

Optimal Subspace-enhanced Dynamic Mode Decomposition-assisted Dissipating Energy Flow-based Forced Oscillation Source Localization Using Synchrophasor Measurement

Lixin Wang, Weijun Suo, Han Gao, Zhenglong Sun, Shiwei Xia, and Tek Tjing Lie

Abstract—Forced oscillation source localization (FOSL) using synchrophasor measurements is critical for mitigating forced oscillations (FOs) in power systems. However, the performance of the conventional dissipating energy flow (DEF) is significantly affected by measurement noise and other irrelevant modal components. To address this challenge, an optimal subspace-enhanced (Os-enhanced) dynamic mode decomposition (DMD)-assisted DEF-based FOSL is proposed in this paper using synchrophasor measurement. First, Os-enhanced DMD is employed to decompose multi-channel measurements into time-domain responses of individual modes. Then, the time-domain components associated with FO are distinguished from the decomposed modes based on the rate of change of modal energy, and are subsequently used to calculate the DEF at all generator buses. Furthermore, the rate of forced energy is introduced as an indicator of energy flow direction to localize the FO source. The performance of the Os-enhanced DMD-assisted DEF is evaluated by the simulated data from IEEE 16-machine 5-area system and field phasor measurement unit (PMU) measurement data from Independent System Operator New England (ISO-NE). Comparative results demonstrate that the Os-enhanced DMD-assisted DEF achieves improved accuracy and robustness for localizing FO sources under noisy conditions.

Index Terms—Forced oscillation (FO), oscillation source localization, dissipating energy flow (DEF), dynamic mode decomposition (DMD), synchrophasor measurement.

I. INTRODUCTION

IN recent years, forced oscillations (FOs) have emerged as a significant threat to the security and stability of modern power systems. FOs are typically induced by external periodic disturbances such as cyclic loads or electrical and mechanical oscillations caused by malfunctioning controllers [1]. FOs often occur abruptly and unpredictably, while conventional controllers are not designed to damp such oscillations. When FOs coincide with a natural system mode, they can potentially cause catastrophic blackouts. In August 2013, an FO with a frequency of 0.9 Hz occurred in the power grid operated by New York Independent System Operator (NY-ISO), resulting in significant power oscillations across interconnections [2]. In January 2019, an FO with a frequency of 0.25 Hz occurred in the Eastern Interconnection, USA, propagating across the entire grid [3]. It is widely acknowledged in both industry and academia that effective mitigation of FOs relies on accurate identification and isolation of their sources. Therefore, developing effective methods for localizing FO sources is of paramount importance.

Extensive research efforts have been devoted to localizing FO sources. In [4], a hybrid dynamic simulation method is proposed to identify the region of the FO source. In [5], a mode shape-based method is employed to localize FO sources using spatial characteristics. However, with the increasing scale and complexity of power systems, acquiring accurate system models and parameters becomes increasingly difficult. As a result, purely model-based source localization methods may suffer from limited practicality and reliability. In recent years, the widespread deployment of wide-area measurement systems (WAMSs) has provided new opportunities for data-based FOSL. In [6], statistical indicators including kurtosis and power spectral density are used to identify FO sources using measurement data. In [7] and [8], damping torque coefficients derived from measured data are employed to localize FO source. However, the torque coefficient may vary across different oscillation modes, making these methods less effective under certain conditions. Artificial intelligence (AI)-based methods have also been proposed for FOSL by establishing mapping relationships between oscillation data and source location. Typically, these

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methods are trained offline using historical datasets and then applied online for real-time source localization [9], [10]. However, their performance heavily depends on the representativeness of the training data, and retraining is often required when system conditions change. In addition, the lack of physical interpretability remains a limitation. In [11], a practical dissipating energy flow (DEF) calculation method using phasor measurement unit (PMU) data is proposed, with extensions to the frequency domain [12] and time-frequency domain [13]. Among the existing methods, the energy-based methods stand out and show promising performance in identifying FO sources due to their clear physical interpretation.

Although the energy-based methods are effective in identifying FO sources, their performance strongly depends on the accurate extraction of FO components from measurement data. To address this issue, variational mode decomposition (VMD) and empirical mode decomposition (EMD) are proposed in [14] and [15], respectively, to extract FO components using intrinsic mode functions. In [16], a band-pass filter method is applied to extract FO components. More recently, time-frequency analysis techniques, including the short-time Fourier transform-based synchrosqueezing transform (FSST) [17] and the synchrosqueezing wavelet transform (SWT) [18], have been employed to extract FO components, which are then used to calculate DEF. However, the aforementioned methods are predominantly designed for single-channel signal analysis, requiring each measurement channel to be processed independently. This leads to low computation efficiency and inconsistent FO component extraction due to the neglect of correlations among multiple channels. Hence, multi-channel decomposition methods such as multivariate EMD (MEMD) [19] and multivariate VMD (MVMD) [20] have been proposed. Nevertheless, these methods remain susceptible to mode mixing, which may result in inaccurate FO extraction, especially in the presence of strong noise.

To effectively localizing FO sources under noisy conditions, this paper proposes optimal subspace-enhanced (Os-enhanced) dynamic mode decomposition (DMD)-assisted DEF. First, we employ the Os-enhanced DMD to extract a state-space matrix from the PMU measurements. Then, eigenvalue decomposition is performed on the extracted matrix to obtain the time-domain responses of individual mode, including active power, reactive power, voltage magnitude, and frequency of each generator. Subsequently, FO components are identified via the rate of change of modal energy (RCME). Using the identified FO components, the DEF and its corresponding rate of forced energy (ROFE) are calculated for all generators. The FO source is intuitively identified using normalized ROFE. The main contributions of this paper are summarized as follows.

1) Os-enhanced DMD is employed to decompose multi-channel measurements into time-domain responses of individual modes, enabling simultaneous FO component extraction from multiple measurement channels. By fully considering the correlations across different channels, the accuracy of FO component extraction is improved.

2) The modal energy of the time-domain components obtained by Os-enhanced DMD is calculated to evaluate the contribution of each mode to the overall system oscillation, and the FO mode is identified based on the relative RCME values and their consistency across all measurement channels. By incorporating subspace optimization on the Grassmann manifold, Os-enhanced DMD effectively concentrates the FO components, mitigating the noise-induced mode distortion and accuracy degradation encountered in conventional DMD.

3) Os-enhanced DMD-assisted DEF is proposed (hereafter termed as the proposed DEF) to effectively localize FO source. By combining the extracted FO components with the conventional DEF, the direction of energy flow injected by each generator is determined, and the FO source is then localized based on normalized ROFE. The integration of Os-enhanced DMD enables rapid and accurate FO component extraction, significantly improving the accuracy and robustness of the proposed DEF.

4) Simulation studies, including FOs triggered by excitation and governor systems under varying disturbance amplitudes, frequencies, and noise levels, demonstrate the effectiveness of the proposed DEF and its superior noise robustness.

