

An Analytical Method for Centroid Computing and Its Application in Wireless Localization

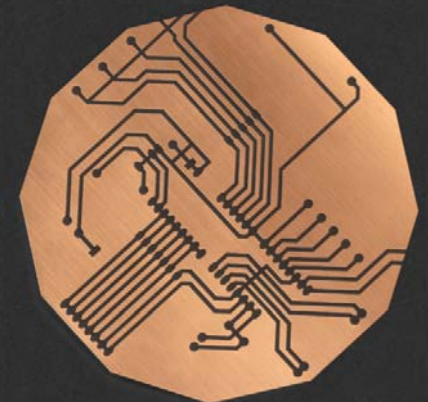
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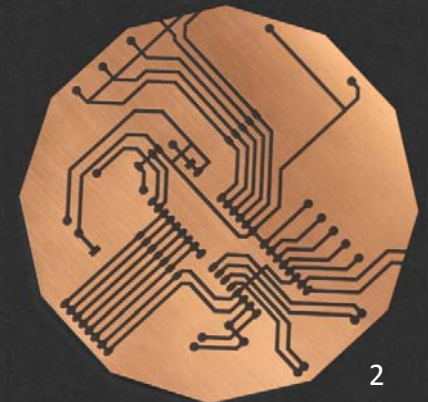
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12 December 2013



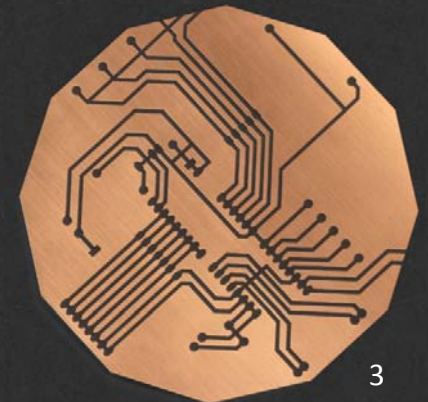
Outline

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- Proposed Analytical Approach
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 - General case
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Background

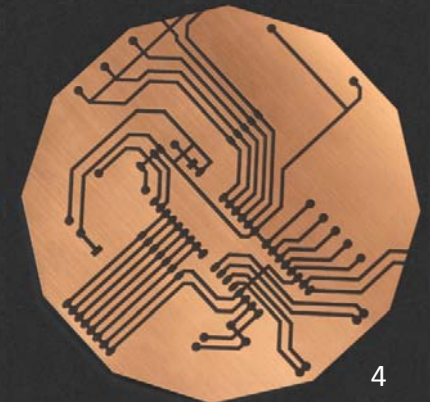
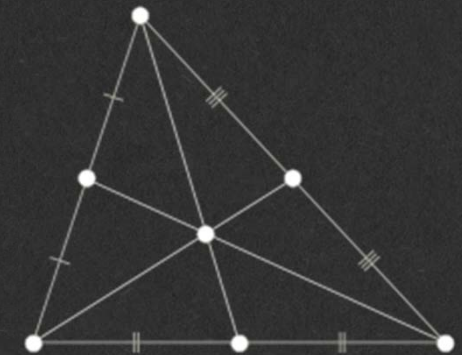
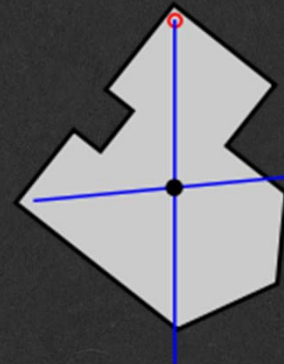
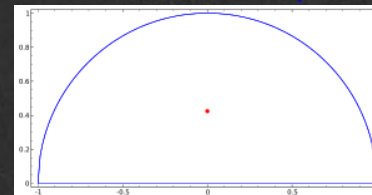
- Wireless Sensor Networks
 - Wide application scenarios
 - Omni-directional transmission gives a coverage roughly like a circle
 - Scarce power resources, usually operating on batteries
- Wireless localization Technique
 - Location information is necessary for meaningful data collection.
 - Localization Techniques
 - Region-based
 - Range-based



