# Modelling of light weight floor/ceiling structures

**Hyuck Chung**

hyuck@math.auckland.ac.nz

Acoustics Research Centre, The University of Auckland, New Zealand

## ABSTRACT

This talk presents a mathematical model of light weight timber floor/ceiling structures. The structures studied here consist of three basic components, upper plate, joist beams and ceiling. The shape of the whole structure is rectangular with the edges simply supported. Connections between the joist beams, the upper plate and the ceiling are considered. This connection conditions include the slippage and the springs. The configuration of the structure is made progressively more complex by adding more components such as cavity air and stiffening battens.

## INTRODUCTION

In this article, we study vibration of a light-weight floor/ceiling structure that has three basic components, upper plate, ceiling and joist beams running parallel in one direction. An advantage of such a lightweight structure is its cheaper cost and faster construction time. However, the structure tends to have poor low-frequency noise insulation performance because of the lightweight components (compare to heavy masonry counterpart). In (Blazier and DuPree, 1994) it is shown that impact noise, particularly in the low frequency range, transmission between the apartments using such floor/ceiling structure can be a major problem.

In recent years there have been numerous theoretical studies on the vibrations of walls, floors and ceilings, because these building components are the primary source of sounds in a room. A difficulty arises from the fact that these building components themselves are made up of many components with widely varying mechanical properties. Furthermore, the connections between different components have not been studied in detail. In order to predict the low frequency vibration of the structure, these components must be individually modelled rather than lumped together. In this paper we also focus on the interaction between the upper plate and the joist beams in particular.

A series of articles by Hammer and Brunskog (Brunskog and Hammer 2002,2003 and Hammer and Brunskog 2000) give detailed studies of the modelling of tapping machines and floor vibration. Their technique is based on Mace's method in (Mace 1980), which deals with a periodically stiffened elastic plate. Mace's method is based on (Evseev 1973 and Lin and Garrelick 1977). Studies of double leaf wall structures using the combination of the above two techniques are given in (Yairi et al 2002). Although a great deal of detail of forces and moments are described in both papers, the behaviour of the two neighbouring components at the joint is not discussed. The papers mentioned above take advantage of periodicity of the structures that have either joists or pins located in a regular interval. In these articles, the components are not physically joined, rather they are simply in contact with each other and free to slip. We address this issue in this paper because the floor upper plate is usually screwed or glued onto the joists.

The floor/ceiling structures, sometimes called double-leaf structure, have also been studied in a different context. In (Craik 1996, 2000), the transmission of vibration across a plate-beam joint is studied. In (Craik 1996, 2000), the transmission and coupling loss factor are used in statistical energy analysis to find the sound transmission through double-leaf structured walls with various connecting methods. The connections between two neighbouring components in these models are rigidly connected, which is not true for timber construction. In more realistic settings the timber-based floor structures have been studied in (Emms and Hallows 2002).

There are various ways to connect the floor to the joists. We here consider simple springs to model the resistance when there is slippage between the components. The amount of the resistance at the connection can be changed by varying the spring constants. The resulting solution is computationally efficient and robust for a wide range of physical parameters. In the following sections, we will show that the interaction conditions play an important role in predicting the low-frequency vibration of the floor/ceiling structure.

## MODELLING PROCEDURE

We use the following four differential equations for two plate, a beam and Helmholtz equations to model the floor/ceiling structure.

$$[D_0\Delta^2_{x,y} - m_0\omega^2]w_0(x,y) = F_0(x,y)$$
$$[D_2\Delta^2_{x,y} - m_2\omega^2]w_2(x,y) = F_2(x,y)$$
$$[EI\frac{d^4}{dx^4} - m_1\omega^2]w_1(x) = P(x)$$
$$[\Delta_{x,y,z} + \frac{\omega^2}{c^2}]p(x,y,z) = 0$$

where $m_0$ ($m_2$) and $F_0$ ($F_2$) are the mass density per unit area and external force amplitude of the upper plate (ceiling), respectively. The derivation of the above equation can be found in many structural acoustics text books such as (Moorse 1968, Cremer 1973, Fahy 1985). The force from the joist beam is denoted by $P_1$. The flexural rigidity, for example $D_0$ for the upper plate, is computed by $E_0h_0^3/12(1-\nu^2)$, where $E_0$ is Young's modulus and $\nu$ is Poisson's ratio for the plate. For the beam equation, $E_1$, $I_1$ and $m_1$ are Young's modulus, moment of inertia and the mass density (per unit length) of the beam, respectively. The moment of inertia is computed by $h_1 d_1/12$, where $h_1$ and $d_1$ are the thickness and the width of the beams, respectively. On the right hand side of the beam equation, $P_1$ denotes the force acting on the beam. Note that there are more than one joist beams, thus there are the same number of separate beam equations in the model. Here we have shown only one of them to avoid the clutter.

The sound pressure in the cavity is denoted by $p(x,y,z)$. In Helmholtz equation $c$ denotes the speed of sound.

In this model we can include the resistance force between the upper plate and the joist beams generated by the bending of the two components. We model the resistance using a spring constant and the slippage distance between the components. Hence the force, $f$, is computed by,

$$f(x) = \sigma[\frac{h_0}{2}\frac{\partial w_0}{\partial x} + \frac{h_1}{2}\frac{dw_1}{dx}]$$

The following sections show that the inclusion of the above force is crucial in predicting the low-frequency vibration of the structures.

