as follows. Section 2 presents the class of exponential-type and GJR-GARCH with mixture distribution for the innovation and discusses the Bayesian MCMC method using the parallel computing technique. Section 3 performs the simulation study. Section 4 shows the empirical study results. Section 5 concludes.

2. Models and Estimation

This section introduces our proposed models where we include the ICAPM structure in the observation equation of GARCH, the exponential-type GARCH, and the GJR-GARCH, with the innovation following the mixture of the Gaussian distribution and the heavy-tailed GPD. In addition, we facilitate the model estimation using the GG algorithm together with the parallel computing technique. To assess the performance of our proposed models and method, we compare them with the EGARCH-ICAPM using the generalized error distribution (GED) innovation as the benchmark model and the quasi-maximum likelihood estimation as the benchmark method.

2.1. Models

We first consider the ICAPM given in Merton [36], which is a linear-in-variance function of a financial return series. For $t = 1, ..., T$, let y_t denote a financial return series. Based on Hafner & Kyriakopoulou [28], the ICAPM is then given by

$$y_t = \lambda_1 + \lambda_2 h_t \ + \varepsilon_t, \tag{1}$$

where t is the time index, λ_1 is the expected risk-free rate with the hypothetical value of zero, λ_2 is the expected (positive) risk premium, h_t is the conditional volatility, $\varepsilon_t = \sqrt{h_t} z_t$ is the innovation with mean zero and the conditional variance (or volatility) h_t, and z_t is the independent and identically distributed (iid) random variable with zero mean and unit variance. Note that we let λ_1 be an unknown and random that needs to be estimated while it is known and fixed in Hafner & Kyriakopoulou [28]. This ICAPM structure is then applied to all GARCH models implemented in this study.

The specifications we used to model the conditional volatility h_t include the typical GARCH(p,q), the exponential-type GARCH that are EGARCH(p,q,r) and Log-GARCH(p,q), and the GJR-GARCH(p,q). Prior works indicated that the order lags for the innovation and volatility terms are best performing when $p = 1$ and $q = 1$. We then follow that in our analysis. For illustrative purposes, we describe below the specification of all GARCH models used here in detail.

The typical GARCH(p,q) model is used as the starting model where we follow Bollerslev [9] and given by

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \nu_j h_{t-j},$$

where $p \geq 0, q > 0,\ \omega > 0,\ \alpha_i \geq 0, \nu_j \geq 0,\ 0 \leq \alpha_i + \nu_j \leq 1,\ 1 \leq\ i \leq\ p, 1 \leq\ j \leq\ q$ and p, q are the discrete value to ensure the conditional variance h_t being an almost surely strictly positive or a strictly stationary process. In this model, the parameter space of volatility coefficients is denoted as $\theta = \{\omega, \alpha_i, \nu_j, 1 \leqslant i \leqslant p, 1 \leqslant j \leqslant q\}$.

We then turn to a popular GARCH model extension, which is an exponential-GARCH(p,q,r) or EGARCH(p,q,r) of Nelson [39] and given by

$$\log h_t = \omega + \sum_{i=1}^{p} \gamma_i z_{t-i} + \sum_{j=1}^{q} \delta_j |z|_{t-j} + \sum_{k=1}^{r} \nu_k \log h_{t-k},$$

where the conditional volatility is now log volatility. The parameter space is then $\theta = \{-\infty \leqslant \omega, \gamma_i, \delta_j \leqslant \infty, |\nu_k| < 1, 1 \leqslant i \leqslant p, 1 \leqslant j \leqslant q, 1 \leqslant k \leqslant r\}$. The "news impact" function $g(z_{t-1})$ of the EGARCH model is $\omega + \sum_{i=1}^{p} \gamma_i z_{t-i} + \sum_{j=1}^{q} \delta_j |z|_{t-j}$ while ν is the volatility persistence parameter, γ and δ are the shock asymmetry and the size effect, respectively. If $\gamma > 0$, that means positive shocks increase the volatility more than negative shocks of the same size and vice versa. For the review of the GARCH family including the exponential-type GARCH and the GJR-GARCH, see, among others, Bera & Higgins [6], Hentschel [29], and Jose Rodriguez & Ruiz [31].

Another popular exponential-type GARCH is the LogGARCH(p,q) from Francq et al. [23] and given by

$$\log h_t = \omega + \sum_{i=1}^{p} \left(\alpha_i^+ I_{\{\varepsilon_{t-i}>0\}} + \alpha_i^- I_{\{\varepsilon_{t-i}<0\}}\right) \log \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \nu_j \log h_{t-j},$$

where I is an indicator function. The parameter space is now $\theta = \{-\infty \leqslant \omega, \alpha_i^+, \alpha_i^- \leqslant \infty, |\nu_j| < 1, 1 \leqslant i \leqslant p, 1 \leqslant j \leqslant q\}$ for the existence of a stationary solution. The news impact function of this LogGARCH model is $\omega + \sum_{i=1}^{p} \left(\alpha_i^+ I_{\{\varepsilon_{t-i}>0\}} + \alpha_i^- I_{\{\varepsilon_{t-i}<0\}}\right) \log \varepsilon_{t-i}^2$. When $\alpha_i^- = \alpha_i^+$, the model becomes the symmetric LogGARCH process. Note that the LogGARCH has a well-known problem, i.e., if the ε_{t-i} term equals zero, the conditional volatility cannot be generated. For the discussion of the LogGARCH process, see, among others, Sucarrat & Escribano [48], and Sucarrat et al. [49]. For the recent studies of the exponential-type GARCH process, see, Hafner & Kyriakopoulou [28], Caporale & Zekokh [10], Francq et al. [23], and Bildirici & Ersin [7], among others.

The GJR-GARCH(p,q) by Glosten et al. [26] is the last model we used in the analysis. This model becomes better known among the GARCH family and given by

$$h_t = \omega + \sum_{i=1}^{p} \left(\alpha_i + \gamma_i I_{\{\varepsilon_{t-i}<0\}}\right) \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \nu_j h_{t-j}$$

where $\omega > 0, \alpha_i, \nu_j \geq 0,\ \nu_j + \gamma_i \geq 0$. α_i and γ_i are the size effect and shock asymmetry, respectively. Hence, the parameter space is $\theta = \{\omega, \alpha_i, \gamma_i, \nu_j, 1 \leqslant i \leqslant p, 1 \leqslant j \leqslant q\}$ while the news impact function of the GJR-GARCH is $\omega + \sum_{i=1}^{p} \left(\alpha_i + \gamma_i I_{\{\varepsilon_{t-i}<0\}}\right) \varepsilon_{t-i}^2$. See, among others, Goel et al. [27], Caporale & Zekokh [10], Oh & Patton [40], Fengler & Okhrin [21], and De Lira Salvatierra & Patton [15], for the recent studies and discussion of this type of model.

2.2. Model Innovations

With all GARCH model specifications mentioned above, we apply two types of innovations including the mixture distribution and the GED. The former is what we expect to perform well in the COVID-19 situation while the latter is our benchmark for comparison. In the literature, the mixture distributions have been used in data-intensive research in many fields such as astrophysics and space, meteorology, machine learning, big data analytics, time-series analytics, and modern finance.

2.2.1. Mixture Distribution

The mixture distribution can be represented by either a finite set of cumulative distribution functions (cdf), $P_k(z_t)$, or probability density functions (pdf), $p_k(z_t)$, for $k = 1, \ldots, n$. Hence, the mixture distribution function and the mixture density function, respectively, are a sum such that

$$F(z_t) = \sum_{k=1}^{n} w_k P_k(z_t),$$

$$f(z_t) = \sum_{k=1}^{n} w_k p_k(z_t).$$

where weight $w_k > 0$ and $\sum w_k = 1$. In financial time series, the mixture distribution is popularly applied due to its flexibility in capturing asymmetrical tails of financial returns. If the mixing weight w_k is pre-specified, it can also represent the quantile level of the mixture distribution.

In this study, we follow Sahamkhadam et al. [45] that use the mixture of Gaussian and GPD densities. Precisely, we have the GPD at the lower and upper tails to capture the extreme (rare) events while the Gaussian can represent the usual (normal) circumstances. We expect asymmetric-GARCH-ICAPM models with this mixture distribution can fit well with the returns under the COVID-19 pandemic. The mixture pdf is then given by

$$p(z_t) = \begin{cases} \frac{1}{\beta^L}\left(1+\xi^L \frac{(z_t-\mu^L)}{\beta^L}\right)^{-\frac{1}{\xi^L}-1} & , z_t \leq \Phi^{-1}(a) \\ \phi(z_t) & , \Phi^{-1}(a) < z_t < \Phi^{-1}(b) \\ \frac{1}{\beta^R}\left(1+\xi^R \frac{(z_t-\mu^R)}{\beta^R}\right)^{-\frac{1}{\xi^R}-1} & , z_t \geq \Phi^{-1}(b) \end{cases} \quad (2)$$

where a and b are a pre-specified quantile, $-\infty < \mu < \infty$ is a location parameter, $0 < \beta < \infty$ is a scale parameter, $-\infty < \xi < \infty$ is a shape parameter, and L and R denote the lower and upper tails, respectively. The support of z_t of GPD is $z_t \geq 0$ when $\xi \geq 0$ and $\mu \leq z_t \leq \mu - \beta/\xi$ when $\xi < 0$, and $0 \leq \xi < 0.5$ represents a fat tail and exists at least up to second moment. The GPD consists of, in the sense of generalization, an ordinary Pareto distribution when $\xi > 0$, an exponential distribution when $\xi = 0$, and a short-tailed, Pareto type II distribution when $\xi < 0$. We denote $X \sim GPD(\mu, \beta, \xi)$, therefore, $E(X) = \mu + \frac{\beta}{1-\xi}$ when $\xi < 1$ and $Var(X) = \frac{\beta^2}{(1-\xi)^2(1-2\xi)}$ where $\xi < 0.5$ to ensure the variance is defined, see, e.g., Hosking & Wallis [30] and Singh & Guo [47]. While $\phi(\cdot)$ and $\Phi^{-1}(\cdot)$ are, respectively, the standard Gaussian pdf and inverse cdf.