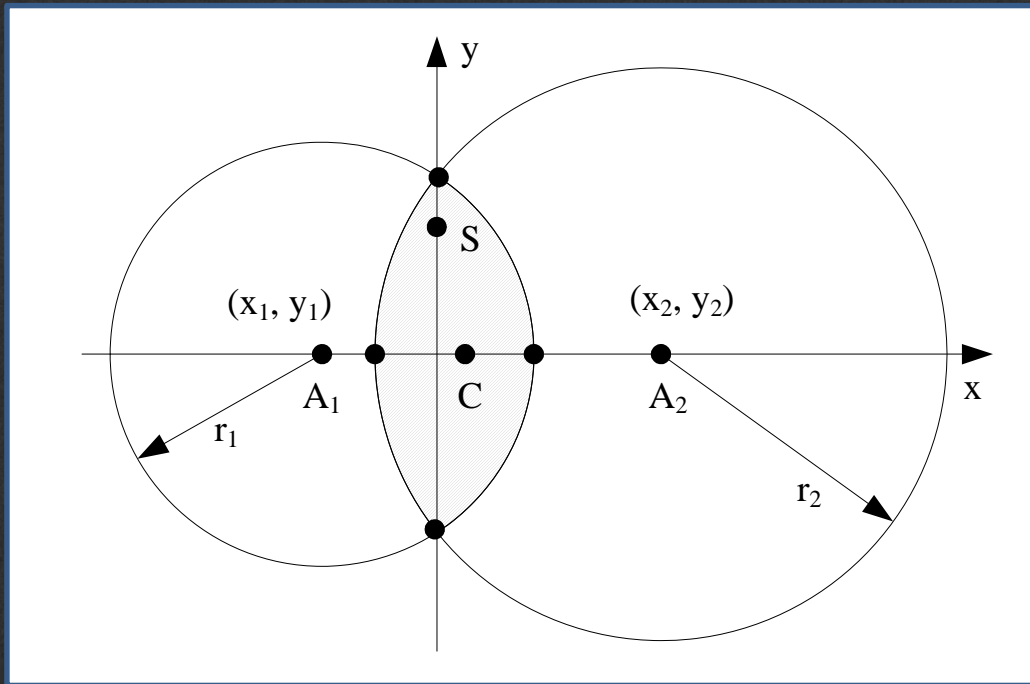
Centroid Computing

- Centroid
 - the point at which a cardboard cut-out of the region could be perfectly balanced on the tip of a pencil, with the assumption of uniform density and uniform gravitational field.
- How to locate the centroid?
 - Plumb line method
 - Geometric decomposition method
 - By integral formula

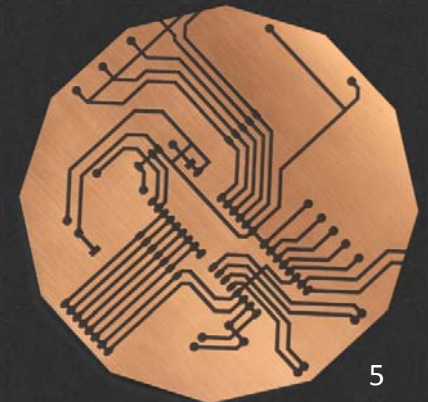
Source: <http://en.wikipedia.org/wiki/Centroid>



Centroid Computing



It is not difficult to compute the centroid of a half circle. But how about the centroid for the overlapped region of two intersecting circles (see the figure on the left)?



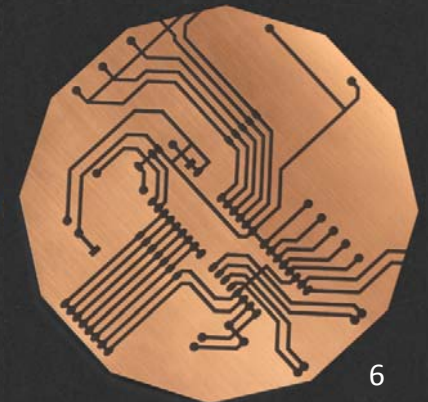
Proposed Analytical Method

- Overlapped area of two intersecting circles

$$\begin{cases} (x-x_1)^2 + (y-y_1)^2 = r_1^2, & (x-x_2)^2 + (y-y_2)^2 = r_2^2 \\ (x_s-x_1)^2 + (y_s-y_1)^2 \leq r_1^2, & (x_s-x_2)^2 + (y_s-y_2)^2 \leq r_2^2 \\ |r_1-r_2| \leq \sqrt{(x-x_1)^2 + (y-y_1)^2} \leq r_1+r_2 \end{cases}$$