The remainder of this paper is organized as follows. Section II presents the problem statement of FO. Section III introduces the theoretical background. Section IV presents the analysis of the proposed DEF. Section V presents the case studies. Section VI concludes the paper.

II. PROBLEM STATEMENT OF FO

A periodic forced disturbance can influence generators and their associated state variables. The schematic diagram of power system with FO sources shown in Fig. 1 illustrates various FO sources including generator governor, exciter, cyclic load, and high-voltage direct current (HVDC) converter, etc., where V^x and θ^x ($x \in \{i, m, j, k, l, n\}$) are the voltage magnitude and phase angle at bus x , respectively. Each component may potentially serve as an FO source. The impact of FOs injected by external sources is analyzed using the following mathematical model as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \quad (1)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \quad (2)$$

where $\mathbf{x} \in \mathbb{R}^{n_x}$ is the n_x -dimensional state variable vector; $\mathbf{y} \in \mathbb{R}^{n_y}$ is the n_y -dimensional observation variable vector; $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$ are the nonlinear differential and algebraic equations, respectively; and $\mathbf{u}$ is the vector of periodic disturbance and is assumed to follow an Ornstein-Uhlenbeck (OU) process, which is given by [21]:

$$\dot{\mathbf{u}} = -\mathbf{E}\mathbf{u} + \mathbf{F}\mathbf{e}^{j\omega t} \quad (3)$$

where $\mathbf{E}$ is the diagonal matrix whose diagonal elements represent the correlation time constants of the disturbances; $\mathbf{F}$ is the amplitude matrix of the periodic disturbance; t is the time step; and ω is the disturbance frequency.

By linearizing (1) and (3) and eliminating $\mathbf{y}$ from (1), the power system dynamic model under periodic disturbances

where X_n is the snapshot matrix formed by arranging state variables at n consecutive sampling instants as columns; and X_{n+1} is the consecutive snapshot matrix lagging one time step behind X_n . In practical applications, the discrete form of (8) is given as:

$$X_{n+1} = A_d X_n \quad (9)$$

where A_d is the high-dimensional state matrix of the system.

To reduce the dimensionality, A_d is projected onto a low-dimensional approximation state matrix $\tilde{A}_d$ using proper orthogonal decomposition (POD) mode U [22]:

$$A_d \approx U \tilde{A}_d U^T \quad (10)$$

U is obtained from the singular value decomposition of X_n using (11).

$$X_n = U \Sigma V^T \quad (11)$$

where Σ is the diagonal matrix containing the singular values; and V is the right singular vector.

$\tilde{A}_d$ is then obtained by solving the following optimization problem:

$$\min_F \|X_{n+1} - U \tilde{A}_d \Sigma V^T\|_F^2 \quad (12)$$

Combining (10)-(12), we can obtain:

$$\tilde{A}_d \approx A_d = U^T X_{n+1} V \Sigma^{-1} \quad (13)$$

C. Os-enhanced DMD

In conventional DMD, the projection subspace is determined by a truncated POD basis and remains fixed during decomposition. Hence, the conventional DMD is sensitive to measurement noise. To overcome this limitation, the Os-enhanced DMD formulates a rank-constrained optimization problem on the Grassman manifold, enabling adaptive subspace optimization. The optimization problem is expressed as [23]:

$$\min \|X_{n+1} - A_d X_n\|^2 = \|X_{n+1} - \tilde{L} \tilde{A} L^T X_n\|^2 \quad (14)$$

where L is the low-dimensional subspace matrix, whose dimension is selected via energy criterion of cumulative singular value to retain at least 95% of the total energy; and $\tilde{A}$ is the low-dimensional approximation state matrix associated with L .

For a given L , the optimal low-dimensional approximation state matrix $\tilde{A}^*$ is expressed as [24]:

$$\tilde{A}^* = L^T X_{n+1} X_n^T L (L^T X_n X_n^T L)^{-1} \quad (15)$$

Substituting (15) into (14) yields:

$$\begin{cases} \min \|X_{n+1} - \tilde{L} \tilde{A}^* L^T X_n\|^2 = \min (\|X_{n+1}\|^2 - \|L^T X_{n+1} M\|^2) \\ M = X_n^T L (L^T X_n X_n^T L)^{-1} L^T X_n \end{cases} \quad (16)$$

Accordingly, the original bi-variable optimization problem in (14), involving both $\tilde{A}$ and L , is reduced to a single-variable optimization problem with respect to L :

$$\begin{cases} \max g(L) = \|L^T X_{n+1} M\|^2 \\ \text{s.t. } M = X_n^T L (L^T X_n X_n^T L)^{-1} L^T X_n \\ L^T L = I \end{cases} \quad (17)$$

where I is the identity matrix; and $g(L)$ is the objective function with gradient G .

To solve (17), a conjugate gradient algorithm on the Grass-

man manifold is employed. The initial gradient G_0 and the initial low-dimensional subspace matrix L_0 are expressed as [25]:

$$\begin{cases} G_0 = (I - L_0 L_0^T) (\partial g(L) / \partial L)_{L=L_0} \\ H_0 = -G_0 \end{cases} \quad (18)$$

During the optimization process, the search direction follows the geodesic path on the Grassman manifold. The updated subspace at the $(k+1)^{\text{th}}$ iteration is expressed as:

$$\begin{cases} L_{k+1} = L_k Q \Gamma Q^T + P Y Q^T \\ H_k = P \Lambda Q^T \end{cases} \quad (19)$$

where L_k and L_{k+1} are the low-dimensional subspace matrices at the k^{th} and $(k+1)^{\text{th}}$ iterations, respectively; P and Q are the left and right singular vectors of the search direction matrix H_k ; Λ is the singular value matrix of H_k ; and Γ and Y are the diagonal matrices containing the cosine and sine of the elements in Λ , respectively.

The gradient and search direction are updated iteratively [25] as:

$$\begin{cases} G_{k+1} = (I - L_{k+1} L_{k+1}^T) (\partial g(L) / \partial L)_{L=L_{k+1}} \\ H_{k+1} = G_{k+1} + \Delta_{k+1} \end{cases} \quad (20)$$

where Δ_{k+1} is the correction term; and G_{k+1} is the projected gradient at the $(k+1)^{\text{th}}$ iteration.

$$\Delta_{k+1} = \frac{\langle G_{k+1} - \tau(G_k), G_{k+1} \rangle}{\|G_{k+1}\|^2} \tau(H_k) \quad (21)$$

where τ is the vector translation operator on the Grassmann manifold; and $\langle \cdot, \cdot \rangle$ denotes the inner product.

$$\begin{cases} \tau(G_k) = G_k - (L_k Q Y_k + P(I - \Gamma_k)) P^T G_k \\ \tau(H_k) = (-L_k Q Y_k + P \Gamma_k) \Lambda Q^T \end{cases} \quad (22)$$

where Γ_k and Y_k are the diagonal matrices containing the cosines and sines of the singular values at the k^{th} iteration, respectively.