2.2.2. GED Innovation

For the sake of comparison, we also apply the GED innovation, proposed by Nadarajah [38], to test its forecasting performance under extreme events. The GED is a useful distribution in time series analysis and is also known as the exponential power distribution. Let $X \sim GED(\mu, \beta, \xi)$. It includes the Gaussian and Laplace distributions, when $\xi = 2$ and $\xi = 1$, respectively. As $\xi \to \infty$, the pdf converges pointwise to a uniform pdf on $(\mu - \infty, \mu + \infty)$. When $\xi < 2$ represents fat-tail distribution and $\xi > 2$ represents lighter-tail distribution than normal.

With the observation equation in (1), the GED pdf for the innovation is given by

$$p(z_t) = \frac{\xi}{2\beta\Gamma(1/\xi)} e^{-(|z_t-\mu|/\beta)^{\xi}}$$

where $-\infty < \mu < \infty$, $0 < \beta < \infty$ and $0 < \xi < \infty$ are the location, scale and shape parameter, respectively. The support of z_t is $-\infty < z_t < \infty$.

2.3. Posterior Inference

2.3.1. Likelihood Function and Prior

In the estimation of all GARCH-ICAPM models in our study, we use the Bayesian MCMC method as we expect it to return better results than those of the traditional method, e.g., quasi-maximum likelihood estimation (QMLE). Results from the simulation study confirm this by showing that our Bayesian method fits better than the QMLE in terms of the Akaike information criterion (AIC) and Bayesian information criterion (BIC), see Section 3 for the estimation results. In addition, all of the model likelihoods are intractable. Using the sampling method from the Bayesian MCMC can conveniently get the parameter estimates than the mountain-climbing QMLE. Hence, with the Bayesian MCMC, we need to estimate the model parameters using the posterior density, which is proportional to the prior and the likelihood as $p(\theta|\ y) \propto p(\theta)\, L(y|\theta)$ where $p(\theta|\ y)$ is the posterior density, $p(\theta)$ is the prior density, and $L(y|\theta)$ is the likelihood function.

Following from the observation equation in (1), let $z_t = (y_t - \lambda_1 - \lambda_2 h_t)/\sqrt{h_t}$. To standardize the innovation with the mixture distribution in Equation (2), we set $\beta = (1-\xi)(1-2\xi)^{1/2}$. Therefore, the log-likelihood function of our proposed model with the mixture of Gaussian and two-sided GPD innovation is then

$$
\begin{aligned}
\ell(\theta) &= \sum_{t=1}^{T} log[q^L p_L^{GPD}(z_t) + (1-q^L-q^R)p^G(z_t) + q^R p_R^{GPD}(z_t)] \\
p_L^{GPD}(z_t) &= \frac{1}{\sqrt{h_t}(1-\xi^L)(1-2\xi^L)^{1/2}}\left(1+\frac{\xi^L(y_t-\lambda_1-\lambda_2 h_t)}{\sqrt{h_t}(1-\xi^L)(1-2\xi^L)^{1/2}}\right)^{-\frac{1}{\xi^L}-1}, \\
p^G(z_t) &= \frac{1}{(2\pi h_t)^{1/2}} exp\left[\frac{-(y_t-\lambda_1-\lambda_2 h_t)^2}{2h_t}\right], \\
p_R^{GPD}(z_t) &= \frac{1}{\sqrt{h_t}(1-\xi^R)(1-2\xi^R)^{1/2}}\left(1+\frac{\xi^R(y_t-\lambda_1-\lambda_2 h_t)}{\sqrt{h_t}(1-\xi^R)(1-2\xi^R)^{1/2}}\right)^{-\frac{1}{\xi^R}-1}
\end{aligned}
\tag{3}
$$

where q is a pre-specified quantile level or weight of a pdf, h_t is the conditional variance for the all GARCH family including GARCH(1,1) and GJR-GARCH(1,1) while h_t of EGARCH(1,1,1) and LogGARCH(1,1) is the conditional log-variance. Note that the pre-specified quantile q can be defined as the left and right boundary of the distribution tail where we follow Sahamkhadam et al. [45] and Wang et al. [50] to set q^L and q^R at 0.10. However, to assess the ability of our models in capturing the extreme events, we also set q at 0.01 and 0.05 in the simulation study.

For the EGARCH(p,q,r)-ICAPM model as that in Hafner & Kyriakopoulou [28], with the standardized GED innovation where we set $\beta = \sqrt{\Gamma(1/\xi)/\Gamma(3/\xi)}$, the log-likelihood function is then given by

$$
\begin{aligned}
\ell(\theta) = (T-1)\log\left(\frac{\Gamma(3/\xi)^{1/2}}{\Gamma(1/\xi)^{3/2}}\xi\right) - 2\sum_{t=2}^{T}\log\left(\sqrt{exp(h_t)}\right) \\
-\sum_{t=2}^{T}\left(\left|\frac{y_t-\lambda_1-\lambda_2 exp(h_t)}{\sqrt{exp(h_t)}}\right|/\sqrt{\frac{\Gamma(1/\xi)}{\Gamma(3/\xi)}}\right)^{\xi}
\end{aligned}
\tag{4}
$$

where $\Gamma(\cdot)$ is a gamma function.

In the posterior inference, prior specifications are a crucial part as it determines the robustness and accuracy of the posterior statistics, e.g., mean and standard deviation. The parameters of interest include the risk premium, which is expected to be positive, the risk-free rate to be very close to zero, and the conditional volatility parameters and their innovation parameters. With different specifications of those parameters, we need to apply different priors to different model parameters. We set priors for the GARCH(1,1)-ICAPM-Mixture process, as

$$
\begin{aligned}
\lambda_1 &\backsim N\left(\mu_{\lambda_1},\sigma^2_{\lambda_1}\right), \\
\lambda_2, &\backsim G\left(\alpha_{\lambda_2},\beta_{\lambda_2}\right), \\
\xi^L &\backsim G\left(\alpha_{\xi^L},\beta_{\xi^L}\right), \\
\xi^R &\backsim B\left(\alpha_{\xi^R},\beta_{\xi^R}\right),
\end{aligned}
\qquad
\begin{aligned}
\omega &\backsim B\left(\alpha_\omega,\beta_\omega\right), \\
\alpha &\backsim B\left(\alpha_\alpha,\beta_\alpha\right), \\
\nu &\backsim G\left(\alpha_\nu,\beta_\nu\right),
\end{aligned}
$$

for the EGARCH(1,1,1)-ICAPM-Mixture process, as

$$
\begin{aligned}
\lambda_1 &\backsim N\left(\mu_{\lambda_1},\sigma^2_{\lambda_1}\right), \\
\lambda_2 &\backsim N\left(\mu_{\lambda_2},\sigma^2_{\lambda_2}\right), \\
\omega &\backsim N\left(\mu_\omega,\sigma^2_\omega\right), \\
\gamma &\backsim N\left(\mu_\gamma,\sigma^2_\gamma\right),
\end{aligned}
\qquad
\begin{aligned}
\delta &\backsim B\left(\alpha_\delta,\beta_\delta\right), \\
\nu &\backsim B\left(\alpha_\nu,\beta_\nu\right), \\
\xi^L &\backsim Unif\left(a_{\xi^L},b_{\xi^L}\right), \\
\xi^R &\backsim Unif\left(a_{\xi^R},b_{\xi^R}\right),
\end{aligned}
$$

for the LogGARCH(1,1)-ICAPM-Mixture process, as

$$
\begin{aligned}
\lambda_1 &\backsim N\left(\mu_{\lambda_1},\sigma^2_{\lambda_1}\right), \\
\lambda_2 &\backsim G\left(\alpha_{\lambda_2},\beta_{\lambda_2}\right), \\
\omega &\backsim B\left(\alpha_\omega,\beta_\omega\right), \\
\alpha^+ &\backsim N\left(\mu_{\alpha^+},\sigma^2_{\alpha^+}\right),
\end{aligned}
\qquad
\begin{aligned}
\alpha^- &\backsim B\left(\alpha_{\alpha^-},\beta_{\alpha^-}\right), \\
\nu &\backsim G\left(\alpha_\nu,\beta_\nu\right), \\
\xi^L &\backsim G\left(\alpha_{\xi^L},\beta_{\xi^L}\right), \\
\xi^R &\backsim B\left(\alpha_{\xi^R},\beta_{\xi^R}\right),
\end{aligned}
$$

for GJR-GARCH(1,1)-ICAPM-Mixture process, as

$$
\begin{aligned}
\lambda_1 &\backsim N\left(\mu_{\lambda_1},\sigma^2_{\lambda_1}\right), \\
\lambda_2 &\backsim G\left(\alpha_{\lambda_2},\beta_{\lambda_2}\right), \\
\omega &\backsim B\left(\alpha_\omega,\beta_\omega\right), \\
\alpha &\backsim B\left(\alpha_\alpha,\beta_\alpha\right),
\end{aligned}
\qquad
\begin{aligned}
\gamma &\backsim G\left(\alpha_\gamma,\beta_\gamma\right), \\
\nu &\backsim G\left(\alpha_\nu,\beta_\nu\right), \\
\xi^L &\backsim G\left(\alpha_{\xi^L},\beta_{\xi^L}\right), \\
\xi^R &\backsim B\left(\alpha_{\xi^R},\beta_{\xi^R}\right),
\end{aligned}
$$

and for the EGARCH(1,1,1)-ICAPM-GED process, as

$$\begin{aligned} \lambda_1 &\backsim B\left(\alpha_{\lambda_1}, \beta_{\lambda_1}\right), \\ \lambda_2 &\backsim N\left(\mu_{\lambda_2}, \sigma^2_{\lambda_2}\right), \\ \omega &\backsim N\left(\mu_{\omega}, \sigma^2_{\omega}\right), \\ \nu &\backsim G\left(\alpha_{\nu}, \beta_{\nu}\right), \end{aligned} \qquad \begin{aligned} \gamma &\backsim N\left(\mu_{\gamma}, \sigma^2_{\gamma}\right), \\ \delta &\backsim G\left(\alpha_{\delta}, \beta_{\delta}\right), \\ \xi &\backsim G\left(\alpha_{\xi}, \beta_{\xi}\right), \end{aligned}$$

where $B(\cdot,\cdot)$ denotes beta distribution, $Unif(\cdot,\cdot)$ denotes uniform distribution, and $G(\cdot,\cdot)$ denotes gamma distribution. These priors and their hyperparameter values are chosen based on their forecasting performance.