- By integration

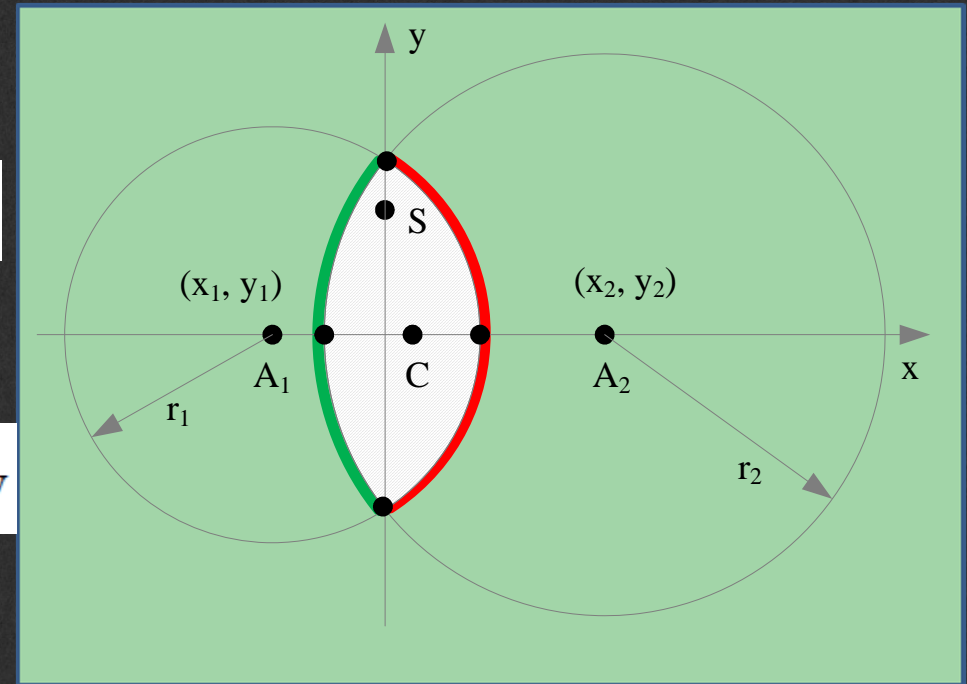
$$(\bar{x}, \bar{y}) = \left(\frac{\iint_R x dx dy}{\iint_R dx dy}, \frac{\iint_R y dx dy}{\iint_R dx dy} \right)$$



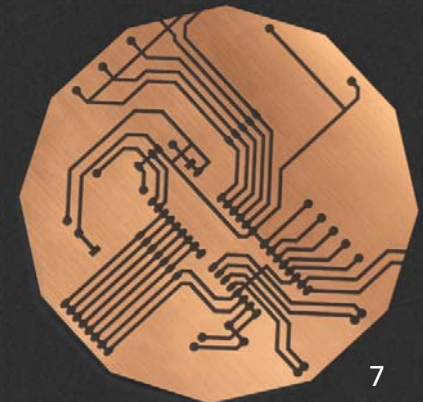
Analytical Solution for Special Case

$$f(x_2) = \pm \sqrt{r_2^2 - (x - x_2)^2}, \text{ for left boundary}$$

$$f(x_1) = \pm \sqrt{r_1^2 - (x - x_1)^2}, \text{ for right boundary}$$



$$\bar{x} = \frac{\int_{x_2-r_2}^0 2x\sqrt{r_2^2 - (x-x_2)^2} dx + \int_0^{x_1+r_1} 2x\sqrt{r_1^2 - (x-x_1)^2} dx}{\int_{x_2-r_2}^0 2\sqrt{r_2^2 - (x-x_2)^2} dx + \int_0^{x_1+r_1} 2\sqrt{r_1^2 - (x-x_1)^2} dx}$$