Based on (20)-(22), L is iteratively updated until the stopping criterion is satisfied: $(g(L_{k+1}) - g(L_k)) / g(L_k) < \varepsilon$, where ε is a predefined convergence tolerance. The resulting optimal subspace matrix L^* is substituted into (15) to obtain $\tilde{A}^*$.

$$\tilde{A}^* = (L^*)^T X_{n+1} X_n^T L^* ((L^*)^T X_n X_n^T L^*)^{-1} \quad (23)$$

Eigenvalue decomposition is then performed on $\tilde{A}^*$ as:

$$\tilde{A}^* = \Phi \Pi \Phi^{-1} \quad (24)$$

where $\Pi = \text{diag}(\lambda_i) \in \mathbb{R}^{n \times n}$ is the eigenvalue matrix, and λ_i is the i^{th} eigenvalue; and Φ is the corresponding right eigenvector.

The mode shape matrix ψ_i is defined as [23]:

$$\psi_i = \lambda_i^{-1} X_{n+1} W S^{-1} \phi_i \quad (25)$$

where W and S are the right singular vector and singular value matrix of X_n , respectively; and ϕ_i is the right eigenvector associated with λ_i .

The time-domain response of the r^{th} mode, i.e., $\Delta y_r(t)$, is reconstructed as [23]:

$$\Delta y_r(t) = e^{\lambda_r t} \psi_r^{-1} x(0) \quad (26)$$

where $\boldsymbol{\psi}_r$ is the shape matrix of the r^{th} mode; and $\boldsymbol{x}(0)$ is the initial system state vector.

IV. PROPOSED DEF FOR LOCALIZING FO SOURCES

Existing FO component extraction methods are mostly based on single-channel analysis, requiring individual decomposition for each measurement channel. This often leads to asynchronous FO mode identification across different channels. Moreover, their limited noise robustness degrades the accuracy of localizing the FO sources. To address these issues, we apply Os-enhanced DMD to jointly decompose oscillation modes from multi-channel measurements of active power P , reactive power Q , voltage V , and frequency f collected at multiple locations, improving the accuracy of the proposed DEF.

A. Multi-channel Decomposition Using Os-enhanced DMD

With PMUs installed at critical buses, multiple measurements can be simultaneously measured at each bus. For the g^{th} bus, deviation signal matrix of each quantity $\mu \in \{P, Q, U, f\}$ are calculated as $\Delta \mathbf{X}_{gu} = \mathbf{X}_{gu} - \boldsymbol{\mu}_0$, where $\mathbf{X}_{gu}$ is the measured signal matrix, and $\boldsymbol{\mu}_0$ is the pre-disturbance. $\Delta \mathbf{X}_{gu}$ is constructed as in (27), and the subscript n denotes the n^{th} sampling point.

$$\Delta \mathbf{X}_{gu} = [\Delta \mathbf{X}_{gP} \quad \Delta \mathbf{X}_{gQ} \quad \Delta \mathbf{X}_{gU} \quad \Delta \mathbf{X}_{gf}]^T = \begin{bmatrix} \Delta P_{g(1)} & \Delta P_{g(2)} & \cdots & \Delta P_{g(n)} \\ \Delta Q_{g(1)} & \Delta Q_{g(2)} & \cdots & \Delta Q_{g(n)} \\ \Delta \ln U_{g(1)} & \Delta \ln U_{g(2)} & \cdots & \Delta \ln U_{g(n)} \\ \Delta f_{g(1)} & \Delta f_{g(2)} & \cdots & \Delta f_{g(n)} \end{bmatrix} \quad (27)$$

Equation (27) is then decomposed using the Os-enhanced DMD. The decomposed $\Delta \mathbf{X}_{gu}$ can be formulated as in (28). Each element corresponds to the time-domain response of a specific oscillation mode reconstructed via (26). Specifically, $\Delta y_{gp, \Delta P(p)}$, $\Delta y_{gp, \Delta Q(p)}$, $\Delta y_{gp, \Delta \ln U(p)}$, $\Delta y_{gp, \Delta f(p)}$ are the p^{th} mode components of ΔP , ΔQ , $\Delta \ln U$, and Δf for the g^{th} bus, respectively.

$$\Delta \mathbf{y}_g = \begin{bmatrix} \Delta y_{g1, \Delta P(1)} & \Delta y_{g1, \Delta Q(1)} & \Delta y_{g1, \Delta \ln U(1)} & \Delta y_{g1, \Delta f(1)} \\ \Delta y_{g2, \Delta P(2)} & \Delta y_{g2, \Delta Q(2)} & \Delta y_{g2, \Delta \ln U(2)} & \Delta y_{g2, \Delta f(2)} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta y_{gp, \Delta P(p)} & \Delta y_{gp, \Delta Q(p)} & \Delta y_{gp, \Delta \ln U(p)} & \Delta y_{gp, \Delta f(p)} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta y_{gn, \Delta P(n)} & \Delta y_{gn, \Delta Q(n)} & \Delta y_{gn, \Delta \ln U(n)} & \Delta y_{gn, \Delta f(n)} \end{bmatrix}^T \quad (28)$$

B. FO Mode Extraction

In (28), Os-enhanced DMD decomposes multi-channel measurements into a set of time-domain components associated with distinct modes. To accurately localizing FO sources, reliable identification and extraction of the FO mode from these decomposed modes are crucial. Physically, the difference between FO and natural oscillation modes lies in their damping characteristics and modal energy evolution. Natural oscillation modes are governed by inherent system damping and exhibit an exponential decay in modal energy once the disturbance subsides. In contrast, FOs are continu-

ously sustained by external periodic disturbances and therefore exhibit near-zero damping. Based on this physical insight, RCME is introduced as a dynamic indicator to characterize the temporal evolution of modal energy. Since FO modes are persistently driven, they exhibit consistently higher RCME values across different channels, i.e., ΔP , ΔQ , $\Delta \ln U$, Δf , compared with natural oscillation modes. Therefore, RCME provides a physically interpretable and theoretically grounded criterion for discriminating FO mode from natural oscillation mode.

For the p^{th} mode component in the $(\Delta \mu)^{\text{th}}$ channel of $\Delta \mathbf{X}_{gu}$, i.e., $\Delta y_{p, \Delta \mu}(t)$, the corresponding modal energy $b_{p, q}$ accumulated up to the q^{th} time sample is calculated as [18]:

$$b_{p, q} = \sum_{t=1}^q (\Delta y_{p, \Delta \mu}(t))^2 \quad (29)$$

The RCME $\eta_{p, q}$ is calculated as:

$$\eta_{p, \Delta \mu} = \left| \frac{b_{p, q} - b_{p, q+1}}{b_{p, q+1}} \right| \times 100\% \quad p \in 1, 2, \dots, n-1 \quad (30)$$

Accordingly, the RCME matrix of $\Delta \mathbf{X}_{gu}$ is defined as:

$$\boldsymbol{\eta}_g = \begin{bmatrix} \eta_{1, \Delta P_g} & \eta_{1, \Delta Q_g} & \eta_{1, \Delta \ln U_g} & \eta_{1, \Delta f_g} \\ \eta_{2, \Delta P_g} & \eta_{2, \Delta Q_g} & \eta_{2, \Delta \ln U_g} & \eta_{2, \Delta f_g} \\ \vdots & \vdots & \vdots & \vdots \\ \eta_{p, \Delta P_g} & \eta_{p, \Delta Q_g} & \eta_{p, \Delta \ln U_g} & \eta_{p, \Delta f_g} \\ \vdots & \vdots & \vdots & \vdots \\ \eta_{n, \Delta P_g} & \eta_{n, \Delta Q_g} & \eta_{n, \Delta \ln U_g} & \eta_{n, \Delta f_g} \end{bmatrix}^T \quad (31)$$

where $\eta_{p, \Delta P_g}$, $\eta_{p, \Delta Q_g}$, $\eta_{p, \Delta \ln U_g}$, and $\eta_{p, \Delta f_g}$ are the RCME values corresponding to the p^{th} mode component of ΔP_g , ΔQ_g , $\Delta \ln U_g$, and Δf_g , respectively.