2.3.2. MCMC Algorithm

With the priors given above, the posterior densities become intractable. Hence, to facilitate the parameter estimation, we employ the GG sampling to obtain the posterior statistics for the model parameters that have different supports. Under the GG, each model parameter is sampled one-by-one directly from the univariate conditional posterior density. This algorithm is similar to Gibbs sampling, which accepts all samples, but it is approximated at a finite number of grid points. See, among others, Wichitaksorn & Choy [51] and Ritter & Tanner [44] for more details on the GG sampling.

Though the GG algorithm fits well with the intractable posteriors for parameters with different constraints, it requires a lot of computational time as it needs to evaluate all grid points; the higher number of grid points, the more accurate the model parameter is. Hence, we follow Wichitaksorn & Choy [51] to parallelize the grid points and evaluate them at the same time. This parallel computing technique can, more or less, fasten the GG estimation procedure. Note that the parallel GG may not be reversible but it is simpler to implement, less computationally expensive, and has significantly smaller autocorrelation coefficients of the Markov chains generation than other variant methods. See more discussion on the convergence of Griddy Gibbs sampling and other perturbed Markov chains in Dinh et al. [16]. The generic algorithm below shows the parallel-GG estimation steps that can be applied to all models.

Parallel-Griddy-Gibbs Sampling Algorithm:

Let $\Theta = \{\theta_1, \ldots, \theta_K\}$ and $s = 1, 2, \ldots, S$, respectively, denote the parameter space and the iterations.

(1) Take an initial vector of the estimates either from a random vector or the QMLE method, $\hat{\theta}_k^{(0)}, k = 1, 2, \ldots, K$.
(2) Repeat for $s = 1, 2, \ldots, S$, and for $k = 1, 2, \ldots, K$.
  (a) Draw candidate $\hat{\theta}_k^{(s)}$ where $\hat{\theta}_k^{(s)}$ is divided into G grid points as $\hat{\theta}_k^{(s,1)}, \ldots, \hat{\theta}_k^{(s,G)}$.
  (b) Calculate full conditional posterior density for each grid point $p(\hat{\theta}_k^{(s,g)}|\hat{\theta}_1^{(s,g)}, \cdots, \hat{\theta}_{k-1}^{(s,g)}, \hat{\theta}_{k+1}^{(s-1,g)}, \cdots, \hat{\theta}_K^{(s-1,g)})$ and let $p(\hat{\theta}_k^{(s,g)}|\cdot) = w_k^g$ where $g = 1, \cdots, G$.
  (c) Normalize w_k^g as $\widetilde{w}_k^g = \frac{w_k^g}{\sum_{g=1}^G w_k^g}$ and cumulative sum $\widetilde{w}_k^g$ implies the empirical cdf $F\left(\hat{\theta}_k^{(s,g)}\right)$ where $F\left(\hat{\theta}_k^{(s,G)}\right) = \widetilde{w}_k^G = 1$
  (d) Draw $u \sim Unif(0, 1)$ and set $\hat{\theta}_k^{(s)} = \hat{\theta}_k^{(s,g)}$ if $F(\ \widehat{\theta}_k^{(s,g-1)}) < u \le F(\widehat{\theta}_k^{(s,g)})$

(3) Repeat Step 2 for $\hat{\theta}_1^{(s)}, \hat{\theta}_2^{(s)}, \ldots, \hat{\theta}_K^{(s)}$.

It is essential to discard an initial set of the posterior draws to remove the effect of the initial conditions. Moreover, the initial range is an important factor to obtain the robustness of model parameters and the initial range can be obtained through some estimation methods. This is known as adaptive GG sampling. In this paper, we use the QMLE method by Hafner & Kyriakopoulou [28] to obtain the initial range of the values. Note in Step (d) that u will always fall into an interval, which is between two grid points, e.g., $g-1$ and g. For example, if u is less than or equal $F(\hat{\theta}_k^{(s,g-1)})$ but greater than $F(\hat{\theta}_k^{(s,g-1)})$, $\hat{\theta}_k^{(s)}$ will be set as $\hat{\theta}_k^{(s,g-1)}$. However, if u is above $F(\hat{\theta}_k^{(s,g)})$ but less than or equal $F(\hat{\theta}_k^{(s,g+1)})$, $\hat{\theta}_k^{(s)}$ will be set as $\hat{\theta}_k^{(s,g+1)}$. Since we parallelize the grid-point evaluations, we call our estimation method the parallelization-adaptive-GG sampling method. Another important component in the GG sampling is the choice of the number of grid points where we need to trade-off between the accuracy and the computational time Dinh et al. [16]. The number of grid points needs not be the same for all iterations. It can be adjusted to be adaptive. In the end, we collect the posterior samples to produce relevant statistics and densities for inference.

3. Simulation Study

In the simulation study, we aim to assess the performance of the proposed GARCH models including the GARCH(1,1)-ICAPM-Mixture, EGARCH(1,1,1)-ICAPM-Mixture, LogGARCH(1,1)-ICAPM-Mixture, GJR-GARCH(1,1)-ICAPM-Mixture and EGARCH(1,1,1)-ICAPM-GED, and the parallel-GG estimation method. In all cases, we compare the estimation results between the parallel-GG with those of the full one-step QMLE using 100 datasets. Each dataset generates 1000 simulated observations for the proposed models. The vector of initial value $\hat{\theta}_k^{(0)}$ is a vector of random numbers where we found there is a convergence issue for the full one-step QMLE method while our proposed parallel-GG method converges well.

Note that all true parameter values in the simulation study were chosen based on the empirical data and prior works. The likelihood function in Equations (3)-(4) is essential for model calibration, either in terms of the full one-step QMLE or the GG method. The QMLE is performed by maximizing the log-likelihood function and this optimization method can be computed using the `fmincon` function in MATLAB.

According to the GG setting, we ran 3000 MCMC iterations with 300 grid points including 500 burn-ins, and then, kept every fifth draw, which yielded 500 posterior samples. We implemented the experiments in MATLAB and performed them on a desktop computer with an Intel Core i7-9700 CPU, 3.00 GHz, and 32 GB RAM. The prior distribution of all proposed models used in the simulation study can be found in Table A1 in the supplementary material. To compare across models, we calculated the simulation inefficiency factor (SIF); see, e.g., Chib & Greenberg [13], the standard deviation (SD), the mean absolute deviation (MAD), and the computational time. Also, AIC, BIC, and deviance information criterion (DIC) (except for the QMLE method) were used to assess the in-sample fit.

The results in Table 1 show that our proposed models using the parallel-GG method are more favorable than those of the QMLE. Our parameter estimates are close to their true values and have reasonable SIFs. Precisely, the MADs are lower for the MCMC estimates. Furthermore, according to the average AICs and BICs, the MCMC algorithm does better, except for the EGARCH(1,1,1)-ICAPM-GED. The one-step QMLE could have a convergence issue as the SDs are significantly variable and of high value, which we chose not to report in Table 1.

[Table 1 about here]

The computational time is higher for the asymmetric-GARCH-ICAPM with mixture innovation than that of the GED as there are more parameters in the model. As an example, the convergence draws in Figure 1 show that the GG-MCMC algorithm for the EGARCH(1,1,1)-ICAPM with mixture innovation process converges well. The convergence draws of other models can be found in the supplementary material.

[Figure 1 about here]

To assess the abilities in capturing the extreme events of our models, we also estimate, as an example, the EGARCH-ICAPM-Mixture model in extreme quantiles including 0.01, 0.05, and 0.10. The results in Table 2 show that for lower quantiles QMLE seems to return lower MAD for some parameters. However, for the overall model, based on the AIC and BIC the parallel-GG method performs better.

[Table 2 about here]

In summary, our proposed models with the parallel-GG estimation are superior in terms of accuracy for parameter values and model performance. In practice, the QMLE estimates may be used as the starting value for the MCMC to achieve faster convergence and less computational time. However, this is absolutely not necessary as the QMLE sometimes does not converge well. Though the asymmetric-GARCH-ICAPM with mixture innovation requires more computational time than that of the GED, however, their property in capturing the extreme event cannot be ignored and should be investigated further in the empirical study.

4. Empirical Study

Based on the simulation study results, we then implemented our proposed GARCH models with the ICAPM structure and mixture innovation for the daily stock returns from various markets around the world between January 2009 and April 2020. The data were obtained from Yahoo Finance and other relevant sources for the US (S&P500), Hong Kong (HSI), Australia (ASX200), New Zealand (NZX50), UK (FTSE100), Japan (Nikkei225), and Thailand (SET). We chose the start date at the beginning of 2009 to avoid the effect of the global financial crisis in 2008. For the out-of-sample forecast evaluation purpose, we split the dataset into the training set that starts from January 2009 to January 2020 and the test set is from February 2020 to April 2020. The test set represents our main aim to assess the forecasting performance of our exponential-type GARCH models during the COVID-19 pandemic, especially in the early period when stock markets around the world tumbled severely. Note that, for each market, the returns in our study are the daily demeaned log returns, $r_t = (ln\,(S_t/S_{t-1}) - \bar{r}_t) * 100$, where S_t is the stock market index at time t.

Table 3 shows summary statistics for the seven markets from both training and test sets. As expected and based on the Jarque-Bera test results, all stock returns are non-normally distributed except the HSI returns in the out-of-sample period. Most of them are the right-skewed distribution, which indicates the mean-adjusted returns are still positive, except the out-of-sample returns from the NZX50 and Nikkei225. This might be due to the prolonged effect of COVID-19 that made them negative returns. The high kurtosis reveals the fat-tailed property of the returns. All these properties can be confirmed by the time series and density plots in Figures 2 and 3. Hence, modelling these returns using our GARCH models with a mixture innovation is justified.

[Table 3 about here]

[Figures 2 and 3 about here]

Regarding the estimation, there are slight modifications for the MCMC settings from the simulation study. In all cases, we ran 10000 MCMC iterations, using 500 grid points, including 2000 burn-ins, and kept every tenth draw that yielded 800 posterior samples. Note that the prior distribution of all proposed models for each stock return can be found in Table A2 in the supplementary material.