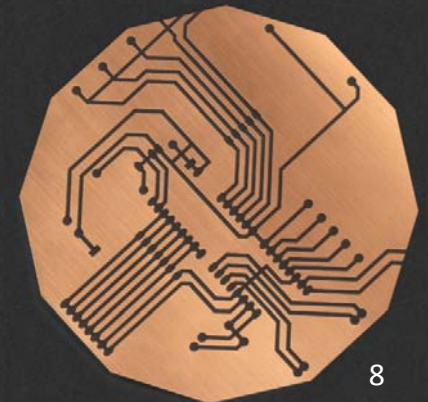


Analytical Solution for Special Case

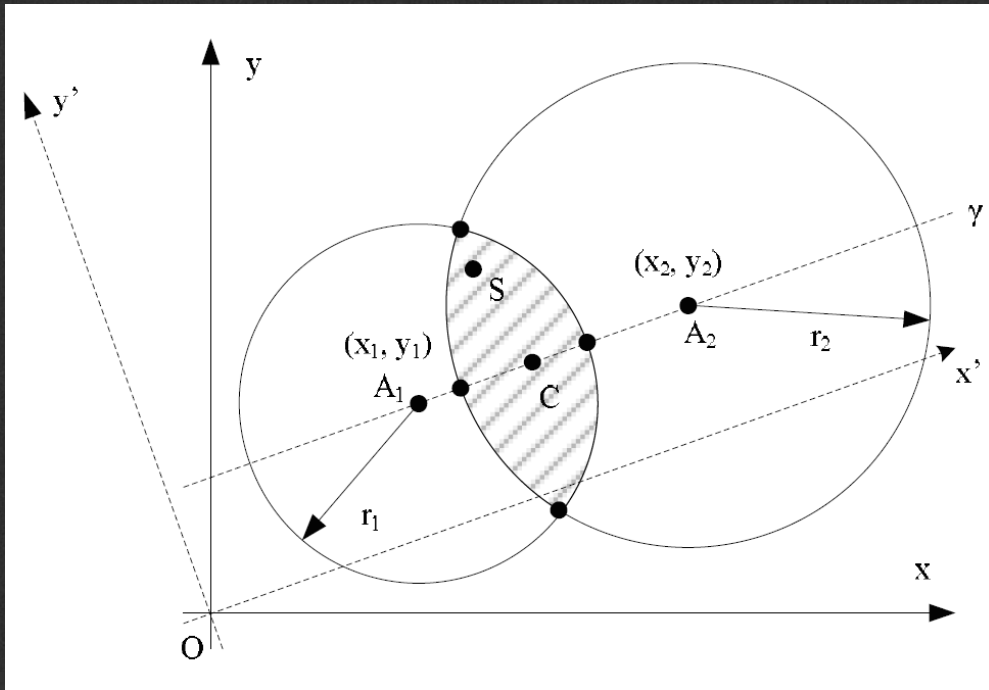
$$\bar{x} = P/Q$$

$$P = \frac{\pi}{4}(x_1 r_1^2 + x_2 r_2^2) + \left(\frac{r_1^2}{3} + \frac{x_1^2}{6} \right) \sqrt{r_1^2 - x_1^2} \\ - \left(\frac{r_2^2}{3} + \frac{x_2^2}{6} \right) \sqrt{r_2^2 - x_2^2} + \frac{r_1^2 x_1}{2} \sin^{-1} \left(\frac{x_1}{r_1} \right) - \frac{r_2^2 x_2}{2} \sin^{-1} \left(\frac{x_2}{r_2} \right)$$

$$Q = \frac{\pi}{4}(r_1^2 + r_2^2) + \frac{x_1}{2} \sqrt{r_1^2 - x_1^2} - \frac{x_2}{2} \sqrt{r_2^2 - x_2^2} \\ + \frac{r_1^2}{2} \sin^{-1} \left(\frac{x_1}{r_1} \right) - \frac{r_2^2}{2} \sin^{-1} \left(\frac{x_2}{r_2} \right)$$

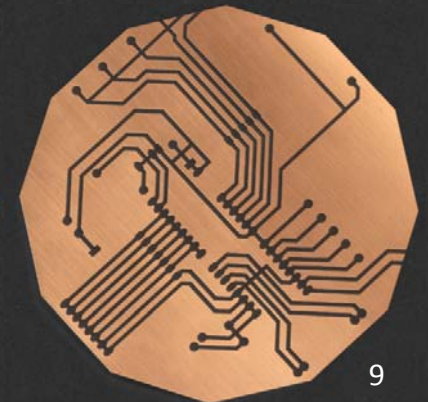


General Case



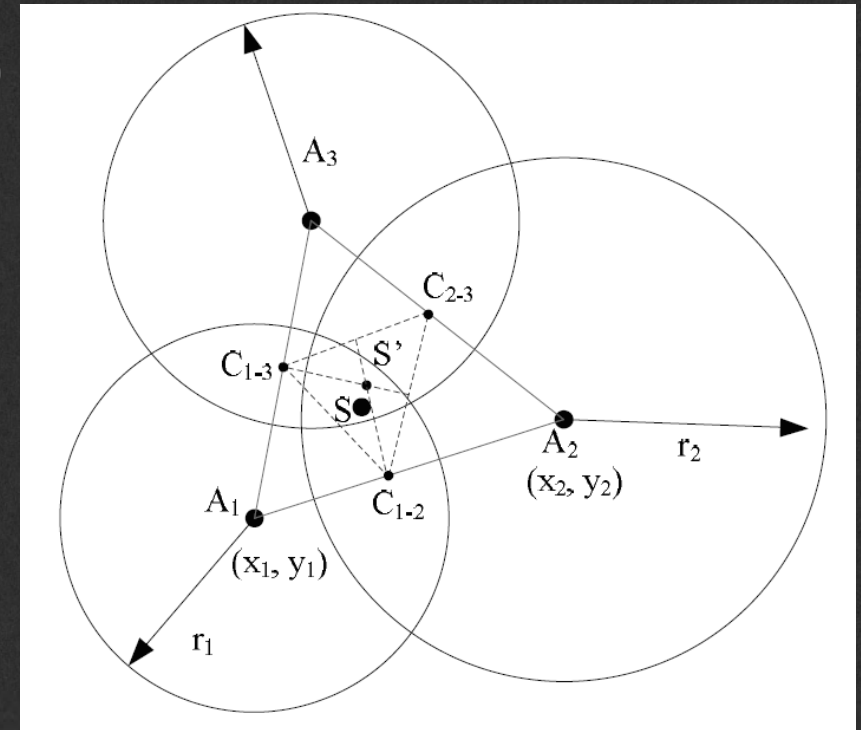
For the general case where the centres of two intersecting circles are not on the same axis, we need to:

- 1) Perform axis rotation
- 2) Perform axis shifting
- 3) Compute the centroid using the solution from special case
- 4) Perform axis shifting
- 5) Perform axis rotation and then obtain the analytical solution

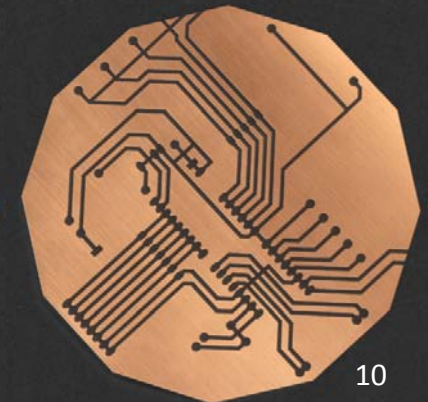


Application of Analytical Centroid Computing

- Centroid-Based Localization (CBL)
 - Anchors deployment
 - e.g., A_1, A_2, A_3
 - Beacon frames broadcast
 - Beacon frames synthesis
 - Compute $C_{1,2}, C_{1,3}, C_{2,3}$
 - Location estimation
 - Estimate sensor location for S

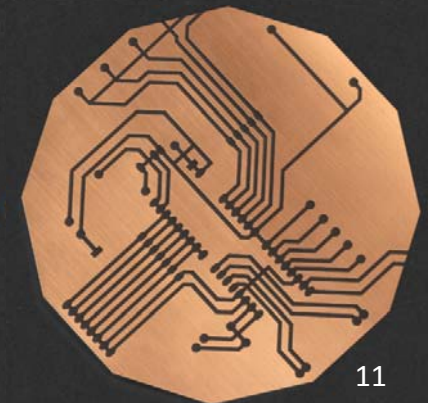
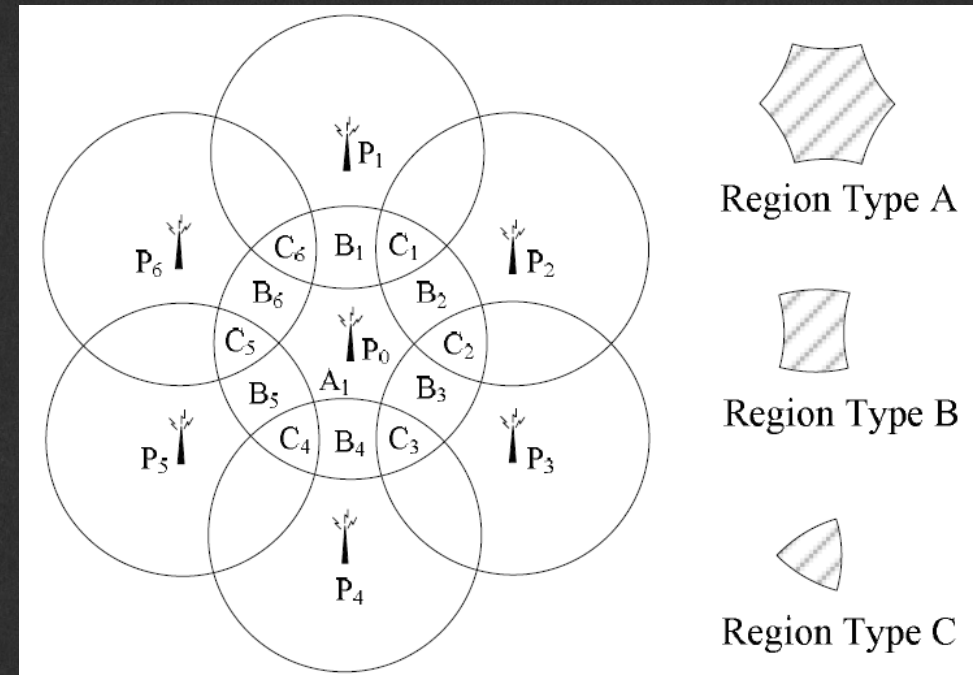


$$S': (x_{s'}, y_{s'}) = \left(\frac{x_{C_{1,2}} + x_{C_{1,3}} + x_{C_{2,3}}}{3}, \frac{y_{C_{1,2}} + y_{C_{1,3}} + y_{C_{2,3}}}{3} \right)$$