The FO mode corresponds to the p^{th} column of $\boldsymbol{\eta}_g$, characterized by significantly higher RCME values across all four channels. If all elements in the p^{th} column of $\boldsymbol{\eta}_g$ exceed a threshold η_0 , the corresponding ω_p is regard as the FO frequency. This criterion enables reliable FO mode identification and allows the extraction of its time-domain components from ΔP_g , ΔQ_g , $\Delta \ln U_g$, and Δf_g . Therefore, the selection of η_0 is critical: a larger value may miss the dominant FO mode, while a smaller value may introduce irrelevant mode. In this paper, a typical value of $\eta_0 = 0.6$ is selected, and its effectiveness is verified in Supplementary Material C.

The extracted time-domain components associated with the identified FO mode are expressed as:

$$\Delta \mathbf{y}_{gp} = \begin{bmatrix} \Delta y_{gp, \Delta P(1)} & \Delta y_{gp, \Delta Q(1)} & \Delta y_{gp, \Delta \ln U(1)} & \Delta y_{gp, \Delta f(1)} \\ \Delta y_{gp, \Delta P(2)} & \Delta y_{gp, \Delta Q(2)} & \Delta y_{gp, \Delta \ln U(2)} & \Delta y_{gp, \Delta f(2)} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta y_{gp, \Delta P(n)} & \Delta y_{gp, \Delta Q(n)} & \Delta y_{gp, \Delta \ln U(n)} & \Delta y_{gp, \Delta f(n)} \end{bmatrix}^T \quad (32)$$

where $\Delta \mathbf{y}_{gp}$ is the time-domain component matrix of the p^{th} mode component at the g^{th} bus.

C. Proposed DEF

The time-domain FO components obtained in (32) are used as inputs to (7) to calculate the DEF flowing from bus

i to bus j for the p^{th} mode component.

$$W_{ij,p}^D = \int (2\pi\Delta y_{gp,\Delta P}\Delta y_{gp,\Delta f}dt + \Delta y_{gp,\Delta Q}d\Delta y_{gp,\Delta \ln U}) \quad (33)$$

To characterize the variation trend of DEF, the ROFE is introduced and defined as the slope of the DEF curve as:

$$ROFE_{ij,p}^D = \frac{W_{ij,p}^D[\tau+1] - W_{ij,p}^D[\tau]}{t[\tau+1] - t[\tau]} \quad (34)$$

where $ROFE_{ij,p}^D$ is the rate of change of DEF from bus j to bus j for the p^{th} mode component.

Notably, although the absolute value of $ROFE_{ij,p}^D$ may vary with oscillation amplitude, the relative ROFE values among different generators tend to remain constant. Therefore, the normalized ROFE, i.e., $ROFE_{ij,p}^{D*}$ is defined as:

$$ROFE_{ij,p}^{D*} = \frac{ROFE_{ij,p}^D}{\max\{ROFE_{ij,p}^D\}} \quad (35)$$

By calculating $ROFE_{ij,p}^{D*}$ for all available PMU buses, the variation trend of DEF is quantified and used for localizing FO sources. The localization criterion is defined as follows: if $ROFE_{ij,p}^{D*} = 1$, the dynamic mode component behind bus j continuously injects energy into the grid, indicating it is an FO source; if $ROFE_{ij,p}^{D*} < 0$, the mode component absorbs oscillation energy and is regarded as an FO sink; and if $0 < ROFE_{ij,p}^{D*} < 1$, a larger value indicates a stronger contribution to the FO.

D. Main Steps of Proposed DEF

The main steps of the proposed DEF are outlined as follows:

Step 1: a smoothing filter is applied to remove noise and steady-state components, while the interpolation handles missing or anomalous data. PMU measurements of P , Q , $\ln U$, and f are collected, and their corresponding deviations are calculated as inputs for the energy analysis of Os-enhanced DMD.

Step 2: Os-enhanced DMD is employed to each deviation to obtain the time-domain response of each FO mode. RCME for each electrical quantity at each bus is then calculated using (30), enabling accurate FO mode identification. Once identified, the corresponding time-domain FO components are extracted from each signal.

Step 3: using the extracted FO components from *Step 2*, the DEF for each branch is calculated using (33), followed by the calculation of $ROFE_{ij,p}^{D*}$ in (35). The FO source is finally localized by comparing the values of the $ROFE_{ij,p}^{D*}$ across all generator buses.

E. Practical Consideration

1) Parameter and Setting of Os-enhanced DMD

The key parameters involved in the Os-enhanced DMD are summarized as follows.

1) ε for the conjugate-gradient optimization on the Grassmann manifold is set to be 10^{-6} , following common practice in optimization-based DMD [26]. This value provides a balanced trade-off between numerical accuracy and computation efficiency, ensuring stable convergence across different noise levels and operation conditions.

2) L_0 is constructed using the first left singular vector of

X_n , corresponding to the dominant POD modes. This initialization captures the principal spatio-temporal structures of the data and is widely adopted in both conventional and optimized DMDs [27], facilitating robust convergence of the subsequent optimization steps.

3) The maximum number of iterations $N_{\max}$. To avoid excessive computation or potential stagnation, $N_{\max}$ is limited to 200. This setting is consistent with studies on optimization-based DMD [28] and is found to be sufficient for all test systems considered in this paper to achieve stable convergence without unnecessary computation overhead.

2) Low-pass Filter Selection

To suppress high-frequency noise while preserving low-frequency electromechanical oscillations, a fourth-order Butterworth low-pass filter is employed due to its flat passband and linear phase response. Since the proposed DEF focuses on FOs in the low-frequency electromechanical range (typically 0.1-2.5 Hz), the cutoff frequency is set to be 3 Hz. In the discrete-time implementation, a 2-s filtering window (200 samples at a sampling rate of 100 Hz) is applied. This configuration provides effective noise attenuation without distorting the low-frequency oscillatory components of interest, thereby accurately extracting FO mode and localizing FO sources.