Figure 4, as an example, shows the convergence draws and the density plots of posterior samples of the EGARCH(1,1,1)-ICAPM with mixture innovation process for the S&P500 returns. See additional empirical study results in the supplementary material. Overall, the convergence draws show that the parameter estimates reach a steady state while the density plots look reasonable.

[Figure 4 about here]

It can be clearly seen from Table 4 that most of our exponential-type-GARCH-ICAPM models, except EGARCH(1,1,1)-ICAPM with GED innovation, return the hypothetical value of zero for $\hat{\lambda}_1$, that confirms the assumption of risk-free rate under the ICAPM. Though the $\hat{\lambda}_1$ from EGARCH(1,1,1)-ICAPM with GED innovation model is significantly different from zero, its value is very tiny and its effect can be negligible. As expected, the parameter estimates of $\hat{\lambda}_2$, which is the risk premium, from all markets are positive and statistically significant. However, there are disagreements on $\hat{\lambda}_2$ across markets. The returns for the S&P500 and NZX50 each have a higher risk premium from two different models including the EGARCH(1,1,1)-ICAPM with mixture innovation and the GJR-GARCH(1,1)-ICAPM with mixture innovation for the former, and the GARCH(1,1)-ICAPM with mixture innovation and the GJR-GARCH(1,1)-ICAPM with mixture innovation for the latter while the FTSE100 and N225 have the higher risk premium from the GARCH(1,1)-ICAPM with mixture innovation and the GJR-GARCH(1,1)-ICAPM with mixture innovation, respectively. Since the higher risk premium indicates a higher risk-based return, that means the S&P500 and NZX50 fare better than the other markets between January 2009 and January 2020.

[Table 4 about here]

All GARCH-ICAPM models from all markets return the very high volatility persistence, assessed by $\hat{\nu}$ for the GARCH and EGARCH, $\hat{\nu} + (\hat{\alpha}^{+} + \hat{\alpha}^{-})/2$ for the LogGARCH, and $\hat{\alpha} + \hat{\nu} + \hat{\gamma}/2$ for the GJR-GARCH. This indicates the longer-lasting effect of the volatility. That means if the volatility happens, it is likely to continue for a couple of days. Based on the $|\hat{\gamma}| > 0$ for the GJR-GARCH, $\hat{\gamma} < 0$ for the EGARCH, $\alpha^{+} < \alpha^{-}$ for the LogGARCH, and $\hat{\xi}^{L} < \hat{\xi}^{R}$ from the GPD, there is a strong leverage effect in all stock markets. This implies the scale of the negative news/shocks is larger than the positive news/shocks, while the estimated constant term in the GARCH family, $\hat{\omega}$, is positive for all series except the EGARCH. This is probably due to the fact that the constant term represents the stability level of unconditional variance that differs in each model.

In summary, we find that all stock markets have a very high volatility persistence effect. Moreover, the (asymmetric-)GARCH-ICAPM family clearly shows a higher persistence effect than the classical GARCH-ICAPM model. There is also strong statistical evidence that the returns from all markets are asymmetrically and non-normally distributed, which is a confirmed fact of financial markets. Precisely, financial stock returns are volatility clustering, heavy-tailed, and asymmetrically distributed, see more details in Caporale & Zekokh [10], Engle [18], Fan & Patton [20], and Patton [42]. This justifies the use of our exponential-type-GARCH-ICAPM with a mixture innovation process here in this study.

To compare across models, we calculate the marginal (log-)likelihood given in Chib [12], the root mean squared error (RMSE), and the mean absolute percentage error (MAPE) for the in-sample fit, while we use the RMSE, MAPE, and the two-sample Kolmogorov-Smirnov (KS) test for the out-of-sample forecasting performance. Note that the KS test is used to assess whether the forecast distribution is close to the empirical one. The higher the p-value, the closer the forecast distribution is.

The results from Table 5 show that the LogGARCH(1,1)-ICAPM-Mixture is the best performing model in terms of marginal log-likelihood for the training period in all markets while the EGARCH(1,1,1) with either the mixture or the GED innovation is the best in terms of the forecast errors (RMSE and MAPE) under the same situation, except the HSI and FTSE100. For the COVID-19 forecasting period, the EGARCH(1,1,1)-ICAPM is still the one that returns the best out-of-sample forecasts either in terms of forecast errors or forecast distribution. Precisely, the EGARCH(1,1,1)-ICAPM with the GED innovation is slightly better as measured by the RMSE and MAPE while that with the mixture innovation is marginally better in terms of the KS test. Hence, we can conclude that the EGARCH-ICAPM model is the main driving model in the early period of COVID-19. This is probably caused by the news impact function, which is the main feature of the model that can capture asymmetric shocks. These findings also confirm the superiority of including the ICAPM structure we proposed in this study.

[Table 5 about here]

As in the simulation study, we also assess our proposed models across markets and quantiles. For example, Table 6 shows the results from the in-sample parameter estimation of EGARCH(1,1,1)-ICAPM with mixture innovation in all markets. In terms of the marginal log-likelihood, we find the models with the lowest mixing probability or quantile of 0.01 are the best in capturing the extreme returns, and hence the volatility.

[Table 6 about here]

5. Conclusions

During the early period of the COVID-19 pandemic (the first quarter of 2020), the plummets of stock markets around the world indicated the unexpected risk from the unforeseeable future. In this study, we analyze the returns from seven stock markets including the US (S&P500), Hong Kong (HSI), Australia (ASX200), UK (FTSE100), Japan (N225), Thailand (SET), and New Zealand (NZX50). The model we used in the analysis is the ICAPM together with the asymmetric-GARCH processes, including standard GARCH, exponential GARCH, log GARCH, and GJR-GARCH, where their innovation is either the mixture of GPD and Gaussian or the GED. Due to the complexity of estimating the parameters of the GPD and the GED, we use parallel computing for the GG algorithm, which is an MCMC method, to facilitate the estimation. Our simulation study shows that the parallel-GG outperforms the one-step QMLE in terms of the mean absolute deviation. We then proceed to the empirical study using the parallel GG to estimate all (asymmetric-)GARCH-ICAPM models. We found in the training (in-sample) period of January 2009 to January 2020 that the LogGARCH(1,1)-ICAPM with mixture innovation and the EGARCH(1,1,1)-ICAPM with either mixture or GED innovation are the best performing models across seven stock markets. For the forecasting (early COVID-19) performance, we found that both EGARCH(1,1,1)-ICAPM with mixtures and GED innovations are the models that return the lowest forecast errors and the closest forecast distribution (to the real data). The outstanding forecasting performance of EGARCH models is possibly caused by the news impact function that can capture the asymmetric shocks existing during the early period of the COVID-19 pandemic.

References

[1] Andrieu, C., Doucet, A., & Holenstein, R. (2010). Particle markov chain monte carlo methods. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *72*.

[2] Ardia, D., Bluteau, K., Boudt, K., & Catania, L. (2018). Forecasting risk with markov-switching garch models: A large-scale performance study. *International Journal of Forecasting*, *34*, 733–747.

[3] Ausín, M. C., & Galeano, P. (2007). Bayesian estimation of the gaussian mixture garch model. *Computational Statistics & Data Analysis*, *51*, 2636–2652.

[4] Bali, T. G., & Engle, R. F. (2010). The intertemporal capital asset pricing model with dynamic conditional correlations. *Journal of Monetary Economics*, *57*, 377–390.

[5] Benczúr, A. A. (2011). Affordable supercomputing for data mining applications. *Procedia Computer Science*, *7*, 136–138.

[6] Bera, A. K., & Higgins, M. L. (1993). Arch models: Properties, estimation and testing. *Journal of Economic Surveys*, *7*, 305–366.

[7] Bildirici, M., & Ersin, Ö. Ö. (2009). Improving forecasts of garch family models with the artificial neural networks: An application to the daily returns in istanbul stock exchange. *Expert Systems with Applications*, *36*, 7355–7362.

[8] Billio, M., Casarin, R., & Osuntuyi, A. (2016). Efficient Gibbs sampling for markov switching garch models. *Computational Statistics and Data Analysis*, *100*, 37–57.

[9] Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*, 307–327.

[10] Caporale, G. M., & Zekokh, T. (2019). Modelling volatility of cryptocurrencies using markov-switching garch models. *Research in International Business and Finance*, *48*, 143–155.

[11] Chang, C.-L., McAleer, M., & Wong, W.-K. (2018). Big data, computational science, economics, finance, marketing, management, and psychology: Connections. *Journal of Risk and Financial Management*, *11*.

[12] Chib, S. (1995). Marginal likelihood from the Gibbs output. *Journal of the American Statistical Association*, *90*, 1313–1321.

[13] Chib, S., & Greenberg, E. (1996). Markov chain Monte Carlo simulation methods in econometrics. *Econometric Theory*, *12*, 409.

[14] Creal, D. D., & Tsay, R. S. (2015). High dimensional dynamic stochastic copula models. *Journal of Econometrics*, *189*, 335–345.

[15] De Lira Salvatierra, I., & Patton, A. J. (2015). Dynamic copula models and high frequency data. *Journal of Empirical Finance*, *30*, 120–135.

[16] Dinh, V., Rundell, A. E., & Buzzard, G. T. (2017). Convergence of griddy Gibbs sampling and other perturbed Markov chains. *Journal of Statistical Computation and Simulation*, *87*, 1379–1400.

[17] Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, *50*, 987–1007.

[18] Engle, R. (2004). Risk and volatility: Econometric models and financial practice. *The American Economic Review*, *94*, 405–420.