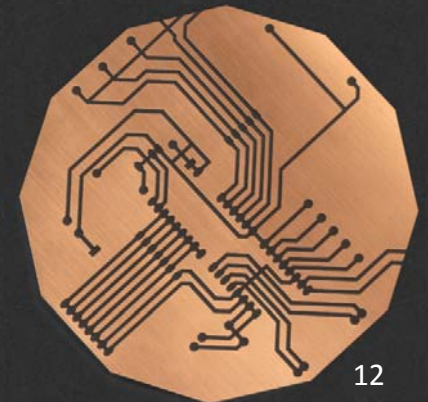
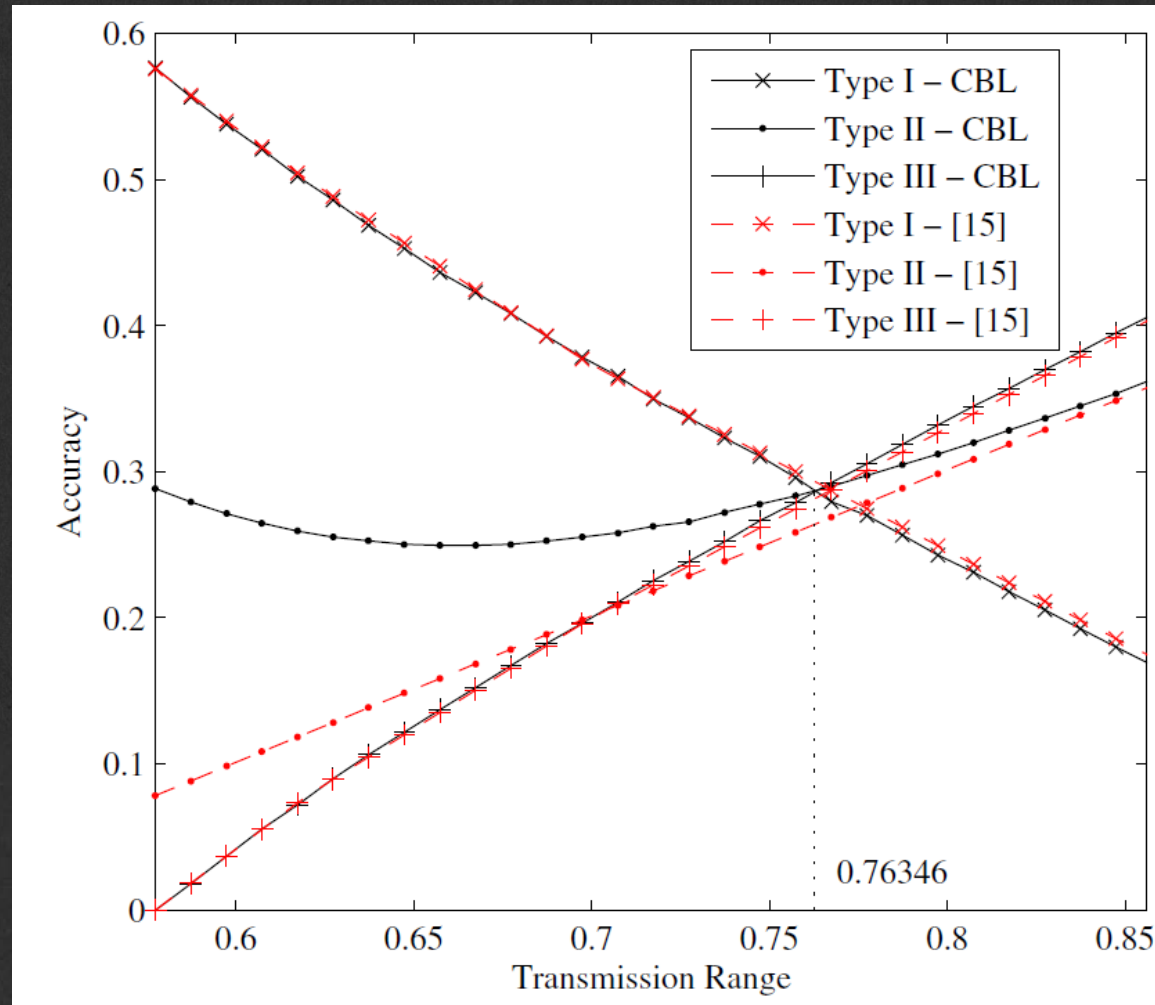