## COMPUTATION RESULTS

### Simple joist floor

We here show the computation results of the solution of a simple joist-floor consisting only of the upper plate and the joist beams. The physical values used in the computation are, $m_0$ = 0.015*500Kg/m$^2$, $m_1$ =0.045*0.3*500Kg/m, $\nu$ = 0.4, $E_0$ = $10^{10}$Pa, $E_1$=1.4*$10^{10}$Pa and loss factor for the Young's moduli is 0.03. Figure 1 shows root mean square velocity of the upper plate when the plate is excited at a point with 1 Newton of force. The slippage resistance constant is varied to show its effect on the structure's behaviour. The figure shows that the slippage coupling in the model makes substantial difference to the location of the resonance frequencies.

In a timber structure, the effects of the damping are important. Although, there are many ways to model the damping, complex modelling of the damping is outside the scope of this paper. We thus consider the damping as complex valued Young's module with the loss factor being the ratio of the imaginary part to the real part. The results in the figures in this section use $E_0$=$10^{10}$(1+0.03i). One may choose frequency dependent damping, that is, the loss factor is a function of the frequency.

### Multi-layered floor/ceiling structure

We here apply the model to more complex structures depicted in Figures 2 and 3. The structures in figures 2 and 3 span 7m in length and 3.2m in width. The upper plate in figure 2 is 15mm plywood and the upper plate in figure 3 has additional 38mm of Gypsum Fibreboard screwed to the plywood. The ceiling consists of two layers of 13mm dense plaster boards. The ceiling is connected via rubber resilient clips, which are clipped on to steel battens. The battens run orthogonal to the joist beams. The cavity is filled with sound control fibreglass, which has an average measured flow resistivity of 7250 Rayls/m. The definition of the flow resistivity can be found in (Bies and Hansen 2003). The authors are not aware of definitive modelling of the porous media, the glass fibre filling in the cavity, for the low-frequency range considered in this paper. One reason may be that models for the porous media assume a rigid media, whereas the whole glass-fibre layer moves with the structure at low-frequencies. Hence the flow resistivity is here linearly increased as the frequency becomes higher until the resistivity reaches the conventionally measured value. The slope of the resistivity is determined by the graphs given in appendix 3 of (Bies and Hansen 1994).

The floor upper and joist slippage resistance was experimentally determined by matching the frequencies of the first and second modes of the floor vibration when the floor consisted only of plywood screwed to joists with no ceiling. This was then assumed to be constant for the other floors tested. Values of other parameters (such as stiffness, density, and loss factor) for the floor components were either drawn from the manufacturers' nominal data or determined from measurement of samples of the material. It should be noted that the plywood and plasterboard have orthotropic bending stiffnesses, whereas the model assumed isotropic bending stiffnesses. The model also assumes that the ceiling battens are attached to every joist by resilient clips, whereas in practice, these clips were attached to every other joist (the stiffnesses of the clips in the model were halved to compensate for this).

The root mean square surface velocities of the ceiling experimental results are compared to those predicted by the model, and are shown in figures 4 and 5 (corresponding to the floors of figures 2 and 3, respectively), which are taken from (Chung and Emms 2006). The measurements are taken using a laser vibr-meter with a sacnning head that can measure the velocity at multiple points on the surface. The entire surface of the ceiling was scanned. The surface velocity of the ceiling is averaged over the area. In figure 4 we see good comparison of the results until about 80Hz. Observation of spatial vibration of the upper surface at frequencies above 80Hz suggested that the plywood was decoupling from the joists between the screws, so that the plywood-joists connection was no longer a continuous line as assumed in the model. In figure 4 we also see resonant peaks in both experimental and model results above 60Hz. These are due to ceiling resonances between the ceiling battens and are, in part controlled by the ceiling batten bending stiffness.

The locations of the first few resonant frequencies can be matched even better by adjusting the slippage resistance. However, the value of the resistance was fixed for the two examples here in order to avoid arbitrary adjustments of the parameter. In some cases the slippage resistance may have to be truly adjusted because of the changing conditions such as the weight of the upper layer. We did not consider such an adjustment of the slippage resistance because we do not know the exact nature of the changes in the contact conditions between the upper layer and the joist beams. The details of the experiment program will be available from the FWPRDC report (Emms et al 2006).

## CONCLUSIONS

We have shown that the slippage between the floor and the joists has a significant effect on the vibration of the whole structure particularly on the locations of the first few resonant frequencies. Therefore, it is inadequate to model the coupling as either completely rigid or free connections. The method is computationally cheap and can be implemented directly into computer codes. The method of solution is then extended to multi-layered structures with a ceiling and a cavity. The equations for the coefficients and the interacting forces are arranged in the same order as the real structure. The comparison between the theoretical and the experimental results show that the trend of the vibration of the structure is well predicted by the model.

**Figure 1.** Root mean square velocity of the floor surface for various $\sigma$ =0 (dash), $1.5*10^6$ (dash-dot), $1.5*10^7$ (solid), $1.5*10^8$ (dotted).

**Figure 2.** Schematic drawing of the floor/ceiling structure.

**Figure 3.** Schematic drawing of the floor/ceiling structure.

**Figure 4.** Comparison of the root mean square velocity of the experimental (solid) and the theoretical (dashed) results from the structure in figure 2.

**Figure 5.** Comparison of the root mean square velocity of the experimental (solid) and the theoretical (dashed) results from the structure in figure 3.

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