[19] Engle, R. F., Lilien, D. M., & Robins, R. P. (1987). Estimating time varying risk premia

in the term structure: The arch-m model. *Econometrica*, *55*, 391–407.
[20] Fan, Y., & Patton, A. J. (2014). Copulas in econometrics. *Annual Review of Economics*, *6*, 179–200.
[21] Fengler, M. R., & Okhrin, O. (2016). Managing risk with a realized copula parameter. *Computational Statistics & Data Analysis*, *100*, 131–152.
[22] Francq, C., Wintenberger, O., & Zakoïan, J.-M. (2013). Garch models without positivity constraints: Exponential or log garch? *Journal of Econometrics*, *177*, 34–46.
[23] Francq, C., Wintenberger, O., & Zakoïan, J.-M. (2018). Goodness-of-fit tests for log-garch and egarch models. *TEST: An Official Journal of the Spanish Society of Statistics and Operations Research*, *27*, 27–51.
[24] Fujiwara, T., Li, J. K., Mubarak, M., Ross, C., Carothers, C. D., Ross, R. B., & Ma, K.-L. (2018). A visual analytics system for optimizing the performance of large-scale networks in supercomputing systems. *Visual Informatics*, *2*, 98–110.
[25] Geweke, J. (1986). Comment. *Econometric Reviews*, *5*, 57–61.
[26] Glosten, L., Jagannathan, R., & Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, *48*, 1779–1801.
[27] Goel, A., Sharma, A., & Mehra, A. (2019). Robust optimization of mixed cvar starr ratio using copulas. *Journal of Computational and Applied Mathematics*, *347*, 62–83.
[28] Hafner, C. M., & Kyriakopoulou, D. (2021). Exponential-type garch models with linear-in-variance risk premium. *Journal of Business & Economic Statistics*, *39*, 589–603.
[29] Hentschel, L. (1995). All in the family nesting symmetric and asymmetric garch models. *Journal of Financial Economics*, *39*, 71–104.
[30] Hosking, J. R. M., & Wallis, J. R. (1987). Parameter and quantile estimation for the generalized pareto distribution. *Technometrics*, *29*, 339–349.
[31] Jose Rodriguez, M., & Ruiz, E. (2012). Revisiting several popular garch models with leverage effect: Differences and similarities. *Journal of Financial Econometrics*, *10*, 637–668.
[32] Kolm, P. N., Tütüncü, R., & Fabozzi, F. J. (2014). 60 years of portfolio optimization: Practical challenges and current trends. *European Journal of Operational Research*, *234*, 356–371.
[33] Lam, H.-F., Hu, J., Zhang, F.-L., & Ni, Y.-C. (2019). Markov chain Monte Carlo-based bayesian model updating of a sailboat-shaped building using a parallel technique. *Engineering Structures*, *193*, 12–27.
[34] Li, F., & Kang, Y. (2018). Improving forecasting performance using covariate-dependent copula models. *International Journal of Forecasting*, *34*, 456–476.
[35] Lopatka, K., & Czyzewski, A. (2014). Acceleration of decision making in sound event recognition employing supercomputing cluster. *Information Sciences*, *285*, 223–236.
[36] Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica*, *41*, 867–887.
[37] Milhøj, A. (1987). *A multiplicative parameterization of arch models*. Department of Statistics, University of Copenhage.
[38] Nadarajah, S. (2005). A generalized normal distribution. *Journal of Applied Statistics*, *32*, 685–694.
[39] Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, *59*, 347–370.
[40] Oh, D. H., & Patton, A. J. (2018). Time-varying systemic risk: Evidence from a dynamic copula model of cds spreads. *Journal of Business & Economic Statistics*, *36*, 181–195.
[41] Pareek, M. K., & Thakkar, P. (2015). Surveying stock market portfolio optimization techniques. In *2015 5th Nirma University International Conference on Engineering (NUiCONE)* (pp. 1–5).
[42] Patton, A. J. (2012). A review of copula models for economic time series. *Journal of Multivariate Analysis*, *110*, 4–18.
[43] Prudencio, E., & Cheung, S. H. (2012). Parallel adaptive multilevel sampling algorithms

for the bayesian analysis of mathematical models. *International Journal for Uncertainty Quantification*, *2*, 215–237.

[44] Ritter, C., & Tanner, M. A. (1992). Facilitating the Gibbs sampler: The Gibbs stopper and the griddy-Gibbs sampler. *Journal of the American Statistical Association*, *87*, 861–868.

[45] Sahamkhadam, M., Stephan, A., & Östermark, R. (2018). Portfolio optimization based on garch-evt-copula forecasting models. *International Journal of Forecasting*, *34*, 497–506.

[46] Sanchez-Vazquez, C., Avila-Costa, M., & Cervantes-Pérez, F. (2014). Implementation of a computational model for information processing and signaling from a biological neural network of neostriatum nucleus. *Journal of Applied Research and Technology*, *12*, 568–584.

[47] Singh, V. P., & Guo, H. (1995). Parameter estimation for 3-parameter generalized pareto distribution by the principle of maximum entropy (pome). *Hydrological Sciences Journal*, *40*, 165–181.

[48] Sucarrat, G., & Escribano, A. (2018). Estimation of log-garch models in the presence of zero returns. *The European Journal of Finance*, *24*, 809–827.

[49] Sucarrat, G., Gronneberg, S., & Escribano, A. (2016). Estimation and inference in univariate and multivariate log-garch-x models when the conditional density is unknown. *Computational Statistics & Data Analysis*, *100*, 582–594.

[50] Wang, Z.-R., Chen, X.-H., Jin, Y.-B., & Zhou, Y.-J. (2010). Estimating risk of foreign exchange portfolio: Using var and cvar based on garch–evt-copula model. *Physica A: Statistical Mechanics and its Applications*, *389*, 4918–4928.

[51] Wichitaksorn, N., & Choy, B. (2015). Bayesian parallel computation for intractable likelihood using griddy-Gibbs sampler. D. K. Dey, & A. Loganathan (Eds.), *Current Trends in Bayesian Methodology with Applications (1st ed.)* (pp. 619–629). Chapman and Hall/CRC.

[52] Zakoian, J.-M. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and Control*, *18*, 931–955.

Table 1. Estimation results of various models in simulation study

Models	Parameters	True Value	Parallel-GG				One-step QMLE	
			Mean	SD	SIF	MAD	Mean	MAD
GARCH-ICAPM-Mixture	λ_1	0.0000	0.0000	0.0012	2.4774	**0.0000**	0.0017	0.0017
	λ_2	0.0426	0.0454	0.0061	2.1778	**0.0028**	0.0909	0.0483
	ξ^L	0.0654	0.0656	0.0258	2.2808	**0.0002**	0.0991	0.0337
	ξ^R	0.3141	0.3080	0.0945	2.2174	**0.0061**	0.2985	0.0156
	ω	0.0080	0.0087	0.0026	2.1067	**0.0007**	0.0099	0.0019
	α	0.0880	0.0861	0.0033	2.1073	**0.0019**	0.0180	0.0700
	ν	0.8980	0.8987	0.0642	2.1594	**0.0007**	0.9787	0.0807
	AIC		**1596.40**				2548.27	
	BIC		**1630.76**				2582.63	
	DIC		2103.64				n.a.	
	Computational time (seconds)		442.51				0.01	
EGARCH-ICAPM-Mixture	λ_1	0.0000	-0.0448	0.0356	2.3378	0.0448	0.0195	**0.0195**
	λ_2	0.0200	0.0466	0.0202	2.2520	**0.0266**	0.0599	0.0399
	ξ^L	0.1500	0.1722	0.0860	2.3734	**0.0222**	0.1246	0.0254
	ξ^R	0.3500	0.2940	0.0838	2.5761	0.0560	0.3624	**0.0124**
	ω	-0.1000	-0.1011	0.0181	5.6055	**0.0011**	-0.0866	0.0134
	γ	-0.1200	-0.1313	0.0206	2.0793	0.0113	-0.1222	**0.0022**
	δ	0.1300	0.1375	0.0223	6.3039	**0.0075**	0.2074	0.0774
	ν	0.9800	0.9603	0.0104	3.3496	**0.0197**	0.8327	0.1473
	AIC		**3091.61**				3141.54	
	BIC		**3130.88**				3180.80	
	DIC		3143.56				n.a.	
	Computational time (seconds)		1582.34				1.04	
LogGARCH-ICAPM-Mixture	λ_1	0.0000	0.0000	0.0001	2.2997	**0.0000**	0.0001	0.0001
	λ_2	0.0450	0.0434	0.0042	2.0423	**0.0016**	0.0330	0.0120
	ξ^L	0.0700	0.0705	0.0084	2.2974	**0.0005**	0.1122	0.0422
	ξ^R	0.2800	0.3243	0.0550	2.1204	0.0443	0.2586	**0.0214**
	ω	0.0320	0.0381	0.0020	2.5199	0.0061	0.0354	**0.0034**
	α^+	0.0030	0.0028	0.0018	2.3736	**0.0002**	0.0066	0.0036
	α^-	0.0530	0.0557	0.0040	2.6059	**0.0027**	0.0563	0.0033
	ν	0.9600	0.9622	0.0028	2.5192	**0.0022**	0.9232	0.0368
	AIC		**2562.25**				2569.77	
	BIC		**2601.51**				2609.03	
	DIC		2547.88				n.a.	
	Computational time (seconds)		2012.44				1.33	
GJRGARCH-ICAPM-Mixture	λ_1	0.0000	0.0000	0.0001	2.5272	**0.0000**	0.0001	0.0001
	λ_2	0.0350	0.0349	0.0035	2.0742	**0.0001**	0.0515	0.0165
	ξ^L	0.0700	0.0703	0.0084	2.4086	**0.0003**	0.1217	0.0517
	ξ^R	0.2800	0.2311	0.0462	2.1888	0.0489	0.2516	**0.0284**
	ω	0.0080	0.0087	0.0006	2.1908	**0.0007**	0.0095	0.0015
	α	0.0310	0.0312	0.0032	2.1718	**0.0002**	0.0294	0.0016
	γ	0.0070	0.0072	0.0021	2.2443	**0.0002**	0.0132	0.0062
	ν	0.9500	0.9494	0.0026	2.2629	**0.0006**	0.9478	0.0022
	AIC		**2307.76**				2340.01	
	BIC		**2347.02**				2379.27	
	DIC		2294.11				n.a.	
	Computational time (seconds)		1452.25				0.68	
EGARCH-ICAPM-GED	λ_1	0.0000	0.0006	0.0002	2.3395	**0.0006**	0.0072	0.0072
	λ_2	0.0540	0.0612	0.0078	2.1796	**0.0072**	0.0702	0.0162
	ξ	1.2500	1.3004	0.0839	3.5385	**0.0504**	1.0075	0.2425
	ω	-0.1020	-0.0976	0.0035	2.7592	**0.0044**	-0.1701	0.0681
	γ	-0.1780	-0.1084	0.0097	2.2352	**0.0696**	-0.1014	0.0767
	δ	0.0550	0.0452	0.0028	2.1275	**0.0098**	0.0258	0.0292
	ν	0.9540	0.9745	0.0029	2.7787	0.0205	0.9488	**0.0052**
	AIC		-1712.20				**-1879.07**	
	BIC		-1677.85				**-1844.72**	
	DIC		-1709.20				n.a.	
	Computational time (seconds)		403.52				0.56	

Notes: n.a. = not available. Bold fonts indicate better results.

