

**Subsumption & Economic Preference Expression: An  
Agent-Based Computational Architecture For Principled  
Exploratory Applications**

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# Abstract

This work focuses on the investigation of economic preferences, particularly within economic systems, using agent-based models (ABMs). Economic systems are a challenging area of research, exhibiting complex adaptive, non-linear, recursive properties. ABMs are now relatively widely used in economics as experimental tools across many economic schools. While these schools are presented as distinct theoretical frameworks, typical experimental approaches are strikingly similar: quantitatively rigorous, highly abstracted models of core preferences and economic behaviours are typically bundled into centralised structures and other behaviours ignored. Vast amounts of data are potentially available from computational models making validation & verification of experimental results problematic. Researchers have relied on replication of time series artefacts yielding little explanatory value, rather than examining agent behaviours themselves.

To date relatively little effort has been made to codify, or critically discuss experimental protocols around these models; to verify and replicate findings; or to develop realistic validation metrics. This is a failing in the literature limiting the value of ABM approaches and leaving them open to criticism. This thesis directly addresses these issues from a practical standpoint, presenting discussion of what approaches are appropriate and what features robust methodologies should exhibit. Situation is presented as a basic validation metric, using agent-level performance measures and exploratory tools for verification and validation. Risk-adjusted, agent-level population relative performance measures are developed and tested on a typical agent-based artificial financial market (AFM) demonstrating their effectiveness, while highlighting potential methodological issues and confirming the importance of agent-level exploration alongside other validation and verification methods.

The case for an overall modelling framework based around a subsumptive meta-heuristic, population-based architecture is presented and discussed. In this framework, the SHaaP architecture, core preferences, preference modifying behaviours, and structural modifiers are unbundled so that they can be investigated systematically, while potentially being subsumed into larger, sophisticated non-linear preference structures. A functional SHaaP architecture is developed & implemented as part of the research. A larger case study examining the role of heuristic preference modifying behaviours in economic agent behaviours is presented, also demonstrating the architecture in operation and exploring its dynamics.

Heuristic preference modifiers are widely observed in real economic entities and systems: they have a potentially critical role in regulation & risk management under

uncertainty but remain relatively unexplored. The case study demonstrates the potential of the SHaaP architecture in progressively developing and testing both particular behaviours and performance measures. Analysis of the dynamics of preference modifying heuristics in a population-based structure illustrates the complexity of such systems, serving to identify areas for future research, particularly for regulatory policy design. Potential deficiencies in the relative performance measures were highlighted leading to proposals for further development and testing. Finally the role of the subsumptive architecture as a key, complementary component to traditional agent-based experimental economics models is discussed, particularly as an exploratory tool in developing & testing dynamic systems for regulatory frameworks subject to uncertainty.

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## Attestation of Authorship

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person (except where explicitly defined in the acknowledgements), nor material which to a substantial extent has been submitted for the award of any other degree or diploma of a university or other institution of higher learning.

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W. E. P. Davidson  
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# 1. Introduction

*'The world is its own best model'* - Brooks 1999

*'There is nothing so disastrous as the pursuit of a rational investment policy in an irrational world.'* - John Maynard Keynes (attrib.)

## 1.1. Overview

This thesis is concerned with agent-based computational models of economic preference expression amongst individuals and entities interacting in situated environments. Agent-based models (ABMs) have become a particularly important tool in experimental economics, allowing researchers to investigate models of preference expression in populations which were previously infeasible or impractical. Research in this area has expanded rapidly since the early 1990's with the advent of increasingly cheap computing power, incorporating inputs from social sciences and cognitive psychology, offering insights into the dynamics of behaviours in economic systems and populations across an enormous range of models, encompassing neoclassical economics, behavioural finance, and other economic schools.

However, there are significant causes for concern in current ABM approaches to modelling economic systems, and agent-based methodologies have been criticised both in terms of validation and verification of experimental findings. The work in this thesis supports those criticisms and presents a new experimental framework for developing principled computational models of such systems which systematically addresses these concerns, recommending an alternate approach where models are situated in real environments with direct operational relevance to their performance. Novel agent-level exploratory performance measures are developed and demonstrated in conjunction with basic validation metrics and a new generic, scalable computational architecture, the 'Subsumptive Heuristic adaptive agent-based Preference architecture'<sup>1</sup> for modelling economic preference expression.

The research presented here draws heavily on practical experience with real-world financial models, trading protocols, and domain knowledge derived from practitioners. As such it contrasts sharply with much current and past ABM economic systems research which has focused largely on artificial financial markets (AFMs) where highly abstracted, unsituated, and stylised models are designed around theoretical neoclassical economic structures often ignoring obvious structural mechanics of actual systems and practical measures of performance and success. Situation - placing models in realistic (ideally real) environments - is an important component in both validating and verifying experimental findings.

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<sup>1</sup>Hereafter referred to as the 'SHaaP architecture', or 'SHaaPa', or occasionally just 'SHaaP'.

Beyond methodological issues, a second key area identified in the thesis is the general approach taken in the literature to modelling economic preferences. Typical academic ABMs treat economic preferences as single, central functional units, whether unboundedly rational, sophisticated mathematical structures or simple rule-based systems, operating in de facto isolation. This approach ignores separate behaviours and structures, widely observed in real environments which can directly affect and form part of overall preference expression. This is a potentially important omission, not just in terms of the interpretation of experimental findings, but also in understanding the processes underlying preference formation and overall expression. This research develops and explores the concept of such structures, described as 'preference modifiers',<sup>2</sup> which may have a significant role in contributing to, and managing risk and uncertainty in economic systems.

The SHaaP architecture developed here is central to the overall approach to modelling economic preferences and systems in the thesis. The architecture is based around subsumption as a structural meta-heuristic, following Rodney Brooks' proposals for developing AI in evolutionary robotics[21, 23, 25]. In conjunction with a population-based structure subsumption allows preference behaviours within agents to be broken down into interacting functional elements, including core preferences and preference modifiers, subsumed within larger structures for expression. Within the SHaaP architecture preferences and modifiers can then be explicitly defined and explored. This has important, immediate and direct benefits in verification and validation of ABMs, and also for practical application in the design of risk management systems, regulatory policy and decision support.

Having established the need for appropriate agent-level performance metrics and analysis, the main body of the research explores the role of simple heuristic preference modifiers as components in economic preference expression. This follows recent work by Gerd Gigerenzer and others<sup>3</sup> which proposes a role for simple heuristics in decision making in uncertain environments. The subsumptive approach presented here extends this work, allowing preference expression to be modelled from simple, well understood (or at least clearly identified) component elements in situated environments not just at an individual level, but also within populations and corporate entities.

The remainder of this chapter presents the background and motivation for the research, the areas of focus for the work, an overview of the main contributions and applications together with future directions and a brief summary of the thesis chapter structure.

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<sup>2</sup>

This term is introduced in the thesis to include both preference subsuming behaviours, and a subset of control actions at an agent level which are described in many academic papers as replicating 'realistic elements' of actual markets. Frequently these modifiers take the form of simple heuristics. In actual trading environments and trading systems they perform important, sometimes vital, functions and are discussed in detail later.

<sup>3</sup>Chapters 2 & 4 provide a review of this work in the context of the SHaaP architecture and case study experiments in this research.

## 1.2. Background & Motivation

An overarching goal of the work presented here is to provide a clear structure for principled methodological processes, which allow bottom-up models of economic preference expression to be developed and explored which have good explanatory value and operational meaningfulness in real world contexts. The framework and the methodologies presented, their context in terms of the literature, alternate approaches and application will inevitably be critically examined throughout the thesis, however before that it is worth giving a brief outline of the underlying motivation for the work and its objectives.

### 1.2.1. Some Practical Background

The original impetus towards this research came from the attempt to better understand and codify fairly substantial practical experience<sup>4</sup> and observation of markets. This was sparked in part by Nicolas Nasim Taleb's writing on characteristic market structures and modelling failings in 'Fooled by Randomness'[137] and 'The Black Swan'[138] - works addressing common misconceptions of trading behaviours, market processes, and risk.

For the most part, as an ex-practitioner reviewing published academic research, there was little or no obvious connection between conventional academic economic models and actual behaviours in real economic systems, either in terms of operational decisions or gross market behaviours. Indeed this has been generally acknowledged by practitioners and academics alike given the evidence of misalignment in forecasting market crashes and volatility - as highlighted by Mandelbröt[94].

Samuelson[114], in the vanguard of mid-20th century neoclassical economics, decried academic economic research of the time for a lack of operational meaningfulness. Typical, conventional economic models since then have approached the problem by simplifying it: taking a top-down approach, where unboundedly rational, representative agents<sup>5</sup> can replace the actual, boundedly rational individual participants in the market, frequently casting economic preferences and demands in the system as a utility maximisation, optimisation problem. Obviously some level of abstraction is necessary and desirable to derive useful, functional models, however, the abstraction in academic work and the validation metrics presented is often so extreme as to deny significant practical relevance or explanatory value.

The emergence of agent-based modelling in the 1990's in tandem with rapid increases (and cost reductions) in readily available computing resource appeared to present a practical, realistic experimental platform to address this disconnection, offering the

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For the first 15 years of my career, after completing an undergraduate degree in Experimental Psychology, I ran a various trading operations in the City of London, mainly concerned with arbitrage and proprietary portfolio management. This entailed the application of a wide variety of trading models, from sophisticated derivative pricing systems to simple, essentially heuristic rules for risk management and strategy.

<sup>5</sup>See Hartley[54] for an overview of rational representative agent roles in macroeconomics and Kirman[73] for an agent-based perspective on this area.

possibility of building bottom-up models of systems with much less abstraction and much more explanatory value.

It was somewhat frustrating then to discover that over 15 years later the main body of academic ABMs for economic systems still had very little operational relevance to actual markets or preference expression, either still being so abstracted as to serve only as illustration of *potential* economic mechanisms in *potential* systems with no attempt to model actual processes, or suffering from severe flaws in methodology and validation criteria, or both. Many recent behavioural models appear to have adopted similar approaches to neoclassical researchers in abstracting their models - casting economic agents as constrained, expected utility maximisers and creating a de facto homuncular fallacy rather than models with significant explanatory value.

### 1.2.2. Beyond Practical

While frustration in trying to make sense of personal experience may be a motivation to begin research, it is clearly not sufficient in itself to justify extended study. However identification of problems extant academic research and applicability does give a basis for examining alternate approaches, pointing to some basic questions, which in turn point to the main areas of interest for the research presented here.

- How are economic preferences & economic systems (not just traded financial markets) different from other complex adaptive systems?
- What are appropriate fitness and performance measures for economic agents?
- How can models be acceptably validated?
- What constitutes operational meaningfulness?
- How can academic observations be practically applied?
- Are there other systems where expression of preferences demonstrate similar features?

These elements will be discussed in detail in later chapters, however it is worth identifying the main elements here before revisiting them.

Unsurprisingly, given availability of data and potential direct commercial relevance, the vast majority of research focuses on models of traded financial markets in one form or another. However the range of application for agent-based models exploring economic preferences in other systems is significant, including corporate governance; strategy; financial regulation; policy formulation; and legal framework effects.

#### 1.2.2.1. Economic Preferences & Economic systems - Differentiating Factors

Economic systems, i.e. systems in which modes of economic preference expression are the principal drivers, represent a uniquely challenging research area. Unlike other naturally occurring systems which exhibit complex non-linear behaviours, in economic systems participants' expectations affect the state of the system: as participants express their preferences and economic choices these expectations are themselves affected by their beliefs about other participants' behaviours and expectations. They may be characterised as complex adaptive systems (CAS), but it is

this 'self-regarding', reflexive aspect which differentiates them from other complex systems.

Economic system behaviours reflect the expression of preferences shaped in the form of demand and constraint on supply. System structure; participant expectations; and the distinction between risk vs. uncertainty are key characteristics affecting behaviour within these systems and attempts to describe them.

SYSTEM STRUCTURE:

- Economic systems are rarely isolated, closed or informationally efficient.
- External, notionally independent, systems may interact and affect each other.
- Participants arrive and leave - they are also typically heterogeneous in terms of preferences and beliefs.
- Economic agents within systems are typically not unboundedly rational, nor do their preferences remain constant.
- Behaviours exhibit path-dependency.