Region-Based Localization

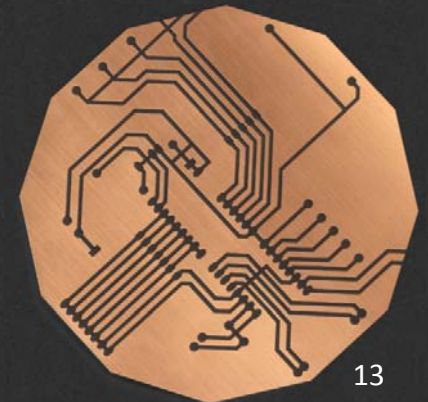
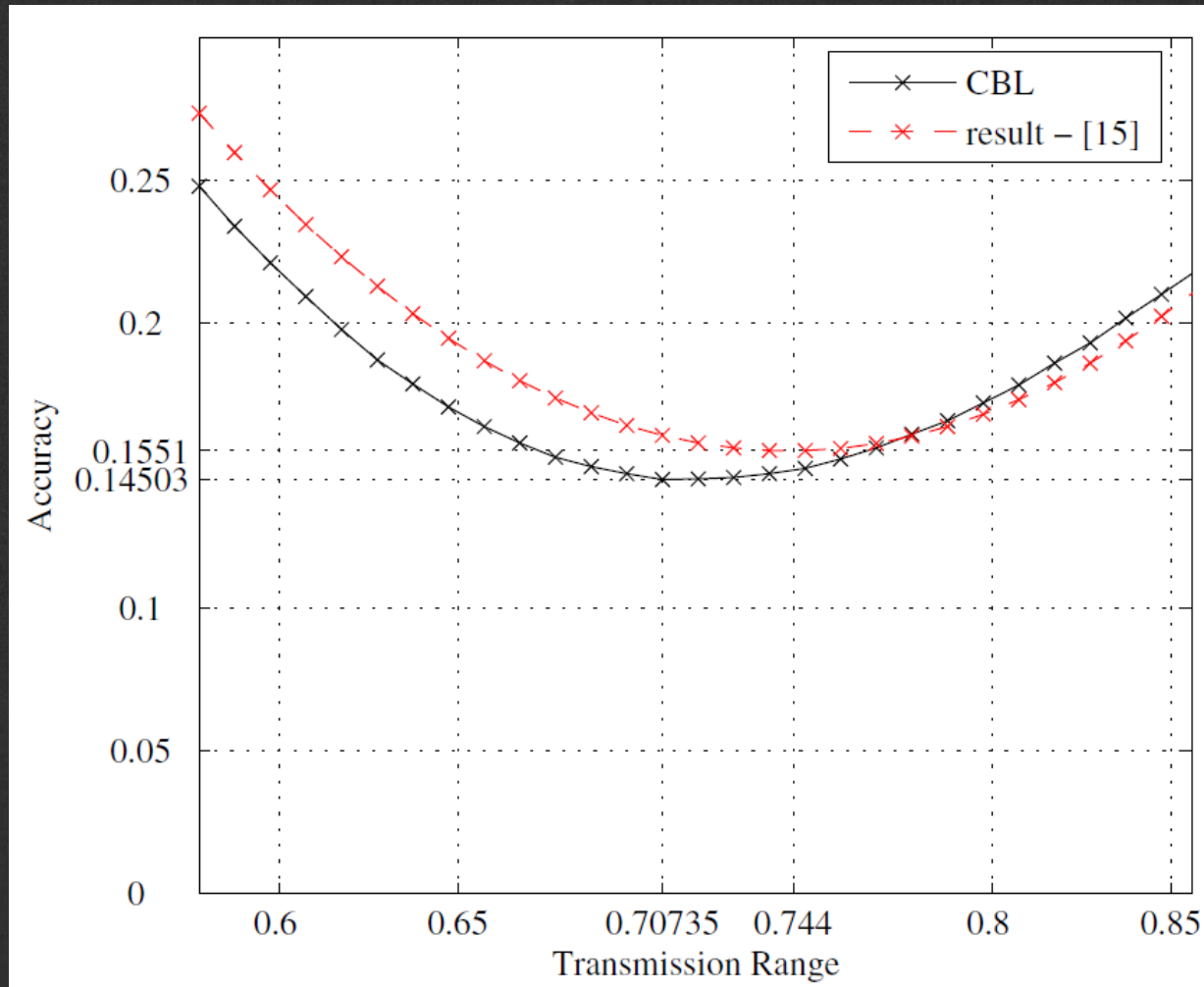
- To facilitate comparison, existing Region-Based Localization (RBL) was revisited here.