Table 2. Estimation results of EGARCH-ICAPM-Mixture model at lower quantiles in simulation study

		Parallel-Computing-GG				1-step QMLE	
0.01quantile							
		Mean	SD	SIF	MAD	Mean	MAD
λ_1	0.0000	-0.0320	0.0359	2.2372	0.0320	0.0006	**0.0006**
λ_2	0.0200	0.0504	0.0220	2.0991	**0.0304**	0.0587	0.0387
ξ^L	0.1500	0.2508	0.1392	2.1993	0.1008	0.1014	**0.0486**
ξ^U	0.3500	0.2606	0.1379	2.1551	0.0894	0.3392	**0.0108**
ω	-0.1000	-0.1060	0.0180	3.3646	0.0060	-0.0948	**0.0052**
γ	-0.1200	-0.1265	0.0191	2.3725	0.0065	-0.1223	**0.0023**
δ	0.1300	0.1387	0.0220	3.7167	0.0087	0.1289	**0.0011**
ν	0.9800	0.9648	0.0091	2.3915	**0.0152**	0.9616	0.0184
AIC		**2959.52**				2970.62	
BIC		**2998.78**				3009.88	
DIC		2956.62				n.a.	
0.05quantile							
λ_1	0.0000	-0.0476	0.0380	3.0571	0.0476	0.0040	**0.0040**
λ_2	0.0200	0.0507	0.0221	2.3280	**0.0307**	0.0612	0.0412
ξ^L	0.1500	0.1995	0.1108	2.2283	0.0495	0.1046	**0.0454**
ξ^U	0.3500	0.2623	0.1117	2.4280	0.0877	0.3324	**0.0176**
ω	-0.1000	-0.1048	0.0180	4.4462	**0.0048**	-0.0949	0.0051
γ	-0.1200	-0.1283	0.0206	3.2226	0.0083	-0.1154	**0.0046**
δ	0.1300	0.1411	0.0218	3.7457	**0.0111**	0.1670	0.0370
ν	0.9800	0.9615	0.0102	3.0885	**0.0185**	0.9065	0.0735
AIC		**3041.28**				3066.29	
BIC		**3080.54**				3105.56	
DIC		3055.58				n.a.	
0.10quantile							
λ_1	0.0000	-0.0448	0.0356	2.3378	0.0448	0.0195	**0.0195**
λ_2	0.0200	0.0466	0.0202	2.2520	**0.0266**	0.0599	0.0399
ξ^L	0.1500	0.1722	0.0860	2.3734	**0.0222**	0.1246	0.0254
ξ^U	0.3500	0.2940	0.0838	2.5761	0.0560	0.3624	**0.0124**
ω	-0.1000	-0.1011	0.0181	5.6055	**0.0011**	-0.0866	0.0134
γ	-0.1200	-0.1313	0.0206	2.0793	0.0113	-0.1222	**0.0022**
δ	0.1300	0.1375	0.0223	6.3039	**0.0075**	0.2074	0.0774
ν	0.9800	0.9603	0.0104	3.3496	**0.0197**	0.8327	0.1473
AIC		**3091.61**				3141.54	
BIC		**3130.88**				3180.80	
DIC		3143.56				n.a.	

Notes: n.a. = not available. Bold fonts indicate better results.

Table 3. Descriptive statistics of the daily returns from seven stock markets

	S&P500	**HSI**	**ASX200**	**NZX50**	**FTSE100**	**Nikkei225**	**SET**
	In-sample data: 01/01/2009-30/01/2020						
Min.	-6.94	-6.04	-4.39	-3.76	-6.22	-10.61	-5.86
Max.	6.79	7.13	3.56	2.69	5.01	7.39	5.71
Mean	-0.0011	-0.0017	0.0001	-0.0004	-0.001	-0.0007	-0.0023
Std.	1.02	1.25	0.91	0.59	0.99	1.34	1.02
Skewness	-0.35	-0.10	-0.31	-0.51	-0.25	-0.46	-0.31
Kurtosis	8.10	5.38	4.86	5.71	6.23	7.43	6.88
JB Test	3075***	649***	458***	1001***	1243***	2338***	1744***
Obs.	2788	2739	2851	2863	2799	2737	2705
	Out-of-sample data: 01/02/2020-30/04/2020						
Min.	-12.60	-4.88	-9.82	-6.21	-11.17	-6.04	-11.18
Max.	9.13	5.03	7.15	13.43	9.01	7.96	7.90
Mean	-0.0145	-0.0046	0.0158	0.0264	-0.0146	0.0132	0.0157
Std.	3.91	1.98	3.21	2.47	3.02	2.61	3.20
Skewness	-0.36	-0.18	-0.54	1.89	-0.75	0.44	-1.15
Kurtosis	4.39	3.54	4.01	13.69	5.77	4.15	6.10
JB Test	6.36*	1.08	5.64*	396.57***	25.58***	5.25**	38.37***
Obs.	62	61	62	74	62	60	62

Notes: JB Test is the test statistic from Jarque-Bera's normality test. *,**,*** indicate the statistical significance at 10%, 5% and 1%, respectively. Obs. = number of observations.

Table 4. In-sample parameter estimates across models and markets using parallel-Griddy-Gibbs method