EXPECTATIONS: a fundamental differentiating factor of economic systems. Beyond the trivial observation that at some level almost any interaction in modern society between members of that society beyond purely social can have economic components, participants' expectation formation and their behaviours based on those expectations are basic to economic interactions. Economic actors within a system may consider their expectations on the future state of the system, and their beliefs about other actors' expectations, adjusting their behaviours to reflect these. As a whole the result is that economic systems are complex and adaptive: as beliefs, actions, and the economic state of elements in the system change so do the economic agents themselves and their expectations.

RISK *vs.* UNCERTAINTY: The distinction between risk and uncertainty is an important one and a key concept in modelling economic systems. This differentiation is often ignored, or overlooked, by practitioners and academics alike, although it has been recognised for a considerable time. Pre-dating the formulation of modern neo-classical economics, Knight[74] classified *risk* as an objectively calculable measure for outcomes where the probability distribution is known, whereas *uncertainty* represents cases where distributions are not known and cannot be reasonably inferred from available information. This is akin to Rumsfeld's[111] rather unfairly maligned statement on 'known unknowns' and 'unknown unknowns',

"...there are known knowns; there are things that we know that we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know."

The practical reality in designing robust models and regulatory systems is that such models have to identify and cater for both the known unknowns and, more problematically, for the fact that there will be things that you don't know that you don't know. Taleb[138] has attempted to describe the impact of both types of unknown in financial systems, describing 'unknown unknowns' in terms of black swans - a creature no one had conceived of till they finally encountered one.<sup>6</sup>

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<sup>6</sup>A recent relevant example is the removal by the Swiss central bank of their currency peg of

In Knight's terms modern neoclassical models generally deal only with risk, ignoring uncertainty or, while allowing that it is present, constraining the frame of reference to systems where assets are assumed to follow known distributions and investors preferences & behaviours are strictly defined. In purely academic models this may be an acceptable abstraction, however given their relative ease of implementation and apparent robustness (at least over short periods), these models and derivatives of them are frequently used in practical applications where their limitations have problematic implications.

Faced with uncertainty it is worth considering what form structures to manage or mitigate this exposure would take and indeed may already be used. Intuitively it seems obvious that such mechanisms cannot focus on optimisation - no distributions are known for uncertain exposures, nor when uncertain events will disrupt otherwise stable systems, nor the extent of their disruption in other systems.

Heuristic structures which promote resilience at the expense of optimisation seem likely candidates: structures which act to limit overall exposure, to aggregate uncorrelated exposures, concentrate expertise in handling and reporting unusual or difficult exposures. Typically as well, it seems likely that such mechanisms take the form of preference modifiers working in conjunction with core preferences which may themselves be neoclassical functions, heuristic, or combinations of the two.

Knight discussed structures for managing uncertainty in his 1921 work. More recently, since the global financial crisis (GFC), researchers and practitioners have begun to explore this area[99]. Many regulatory and risk management processes observable in economic systems show characteristics which can be identified in terms of heuristic uncertainty management, however there has been little specific experimental work reported in this area. Other than lack of differentiation between uncertainty and risk, it seems likely that this is at least in part due to the absence of an effective experimental framework - the SHaaP architecture addresses this issue. Heuristic preference modifiers and their potential role in managing uncertainty form the basis of the primary case study and research area of the thesis.

### 1.2.2.2. Validation and Performance Measures

The selection of appropriate validation criteria is fundamental to both the design of experiments and inferences based on them. The choice of suitable fitness and performance measures is a basic starting point in this process where models of complex adaptive systems are themselves inherently complex.

Typical ABM research has been designed around theoretical, neoclassical economic structures with utility maximizing, risk-averse agents whose preferences are clearly specified.<sup>7</sup> Such 'traditional' agent-based models rely on identifiable theoretical equilibria and replication of gross statistical artefacts observed in actual markets for validation.

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the Swiss Franc to the Euro in January 2015. As an event this was certainly unforeseen and arguably considered so incredibly unlikely as to be a black swan. The resultant immediate volatility bankrupted a number of companies and hedge funds, while its longer term impact will affect many businesses and individuals who previously believed or behaved as if they had no foreign exchange risk in operating between the Swiss and European economies.

<sup>7</sup>See LeBaron [84] and Tesfatsion [142] for comprehensive reviews of these approaches.

From a practitioner's perspective this is problematic: it is difficult to see how operationally meaningful models can be derived from such studies as the models themselves are completely unsituated, and as there is little alignment of either performance measures or preferences to real world equivalents or actual behaviours. Even in cases where realistic fitness measures are used (such as Chen & Yeh [30] and Brock & Hommes [16]) there appears to be little attempt to evaluate models using realistic, agent-level performance measures.

Definitions of experimental success need realistic criteria appropriate to the domain: for economic systems recreating statistical artefacts in financial time series, or cherry picking best performing exemplars of agent behaviours are unlikely in themselves to be sufficient - better measures are necessary to validate and usefully explore computational models. These are discussed and developed in the case studies presented in the research. A further compelling validation criterion is situation[25]. Beyond academically abstracted models which are internally consistent, if a model or agents in the model can function successfully when situated in realistic environment this gives a baseline for validation. Taken together with realistic performance measures, effectively 'situated' measures themselves, this provides a basic level of validation.

### **1.2.2.3. Operational Meaningfulness and Practical Applications**

While Samuelson might have claimed success for neoclassical economists in producing operationally meaningful theorems this has been largely at a policy level. From a practitioner's point of view, except as a central banker, or perhaps as a global macro fund manager, neoclassical economic theories yield little in the way of practically applicable operational benefits.

Definitionally this is unsurprising given a top down modelling approach, however more disappointing is that behavioural finance, bottom-up modelling and experimental economics have not been significantly more successful. This is not for want of claims to the contrary, but rather lack of logical validation of those claims. Such validation needs consistent analysis of agent-level behaviours and that, as suggested in the previous section, requires functionally meaningful performance measures and operational analysis.

'Operational meaningfulness' in this thesis then will mean logically valid and supportable inferences about functional behaviours and preference expression - this directly contributes to applications and decision support. Situation is, to state it again, of key importance not simply for validation, but also to allow direct application of experimental findings without 'concretization' or 'de-abstraction'. This is important for regulators, risk-managers, and economic systems in general.

## **1.3. Organisation of the Thesis**

This section provides an overview of the main areas to be discussed, briefly describing the content of each chapter set in the context of the overall structure of the research. The aim here is to give a forward looking view, highlighting the key issues addressed, contributions, and, given the multidisciplinary aspects of the research,

some questions (and answers) which are effectively out of sequence, but are present throughout the work.

Overall the research presented in the thesis effectively falls into three main areas:

- Methodological approaches and issues in experimental economics as applied by different schools, particularly focusing on validation and case study experiments supporting the design of realistic performance measures.
- Development of the SHaaP architecture as a new, alternative experimental platform and methodological approach, incorporating heuristic, subsumptive & population based structures.
- The problem domain in the context of the main challenges and issues in practical application, particularly for uncertainty prone systems and preference expression. Case study experiments exploring simple heuristic preference modifying behaviours within the SHaaP architecture and situated environments.

**Chapter 2 - Economic Preferences, Expression & Economic Systems** This chapter presents economic preferences and systems as a research domain in the context of the computational architecture developed in the thesis and the overall motivation for the research. The chapter gives a brief overview of major modern economic schools and alternate approaches to modelling economic systems before moving on to critically examine the implications of the various theoretical assumptions and approaches in research and application together with their relevance to this research. The main area addressed is the limitation these assumptions impose on the strength of inferences which may be drawn from experimental observations, where these are possible, and the degree of operational meaningfulness of these approaches.

The emergence of behavioural finance and agent-based computational economics (ACE) are discussed, particularly their contribution to the field of experimental economics, facilitating the resurgence of non-equilibrium based economic schools, alongside some failings in terms of scope and direction.

The potential for fast & frugal heuristics as viable processes in preference expression is introduced. Increased interest in simple heuristics as policy tools since the GFC highlights the importance of developing a robust methodological basis for ABMs exploring these systems, although the case is made that such heuristics are unlikely to be sufficient in themselves when modelling actual systems

The distinction between risk vs. uncertainty is revisited. The apparent associated failure of behavioural finance models to shake off the baggage of neoclassical model abstraction and tractability is an important topic. The potential for simple heuristic regulatory structures to address at least some of these issues takes up the latter part of the chapter and sets the backdrop for discussion of the experimental architecture, SHaaPa, presented in Chapter 4.

**Chapter 3 - Agent-Based Computational Models & Economic Preferences** Current agent-based approaches to modelling economic systems and preferences are presented and critically examined as literature examples of both simple and rich artificial models are reviewed. Problems with existing approaches are discussed -

particularly lack of situation, and the use of unsuitable performance measures in model validation.

Practical and methodological issues which must be addressed in designing, implementing and using these models are reviewed. Validation is in the end the key focus of this discussion: the case is made for a principled approach not only to developing experimental protocols, but also to verification of overall model design.

Situation and the need for appropriate agent-level performance measures and exploratory processes are core components to the validation methodology proposed. These form the basis to the design of the SHaaP architecture in Chapter 4 and the case study analyses in Chapters 5 & 6.

Taken together the discussion and arguments presented in this and the preceding chapter form an extensive critical review of the problem domain and current approaches. This represents a significant piece of work in its own right - as a contribution these chapters argue for operationally meaningful, situated approaches to modelling economic preferences where practitioners and academics are better aligned. As context for the research, the chapters serve to establish the rationale and case for approach developed in the rest of the thesis.

**Chapter 4 - SHaaPa - An Experimental Platform & Architecture** This chapter presents the SHaaP architecture: this is a new experimental platform developed and used in the main case study experiments in the thesis. Academic research to date has centred largely on recasting economic systems in terms of utility maximisation, optimisation and forecasting problems. That is fundamentally different from aggregate behaviours observed in real world systems: while appropriate to some research topics, this level of abstraction yields little explanatory value and has limited practical application. The SHaaP architecture is novel in that it begins by building operationally meaningful behaviours from the bottom up, validated in situated experiments.

Central to the overall thesis, SHaaPa provides the basis for a principled approach to addressing the questions set out in the previous sections: applying domain knowledge with appropriate fitness and performance measurement, exploratory and confirmatory analysis of economic preference expression becomes a realistic objective. The architecture and performance measures developed in Chapter 5 support operationally meaningful inferences from experiments; validation; verification; and potential practical decision support applications.

SHaaPa is an extensible, generic approach to modelling preference behaviours in reflexive systems. SHaaPa applies subsumptive principles identified in the evolution of complex organisms in nature[47] and extends concepts developed in evolutionary robotics presented by Rodney Brooks[21, 24, 22]. The overall structure of the architecture is set out in the chapter, explaining its rationale particularly in terms of subsumption and its contributions to agent-based models of economic systems and preference expression.

Economic preference expression behaviours rather than either classification or forecasting are the main focus of the architecture: while encapsulated components of the system may in fact use sophisticated quantitative, optimised algorithms and machine learning, their outputs are subsumed and may be used heuristically. This

focus on decisions and behaviours measured against realistic performance measures is important.

In Chapter 6 the SHaaP architecture is demonstrated in the main case study drawing together the performance measures developed in Chapter 5 in a series of experiments exploring simple heuristic preference modifiers in a situated environment. The importance of clearly defined experimental platforms and modelling environments has been stressed in the discussion in earlier chapters and is emphasised by the findings in Chapter 5. The SHaaP architecture provides the basis for principled development of new platforms of this type specifically incorporating practical domain knowledge of economic preference expression and behaviours.

**Chapter 5 - Performance & Fitness Measures: Verification & Validation** In this chapter novel, agent-level, situated population relative performance (PRP) measures are proposed and developed aimed at supporting exploratory analysis of agent and model behaviours in economic preference expression. This follows the discussion on validation and methodological issues in ABMs in Chapter 3 where the importance of such measures was emphasised. The apparent lack of these types of measure as exploratory tools in the literature is a serious methodological failing. As part of the overall research presented here an important step was to develop appropriate measures to test and validate experimental models.