3) Enhancement of Data Quality

The proposed DEF is fully data-driven, and its performance is inherently influenced by the data quality. In practice, PMU data may contain missing samples or anomalous measurements, which can degrade localization performance. An in-depth treatment of data quality enhancement is beyond the scope of this paper. However, several general recommendations are provided. For missing data, established detection and estimation methods can be applied to restore data completeness and accurately localize the FO sources [29]. In addition, various outlier detection and replacement methods have been extensively investigated and can be applied to alleviate the impact of anomalous measurements [30]. Considering the scope of this paper, these recommendations are not investigated in detail but represent important directions for future work to further improve the performance of the proposed DEF.

4) Window Length Determination

Selecting an appropriate window length is essential for balancing frequency resolution and the ability to capture time-varying oscillatory behavior. In this paper, MATLAB/Simulink simulations are conducted with a sampling interval of $T_s = 0.01$ s, corresponding to a sampling rate of $F_s = 100$ Hz. Since the proposed DEF focuses on low-frequency FOs, Δf determines the required window length $N = F_s / \Delta f$. For electromechanical oscillations in power systems, a practical minimum frequency resolution of $\Delta f = 0.05$ Hz is generally sufficient to distinguish dominant modes, corresponding to a window length of 2000 samples (i.e., 20 s). To further enhance frequency discrimination while maintaining reasonable temporal resolution, a slightly longer window of 30 s is adopted, providing adequate frequency resolution for the test cases, in which the oscillation modes are not closely spaced. It should be noted that for systems containing multiple close-

ly spaced low-frequency modes, a smaller Δf (and hence a longer window) may be required to ensure sufficient frequency resolution.

V. CASE STUDIES

In this section, the performance of the proposed DEF is evaluated using simulation data from IEEE 16-machine 5-area system and field PMU measurement data from Independent System Operator New England (ISO-NE). In addition, comparative analyses with VMD, MEMD, Hankel block-enhanced DMD (HeDMD), and conventional DMD are conducted to demonstrate the effectiveness and superiority of the proposed DEF.

A. IEEE 16-machine 5-area System

The schematic diagram of the IEEE 16-machine 5-area system is shown in Supplementary Material A Fig. SA1. The detailed system parameters are available in [31].

1) Case 1: Governor-induced FO

Small-signal stability analysis reveals the presence of an inter-area oscillation mode at 0.532 Hz. To excite resonance with this mode, a sinusoidal disturbance with a frequency of 0.53 Hz is injected into the governor of generator G1. To mimic the actual operation conditions, Gaussian white noise is added to the simulated data, with a signal-to-noise ratio (SNR) of 50 dB. The simulation is conducted over a duration of 30 s with a time step of 0.01 s. The resulting trajectories of P , Q , $\ln U$, and f are illustrated in Supplementary Material A Fig. SA2.

Using the proposed DEF, a multi-channel measurement matrix is constructed from ΔP , ΔQ , $\Delta \ln U$, and Δf . The matrix is then analyzed to extract the modal parameters and compute RCME for each electrical increment in each mode. Due to space limitations, only the RCME values for one representative generator from each area are presented in Supplementary Material A Fig. SA3. It can be observed from Fig. SA3 that, Mode 28 exhibits the largest RCME, which exceeds η_0 and is significantly higher than those of the other modes. Following the FO mode extraction described in Section IV-B, Mode 28 is therefore identified as the FO mode. Figure 2 presents the extracted time-domain response of ΔP , ΔQ , $\Delta \ln U$, and Δf for all 16 generators (i. e., G1-G16). These extracted FO components are then substituted into (33) to calculate the DEF (E), with the results shown in Fig. 3. $ROFE_{ij,p}^{D*}$ for each generator in Case 1, calculated using (35), is given in Table I.

As shown in Fig. 3 and Table I, the DEF of G1 is negative and exhibits a significant and consistent downward trend over the 0-30 s interval with $ROFE_{ij,p}^{D*}=1$, while the DEFs of the remaining generators exhibit positive or nearly-zero slopes, corresponding to varying degrees of energy dissipation. According to the localization criterion in Section IV-C, G1 is identified as the FO source, which is consistent with the actual source. These results confirm that the proposed DEF can successfully localize the FO source.

To further verify the superiority of the proposed DEF, its performance is compared with those of MEMD [19] and VMD [15].

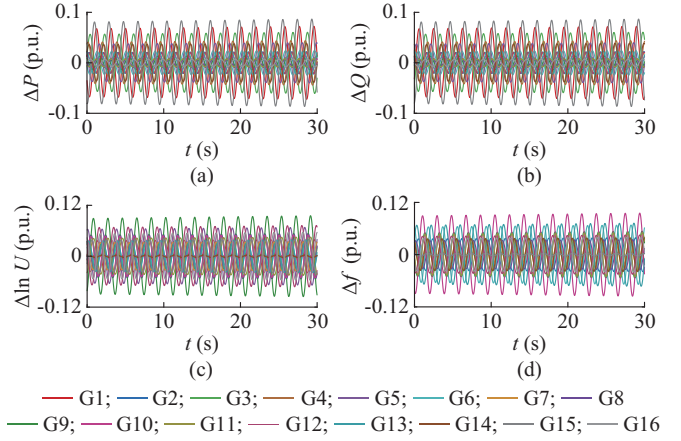


Fig. 2. Extracted time-domain response of ΔP , ΔQ , $\Delta \ln U$, and Δf for G1-G16. (a) ΔP . (b) ΔQ . (c) $\Delta \ln U$. (d) Δf .

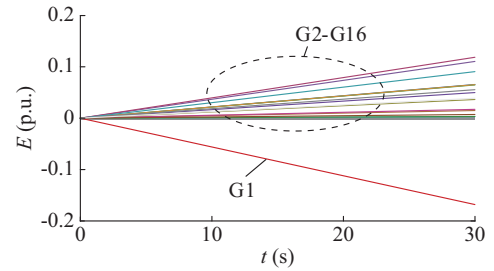


Fig. 3. DEF result in Case 1 for G1-G16.

TABLE I
 $ROFE_{ij,p}^{D*}$ IN CASE 1 FOR G1-G16

Generator	$ROFE_{ij,p}^{D*}$ under different methods		
	Proposed	VMD	MEMD
G1	1.000	1.000	1.000
G2	-0.390	-0.024	-1.373
G3	-0.385	-0.344	-0.117
G4	-0.088	-1.481	-0.959
G5	-0.657	-1.883	-1.698
G6	-0.018	-0.940	-0.779
G7	-0.388	-1.491	-0.539
G8	-0.297	-1.499	-1.423
G9	-0.020	-0.001	-0.094
G10	-0.102	-0.012	-0.090
G11	-0.216	-0.001	-0.010
G12	-0.705	-2.057	-1.770
G13	-0.539	-0.009	-1.529
G14	-0.043	-0.005	-0.019
G15	-0.330	-0.002	-0.009
G16	-0.231	-0.001	-0.001

For VMD, the number of decomposition modes and the penalty factor are selected according to [15]. VMD is performed on the oscillation signals shown in Supplementary Material A Fig. SA2, and the RCME values for selected generators (G1 and G13-G15) are presented in Supplementary Material A Fig. SA4. It can be observed that the FO components extracted from G1 and G15 exhibit noticeable inconsis-

tency, which can be attributed to the independent single-channel decomposition of VMD. Such independent processing makes it challenging to account for inter-channel observability, thereby resulting in inconsistent extracted FO frequencies across different measurement channels.