Model	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
S&P500																										
	λ_1		λ_2		ξ^L		ξ^R		ξ		ω		γ_1		δ_1		α_1^+		α_1^-		α_1		γ_1		ν_1	
EGARCH(1,1,1)-ICAPM-Mixture	-0.003	0.0125	0.0346***	0.0089	0.0807	0.0686	0.2786***	0.0579	-	-	-0.1668***	0.0176	-0.1632***	0.0095	0.1813***	0.0213	-	-	-	-	-	-	-	-	0.9662***	0.0053
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0366***	0.0020	0.0675***	0.0080	0.3030***	0.0370	-	-	0.0383***	0.0018	-	-	-	-	0.0049**	0.0020	0.0597***	0.0013	-	-	-	-	0.9589***	0.0004
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0734***	0.0057	0.0705***	0.0084	0.2847***	0.0380	-	-	0.0080***	0.0007	-	-	-	-	-	-	-	-	0.0106***	0.0026	0.0314***	0.0025	0.9467***	0.0005
GARCH(1,1)-ICAPM-Mixture	-0.0051	0.0087	0.0229*	0.0128	0.2998***	0.0831	0.3560***	0.0353	-	-	0.0181***	0.0050	-	-	-	-	-	-	-	-	0.1475***	0.0197	-	-	0.8450***	0.0175
EGARCH(1,1,1)-ICAPM-GED	0.0002*	0.0001	0.0198***	0.0019	-	-	-	-	1.7341***	0.0524	-0.1684***	0.0004	-	-	0.1805***	0.0001	-	-	-	-	-	-	-0.1222***	0.0104	0.9608***	0.0003
HSI																										
EGARCH(1,1,1)-ICAPM-Mixture	0.005	0.0271	0.0248**	0.0116	0.0867	0.0688	0.2552***	0.0697	-	-	-0.0898***	0.0124	-0.0520***	0.011	0.1152***	0.0161	-	-	-	-	-	-	-	-	0.9825***	0.0045
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0299***	0.0017	0.0682***	0.0081	0.3537***	0.0284	-	-	0.0402***	0.0019	-	-	-	-	0.0042*	0.0022	0.0568***	0.0030	-	-	-	-	0.9609***	0.0018
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0369***	0.0032	0.0693***	0.0081	0.2606***	0.0379	-	-	0.0088***	0.0006	-	-	-	-	-	-	-	-	0.0087***	0.0022	0.0373***	0.0026	0.9488***	0.0020
GARCH(1,1)-ICAPM-Mixture	0.0009	0.0092	0.0285**	0.0136	0.0986	0.0711	0.3069***	0.0449	-	-	0.0171***	0.0059	-	-	-	-	-	-	-	-	0.0594***	0.0084	-	-	0.9270***	0.0103
EGARCH(1,1,1)-ICAPM-GED	0.0002*	0.0001	0.0197***	0.0020	-	-	-	-	1.2093***	0.0418	-0.0514***	0.0008	-	-	-0.0242	0.0155	-	-	-	-	-	-	0.1082***	0.0001	0.9320***	0.0003
ASX200																										
EGARCH(1,1,1)-ICAPM-Mixture	0.0025	0.0151	0.0306***	0.0105	0.0458	0.0382	0.2495***	0.0595	-	-	-0.1062***	0.0136	-0.0901***	0.0124	0.1177***	0.0159	-	-	-	-	-	-	-	-	0.9812***	0.0044
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0260***	0.0027	0.0693***	0.0083	0.3684***	0.0256	-	-	0.0349***	0.0018	-	-	-	-	0.0018	0.0011	0.0584***	0.0027	-	-	-	-	0.9608***	0.0019
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0548***	0.0038	0.0692***	0.0076	0.2579***	0.0381	-	-	0.0083***	0.0007	-	-	-	-	-	-	-	-	0.0088***	0.0022	0.0341***	0.0023	0.9473***	0.0009
GARCH(1,1)-ICAPM-Mixture	-0.0032	0.0088	0.0523**	0.0262	0.0433	0.0377	0.2879***	0.0456	-	-	0.0108***	0.0029	-	-	-	-	-	-	-	-	0.0758***	0.0116	-	-	0.9080***	0.0128
EGARCH(1,1,1)-ICAPM-GED	0.0002**	0.0001	0.0225***	0.0021	-	-	-	-	1.5872***	0.0618	-0.1030***	0.0002	-	-	-0.0408***	0.0102	-	-	-	-	-	-	0.1235***	0.0001	0.9749***	0.0029
FTSE100																										
EGARCH(1,1,1)-ICAPM-Mixture	-0.0067	0.0183	0.0346***	0.0091	0.0471	0.0404	0.2665***	0.0609	-	-	-0.1454***	0.0176	-0.1402***	0.0148	0.1617***	0.0205	-	-	-	-	-	-	-	-	0.9689***	0.0065
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0175***	0.0067	0.0683***	0.0082	0.3350***	0.0307	-	-	0.0288***	0.0017	-	-	-	-	0.0018	0.0011	0.0487***	0.0024	-	-	-	-	0.9614***	0.0023
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0313***	0.0038	0.0697***	0.0081	0.2611***	0.0383	-	-	0.0085***	0.0007	-	-	-	-	-	-	-	-	0.0104***	0.0027	0.0342***	0.0024	0.9471***	0.0008
GARCH(1,1)-ICAPM-Mixture	-0.0056	0.0100	0.0586**	0.0250	0.0542	0.0471	0.3051***	0.0495	-	-	0.0196***	0.0060	-	-	-	-	-	-	-	-	0.1096***	0.0160	-	-	0.8666***	0.0189
EGARCH(1,1,1)-ICAPM-GED	0.0002*	0.0001	0.0135***	0.0015	-	-	-	-	1.4982***	0.0551	-0.1204***	0.0002	-	-	-0.0705	0.0105	-	-	-	-	-	-	0.1498***	0.0001	0.9749***	0.0029
N225																										
EGARCH(1,1,1)-ICAPM-Mixture	-0.0139	0.0223	0.0315***	0.0103	0.1229*	0.0748	0.2946***	0.0589	-	-	-0.1521***	0.0189	-0.1376***	0.0177	0.2200***	0.0252	-	-	-	-	-	-	-	-	0.9301***	0.0122
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0230***	0.0047	0.0688***	0.0082	0.3542***	0.0292	-	-	0.0437***	0.0016	-	-	-	-	0.0045**	0.0021	0.0557***	0.0025	-	-	-	-	0.9597***	0.0011
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0316***	0.0031	0.0707***	0.0093	0.2533***	0.0453	-	-	0.0097***	0.0007	-	-	-	-	-	-	-	-	0.0087***	0.0017	0.0373***	0.0021	0.9474***	0.0010
GARCH(1,1)-ICAPM-Mixture	-0.0045	0.0089	0.0287**	0.0126	0.1195	0.0786	0.3349***	0.0424	-	-	0.0644***	0.0147	-	-	-	-	-	-	-	-	0.1283***	0.0180	-	-	0.8340***	0.0209
EGARCH(1,1,1)-ICAPM-GED	0.0002*	0.0001	0.0254***	0.0021	-	-	-	-	1.5815***	0.0523	-0.0975***	0.0023	-	-	-0.0778***	0.0117	-	-	-	-	-	-	0.1499***	0.0001	0.9320***	0.0004
SET																										
EGARCH(1,1,1)-ICAPM-Mixture	-0.0073	0.0172	0.0292***	0.0113	0.0649	0.0563	0.2463***	0.0638	-	-	-0.1654***	0.0182	-0.0565***	0.0131	0.1959***	0.0221	-	-	-	-	-	-	-	-	0.9762***	0.0057
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0461***	0.0024	0.0683***	0.0081	0.2955***	0.0395	-	-	0.0373***	0.0018	-	-	-	-	0.0044**	0.0020	0.0574***	0.0030	-	-	-	-	0.9591***	0.0007
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0352***	0.0033	0.0698***	0.0085	0.2783***	0.0340	-	-	0.0081***	0.0008	-	-	-	-	-	-	-	-	0.0084***	0.0021	0.0342***	0.0022	0.9469***	0.0007
GARCH(1,1)-ICAPM-Mixture	-0.0032	0.0099	0.0429**	0.0198	0.0678	0.0562	0.3149***	0.0389	-	-	0.0090***	0.0025	-	-	-	-	-	-	-	-	0.0904***	0.0122	-	-	0.8967***	0.0116
EGARCH(1,1,1)-ICAPM-GED	0.0002**	0.0001	0.0195***	0.0019	-	-	-	-	1.7098***	0.0416	-0.1685***	0.0003	-	-	-0.0417***	0.0099	-	-	-	-	-	-	0.2036***	0.0001	0.9749***	0.0030
NZX50																										
EGARCH(1,1,1)-ICAPM-Mixture	0.0091	0.01	0.0279**	0.0114	0.0433	0.0352	0.2367***	0.0629	-	-	-0.1486***	0.0337	-0.0327***	0.0112	0.1233***	0.0245	-	-	-	-	-	-	-	-	0.9598***	0.0143
LogGARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0496***	0.0038	0.0700***	0.0084	0.3370***	0.0327	-	-	0.0264***	0.0019	-	-	-	-	0.0018	0.0011	0.0561***	0.0024	-	-	-	-	0.9602***	0.0015
GJR-GARCH(1,1)-ICAPM-Mixture	0.0000	0.0001	0.0536***	0.0043	0.0691***	0.0079	0.2489***	0.0377	-	-	0.0080***	0.0008	-	-	-	-	-	-	-	-	0.0072***	0.0021	0.0239***	0.0025	0.9469***	0.0007
GARCH(1,1)-ICAPM-Mixture	-0.0033	0.0093	0.0957**	0.0464	0.0473	0.0357	0.2627***	0.0509	-	-	0.0194**	0.0078	-	-	-	-	-	-	-	-	0.0734***	0.0158	-	-	0.8614***	0.0375
EGARCH(1,1,1)-ICAPM-GED	0.0002*	0.0001	0.0158***	0.0018	-	-	-	-	1.5939***	0.0598	-0.1204***	0.0002	-	-	-0.0178*	0.0093	-	-	-	-	-	-	0.1235***	0.0001	0.9751***	0.0029

Note: *,**,*** indicate the statistical significance at 10%, 5% and 1%, respectively.

Table 5. Model comparison and performance across markets

	Marginal Log-Likelihood	In-Sample		Out-of-Sample		KSTEST
		RMSE	MAPE	RMSE	MAPE	P value
S&P500						
EGARCH-ICAPM-Mixture	22460	1.43	6.64	4.13	18.17	**0.08*****
LogGARCH-ICAPM-Mixture	**23938**	1.41	6.77	4.00	10.47	0.05***
GJR-GARCH-ICAPM-Mixture	23451	1.35	6.45	3.99	10.44	0.04***
GARCH-ICAPM-Mixture	19682	1.64	7.78	4.04	14.18	0.06***
EGARCH-ICAPM-GED	4500	**1.29**	**5.81**	3.97	10.06	0.02***
EGARCH-GED	2943	1.30	5.98	**3.96**	**9.68**	0.02***
EGARCH-Mixture	16066	1.48	7.25	4.12	17.23	0.03***
HSI						
EGARCH-ICAPM-Mixture	27874	1.77	7.53	**2.09**	**1.65**	0.34***
LogGARCH-ICAPM-Mixture	**28321**	**1.73**	**6.83**	2.24	1.99	0.21***
GJR-GARCH-ICAPM-Mixture	28139	2.06	8.58	2.43	2.42	0.19***
GARCH-ICAPM-Mixture	24016	2.05	8.66	2.54	2.74	0.22***
EGARCH-ICAPM-GED	16531	1.77	7.65	2.30	2.31	**0.54*****
EGARCH-GED	10982	1.78	7.61	2.34	2.27	0.47***
EGARCH-Mixture	19920	1.81	7.56	2.14	1.74	0.18***
ASX200						
EGARCH-ICAPM-Mixture	22657	1.29	15.89	**3.19**	**2.44**	0.27***
LogGARCH-ICAPM-Mixture	**23419**	1.32	14.60	3.30	5.06	0.10***
GJR-GARCH-ICAPM-Mixture	23167	1.35	15.21	3.34	5.44	0.23***
GARCH-ICAPM-Mixture	19680	1.49	15.82	3.40	5.96	0.18***
EGARCH-ICAPM-GED	5804	**1.28**	**14.12**	3.30	5.43	**0.30*****
EGARCH-GED	3808	1.30	16.85	3.31	5.15	0.19***
EGARCH-Mixture	16197	1.35	16.66	3.20	2.54	0.16***
FTSE100						
EGARCH-ICAPM-Mixture	23594	1.43	12.72	3.21	1.96	**0.34*****
LogGARCH-ICAPM-Mixture	**24669**	**1.38**	**12.56**	3.15	1.67	0.18***
GJR-GARCH-ICAPM-Mixture	24253	1.44	13.57	3.18	1.75	0.22***
GARCH-ICAPM-Mixture	20607	1.60	15.27	3.26	1.89	0.26***
EGARCH-ICAPM-GED	7433	1.39	13.71	**3.14**	**1.70**	0.33***
EGARCH-GED	4905	1.41	13.05	3.16	1.73	0.26***
EGARCH-Mixture	16872	1.49	14.40	3.22	1.92	0.18***
N225						
EGARCH-ICAPM-Mixture	28909	1.92	7.26	2.87	2.071	0.25***
LogGARCH-ICAPM-Mixture	**29597**	1.95	7.65	2.92	2.37	0.18***
GJR-GARCH-ICAPM-Mixture	29497	1.92	7.24	2.91	2.30	0.19***
GARCH-ICAPM-Mixture	25064	2.11	7.92	2.92	2.20	0.21***
EGARCH-ICAPM-GED	16785	1.79	6.80	**2.82**	**2.067**	**0.31*****
EGARCH-GED	11032	**1.78**	**6.75**	2.87	2.10	0.23***
EGARCH-Mixture	20661	1.98	7.66	2.93	2.18	0.10***
SET						
EGARCH-ICAPM-Mixture	22895	1.46	7.38	**3.27**	**1.42**	**0.28*****
LogGARCH-ICAPM-Mixture	**23422**	1.44	7.16	3.35	1.57	0.27***
GJR-GARCH-ICAPM-Mixture	23341	1.41	6.91	3.33	1.50	0.21***
GARCH-ICAPM-Mixture	19726	1.55	7.59	4.07	2.03	0.22***
EGARCH-ICAPM-GED	6494	**1.37**	**6.85**	3.29	1.48	0.24***
EGARCH-GED	4305	1.38	6.87	3.32	1.46	0.16***
EGARCH-Mixture	16358	1.52	7.66	3.38	1.48	0.15***
NZX50						
EGARCH-ICAPM-Mixture	15259	0.844	8.74	2.52	5.21	**0.32*****
LogGARCH-ICAPM-Mixture	**15878**	0.91	9.31	2.54	5.92	0.04***
GJR-GARCH-ICAPM-Mixture	15565	0.88	9.18	2.55	6.03	0.10***
GARCH-ICAPM-Mixture	13177	0.86	8.90	2.54	5.78	0.30***
EGARCH-ICAPM-GED	-6150	0.86	8.94	2.54	5.86	0.09***
EGARCH-GED	-4073	0.87	9.03	2.54	5.89	0.08***
EGARCH-Mixture	10918	**0.843**	**8.58**	**2.51**	**5.04**	0.09***

Notes: *** indicates the statistical significance at 1%. Bold fonts indicate better results.