The measures developed here are derived from risk-adjusted return calculations commonly found in practical application. Risk-adjusted measures, such as Sharpe ratios[117, 118], are used in finance as a means of differentiating between the quality of investment returns. However some measures may be unstable and noisy in low volatility environments so care must be taken in their use and interpretation[34, 118].

This chapter describes and extends a Sharpe-like relative performance measure proposed by Benink et al[8, 9]. Recognising that this measure suffers inherent instability, a new measure incorporating a risk-adjusted effective returns measure,  $X_{eff}$ , used in high-frequency trading models[34] is proposed and tested. These measures are used to produce aggregate population, relative risk-adjusted return statistics and graphical relative return surfaces across agent populations.

The final part of the chapter describes a case study in which a version of the Santa Fe Artificial Stock Market (SFASM) is reconstructed and its performance analysed. The SFASM is widely cited in the literature, and frequently used in teaching programmes as an exemplar of an agent-based AFM[142, 141, 83]. As such it was selected here as a benchmark case for testing the new agent-level measures, since the model dynamics have been well reported, while to date no detailed, direct analysis of agent-behaviours has been reported.

The new PRP measures proved highly effective as exploratory tools in this case study. However the reconstruction process documented in the chapter serves as an excellent, though unexpected illustration of the problems in ABM modelling discussed in Chapter 3. The SFASM implementation in MatLab proved extremely problematic and the results reported in the original studies were never satisfactorily replicated.

The analysis presented demonstrates the effectiveness of the performance measures developed here and the importance of such agent-level measures in validating ex-

perimental models and conclusions. The results point to some possible problems in the SFASM, whilst noting that these may be local to the MatLab implementation here rather than the originally reported findings.

These findings add to the literature supporting the case for more principled experimental methodologies in agent-based models of economic systems, however the principal contribution in this chapter is the development, demonstration, and testing of the new population relative performance measures.

**Chapter 6 - Simple Heuristics vs. Risk & Uncertainty - A Comparative Case Study** This chapter sets out the main body of experiments carried out using the SHaaP architecture. Nominally situated<sup>8</sup> economic agent populations are established and their behaviours systematically explored, specifically in terms of risk and uncertainty mitigation. By decomposing economic preferences into subsumptive structures and functional components, these experiments represent an original approach to examining the role of simple heuristic preference modifying behaviours in economic preference expression.

This forms part of the larger context of the work presented in the thesis, where subsumption as a structural meta-heuristic, in which component behaviours are subsumed within other behaviours and structures during expression, is regarded as a key component to describing and exploring economic preference expression, particularly for developing operationally meaningful models in uncertainty prone systems.

As described in Chapter 4 simple heuristic modifiers, such as 'stop-loss', are commonly observed in practical application, however, as noted by Kaminsky & Lo[65], there is little research in this area reported in the literature. Agent-based models of preference modifying behaviours are notable mainly by their apparent absence. This may be due to the difficulty in usefully analysing agent-level performance, or as suggested by Berg & Gigerenzer[11]<sup>9</sup>, the focus of much current behavioural finance research on 'as-if' models, where agent preferences are homuncular versions of neoclassical preference representations. Typical academic approaches have limited scope, focusing on empirical comparisons of particular investment strategies over single holding periods rather than portfolio investment over time when stop-loss rules are applied, such as in Lei & Li[86], or behavioural finance analyses of investor biases, crowd behaviours and market effects, as in Osler[101].

The experimental designs in this chapter seek to address this shortcoming in the literature and have several objectives: to illustrate subsumption in action as a functional, structural meta-heuristic in economic preference expression; to systematically explore specific preference modifier behaviours; and to provide a working demonstration of the SHaaP architecture itself, providing the basis for ongoing development of this approach.

In the experiments economic agents (iAgts) are specified with deliberately minimal core (MC) preferences, exploring several different preference modifying behaviours

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<sup>8</sup>In this case the 'nominal' qualification refers to the fact that no actual trades were carried out, however the models are situated insofar as agents experience and must perform in actual historic financial time series rather than generating synthetic time series.

<sup>9</sup>See Chapter 2

before introducing learning, and further subsumptive layers and agent-subsuming agents (sAgtS). Populations of agents make investment decisions in two situated environments, using historic time series from the UK stock market and from foreign exchange.

The designs recognise pitfalls in model development seen elsewhere, where computational structures are often ignored or poorly documented making replication difficult or impossible. At the same time they explicitly acknowledge the exploratory nature of ABMs, where abstraction may render research conclusions operationally meaningless. A possible weakness in the methodology is the lack of an overarching theoretical framework, however this is an inevitable problem where combinatorial complexity rapidly makes even simple ABMs intractable.

The experimental findings are quite striking, not only demonstrating clear expected return profile effects for simple heuristic rules, but periods in which simple heuristic preference modifiers with minimal core preferences and no explicit learning or adaptation mechanisms can generate positive returns. This conflicts with theoretical model predictions such as Kaminsky & Lo's[65]. These findings are systematically investigated and discussed using the SHaaP architecture and performance measures developed in earlier chapters.

A simple particle swarm optimisation algorithm was investigated in both a basic and modified forms as part of the investigation. Although intuitively attractive as a mechanism to aid parameter setting and introduce adaptivity into SHaaP agent populations, these were found to have little overall effect on experimental outcomes. This is an interesting finding in itself, suggesting structural resilience in the population-based agent preference structure in the architecture. Potential implications about underlying functional processes of preference modifiers are discussed in the chapter suggesting possibilities for extended further research.

Overall the studies provide important evidence supporting the subsumption and population-based meta-heuristics applied in the SHaaP architecture and experimental methodologies. At the same time the findings offer potentially significant insights into the role of heuristic preference modifiers in actual application. The chapter concludes by critically reviewing the experimental findings, possible weaknesses in the approach and architecture, and contributions of the work.

**Chapter 7 - Discussion & Conclusions** Having presented extensive, critical discussion of the problem domain and academic approaches in Chapters 2 & 3; developed a viable, novel experimental architecture, together with new agent-level performance measures in Chapters 4 & 5; and demonstrated their effectiveness in the case study experiments in Chapters 5 & 6, there is a need to step back and appraise the overall work.

This chapter presents a brief review of the work presented in the thesis, discussing its relevance and its potential range of application. Inevitably, and as a further subsumptive behaviour, the work here is seen as offering an exploratory experimental framework for use alongside and incorporating other established computational models and methods. Investigation of subsumption and population-based structural meta-heuristics are significant contributions of the work incorporated in the experimental architecture. It addresses significant weaknesses in extant ABM

approaches, while offering novel tools to investigate economic preference expression and develop models for decision support and policy design.

It is again emphasised that the principal range of application is in exploring processes underlying economic preference expression and the systems in which these preferences are expressed, where SHaaP models could be applied to model system behaviours and provide decision support. However this represents a vast range of problems for study: future areas for work are proposed within the chapter, including on policy design, regulatory and legal frameworks.

**Appendices** Further details of the main experiments, SHaaP architecture implementation, performance measure definitions, nomenclature and economic terms are given in the appendices.

## 1.4. Main Contributions

This section sets out the main contributions of the work presented here.

1. Extensive critical review of models of economic preference expression in economic systems, and computational models of such systems, from a practitioner's perspective. Identifying risk vs. uncertainty as a key differentiation in designing both theoretical & practical approaches to manage economic exposures, highlighting shortcomings in many current models.
2. Identification of verification and validation issues in the literature for extant methodologies & approaches to ABM models investigating economic preference expression. Solutions to these issues are proposed including,
  - Situation as a key validation metric.
  - Agent-level analysis of model behaviours.
  - Realistic performance measures.
3. Development and testing of new agent-level, situated performance measures for models exploring economic preference expression. These recognise the importance of realistic, agent-level and population-based performance and fitness measures, both in investigation of agent behaviours and overall validation & verification of experimental observations.
4. Identification & description of subsumption as a key structural meta-heuristic allowing economic preferences to be decomposed into component functional behavioural elements. Functional decomposition is an important stage in developing operationally meaningful models of preference expression within larger systems which can then be tested, explored and extended systematically.
5. Differentiation and identification of functional economic preference components in subsumptive structures. Economic preferences are rarely if ever expressed in isolation from their environment - rather they are subject to modifying behaviours and structures while being subsumed into more complex overall preference expression. Preferences are expressed using simple, subsumptive heuristic layers in a population-based structure. This allows,

- traditional explicit *core preference* functions, simple heuristic preferences and compound preferences to be systematically explored.
  - *preference modifiers* - structures which modify core preferences and compound preferences - are introduced and developed with particular relevance to regulatory applications and subsumption.
  - complex, and potentially sophisticated, non-linear preference behaviours to emerge through subsumed clearly defined component structures.
6. The design and implementation of the SHaaP architecture, a novel agent-based subsumptive, population-based experimental architecture.
- Specification and development of an open, transparent, and secular methodological architecture, explicitly recognising & specifying both structural and economic preferences throughout systems.
  - The architecture explicitly allows preferences to be decomposed into functional units and explored in a principled manner - this is an important difference from traditional models where preference modifiers and non-core structures are often simply ignored.
7. A series of original experiments systematically exploring and analysing the role of simple heuristic preference modifying behaviours in economic preference expression. Preference modifying behaviours are prevalent in actual economic systems, but there is little research described in the academic literature on their effects.

These experiments,

- demonstrate the effect of such modifiers on core preferences.
- point to subsumption as an important structural meta-heuristic in describing and developing models of sophisticated non-linear preferences in uncertainty prone environments.

The experimental results are striking in themselves and suggest substantial scope for future work on subsumptive preference structures, uncertainty mitigation behaviours, and simple heuristics in operationally meaningful models of economic preference expression.

8. Application and relevance to regulation and risk management. The SHaaP architecture is potentially attractive both as an academic framework and as a practical simulation tool for decision support: allowing rapid development of operationally meaningful models when combined with the new performance measures and validation metrics developed in the research.

## 1.5. Publications Forthcoming

An issue with completing a substantial, multi-disciplinary piece of research as a part-time student, and emigrating to a new country in the middle of a global financial crisis in which uncertainty effects abounded, is that the main priority here has had to be completing the research and work on the thesis ahead of journal publication. The

result is that paper preparation has been somewhat sidelined till after submission itself.

It is certainly the case that I believe the work here is worthy of publication and offers significant contributions in terms of application, methodologies, and the experimental architecture to the principled design of agent-based models of economic preference expression. In any case this omission in my research is now in the process of being remedied.

Two to three papers from the work presented in the thesis are now in preparation for submission, these papers,

- Introduce the SHaaP architecture as a novel, agent-based approach to modelling economic preference expression. Reporting the preliminary findings on preference modifier behaviours for economic agent populations with heterogeneous preferences in Chapter 6.
- Present the results of the comparative study of agent preference expression in Chapter 6, with and without active adaptation using a particle swarm optimisation algorithm, discussing the observed robustness of the population-based subsumptive agent structure.
- Present and describe the population relative performance measures developed in Chapter 5, reporting on their use in reconstructing the SFASM model. This directly addresses the methodological issues in ABMs which gave rise to the frustration driving the initial research in the thesis, and finally the SHaaP architecture design.

Further papers exploring deeper subsumption hierarchies and reporting the results of agent-subsuming sAgs are planned as the future work proposed in Chapter 7 is rolled out, extending the scope of the research beyond this thesis.

## 2. Economic Preferences, Expression & Economic Systems

*'Only the smallest fraction of economic writings, theoretical and applied, has been concerned with operationally meaningful theorems.'*

Samuelson 1947

### 2.1. Overview

Neoclassical economic models have dominated economic thinking and research for most of the latter part of the 20th Century: an important component in this dominance is their analytical tractability and ease of application driven by central simplifying assumptions over how economic systems can be described. More recently, the emergence (and re-emergence) of alternate schools supported by advances in computing power and modelling platforms has enlivened debate over the operational meaningfulness of neoclassical approaches.