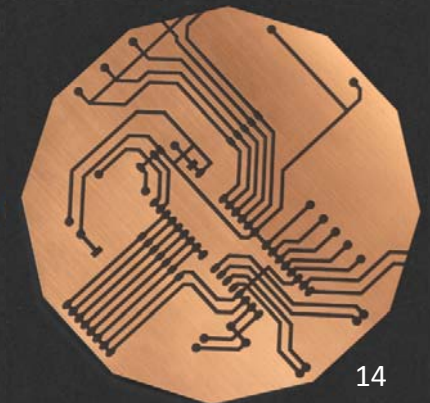
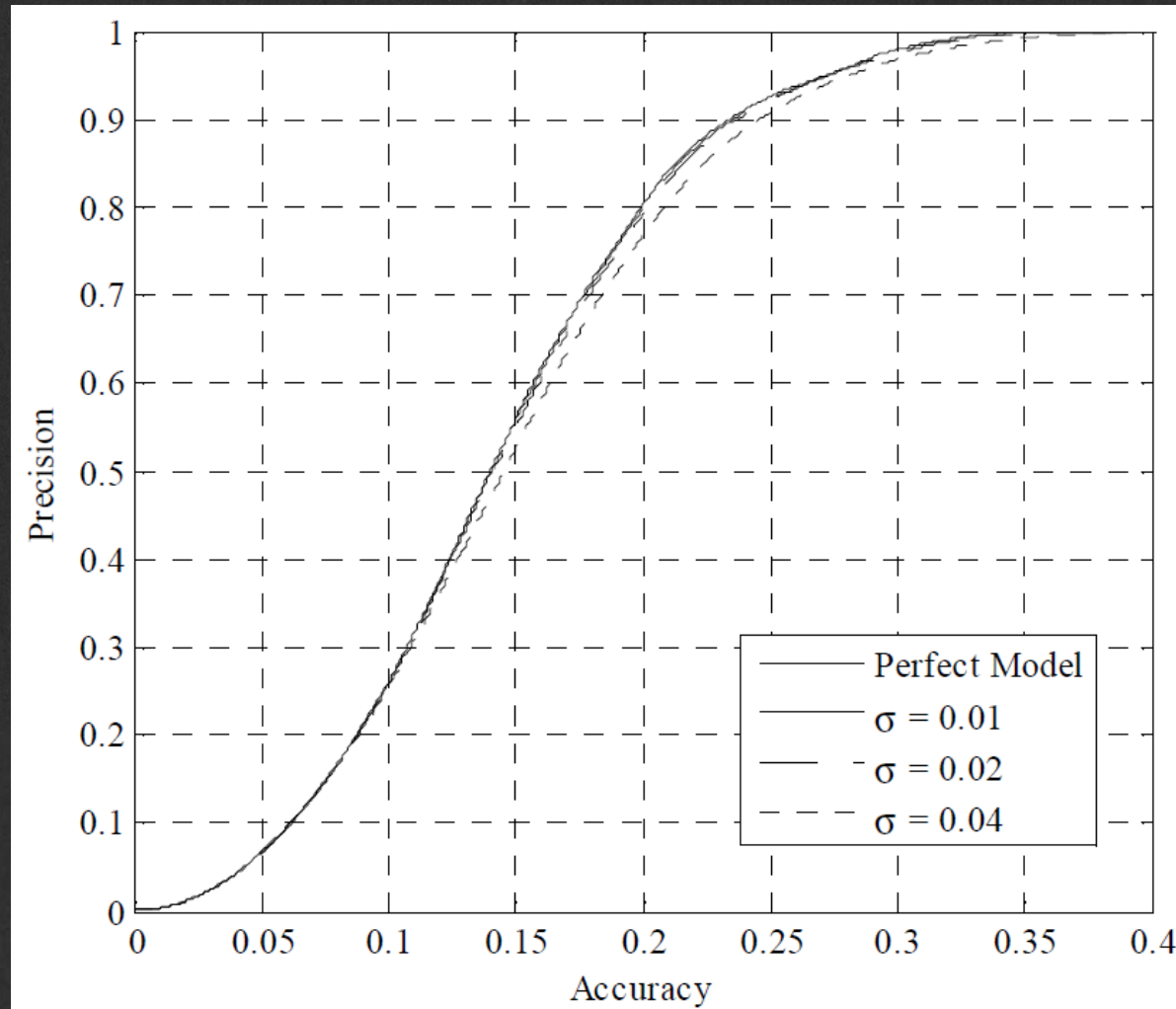
Worst-case Performance Comparison



Average Accuracy Comparison

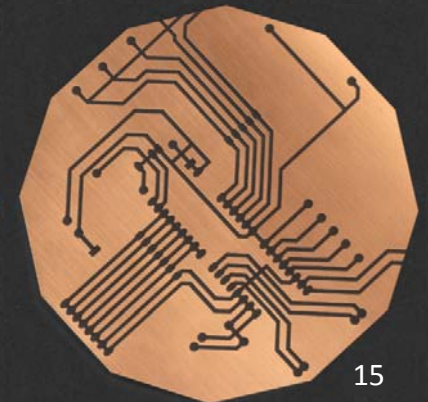


Effect of Mismatch in Anchor Transmission Power Levels



Conclusion

- An analytical approach of computing the centroid for the overlapped area of any two intersecting circles was presented.
- Centroid Based Localization (CBL) was proposed and its performance was compared with existing region based localization technique through computer simulations.
- CBL is able to improve localization accuracy while reducing transmission power of anchor nodes.
- The effect of power mismatch was also studied.



Q&A

Thank you for your attention!

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