A further comparison is conducted using MEMD. Both MEMD and the proposed DEF operate on multi-channel measurements. MEMD is applied to the multi-channel signals in Supplementary Material A Fig. SA2 to obtain IMF components, and RCME values are then calculated using (30), and the results are shown in Supplementary Material A Fig. SA5. It can be observed that the RCME value of the FO components is not significantly higher than that of other components, indicating the presence of mode mixing, which degrades the accuracy of FO extraction. In contrast, the proposed DEF yields FO components with more concentrated energy, demonstrating superior performance in extracting FO modes.

Then, the FO components extracted using VMD and MEMD are used to calculate DEF, as shown in Supplementary Material A Fig. SA6. It can be observed that DEFs derived from MEMD and VMD both show a downward trend for G1, correctly pinpointing the FO source at 0.53 Hz. However, these trends are less pronounced than those obtained using the proposed DEF in Fig. 3. Specifically, over the 0-30 s time interval, the DEF of G1 decreases by 0.17 p.u. using the proposed DEF, whereas only decreasing by 0.04 p.u. and 0.07 p.u. with VMD and MEMD, respectively. This discrepancy is primarily attributed to the difficulty of VMD in consistently extracting FO components from multi-channel measurements. In addition, although MEMD offers improved mode separation, the presence of redundant components in the extracted FO signals compromises localization accuracy.

This paper further investigates the robustness of the proposed DEF under variations of FO amplitude and frequency, with detailed analyses provided in Supplementary Material A Figs. SA7 and SA8, as well as Tables SAI and SAII. The results demonstrate that the proposed DEF exhibits strong adaptability to variations of FO amplitude and frequency, and can accurately localize the FO sources over a wide range of operation conditions.

2) Case 2: Exciter-induced FO

The injection of a forced disturbance into the excitation system of a synchronous generator makes it difficult to estimate FO frequency and localize its source using the energy-based methods. Therefore, a case of exciter-induced FO is further tested to demonstrate the effectiveness of the proposed DEF.

In this case, a sinusoidal disturbance with a frequency of 0.53 Hz is continuously applied to the excitation system of G1, and Gaussian white noise with an SNR of 50 dB is added to the resulting time-domain signals. By applying the proposed DEF, the FO mode at 0.53 Hz is successfully identified based on RCME, with the RCME values of ΔP , ΔQ , $\Delta \ln U$, and Δf all equal to 1. The extracted FO components of ΔP , ΔQ , $\Delta \ln U$, and Δf for buses of G1-G16 are shown in Supplementary Material A Fig. SA9. Based on these FO components, the DEF for each generator bus is calculated.

The DEF result in Case 2 for G1-G16 is presented in Fig. 4. As illustrated in Fig. 4, the DEF of G1 exhibits a noticeable downward trend, indicating that G1 injects the largest oscillatory energy into the grid and is thus correctly identified as the FO source. This result is consistent with the simulation setup, further confirming that the proposed DEF can accurately localize the FO sources under exciter-induced disturbance conditions.

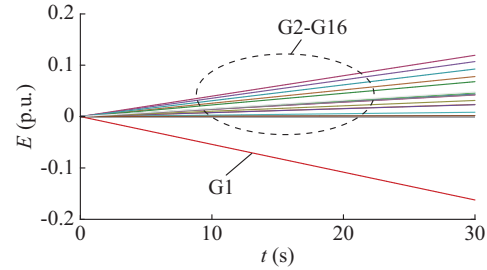


Fig. 4. DEF result in Case 2 for G1-G16.

3) Case 3: Single FO Source

In Case 3, the accuracy of the proposed DEF is further validated under a single FO source that excites two FO modes simultaneously.

- 1) Mode 1: FO mode at 0.36 Hz.
- 2) Mode 2: FO mode at 0.53 Hz.

Specifically, a disturbance signal containing two FO components of modes 1 and 2 is continuously injected into the governor of G1. The frequency components of the signal are close to the system natural oscillation modes at 0.364 Hz and 0.532 Hz, thereby inducing FOs at two distinct frequencies.

Following the same procedure as in the previous cases, the RCME values of all generators are calculated for each mode, and the results are shown in Supplementary Material A Fig. SA10. In Fig. SA10, Mode 36 and Mode 39 are identified as FO modes, as the RCME values of ΔP , ΔQ , $\Delta \ln U$, and Δf at the two frequencies all exceed η_0 . Subsequently, the corresponding FO components are extracted, and the results are presented in Supplementary Material A Fig. SA11. Using the extracted FO components, the DEF for each generator under the two modes is calculated. The DEF result in Case 3 under two modes for G1-G16 is shown in Fig. 5. It is evident that, for both modes, only the DEF of G1 is negative and exhibits a clear downward trend, indicating continuous injection of energy into the grid. Therefore, G1 is identified as the FO source for both modes. This result is fully consistent with the preset simulation conditions, further demonstrating the accuracy of the proposed DEF in multi-mode resonance scenarios.

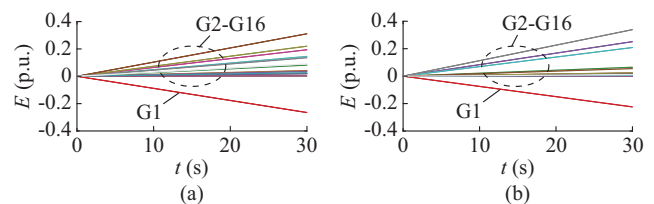


Fig. 5. DEF result in Case 3 under two modes for G1-G16. (a) Under mode 1. (b) Under mode 2.

4) Case 4: Double FO Sources

To evaluate the capacity of the proposed DEF in localizing multiple FO sources, a case with two independent disturbance sources is considered. Specifically, two sinusoidal signals at 0.36 Hz and 0.53 Hz are continuously injected into the governor of G15 and G13, respectively, thereby triggering FO modes at 0.36 Hz (mode 1) and 0.53 Hz (mode 2).

Similarly, a multi-channel measurement matrix is constructed and analyzed using the proposed DEF to extract oscillation parameters and compute RCME values for each quantity in each mode. The RCME values for representative generators are shown in Supplementary Material A Fig. SA12. Notably, pronounced peaks appear around Mode 53 (0.36 Hz) and Mode 57 (0.53 Hz) across various electrical quantities, and the corresponding RCME values exceed η_0 , confirming the presence of two modes at these two frequencies.