In particular, behavioural finance and computational economics have offered the potential to explore and critically examine economic systems using bottom-up, agent-based models with more direct operational meaningfulness than neoclassical models. However, bottom-up models are frequently highly parameterised and subject to combinatorial complexity with the result that they have been, and continue to be, extensively criticised both in terms of methodological issues and validation. Given these issues it is debatable just how far computational economic models and behavioural finance models have actually progressed debate.

Financial markets represent a particularly useful subset of economic systems. While the research presented here is applicable to more general economic systems, models of financial markets allow situated experiments to be designed and evaluated rapidly, with large amounts of data available for examination and comparison. In the thesis therefore, and in keeping with the majority of computational economics research, the focus when developing case studies will be on financial systems, discussing these in terms of more general systems and applications.

Markets are of course, except in laboratories, never totally separate or separable from larger economic systems and, as the GFC and other market crashes have amply demonstrated, ultimately all are interconnected and have the potential to directly affect each other. These factors and the general application of the work presented in the research will be discussed in developing the experimental architecture, case studies, and future directions.

This chapter provides an overview of some core elements of some main economic schools, specifically in terms of criticisms and methodological issues present in experimental and computational economics. The emergence of experimental economics as a general platform across schools is discussed.

Key aspects of neoclassical, neo-Austrian, and behavioural finance schools of economic theory are reviewed together with alternate, agent-based approaches. The chapter provides general background to these schools, presenting concepts used to frame and test economic models relevant to the agent-based systems in this thesis, however it is not an attempt to argue the relative merits of the different schools outside this arena.

Risk and uncertainty as critical characteristics of economic systems and preference expression are presented in some detail. Specific examples of market behaviours as complex adaptive systems are used to illustrate uncertainty exposures and policy responses. The importance of recognising the impact of these types of exposures in developing operationally meaningful theories of economic preference expression is a significant contribution of this discussion.

Overall the chapter sets the context for the research presented in the remainder thesis, particularly in the design of the new experimental architecture presented in Chapter 4 and the case study experiments exploring uncertainty mitigation structures and heuristics in Chapter 6.

## 2.2. Experimental Economics & Operational Meaningfulness

### 2.2.1. Some Context

Despite the temporal distance between Herbert Simon's 1986[129] assertion that economics does not concern itself with processes of decision making and Samuelson's 1947[114] statement that 'operational meaningfulness' has rarely been directly addressed in economic research, the sentiments embodied remain important in capturing basic factors behind much late 20th century economic research.

In the absence of a suitable experimental platform, economic research centred on deductive analysis of theoretical postulates. Validation of proposed theories beyond this analysis relied on matching theoretical implications to observations in actual markets. Academic models bore little or no structural or operational similarity to the markets they described, making extensive use of rational representative agents and top down approaches. Despite Samuelson's statement, beyond gross behaviours at the overall system level, neoclassical models developed subsequently have had little explanatory value - they were not *operationally meaningful* in anything more than a loose descriptive sense and then typically only at the level of economic policy. This is as much a failing due to the lack of feasible experimental frameworks as an absence of will or the belief that top-down approaches were sufficient. Naturally parsimony and appropriate levels of abstraction are important - but if models are to have direct operational relevance beyond economic policy formulation then less abstraction and more attention to underlying processes are equally important.

In the last two decades computational economics has emerged as the principal experimental framework for economic research and as a field of study in its own right. Increasingly cheap computing power and the application of technologies from other social and computational sciences made it possible not only to design, but to actually build and test bottom up models of economic systems. Processes as well as choices could be practicably explored on a much larger scale than possible before.

The use of simulation as a means for model evaluation has progressed debate over neoclassical economic theories and promoted investigation of alternative theories from behavioural and computational finance, while at the same time yielding benefits and insights into systems made up of interacting economic agents. Academic studies have ranged from simple systems made up of small numbers of rudimentary economic agents to relatively 'rich' artificial financial markets (AFMs) where agents have well specified preferences and operating over extended time periods. Agent-based computational economics (ACE) has expanded rapidly, attracting researchers from many disciplines.

Despite, or perhaps because of, its popularity, agent-based research has attracted significant criticism: much of this is acknowledged by practitioners as having at least some validity[142, 84, 40, 13]. Concerns have ranged from basic problems in experimental design and replicability to caution over the extent and the validity of inferences drawn from experimental findings. Methodological issues are critically important in any research, however for 'rich' agent-based markets presented as complex adaptive systems they are especially problematic. Any 'rich AFM'<sup>1</sup> requires a huge degree of parametrization and many computational choices: directly or indirectly these must all be addressed and documented within the experimental design. These issues and their relevance to this research will be discussed in Chapter 3, and again in the development of the experimental framework and methodologies presented in Chapters 4 & 5.

**ACE & Neoclassical Challenges** Neoclassical economic theories of rational expectations and efficient markets dominate much economic thinking in both academia and applied financial modelling. This is perhaps unsurprising, insofar as they appear to fit many observed macro aspects of market behaviours, and given the lack of computationally efficient alternatives, or a useful experimental framework to investigate alternative approaches. Relative ease of implementation, parsimony and testable predictions found favour both with academics and financial practitioners, fostering a deductive approach to development of economic theory.

As a result the main body of reported rich AFM research has focused on experimental protocols either directly centred on exploring neoclassical frameworks, or on structures constructed to allow direct comparison with neoclassical benchmark measures. This has the benefit of allowing a common means of comparison between models and discussions of outcomes, however it carries with it a number of potential problems above and beyond those commonly levied at agent-based paradigms.

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<sup>1</sup>

Here *rich* simply means rich in potential complexity. Numbers of economic agents are relatively large, heterogeneous in beliefs (if not preferences), market clearing processes are relatively sophisticated and agents may interact via social learning and economic transactions.

Given the central, simplifying assumption that markets could be treated 'as if' made up of unboundedly rational, representative economic agents, large scale empirical studies of financial time series appeared to support neoclassical descriptions of market efficiency in terms of continuous equilibria. Building on Arrow & Debreu's general equilibrium framework[2], the development of the efficient markets hypothesis (EMH) presented by Fama in 1970 [42], Muth's rational expectations hypothesis (REH - [97]) and Markovitz's modern portfolio theory (MPT -[95]) in 1950's and 60's provided much of the supporting framework for research and found widespread practical application in finance. Tests for market efficiency have been, and continue to be, seen as important elements in validating new research, including work challenging the EMH and ideas of unbounded rationality.

Other economic schools advocating bottom-up models have a stronger claim to explanatory insight, much better aligned with other social sciences[129]. Modelling systems in terms of the interactions of the individual participants has a strong intuitive appeal, and neoclassical economists have long acknowledged the truism that markets are made up of their participants ( Keynes [72] described them as 'the outcome of the mass psychology of a large number of ignorant individuals'). Hayek [57], as part of the Austrian School, argued the case for such treatments, while careful to point out that a theory tested using such models 'could never be verified only tested for its consistency'. It is important to recognise that this criticism is common to all agent-based approaches<sup>2</sup>.

However Hayek's approach was computationally costly and, in the absence of a suitable experimental platform plus the apparent success of neoclassical paradigms, relatively little progress was made in this direction until the late 1980's.

This situation has radically changed with increasing availability of vast quantities of raw, near real time data; cheap computing power and (relatively) robust modelling platforms. Adopting insights and technologies from other social sciences, game theory, AI research, and the emerging field of *behavioural finance*, bottom-up models allowed markets to be modelled as complex adaptive systems and aspects of bounded rationality in economic decision making to be investigated.

The processes employed by economic agents and their interactions over time have been central to this research. As such, the division noted by Simon has shifted somewhat, so that it became a point of differentiation between economic schools rather than between economics and the other social sciences. The result has been a critical re-examination of some central tenets of neoclassical economics, questioning both the description of markets in terms of rational expectations and empirical evidence that markets are 'efficient'<sup>3</sup> [41]. While this debate continues with some

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The epistemological issue addressed by this qualification is an overriding concern with the interpretation of experimental, computational models in finance - similar issues have been raised in A-Life studies [103]. In agent-based models where the re-creation of gross statistical artefacts are often used as support for model validity, the direction of the logical construction seems inverted - rather the presence of the artefacts needs to be supported by behavioural phenomena which have been validated in some way. The importance of validation as an issue goes hand in hand with 'operational meaningfulness'.

3

Section 2.5 provides a definition of 'efficiency' and a brief overview of neoclassical & other

intensity, the increasing volumes of research engendered into the processes and structures underlying market behaviours have led to effectively new schools of economic research - 'behavioural economics' and 'computational economics' or 'agent-based computational economics' (ACE) [80, 142].

By allowing large scale simulations testing proposed economic structures computational economics has effectively provided an experimental framework previously unavailable to economists, and with it to move away from the almost entirely deductive approach noted by Samuelson [114]. These models at least allow a starting point to explore possible underlying behaviours and market structures responsible for their presence. Extensive simulations of both top-down *and* bottom-up models, have extended financial modelling technologies into practical aspects of risk management, market regulation, derivative structuring and fund management. Ease of construction, flexibility and replication have led to a very rapid increase in research in a laboratory environment across a huge diversity of topics, with some success in exploring basic market behaviours and demonstrating complex behaviours arising out of superficially 'simple' systems.

### 2.2.2. Protocol & Design Issues

An exploratory approach using computational economic models, with the potential explanatory power these bring, also introduces significant issues in supporting experimental findings and models in terms of methodological rigour and validation. Generally criticism of agent-based models has focused on validation of experimental findings and conclusions based on these findings, rather than on methodologies. However both components are important to principled computational models of economic systems, particularly given their recursive non-linear behaviours.

Practical issues with implementing computational models have long been identified by researchers developing such models in economics and other fields relating to their own work[48, 60]. However although widely acknowledged as a significant problem for computational economics, whether agent-based or not, it appears that no common standards or approach have been agreed and these issues remain current. Recent papers by Binmore & Shaked[13] and by Fagiolo et al[40] serve to illustrate that this remains an active debate. Binmore in particular is highly critical of academic practice, identifying problems with replicability; model documentation; cherry picking observations; logical failings; and over-fitting of models as significant issues.

As a separate issue they also suggest that there is a lack of appropriate active, critical review of published research in the literature. While this observation may itself be a part of the 'normal' academic debate it also largely agrees with findings reported in Chapter 5, where reconstruction of the Santa Fe Artificial Stock Market (SFASM) as a 'calibration benchmark', rich AFM while developing performance and fitness measures proved to be profoundly difficult.

Protocol design, methodologies and validation are vitally important to any architecture, particularly in the motivation for SHaaPa, and will be discussed in detail

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economic schools.

in Chapter 3.

### 2.3. Why Are Economic Systems Different?

'The market can remain irrational longer than you or I can remain solvent', Keynes (attrib)<sup>4</sup>

Before considering current economic schools and ACE research, it is worth revisiting why economic systems are problematic as topics for study.

Financial markets in particular are somewhat unique: large volumes of data are readily available; whole industries are devoted to searching for and exploiting any perceived inefficiencies, analysing and modelling market structures; directly or indirectly every individual is both affected by and affects these industries; governments can stand or fall based on their perceived effects on markets, and yet many participants accept that established macro and microeconomic models have only limited explanatory value.

A defining characteristic, highlighted by Hommes[59], is that unlike natural systems, participants' expectations about the system and behaviours based on those expectations affect the state of the system itself.<sup>5</sup> However expectation formation requires some form of knowledge about the current, and, possibly, the historic state of the system. So expectations and market states are interlinked - as such markets have been characterized as *complex adaptive systems* (CAS). The impact, for neo-Austrian adherents (see below), if one accepts that expectation formation is not unboundedly rational, is to allow markets to exist in inefficient, non-equilibrium states, which also then allows for crashes and manias.

A second key aspect is that, as far as trading and investment goes, the 'right' answer may give the wrong result. Similarly the right result does not necessarily indicate the right answer or reasoning, nor is the right answer necessarily 'right' over all investment horizons.

- Sophisticated cashflow modelling approaches can be wrong for extended periods of time during mania's and crashes, while heuristic, trend following rules may be successful throughout. Good, rational models of markets which are correct in the long term may be so unprofitable short term that a highly rational trader or investor ceases to exist in market terms.

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Like many quotes for John Maynard Keynes there is no definitive proof that he ever actually said this, or, if so, in this form. That said, the ideas behind it are relevant to market structure and economic behaviour.