Table 6. In-sample parameter estimates of EGARCH-ICAPM-Mixture model at lower quantiles in all markets

Quantile		λ_1	λ_2	ξ^L	ξ^R	ω	γ	δ	ν	**Marginal Log-Likelihood**
					S&P500					
0.01	Mean	-0.0155	0.0330	0.0790	0.3735	-0.1680	-0.1625	0.1932	0.9635	**22788**
	SD	0.0167	0.0111	0.0835	0.1186	0.0183	0.0119	0.0230	0.0050	
0.05	Mean	-0.0047	0.0337	0.0626	0.3596	-0.1644	-0.1627	0.1821	0.9659	22568
	SD	0.0125	0.0094	0.0583	0.0657	0.0159	0.0095	0.0192	0.0049	
0.10	Mean	-0.0030	0.0346	0.0807	0.2786	-0.1668	-0.1632	0.1813	0.9662	22460
	SD	0.0125	0.0089	0.0686	0.0579	0.0176	0.0095	0.0213	0.0053	
					HSI					
0.01	Mean	-0.0274	0.0262	0.1389	0.2036	-0.0902	-0.0546	0.1210	0.9797	**28042**
	SD	0.0241	0.0113	0.1104	0.1237	0.0124	0.0104	0.0167	0.0047	
0.05	Mean	-0.0069	0.0250	0.1085	0.2968	-0.0901	-0.0524	0.1183	0.9816	27923
	SD	0.0275	0.0114	0.0921	0.0930	0.0131	0.0105	0.0176	0.0046	
0.10	Mean	0.0050	0.0248	0.0867	0.2552	-0.0898	-0.0520	0.1152	0.9825	27874
	SD	0.0271	0.0116	0.0688	0.0697	0.0124	0.0110	0.0161	0.0045	
					ASX200					
0.01	Mean	-0.0188	0.0293	0.1257	0.3141	-0.1011	-0.0941	0.1168	0.9804	**22710**
	SD	0.0158	0.0110	0.0986	0.1245	0.0129	0.0113	0.0152	0.0044	
0.05	Mean	-0.0044	0.0294	0.0722	0.3188	-0.1038	-0.0913	0.1176	0.9811	22650
	SD	0.0142	0.0105	0.0625	0.0739	0.0133	0.0119	0.0158	0.0042	
0.10	Mean	0.0025	0.0306	0.0458	0.2495	-0.1062	-0.0901	0.1177	0.9812	22657
	SD	0.0151	0.0105	0.0382	0.0595	0.0136	0.0124	0.0159	0.0044	
					FTSE100					
0.01	Mean	-0.0358	0.0339	0.1108	0.3110	-0.1455	-0.1406	0.1716	0.9644	**23724**
	SD	0.0166	0.0092	0.0975	0.1260	0.0176	0.0139	0.0211	0.0064	
0.05	Mean	-0.0177	0.0338	0.0609	0.3216	-0.1395	-0.1387	0.1594	0.9689	23623
	SD	0.0176	0.0094	0.0547	0.0715	0.0169	0.0141	0.0200	0.0065	
0.10	Mean	-0.0067	0.0346	0.0471	0.2665	-0.1454	-0.1402	0.1617	0.9689	23594
	SD	0.0183	0.0091	0.0404	0.0609	0.0176	0.0148	0.0205	0.0065	
					N225					
0.01	Mean	-0.0283	0.0282	0.1283	0.3681	-0.1462	-0.1365	0.2193	0.9293	**29092**
	SD	0.0251	0.0111	0.0890	0.1251	0.0167	0.0159	0.0231	0.0109	
0.05	Mean	-0.0185	0.0305	0.1302	0.3778	-0.1466	-0.1362	0.2167	0.9302	28955
	SD	0.0228	0.0110	0.0900	0.0633	0.0179	0.0173	0.0243	0.0117	
0.10	Mean	-0.0139	0.0315	0.1229	0.2946	-0.1521	-0.1376	0.2200	0.9301	28909
	SD	0.0223	0.0103	0.0748	0.0589	0.0189	0.0177	0.0252	0.0122	
					SET					
0.01	Mean	-0.0254	0.0268	0.0952	0.2176	-0.1623	-0.0585	0.1995	0.9769	**23089**
	SD	0.0167	0.0111	0.0835	0.1186	0.0183	0.0119	0.0230	0.0050	
0.05	Mean	-0.0154	0.0287	0.0743	0.3008	-0.1625	-0.0569	0.1961	0.9763	22949
	SD	0.0168	0.0112	0.0693	0.0766	0.0182	0.0125	0.0223	0.0054	
0.10	Mean	-0.0073	0.0292	0.0649	0.2463	-0.1654	-0.0565	0.1959	0.9762	22895
	SD	0.0172	0.0113	0.0563	0.0638	0.0182	0.0131	0.0221	0.0057	
					NZ50					
0.01	Mean	-0.0045	0.0268	0.0838	0.3015	-0.1492	-0.0370	0.1282	0.9583	**15397**
	SD	0.0115	0.0118	0.0690	0.1310	0.0298	0.0104	0.0218	0.0135	
0.05	Mean	0.0043	0.0272	0.0547	0.2955	-0.1504	-0.0348	0.1257	0.9577	15286
	SD	0.0099	0.0116	0.0458	0.0842	0.0338	0.0114	0.0234	0.0157	
0.10	Mean	0.0091	0.0279	0.0433	0.2367	-0.1486	-0.0327	0.1233	0.9598	15259
	SD	0.0100	0.0114	0.0352	0.0629	0.0337	0.0112	0.0245	0.0143	

Note: Bold fonts indicate better results.

Figure 1. Convergence draws and density plots of posterior samples for estimated parameters from EGARCH-ICAPM-Mixture process

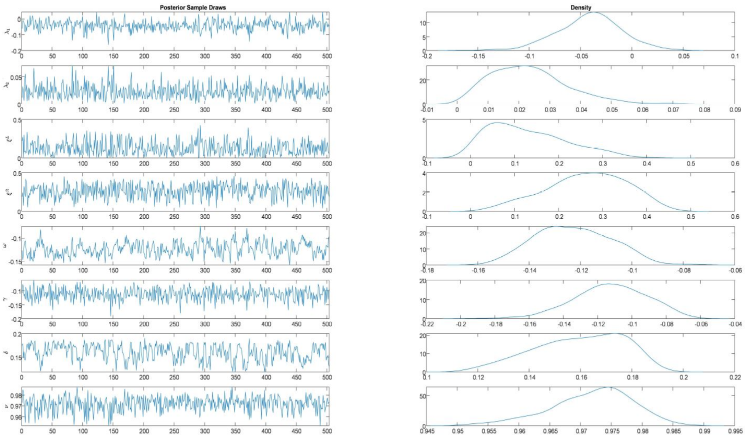

Figure 2. Time series and density plots of the daily returns from seven stock markets between 01/01/2009 and 31/01/2020 (In-sample data set)

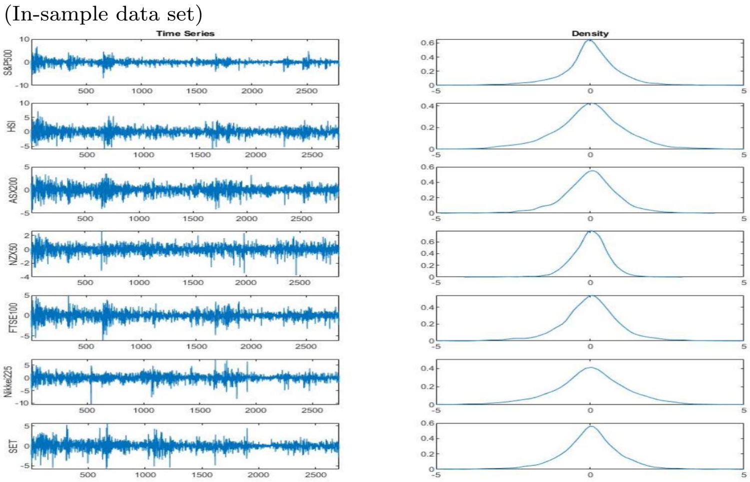

Figure 3. Time series and density plots of the daily returns from seven stock markets between 01/02/2020 and 30/04/2020 (Out-of-sample data set)

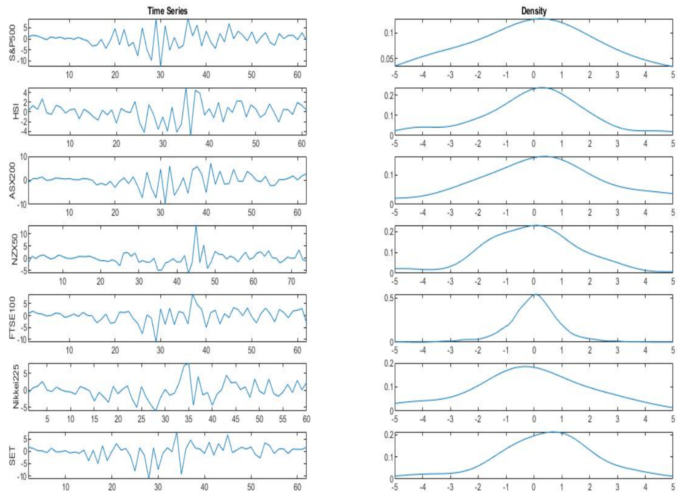

Figure 4. Convergence draws and density plots of posterior samples for EGARCH-ICAPM-Mixture process from the S&P500 returns