The FO components of modes 1 and 2 are extracted using the proposed DEF, and the results are shown in Supplementary Material A Fig. SA13. Finally, based on the extracted components, the DEF of all generators of both modes is calculated, and the DEF result of two modes in Case 4 for G1-G16 is illustrated in Fig. 6. It is observed that under mode 1, G15 exhibits a noticeable downward trend in DEF. Consistently, $ROFE_{ij,p}^{D*}$ also attains a value of 1 for G15. Thus, G15 is identified as the FO source under mode 1, which agrees with the simulation setup. Similarly, under mode 1, G13 shows the most significant DEF decrease, with $ROFE_{ij,p}^{D*} = 1$, identifying it as the FO source for mode 2.

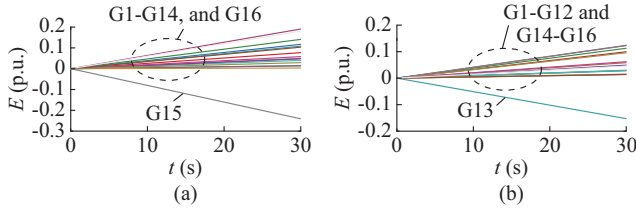


Fig. 6. DEF result under two modes in Case 4 for G1-G16. (a) Under mode 1. (b) Under mode 2.

5) Case 5: Robustness to Measurement Noise

To test the robustness of the proposed DEF against measurement noise, Gaussian white noise with SNRs of 45 dB, 40 dB, 35 dB, and 30 dB is added to Cases 1 and 4. For comparison, MEMD, VMD, HeDMD [22], and conventional DMD are also employed to calculate DEF. $ROFE_{ij,p}^{D*}$ in Case 1 and Case 4 under different SNRs are summarized in Table II and Table III, respectively. In addition, Supplementary Material A Fig. SA14 compares the DEFs obtained using the five methods in Case 1 under SNRs of 45 dB and 30 dB.

From Table II, it can be observed that with an SNR of 45 dB, all five methods can successfully localize the FO source, with the proposed DEF exhibiting a more pronounced declining trend. Specific details can be observed in Supplementary Material A Fig. SA14. When the SNR decreases to 30 dB, the proposed DEF still accurately localizes the FO sources. In contrast, MEMD and conventional DMD incorrectly localize G11 and G8 as the FO sources, respectively. Meanwhile, VMD and HeDMD completely fail to localize the FO sources, as all generators display upward DEF trends.

TABLE II
 $ROFE_{ij,p}^{D*}$ IN CASE 1 UNDER DIFFERENT SNRS

Method	$ROFE_{ij,p}^{D*}$ under different SNRs			
	45 dB	40 dB	35 dB	30 dB
MEMD	1 (G1)	1 (G1)	1 (G1)	1 (G11), -0.298 (G1)
Conventional DMD	1 (G1)	1 (G1)	1 (G8), -0.253 (G1)	1 (G8), -0.438 (G1)
VMD	1 (G1)	1 (G1)		
HeDMD	1 (G1)	1 (G1)	1 (G1)	
Proposed	1 (G1)	1 (G1)	1 (G1)	1 (G1)

TABLE III
 $ROFE_{ij,p}^{D*}$ IN CASE 4 UNDER DIFFERENT SNRS

Method	Mode	$ROFE_{ij,p}^{D*}$ under different SNRs			
		45 dB	40 dB	35 dB	30 dB
MEMD	1	1 (G15)	1 (G15)	1 (G15)	1 (G14), -0.496 (G15)
	2	1 (G13)	1 (G13)	1 (G13)	1 (G12), -0.268 (G13)
Conventional DMD	1	1 (G15)	1 (G15)	1 (G14), -0.368 (G15)	1 (G14), -0.518 (G15)
	2	1 (G13)	1 (G13)	1 (G11), -0.145 (G13)	1 (G11), -0.302 (G13)
VMD	1	1 (G15)	1 (G15)	1 (G16), -0.134 (G15)	1 (G16), -0.153 (G15)
	2	1 (G13)	1 (G13)	1 (G2), -0.329 (G13)	1 (G2), -0.518 (G13)
HeDMD	1	1 (G15)	1 (G15)	1 (G15)	
	2	1 (G13)	1 (G13)	1 (G13)	1 (G13)
Proposed	1	1 (G15)	1 (G15)	1 (G15)	1 (G15)
	2	1 (G13)	1 (G13)	1 (G13)	1 (G13)

Furthermore, as shown in Table III, the proposed DEF consistently localizes FO sources for both modes in Case 4 under different SNRs. By comparison, the other methods exhibit varying degrees of localization errors or fail to localize one or both FO sources as the SNR increases. These results clearly demonstrate that the proposed DEF exhibits superior robustness to measurement noise and is reliable for localizing FO sources.

B. Field PMU Measurement from ISO-NE

To further evaluate the performance of the proposed DEF on field PMU measurement data, two real FO events occurred in ISO-NE [32] are studied in this paper. The partial map of the studied system is shown in Supplementary Material B Fig. SB1. The study area includes three generators: G1 at substation 6, G2 at substation 7, and G1 at substation 8, and it is connected to two external areas, referred to as area 2 and area 3. In addition, two boundary buses, i.e., bus 7 at substation 3 and bus 21 at substation 9, are employed to represent the aggregated connections to the external areas. The two real FO events occurred in ISO-NE are investigated

to validate the applicability of the proposed DEF in practical power systems.

1) Event 1: the first FO event occurred on January 29th, 2018. Post-event analysis reported that the oscillation had a frequency of 1.57 Hz and was originated from G2 at substation 7. The PMU recordings of P , Q , U , and f during the event are shown in Supplementary Material B Fig. SB2.

Using the proposed DEF, the FO mode is first identified based on the RCME values, and the corresponding time-domain components of each measurement are extracted. Using these FO components, the DEF of each generator is calculated, as shown in Fig. 7. It can be observed that G2 exhibits a negative DEF with a significant downward trend, indicating continuous energy injection into the grid and confirming its role as the FO source. In contrast, the generators connected to transmission lines L15 and L16 show upward or nearly flat DEF trends, signifying their behavior as FO sinks. This localization result is consistent with the actual event records, thereby validating the accuracy of the proposed DEF. In addition, Fig. 7 compares the DEF results calculated by the proposed DEF with those derived from conventional DEF, VMD, MEMD, HeDMD, and conventional DMD. It is observed that the comparison methods produce downward trends for both G2 and L15, which may lead to the mislocalization of L15 as the FO source. In contrast, the DEF trends with MEMD, VMD, and HeDMD correctly exhibit a downward tendency only for G2. However, the declining trends are less pronounced than those obtained using the proposed DEF.