5

In its most extreme form with recursive feedback loops leading to sustained inefficiencies, excluded by rational equilibrium-based theories, this has been characterised in terms of *reflexivity*. George Soros as a practitioner has written extensively about its implications.[133] It is open to debate how useful or testable Soros's definition of reflexivity actually is, however Hommes' observation of expectations is fairly uncontentious - allowing both neoclassical and alternate interpretations depending on ones assumptions of agent rationality and behaviour.

- In traded financial markets changes of price are often the only immediately, transparently available information for an asset. This information can be distorted by large transactions driven by non-economic preferences, such as regulatory requirements; economic impacts from other markets; or non-public information.
- Short term trading movements, reflecting large transactions or lack of liquidity, may benefit, or impair, traders with short investment horizons and limited capital, while investors with longer horizons or larger capital bases may be unaffected or able to take advantage of apparent mispricings.

This is somewhat different from saying that the environment is noisy and affects future outcomes so that 'bad' models may persist and grow in influence. Nicolas Taleb, as a 'practitioner, turned academic, turned academic-practitioner', in *Fooled by Randomness*[137] identifies an effect of this as survival bias amongst participants. Taleb criticises poor logical connections between survival in successful investors and the inference that they have good market models: it may be that they have a good trading model, which is a different thing, or they may just have been lucky.

Performance measures and definitions of success are therefore challenging. It is not sufficient in itself to consider only profitability on individual transactions or even cumulative profitability. Performance measures and model validation also need to address resilience in behaviours and strategies as well as risk-adjusted returns over different time horizons.

**Academic vs. Practitioners' Models** In defining market and trading models financial industry practitioners face the same set of problems as academia. However, in actually implementing functional trading models additional considerations have to be addressed simply because their models cannot be neatly stylized to fit limited research questions - they must function in real markets and solve real problems. Brooks[24] describes this as being 'situated', whereas academic models are typically largely 'unsituated'.

Some features situated models need to consider, or at least acknowledge,

- Market infrastructure is important: participants do not all share the same barriers to entry (such as brokerage costs, costs or sources of capital or regulatory requirements).
- Participants are heterogeneous in investment horizons, core preferences and risk tolerances.
- Financial systems are not closed
  - Participants may arrive or leave a market over time.
  - Participants may act in more than one market at any time and experienced global returns may affect local preferences.
  - Assets and asset availability can change.
  - External economic factors may change affected by external events.
- Preference modifiers at trade level may interact with changes in market conditions and infrastructure.

- Information arrives stochastically and may be inefficiently distributed. Despite the best efforts of regulators and whatever claims the banking industry may make, not all information is readily available to all participants - new information arrives at irregular intervals; and may be costly to acquire.

Put simply, actual markets are typically ill-defined. Only in relatively rare, highly specific circumstances, such as true arbitrage, or in a stylized and abstracted laboratory system, is it safe to assume that participants are homogeneous in preferences and trading constraints, or that only single assets are available for trade.

The problem of designing appropriate experimental protocols for situated models is therefore considerable. What form should reasoning processes behind expectation formation and preferences take? Where should these processes come from and how are they updated if at all? Are structural elements relevant? What is actually realistic?

Unsituated models largely sidestep or ignore many of these questions. Many academic models appear to reduce to sets of representative agents operating with neo-classical preferences and rationality - a homuncular representation which may give interesting demonstrations of neoclassical interactions, but brings little practical explanatory value.

**Timeseries Artefacts** Typically financial markets have been described by reporting common statistical features - kurtosis, clustered volatility and cross-correlation of volume, volatility and returns. Despite rapid changes in market practices, global trading, regulation, electronic execution and real-time data feeds and apparently new types of investor, it appears that these features are remarkably persistent (see Fama[41] and LeBaron [79] for typical examples of such studies).

Although some evidence may point to increased efficiency (see below, Section 2.5) principal reported features remain. Indeed, as the granularity of available data has increased, these features have an apparent fractal nature [94]. Explaining the processes underlying these effects and the conditions necessary for their presence are important questions for economic research and are not directly addressed by traditional neoclassical approaches.

## 2.4. Risk & Uncertainty in Economic Systems

Risk and preference behaviours are fundamental to understanding economic systems in which economic agents either take on deliberate exposure to risky assets, or attempt to manage unavoidable exposures according to their preferences, or both. A starting point to understanding requires some definition or categorisation of types of risk in economic exposure.

Neoclassical equilibrium-based models have mainly focused on strict formal definitions of risk considering market states and investors' preferences in terms of utility functions and utility maximising behaviours. Other than questionable assumptions of investor rationality in preference expression, these definitions effectively assume that all future states of asset values and their probability distributions are known

or can be inferred, typically being assumed to be normally distributed random variables. This treats markets in the same way as games of chance or lotteries where all possible outcomes are likelihoods known in advance.

Unfortunately empirical evidence, and common sense do not support such a definition, despite its widespread use by regulators, practitioners, and academics. While these problems are widely acknowledged, generally they are addressed by attempting to fit and tune distributions to better fit historic or newly emerging data. This is important, particularly in the context of this thesis, in that it means that research into risk management structures and behaviours is highly constrained. The solutions investigated become optimisation problems even where such solutions are evidently not applicable or appropriate.

Less common is to recognise fundamental complexity and intractability in economic systems and address this in terms of defining types of risk rather than simply applying quantitative definitions. Alternate risk definitions and categorisation are discussed below. These distinctions are important both to the case studies presented in the thesis, and to the potential contributions of the SHaaP architecture and methodologies developed here.

### 2.4.1. Risk vs. Uncertainty

Unlike quantitative, neoclassical definitions of economic exposure, Frank Knight[74] attempted to distinguish different types of risk, categorising risk in terms of measurable and unmeasurable uncertainty.

- *Risk* is defined according to objectively quantifiable measures, where expected values and distributions are either known or can be established.
- *Uncertainty* is applied to situations where the distributions are not known, or subject to some form of stochastic flux, so that they cannot be reasonably inferred or measured.

ACE researchers, Austrian & neo-Austrian economists and behavioural finance economists have acknowledged decision making under incomplete information as an area for research in their models, whereas most modern neoclassical risk models address only measurable uncertainty, i.e. *risk* in Knight's terms.

This is an intuitively obvious distinction often simply ignored in modern economics. Once recognised, however, it forms a basic first step in classifying types of market exposure and modelling approaches. This distinction between *risk* and *uncertainty* has particular relevance when considering practical, subsumptive heuristic structures, and their role in exploring & designing resilient regulatory and economic entities. *Risk* could reasonably be argued to be a subset of general uncertainty, i.e. measurable uncertainty. However given its relevance as functional distinction in this thesis, building on Knight's work, in referring to economic exposures, *risk* will generally be used to refer to 'measurable uncertainty'; *uncertainty* applied to cases where future asset states cannot be known or reasonably inferred; and 'economic exposure' encompassing both *risk* and *uncertainty*.<sup>6</sup>

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Although what constitutes *risk* is relatively clear, *uncertainty* is a very broad, loose definition - effectively a catch-all for anything which is not *risk*. It makes no distinction between Rumsfeld's[111] '*known unknowns*' and '*unknown unknowns*', nor does it identify Taleb's[138] 'black swans'. This however is symptomatic of economic systems as an overall domain, and the types of distribution and complexity that are present: beyond identifying the possibility that *unknown unknowns* exist, a strict definition is a logical impossibility, while for *known unknowns* the situation is easier to specify though still challenging to address quantitatively.

Classifying economic exposures according to risk and uncertainty is a basic starting point to characterising problem domains and appropriate modelling approaches - in the context of this research it is seen as essential. Without such classifications it is difficult to understand how models can be critically examined and modelling assumptions challenged. The alternative is to simply attempt to fit neoclassical risk models to everything and try modifications to make them fit better when they fail - an approach which yields little explanatory value.

Having described systems as subject to risk or uncertainty, what then?

For risk-prone systems or sub-systems, modelling approaches are relatively straight forward in principle: models can be chosen, designed and tested against empirical data and optimisation approaches applied. By casting economic systems in terms of risk, and making a broad raft of assumptions, neoclassical economists can then produce tractable models. Effectively this takes a normative view of the world and reduces its complexity, ignoring uncertainty altogether. Though such models might be expected to have a strictly limited role in actual market applications, their ease of implementation and the lure of prediction and efficiency via optimisation means that they are in widespread use - albeit with modifications and, ironically, heuristic subsuming control processes. These models, their assumptions and criticisms will be discussed in more detail in the following sections in this chapter.

For uncertainty-prone systems, the modelling approach is less clear. Where asset distributions are unknown and subject to stochastic, non-ergodic change, optimisation approaches are likely to be at best ineffective. Indeed, given the structure of economic systems for participants, optimisation in the face of uncertainty can produce brittle, unstable behaviours. This has been seen in various recent market crashes where program trading, algorithmic arbitrage and relative value trading have exacerbated market volatility after periods of apparent relative stability and efficiency.

The collapse of Long-Term Capital Management (LTCM) discussed in the following section is a particularly clear example of this in practice. LTCM's activities were large enough to change the behaviour of the entire market in the securities it traded and affect other markets as its models drove asset prices towards theoretical model values, and again later as instability forced the same trades to be unwound, increasing volatility and creating large scale misalignment in asset values. Such periods

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Unfortunately when referring to risk management and risk management systems it becomes too cumbersome to attempt to maintain this naming rigour. Risk management systems in any case report in terms of risk, but are required to produce outputs to deal with both risk and uncertainty.

of stability followed by highly volatile disruption are relatively common in financial systems, certainly more much more common than neoclassical models predict[94].

More recently high frequency program trading models have contributed to periods of excess stock market volatility and instability, forcing regulatory responses in attempts to limit potentially destructive cascade effects. The 'Flash Crash' of 2010 described in Section 2.4.3 provides a good example of this effect. Given the prevalence of such trading models now this kind of instability is likely to be an ongoing problem warranting detailed study.

### **2.4.2. Long-Term Capital Management - Rational Exuberance? An Exemplar.**

In practical terms robust, arbitrage-based models can lead to overconfidence with significant consequences, for markets, for policy makers, and investors. The history of Long Term Capital Management (LTCM) and its collapse is a salutary case study[37, 89, 55]. LTCM was one of the most prominent hedge funds of the 1990's focusing on mathematically driven arbitrage and relative value trading models. The fund employed not only some of the most successful proprietary traders of the time, but also Nobel Prize-winning economists Robert Merton and Myron Scholes. Throughout the 1990's this form of trading became widespread amongst financial institutions ranging from hedge funds to proprietary trading desks in investment banks: real-time data feeds to spreadsheet models and trading systems; rapid expansion of financial derivative, interest rate and asset swap markets; and regulatory sanction all contributed to this development. The resulting market dynamic eliminated many arbitrage opportunities and reduced volatility substantially in relative value trading. This had two consequences: firstly falls in volatility meant that risk managers and regulators sanctioned increased trading limits, secondly it meant that to maintain return on capital larger, more leveraged trading positions were required. Both effects fed back into asset prices and volatility in a seemingly benign cycle - for a time.

By 1998 LTCM was running its portfolio with 25x leverage on capital: against US\$4.8 billion of capital LTCM had approximately US\$120 billion of assets. In 1997 LTCM had returned US\$2.7 billion of capital to shareholders having decided that its capital base was too high.<sup>7</sup> Political instability in both South East Asia and Russia served as trigger events for substantial losses on arbitrage positions as this fed through into wider uncertainty, volatility and widening of spreads on speculative positions and on large Russian debt arbitrage holdings when, as a 'Black Swan' event, the Russian government declared a moratorium on future repayments, effectively a default.<sup>8</sup> The impact for LTCM was profound as lenders sought additional collateral

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<sup>7</sup>Dowd[37] reports this as a calculated gamble to increase returns to shareholders in the face of limited investment opportunities. However a more cynical view in keeping with Shleifer & Vishny's[121] analysis of context and preference behaviours is that it was driven by LTCM's bonus scheme incentives - higher IRR (internal rates of return) generate larger bonuses for the fund managers.

<sup>8</sup>

Arguably this was not really a Black Swan if Russia's previous behaviour were taken into account, but markets, or rather market participants, have short memories and at the time such actions

and the company's funding dried up, forcing it to attempt to liquidate positions in increasingly illiquid and volatile markets. The impact for other participants with similar strategies was equally large as previously highly profitable, low-risk operations became significantly loss making.

In 1998, the US Federal Reserve Bank (the Fed), specifically the Federal Reserve Bank of New York, intervened citing concerns over contagion and market stability. Interestingly in later testimony Alan Greenspan, then chairman of the board of Federal Reserve Banks, stated that it was not a case of LTCM being too big to fail, but rather worry that markets would not know which counterparts had significant exposure to the company. The New York Fed brokered a deal in which 14 creditor institutions bailed LTCM out, injecting new capital and allowing its trading positions to be unwound in an orderly fashion. By the time the crisis was resolved losses for LTCM amounted to an estimated US\$4.6 billion[89], essentially all its capital, as attempts to unwind trades further exacerbated volatility and spreads.

The concept of 'too big to fail' whether it was the case or not for LTCM, or whether the Fed actually believed that, has carried through into the GFC and approaches to dealing with that, though the total sums involved for LTCM were tiny in comparison to recent events and, in hindsight certainly did not represent a systemic risk in themselves. The Fed's involvement itself represents an uncertain event, though as Taleb would assert, once known cannot be unknown and has acted to modify bank and investor behaviour since.

### 2.4.3. The 'Flash Crash' - High Frequency Trading & Market Instability

On 6th May 2010 a large automated sell order exacerbated by high frequency trading programs (HFT) caused a sudden sharp sell off in US stock markets. Major US equity indices dropped 5-6% in minutes with no apparent new news in the market, having already fallen over 4% in the face of international market turbulence, setting lows around 10% down from the previous day's close before recovering to finish only 3% lower on the day. Some individual stocks and exchange traded funds fell by as much as 15% before recovering most of their losses by close of business. The CFTC-SEC<sup>9</sup> investigative report produced some months after the incident, though it agreed that HFT activity added to the severity of the move, identified a single large institutional trade as the main trigger, while also describing the market itself as fragmented and fragile[26].

Trading curbs were introduced shortly after, extending to cover a broadening number of companies' shares making up the Dow Jones and S&P indices. Such curbs, which are designed to act as circuit breakers for markets, have been used in exchanged traded futures markets for some time and in stock markets since the stock market crash in 1987. By effectively stopping all trading for a prescribed period they are

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were not considered even likely given the country's political ambitions internationally and attempts at reconciliation.

<sup>9</sup>The U.S. Commodities Futures Trading Commission and the U.S. Securities & Exchange Commission have joint regulatory responsibility for securities and futures markets in the United States.

intended to stabilise order activity, allow information to be distributed and reduce volatility.

As a crude, heuristic process it is debatable how much effect they have and how beneficial they are given that large over-the-counter (OTC) markets exist away from regulated exchanges where transactions cannot be stopped. However for electronic exchanges and program trading it seems to be a reasonable measure, especially if as recently reported around 60% of HFT trading takes a semi-passive market making role[51]<sup>10</sup>. Other measures introduced include rules to break trades which happen at prices too far away from earlier trades.

Published analysis of the events has been relatively limited and slow. The CFTC and SEC have been criticised for the length of time taken to produce their report, but this is symptomatic as much as anything of the evolutionary arms race between regulators and market participants - unsurprising in a competitive economic system where regulators are frequently playing catch-up on new business and trading models as they are created. An interesting feature of the crash and of electronic markets in general is the availability of large amounts of accurate trade and pricing information. A number of empirical analyses of this data have been carried out with conclusions both agreeing with the CFTC-SEC conclusions, and arguing strongly against it - the latter largely denying, or at least minimising, the effect of HFT operations in causing the crash. This may of course represent a normal lobbying behaviour between regulators and practitioners.

An interesting agent-based simulation of HFT effects in an AFM has been published recently by Vourenmaa & Wang[147]. This study, based on an earlier market model by Chiarella et al[31], largely supports the CFTC-SEC conclusions, describing cascade effects and feedback loops as HFT algorithms rapidly readjust their market prices in the face of selling without actually transacting - a behaviour which becomes significant if HFT operations have taken over and automated a large part of equity market making. Vourenmaa's model is however highly abstracted and unsituated - the agents while modelled on descriptions of HFT behaviours have not been tested in real markets and also ignore OTC market activity. It would be interesting to incorporate the actual data from the flash crash in this type of simulation giving some degree of situation, especially given its availability. The original model designed by Chiarella et al also bases core agent preferences on unboundedly rational, CARA<sup>11</sup> utility maximising behaviours: as discussed in Chapter 3 this is a potentially significant modelling choice, although given these are algorithmic trading models, unbounded rationality, if not constant absolute risk aversion, is at least defensible. All that said, as a demonstration of potential market effects it is useful, particularly as it considers agent-level behaviours and as an input to policy formulation.

In the build up to the 2010 flash crash and subsequently HFT and algorithmic

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<sup>10</sup>

Essentially aiming to make profits by accepting retail orders to make a bid-offer spread. This has an ecological rationality in that trades from such activities would be very short term and high quality in risk-adjusted terms.

<sup>11</sup>

Constant Absolute Risk Aversion - see Chapter 3 and Appendix A.3 for definitions and discussion.

trading have been widely discussed particularly in the media where these activities have been dramatised and, to some extent vilified, most recently by Michael Lewis in 'Flash Boys'[87]. As a market behaviour it represents another example of potential instability in financial systems and an uncertainty exacerbating mechanism. The Bluecrest example in Section 2.5 illustrates the scale of algorithmic trading funds in today's markets and as a rational, arbitrage focused behaviour this is unlikely to disappear. As with LTCM however efficiency and optimisation in uncertainty prone environments are potentially destabilising themselves.

As some commentators have noted[112], HFT's are not necessarily bad for the market in general, by driving efficiency and proliferation of electronic trading mechanisms retail investors in particular have benefited significantly (except of course if they happen to be exposed during the small window of flash crashes). Bid-offered spreads have narrowed, market prices are nearly continuous and broking charges collapsed in the last decade. The real issue is not if HFT and algorithmic trading are a 'bad thing', as with any other economic actor in an economic system, it is where their behaviours are exposed to, or exacerbate, uncertainty that is of interest.

Policy and regulatory responses are important, and Haldane's proposals in the following sections are highly relevant. A policy recommendation by Vourenmaa & Wang in their paper is effectively that HFT and algorithmic traders self-certify their models, but this remains a risk-based approach where the temptation to optimise and game regulation is present. It ignores uncertainty effects and directly contradicts Haldane's heuristic approach. In fact the gross trading curbs implemented after the flash crash represent a simple, direct non-arbitragable approach to mitigating uncertainty effects. As yet however, the modelling approach to studying such curbs remains stubbornly unsituated and risk-based.

### 2.4.4. Uncertainty Mitigation & Policy Design

Beyond phenomenological attempts to develop models to replicate time series properties, a pragmatic starting point to addressing uncertainty exposures is to first consider how the effects of uncertainty can be mitigated. Given that at some level economic systems are typically driven by competition over constrained supply or demand, with survival and resilience over extended periods as key performance measures, then one would reasonably expect some such structures to already exist amongst successful entities and economic systems. Examining these and incorporating them in situated models offers a potential domain knowledge driven route both to practical, useful models and to better of understanding system behaviours & uncertainty adaptations.

Current and historic regulatory and governance controls offer many potential examples of high level uncertainty management structures, as do internal risk management functions in financial institutions and treasuries. There are examples of active and passive structures which appear to limit the effects of uncertainty on institutions and individuals either directly or indirectly.

- Regulation
  - Basel III, international banking accords

- Regulatory bodies & regulatory sanction
- Regulatory reporting requirements
  - \* Market exposures & stress tests
  - \* Counterparty credit limits
- Regulatory Capital Limits
  - \* Passive, active & pro-cyclical
- Legal business segregation & certification
  - \* Glass-Steagall Act, Dodd-Frank
  - \* Bank licensing
- Market Driven
  - Internal risk limits & business segregation
    - \* Internal & external risk models, e.g. Value at Risk (VaR)
  - Heuristic active controls, e.g. P&L stop-loss & trading curbs
  - Large sample data collation & analysis
  - Counterparty exposure limits, credit control
  - Specialisation by expertise
  - Consolidation & aggregation of uncertain exposures, e.g. insurance
- Legal
  - Limited liability corporate structures
  - Limited partnership structures
  - Governance, e.g. board structures
  - Insolvency legislation & enforcement

Knight identified a number of such structures in his work. More recently, particularly since the GFC, regulators and policy makers have focused on reviewing existing regulatory structures; policy decisions; and interventions in terms of their efficacy and market outcomes. The Basel II & III banking accords have come under particular scrutiny, given evident failures of local and international regulatory controls to limit market instability and contagion. Economist Andrew Haldane<sup>12</sup> has been particularly critical of current regulatory responses to the GFC, presenting analyses of complexity issues and simulations testing various modelling approaches[53]. These analyses appear to provide evidence that increased complexity in a system contributes to uncertainty and potential instability - a conclusion which intuitively fits with observations of non-linear system dynamics in such systems.

Haldane argues that current frameworks attempt to deal with uncertainty by applying, in Knight's terms, risk-based models, and that this approach can be seen to

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<sup>12</sup>

Executive Director of the Bank of England's Financial Stability and Financial Policy Committee.

be both costly and ineffective when compared to simpler heuristic models. Haldane presents evidence showing that simpler models produce better market exposure forecasts than current approaches in many circumstances and timeframes. In a return on equity, performance-driven business culture, allowing reporting institutions to produce and manage their exposure based on quantitative risk calculations encourages regulatory optimisation. Managers target maximally efficient balance sheet usage, rather than prudent, resilient performance over time. Similar behaviours are seen in corporations as they seek to maximise return on equity using highly leveraged balance sheets and lean manufacturing processes. Manufacturing itself is less exposed to uncertainty as directly controlled processes can be properly mapped and analysed in terms of risk, however there are clearly uncertain elements at the interface between companies and their funding needs as was seen again in the GFC and subsequently.

### 2.4.5. Policy, Uncertainty & Heuristics

Haldane's work follows research from cognitive psychology on fast and frugal heuristics as potentially effective alternatives to computationally intensive, costly neoclassical frameworks for decision making under uncertainty. Gerd Gigerenzer's group<sup>13</sup> has proposed a key role for such heuristics in policy making and implementation[99]. Much of Gigerenzer's research concerns heuristics as processes for expressing individual preferences under uncertainty and bounded rationality (see [49] for a comprehensive introduction). However, as with the preference modifying structure examples above, simple heuristic approaches are attractive as uncertainty mitigating mechanisms. The case they make is that heuristic rules are not necessarily just simple, rough rules of thumb, but when adapted to noisy, uncertain environments with sparse information simple heuristics can outperform sophisticated, costly and slower quantitative, algorithmic approaches. More significantly, applied collectively as core preferences and preference modifiers over extended periods the resultant behaviours and performance are likely to be more resilient than optimised, efficient models.

The attractions of heuristic risk and uncertainty management rules are twofold.

*Computational cost and speed.* Quantitative, risk-based responses to economic instability appear to have led to a quasi-exponential growth in regulatory frameworks - as an example the original Basel banking accord in 1988, Basel I, was a 30 page document, Basel II in 2004 was 347 pages, while Basel III as a revised framework in 2010 is 616 pages long. Haldane[52, 53] emphasises that this is only the tip of the iceberg in terms of increased complexity - the number of parameters to be estimated for a large bank's regulatory capital requirement is in the order of tens of thousands, while for VaR models there will be several thousand risk factors and covariance matrices requiring several million individual risk parameter estimates. These models are data hungry, requiring very large data sets to develop reasonable forecasts, so large, according to Haldane's comparative research, as to be impractical. In his

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The Centre for Adaptive Behaviour & Cognition (ABC) Max Plank Institute for Human Development, Berlin.

studies of market loss forecasting, complex models required data sets for over 300 years in some instances before they could outperform simpler models. Similar outcomes have been found when comparing simple heuristic portfolio allocation rules to Markovitz's more complex mean-variance portfolio allocation models[36, 63]. Heuristic approaches are, by definition, low cost in terms of computational requirements and cost to implement and supervise.