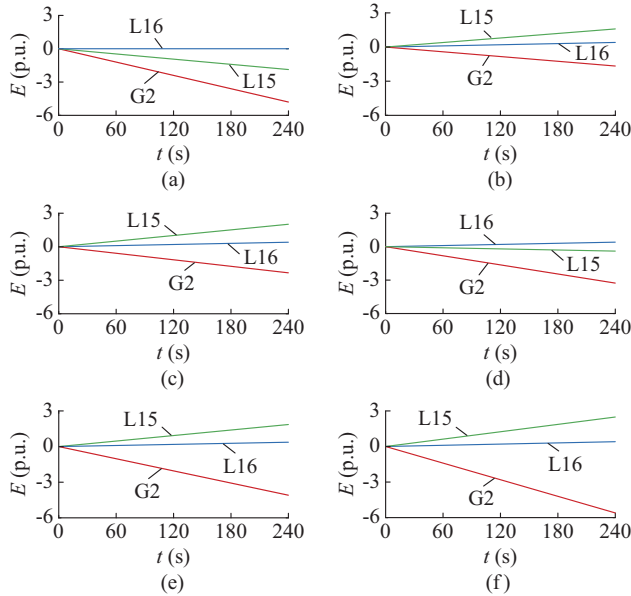


Fig. 7. DEF results calculated by different methods in Event 1. (a) Conventional DEF. (b) VMD. (c) MEMD. (d) Conventional DMD. (e) HeDMD. (f) Proposed DEF.

2) Event 2: the second event occurred on June 20th, 2019. The event was observed in substation 1, where four transmission lines (L1 to L4) connected to the substation exhibited oscillations with significant amplitudes. Post-event analysis indicated that the oscillation was caused by L2, with an FO frequency of approximately 0.14 Hz. The PMU measure-

ments of P , Q , U , and f for L1 to L4 during the event are shown in Supplementary Material B Fig. SB3.

Similarly, the DEF results calculated by different methods in Event 2 are shown in Fig. 8. The conventional DEF indicates that, in addition to L2, the generator connected to L4 also exhibits a downward DEF trend, which may mislead system operators and result in incorrect control actions of FO source. By contrast, the other four comparison methods correctly show a negative and decreasing DEF trend only for L2, successfully localizing the FO source. Nevertheless, the downward DEF trends obtained using these methods are noticeably weaker than those achieved by the proposed DEF. Overall, the results confirm that the proposed DEF can accurately localize FO sources using field PMU measurements, effectively validating its applicability in practical power systems.

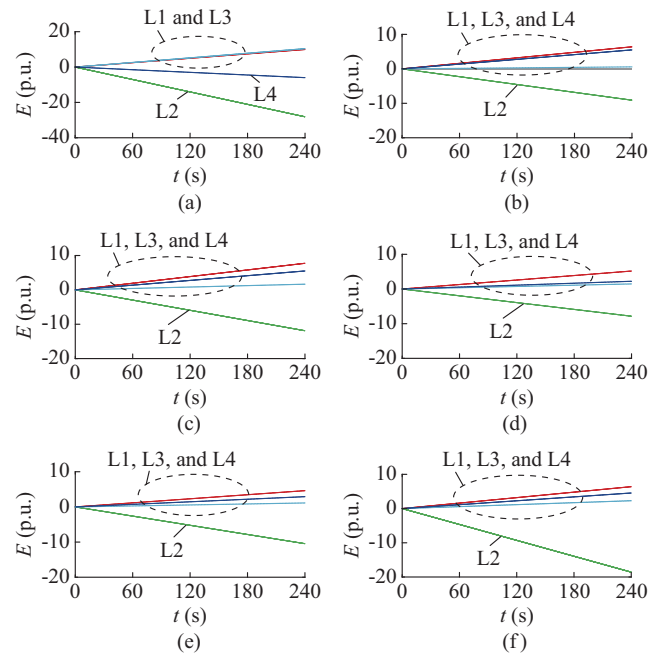


Fig. 8. DEF results calculated by different methods in Event 2. (a) Conventional DEF. (b) VMD. (c) MEMD. (d) Conventional DMD. (e) HeDMD. (f) Proposed DEF.

C. Comparison of Computation Efficiency

Table IV summarizes the computation time of different methods. All simulations were conducted in MATLAB version 2021a on a computing platform equipped with an AMD Ryzen 7-4800H @ 2.90 GHz processor and 8 GB RAM. Note that all methods are implemented under a sequential execution framework to ensure a fair comparison of computation performance. The results indicate that the proposed DEF achieves competitive computation efficiency. Although the conventional DEF yields the shortest computation time with a 34.98% and 26.48% improvement over the proposed DEF for Case 1 of IEEE 16-machine 5-area system and Event 1 of ISO-NE, respectively, it suffers from inferior localization accuracy. Similarly, conventional DMD reduces computation time by 24.34% and 16.92% for Case 1 of IEEE 16-machine 5-area system and Event 1 of ISO-NE, while HeDMD achieves corresponding reductions of 14.76%

and 9.49%. However, both methods are less robust to measurement noise and exhibit degraded accuracy for localizing FO sources compared with the proposed DEF. In contrast, VMD and MEMD require significantly high computation costs due to the iterative decomposition nature. As shown in Table IV, both methods are considerably slower than the proposed DEF. Specifically, VMD requires 32.235-s and 44.324-s computation time for Case 1 of the IEEE 16-machine 5-area system and Event 1 of ISO-NE, respectively. This high computation cost arises from its iterative optimization based on the alternating direction method of multipliers, which involves repeated updates of mode spectra and center frequencies until convergence. MEMD requires 18.384-s and 32.254-s computation time for Case 1 of IEEE 16-machine 5-area system and Event 1 of ISO-NE, as it involves multi-directional projections, iterative sifting for multi-channel mode alignment, and high-dimensional signal operation.

TABLE IV
COMPUTATION TIME OF DIFFERENT METHODS

Method	Computation time for Case 1 (s)	Computation time for Event 1 (s)
Proposed	8.241	10.245
HeDMD	7.025	9.273
VMD	32.235	44.324
MEMD	18.384	32.254
Conventional DMD	6.235	8.512
Conventional DEF	5.358	7.532

VI. CONCLUSION

To accurately localizing the FO sources, this paper proposes an Os-enhanced DMD-assisted DEF using synchrophasor measurement. The Os-enhanced DMD is employed to perform modal decomposition on the multi-channel measurements, significantly enhancing noise-handling capability. The FO mode is identified based on RCME, which enables accurate extraction of the corresponding FO components and provides a reliable foundation for subsequent energy flow-based analysis. Using the extracted FO components, the DEF and the corresponding forced energy rate are calculated to localize the FO source. Simulation results on the IEEE 16-machine 5-area system, as well as field PMU measurement on ISO-NE, demonstrate that the proposed DEF can accurately localize FO sources under various operation conditions. Comparative studies with existing methods under different SNRs further confirm that the proposed DEF exhibits superior capability in extracting FO components under high-noise conditions. In addition, compared with VMD and MEMD, the proposed DEF effectively mitigates mode mixing, thereby enhancing the localization accuracy of FO sources.

Although the proposed DEF has shown effective performance, its capability could be further improved through integration with advanced DEF such as the complex DEF (CDEF). Owing to its complex-domain formulation, CDEF has the potential to enable more compact and efficient energy flow computation. Future work will focus on investigating such integration to further improve the robustness, com-

putation efficiency, and applicability of the proposed DEF in challenging FOs where conventional methods may fail.

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