*Transparency.* With massively complex models comes opacity - the regulatory infrastructure itself becomes a complex system with non-linear outcomes. Day to day operations are vulnerable to unforeseen regulatory model impacts. More concerning, in competitive environments comes the potential and pressure for 'regulatory optimisation' and gaming the system.<sup>14</sup> Even more concerning, as evidenced by the number of rogue trader and fraud cases exposed since the GFC, criminal behaviours appear relatively easy to hide within large bank balance sheets and risk management models. Heuristic approaches, typically, will at least be more transparent in terms of understanding where regulatory breaches are occurring and why.

Overall Haldane proposes a move away from a strategy based around attempts to price risk, leading to optimisation and efficiency behaviours unsuited to uncertain exposures. Instead he suggests the use of what he describes as 'regulatory commandments' which constrain particular behaviours and businesses. Historically the separation of US banking into commercial and investment banking via the Glass-Steagall Act is a good example of this type of rule. Dating from 1933, it was structured to create a firewall between speculative securities activities and commercial banking vital to the economy in general. Simple heuristic commandments on minimal capital requirements fall into the same category, prescribed at levels which are not negotiable, or arbitragable using internal, quantitative risk models.

The use of simple heuristics as components of policy tools is attractive since they can also be used to proactively address particular observed behaviours[99]. Practitioners such as Taleb have also proposed examples of potential heuristic rules which may have a normative function in shaping both individual and institutional behaviour[139]. In Europe post-GFC a number of restrictions on bonus incentives have been legislated which capture some elements of this.

However some market commentators have noted that, somewhat paradoxically, it may be challenging to get financial institutions to accept a simpler, heuristic driven regulatory framework. Felix Salmon[113] writing for Reuters in 2012 suggested that although burdensome, the cost of satisfying complex regulatory requirements acts as a barrier to entry for new institutions and a justification for passing on high charges to their customers.

While the underlying practical arguments clearly support investigation of more simple policy and preference modifying regulatory structures, some of the evidence requires further critical examination. Haldane's case studies may simply demonstrate de-tuning over-optimised models and should be treated as basis for further research rather than definitive proof of heuristic superiority. Similarly Gigerenzer's

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This has been reported as a significant cause for enormous derivative losses from CDOs (collateralised debt obligations), where a large part of their structuring and (initial) profitability was a form of rating agency & regulatory arbitrage.

work takes a typically partisan, somewhat evangelical academic approach, seeming to advocate simple heuristics as the *only* rational solution to uncertainty and boundedly rational behaviours.

In contrast in this research, and in the design of the experimental architecture, neither algorithmic nor heuristic structures are seen as sufficient in themselves, particularly as new black box, machine learning based funds come to make up larger and larger portions of daily trading volumes in financial markets.

It seems reasonable for regulatory and risk management models to combine elements of both schools of thought to produce more workable, robust and resilient frameworks going forward. Indeed, as discussed in the introduction, if current frameworks and market structures are examined they already combine both heuristic and algorithmic preference and modifying structures, the difference being that till now there has been little systematic analysis of the component elements, their behaviours and their interactions. The role of simple heuristics in cognition and preference expression is discussed further in Section 2.6 with respect to behavioural finance, and in the implementation and development of case studies using the SHaaP architecture in this research.

## 2.5. Neoclassicism, The Efficient Markets Hypothesis & Efficiency

The remaining sections in this chapter provide a brief overview and critical discussion of various economic schools as a backdrop to the modelling approaches and case studies presented in the rest of the thesis.

Neoclassical models are generally formulated in terms of rational, utility maximizing representative agents where markets are in continuous equilibrium, and market prices fully reflect all available information, in which case they are taken to be *efficient*.<sup>15</sup> In these models non-rational behaviours are assumed to net out, so that the system can then be treated 'as-if' a single representative economic agent or group of representative agents were present. The supporting theoretical arguments for this framework originate from the rational expectations hypothesis (REH) [97], which provided the basis for modern portfolio theory [95] and the efficient market hypothesis (EMH) presented by Fama in 1970 and revisited in 1991 [42, 43].

The EMH is presented in three main forms varying according to the strength of the inferences made about underlying market processes,

- *Weak-form efficiency*. This states that it is not possible to make excess returns based on historic share prices or publicly available data. As such current share prices are the best, unbiased estimate of the fair value of a stock.

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As noted by Hayek[56] , Benink et al (hereafter Benink) [9] and elsewhere, this definition is rather arbitrary - a definition of a particular state without reference to the causes of the state. Despite this problem it has become the de facto standard, reflecting the dominance of neoclassical models.

Price movements are assumed to be caused only by the arrival of new information and, as such information is assumed to occur randomly, so stock price movements will be random. Thus *technical analysis*, popular as a trading tool among many market practitioners should not be able to allow profitable trading on average. Similarly, while some investors are famously successful, their presence is not of itself enough to reject efficiency - it is necessary to look at the whole population - Taleb [137] argues that survival bias may be the key explanation to the presence of market participants generating apparently consistent excess returns.

- *Semi-strong form efficiency*. Share prices adjust *instantaneously* to new publicly available information. This has the implication that *fundamental analysis* will also be unable to produce excess returns. Tests for the presence of semi-strong form efficiency involve analysis of market adjustment to newly available news - market movements should be instantaneous, gradual change would be taken to imply inefficiency and investor bias. Some studies support at least limited existence of semi-strong form efficiency.
- *Strong-form efficiency*. Stock prices efficiently reflect all information public or not, and no one can earn excess returns. In this case even insider trading cannot be seen to lead to excess returns in the long term.

A key element in the EMH is that security prices 'properly' reflect available information - weak or strong form depending on the degree of availability. A consequence of this is the effect that, if such information is properly reflected, then it will not be possible to generate excess returns systematically. While weak-form efficiency is generally taken to be present and is supported by a large body of work, even proponents of EMH acknowledge that strong form efficiency seems unlikely to hold in general[43]. In the 1980's and 90's some considerable pressure was put on even the weak-form of the EMH both by academics and practitioners, driven both theoretical debate and the profit motive.

MPT has been a core methodology in asserting a rational view of investment and economic systems, allowing mean-variance optimisation of expected returns over risk (in the form of variance of returns) for portfolios of risky assets. However this is at the cost, in common with the EMH, of a number of significant, challenging assumptions. These fall into 3 main areas: assets, investor behaviours, & market structure.

- Assets
  - asset returns are assumed to be normally distributed random variables
  - return distributions are assumed to be known in advance
  - correlations between assets are assumed to be constant
  - past history is assumed to be a fair basis for forming these expectations
- Investors
  - investors are unboundedly rational, risk-averse utility maximisers
  - investors can all accurately infer asset distributions
  - investors are unconstrained by economic limits such as credit availability

- Market structure
  - markets are infinitely liquid, i.e. all transactions can be completed at any time
  - markets are normally assumed to be frictionless, i.e. there are no transaction costs or taxes
  - all information is available to all investors at the same time

### Tests & Criticisms of MPT and the EMH

Critical discussion of MPT and the EMH takes two broad forms: empirical tests of model assumptions & predictions (specifically efficiency and statistical properties), and demonstrations from behavioural finance of persistent bounded rationality at the level of market participants. In presenting a model of markets and investor behaviour reliant on unboundedly rational representative agents and assumed forms of underlying asset distributions, neoclassical economists fail to address observed market micro-structure, particularly relatively infrequent, disruptive behaviours exhibited in market-mania and crashes.

The problem with such 'infrequent' events is that they include the most damaging or risky elements observed in financial markets. They are also not infrequent as the models predict. As pointed out by Mandelbrot[94] and Sornette[132] amongst others, the frequency and structure of these phenomena simply do not fit inside a rational economic framework. MPT effectively allows risk to be characterised as a probabilistic function of expected values equivalent to lotteries where all outcomes are prescribed - this highly abstracted view of market risk which effectively ignores uncertainty as discussed earlier in this chapter. Mandelbrot[94] argues the case for a multi-fractal model of market behaviours, but while some recent work has produced interesting models of market volatility based on this class of model [90] these remain phenomenological in essence.<sup>16</sup> Alternative approaches are required.

Empirical evidence for and against models such as MPT and the EMH comes from statistical tests for and observation of characteristic time series properties which might be predicted or explained by such models. Common feature sets reported include the degree to which time series are serially auto-correlated (and therefore the degree to which future price movements are predictable based on historic data), auto-regressive conditional heteroskedasticity (ARCH - excess or persistent volatility and volatility clustering), excess kurtosis (fat-tailed distributions) and phenomena associated with so-called manias (bubbles and crashes).<sup>17</sup> Typically these features are discussed in terms of stationarity: time series of market returns are generally

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Koehl [75] uses this description in the context of ecological, A-Life models of macroscopic system properties and it seems appropriate here too. Neoclassical models are typically constructed considering analysis of time series properties and differential equations of varying complexity and fractal models using 'fractal generators': they share the feature that they aim to replicate the phenomenology of the time series modelled. Koehl points out that this has the twin implications that it assumes the modelled parameters do not change and it does not allow for the emergence of new parameters, or behaviours, as relevant in a given model.

<sup>17</sup>

observed to be stationary, supporting weak-form efficiency, however the volatility (variance) of these returns for a time series generally is not found to be stationary, with evidence of volatility clustering and persistence present.

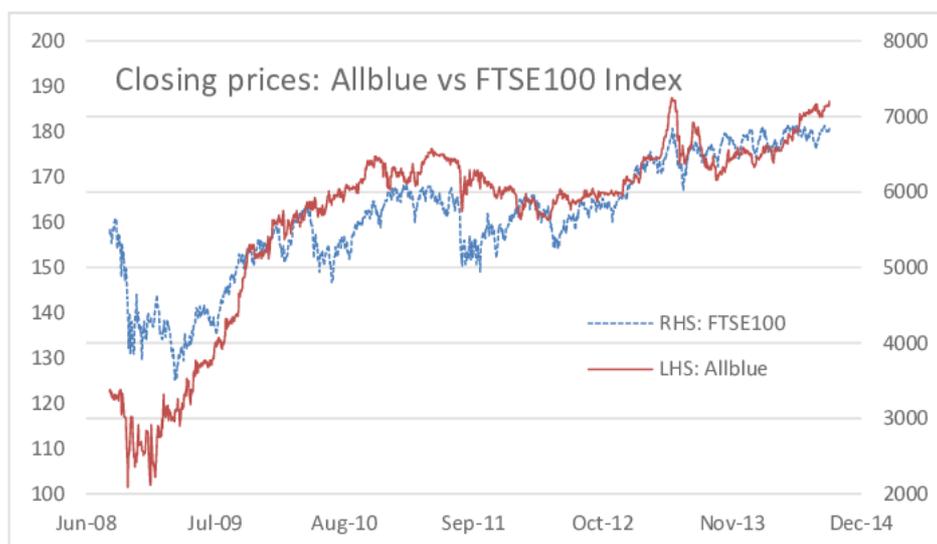
Cheap computing power and abundant sources of real time, high quality market data have allowed extensive empirical analysis of financial asset time series - both in terms of statistical analysis of time series for regularities and on the use of technical trading strategies, i.e. rules which use only historic information to forecast future price movements or make trading decisions. As well as stocks and stock indices, commodity and foreign exchange rates have been examined. A useful overview of this general body of work can be found in papers by Brock et al [17], Boswijk et al [14], De Long et al [35] and Fama [41]. Such studies identify evidence of regularities in stock price behaviour, variously describing significant serial positive or negative correlation of returns at different investment horizons for different stocks, indices and portfolios. However it also became apparent that significant care was needed to avoid data-snooping issues in testing trading strategies. In a later paper considering these findings, Fama [44] is heavily critical of the validity and interpretation of the empirical evidence amassed from these studies cited in support of behavioural economics as an area for research.

One of the more interesting findings comes from a set of papers examining simple technical trading rule performance when applied to daily data of the Dow Jones Industrial Average (DJIA) over various periods. In 1992 Brock et al [17] presented a study of 26 simple technical trading rules applied to daily DJIA data for the period from 1897-1986. Their results were inconsistent with EMH predictions that it should not be possible to derive excess profits from such rules. They showed greater returns and lower volatility of return on buy signals than sells. However re-running the analysis for the period from 1988 to 1999 LeBaron [79] found the return performance much diminished, although a volatility differential remained.

In a separate paper Sullivan et al [136] highlighted potential problems with data-snooping in searching for successful trading strategies and in choice of studies for analysis which may result in survival bias. In their paper they re-run Brock et al's earlier study correcting for data snooping and considering the period from 1897-1986 and then the period from 1987 to 1997. Their results again supported Brock et al's findings for the period to 1986, i.e. that there is no evidence of data snooping. However for the second period, after correcting for data snooping they found no significant outperformance by the trading rules. Further they reported that, for a much larger rule set of nearly 8,000 rules, their findings are the same across the periods. While specific interpretation of the causality behind this would be purely speculative, it does not seem overly surprising in an adaptive, competitive world and might be taken to give some insight into the underlying dynamics which can be seen to drive market behaviours. This fits with the practical reality and experience of trading markets and may be apparent in the published performance of some algorithmic hedge funds, where initial high returns diminish over time, possibly as increasing numbers of funds attempt to exploit the same inefficiencies.

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A good overview of general properties and anomalies commonly observed can be found in Siegel's book [122] while more detailed discussion of the statistical properties of time series can be found in Fama's papers [43, 44] and many others.

**Figure 2.1.:** Allblue vs. FTSE100 Prices. (Source: London Stock Exchange)

As a somewhat anecdotal example, since without access to detailed trading records there may be other explanations, Figure 2.1 shows the historic price of Bluecrest Capital Management's (Bluecrest) algorithmic fund of funds, Allblue, vs the FTSE100 stock index. Bluecrest is one of the largest so called 'black box' hedge funds in the world managing over US\$30 billion in assets. Figure 2.2 shows the exponential moving average (ema) annualised risk-adjusted returns<sup>18</sup> for the same time series over the same period. From the charts it can be seen that although risk-adjusted returns through the GFC were consistently and significantly better than stock market returns, recently there has been significant convergence. In absolute terms, Allblue has ceased to outperform and on a risk-adjusted return basis has moved from strong outperformance to relative under-performance.<sup>19</sup>

The presence of excess volatility and its auto-regressive properties forms a key focus in debate over the EMH, behavioural economics and ACE. Generalized versions of the EMH appear to underestimate expected volatility while giving no basis to explain processes underlying it or non-stationary behaviours [120]. Much recent work in

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See Chapter 5 and Appendix A.2 for discussion and definitions of risk-adjusted return measures.

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It should be noted that the Allblue fund is a 'fund of funds' made up of investments in Bluecrest's individual, specialist funds, which trade not only equity indices but also interest rates, commodities and other international markets. This means direct comparison to FTSE100 returns is not strictly relevant except insofar as the pattern of changes in relative return over the same period of market disruption during and after the GFC.

**Figure 2.2.:** Allblue vs. FTSE100 Returns

computational economics and in multi-fractal studies (see [85, 94, 90, 30] among others) has concentrated on demonstrating that it is possible to create artificial markets exhibiting similar volatility and time series characteristics to real markets and citing these findings in attempts to explain underlying causality. This remains a tenuous logical argument as, particularly in complex systems, many structures could mimic statistical time series properties without having any relation to actual behaviours in real systems, so it is difficult to see what explanatory value such models provide without other sources of validation.

## 2.6. Behavioural Finance, Heuristics & Adaptive Markets

Evidence of cognitive biases and bounded rationality in individual reasoning has provided a second main source of criticism for neoclassical models. Work in this area has formed the basis of behavioural finance as a new economic school. This approach has significant crossover with computational economics, indeed in many examples of agent-based models it is difficult to clearly categorize them as behavioural finance, neo-Austrian or ACE except perhaps by the publication they appear in.

### 2.6.1. Procedural, Ecological & Bounded Rationality.

Reviewing definitions of bounded rationality in behavioural research, Gigerenzer [49] noted two important elements,

1. Bounded rationality is not 'optimisation under constraints'.
2. In Simon's original discussion 'bounded rationality' was not presented as an isolated concept describing cognitive constraints. Rather, it was intimately associated with the viability of the organisms in particular environments and the level of rationality required to be competitively successful - in this form described as '*ecological rationality*'.

In his 1986 paper, 'Rationality in Psychology and Economics' [129], Simon describes an essential difference between neoclassical economic models and alternate models of rationality. He terms the traditional view of rationality in economics as '*substantive rationality*' and that of other social sciences, '*procedural rationality*'.

In contrast to substantive rationality, Simon argues that procedural rationality is developed around the joint propositions that

- Agents (representative or otherwise) do not have unbounded computational ability
- Agents' knowledge of the world is severely limited

The result is that while a rational neoclassical agent makes decisions that are best given known preferences and full objective knowledge of the world, under procedural rationality an agent makes decisions based on what information is available while recognising limitations on computational ability. Thus procedural rationality expresses the concept of bounded rationality in the context of agents' knowledge of the world - basically agents make boundedly rational decisions with incomplete knowledge.

Under procedural rationality as well, decisions do not have to be optimal, rather they only need to be 'good enough'. This is linked to ecological rationality, where the cost involved in decision making means that under competitive constrained conditions, lower cost 'good enough' choices will enable agents to survive without optimisation, particularly under uncertainty where new information and agent decisions mean future market states are in constant flux.

Within finance and trading, this idea of ecological rationality is highly relevant. Where readily exploitable inefficiencies are present computationally expensive trading and investment strategies will generally be rejected in favour of simpler, successful behaviours. As markets evolve and inefficiencies become less easy to identify or exploit competition tends towards more sophisticated models and strategies. However these models are seldom present in isolation - they are typically subsumed by other risk management structures. Subsumptive structural efficiencies as identified by Gee[47] in natural systems are frequently seen in economic systems - concentrating specialisations and managing structures in larger entities. Subsumption of specialisations both of risk & uncertainty management appear to yield competitive advantage over time in the form of resilience and ecologically viable performance. These behaviours and structures can be seen as responses to uncertainty as described in Section 2.4.

Modern financial traded options markets are a good example of this evolution: practitioners having now taken the use of Black-Scholes option model to a commoditised state where individual traders support trading decisions using a readily available collection of highly sophisticated mathematical and computational systems. Few of these traders will ever work through or claim to fully understand the underlying mathematics involved, but are happy to trade, profitably or otherwise, using the models effectively as heuristic tools. Similarly, the dominant organisational structure in these markets is not for individual traders acting independently, rather the most common structure is for groups of traders to be subsumed into trading houses and banks, aggregating risk and operational management.

Examples of simplification in procedural and ecological rationality are also frequent. Pure broking & sales behaviours are present, particularly in fragmented markets, where intermediaries act as introducing agents for transactions. While taking no market risk and only limited, very brief counterparty risk brokers may take a fixed percentage by value of any transaction they mediate. Such behaviours provide very high quality risk-adjusted returns and require relatively little capital or expertise. As with option trading, economies of scale may mean broking and sales businesses are subsumed within larger operational structures. In terms of ecological rationality, with sufficient volumes of business, broking and sales are highly profitable and can provide significantly better risk-adjusted returns than more sophisticated investment management operations.

These examples demonstrate procedural and ecological rationality in action alongside uncertainty mitigation structures. They also support Gee's contention that evolution can produce simplification within subsumption architectures: option traders can specialise and trade only a single derivative product set without their own, independent complex operational structures; while brokers, no longer taking material risk beyond operational errors, act only as settlement and clearing facilitators and do not need sophisticated preference and risk management behaviours.

Some studies identify forms of procedural and bounded rationality, such as Arthur et al [3] who describe rationality in terms of inductive reasoning. However having allowed this uncertainty, agents in their framework are unboundedly rational in expressing their preferences, so they may simply be optimising under constraints. This is a persistent problem in academic models: there is frequently a confusion between fitting a mathematical model of preferences to observed behaviours and the decision processes expressing those preferences. As with the options trading example, the trading processes and the models informing them are typically quite distinct. The 'prospect theory' model described in the next section highlights this issue.

### **2.6.2. Cognitive Psychology & Behavioural Finance**

Evidence from cognitive psychology has demonstrated that even in simple experimental situations, individuals are apparently unable or unwilling to act as if fully rational. This provided a strong impetus to question the underlying assumptions of full economic rationality key to the REH and EMH, generating considerable research interest (Barberis & Thaler [7] provide a good overview of work in this area). This sits well with models of inductive reasoning in behavioural finance and explanations

invoking widespread use of heuristics in decision processes. Gigerenzer [49] provides evidence not only that simple heuristics are used by people in at least some circumstances, but that they can be highly successful as reasoning mechanisms, fitting well with the concepts of bounded and ecological rationality presented by Simon [126, 127]. This is not limited to individual preferences but may also be applied to economic interactions between individuals and other economic entities - agent-based models are ideally suited to exploring such models in a laboratory environment.

Seminal work on individual preferences by Tversky and Kahneman in the late 70's and early 90's provided strong evidence supporting concepts of bounded rationality. Tversky & Kahneman's work has most famously formed the basis for 'prospect theory' [144], a consolidated and coherent attempt to map and describe observed individual preferences - an approach somewhat at odds with the elegance of classical models such as von Neumann-Morgenstern (vNM) expected utility theory which assume rationality and continuity. Yet in the absence of sufficiently strong models to show that in large populations the behaviour of individuals cannot still be treated 'as if' they are rational as a group, i.e. that the group or groups can be replaced by representative agents, such evidence has been frequently dismissed by neoclassical researchers.

However, models simply mapping the shape of a decision process, while useful, do not in themselves give an explanation. Prospect theory models are just as vulnerable to this criticism as vNM models: their popularity may be explained much in the same way as vNM expected utility - they are easy to apply using fitted functions modelling observed preferences. In both cases when included in AFM design there is the ongoing exposure to a homuncular fallacy, with an unboundedly rational homunculus encapsulated within agents to represent their preferences.

As an illustration Table 2.1 shows a probabilistic prospect theory model proposed by Tversky & Kahneman.

Taken together these are not simple sets of formulae. If people then must maximize utility across *these*, we do not seem to have progressed far and with little true added explanatory gains. The cognitive challenge is arguably greater under these prospect theory type preferences than maximizing 'simple' expected utility functions. The level of rationality required, if not unbounded, is certainly considerable and conflicts with observations of actual ability in individuals. Of course if the model were tested in a situated environment as proposed in Chapter 3 and performed well then this would be an interesting result potentially worth pursuing.

This remains an area of active debate, Berg & Gigerenzer [11] present a highly critical review of the problem as it applies to behavioural finance (and at the same time raising many methodological concerns). They make the case that much research in the area is really 'neoclassical economics in disguise'. Although the core preference functions may have changed, economic agents are still presented as utility maximisers, vulnerable to the same criticisms as neoclassical preferences. In this way prospect theory is an attempt to fit a complex utility function to observed behaviours, adding extra parameters as necessary to improve fit. Although individual behaviours are more accurately mapped, the underlying processes are not addressed and the models add little explanatory value.

Berg & Gigerenzer's [11] contention is that after an initial period challenging neoclas-

Facing a gamble giving outcome  $x_i$  with probability  $p_i$ , with multiple outcomes people assign it a value,

$$\sum_i \pi_i \nu x_i$$

where

$$\begin{aligned} \nu &= x^\alpha \quad \text{if } x \geq 0 \\ &= -\lambda(-x)^\alpha \quad \text{if } x < 0 \end{aligned}$$

and

$$\begin{aligned} \pi_i &= w(P_i) - w(P_i^*), \\ w(P) &= \frac{P^\gamma}{(P^\gamma + (1 - P)^\gamma)^{\frac{1}{\gamma}}}. \end{aligned}$$

Where  $P_i$  is the probability that the gamble will give an outcome at least as good as (and  $P_i$  strictly better than)  $x_i$ ,  $\lambda$  is the coefficient of loss aversion,  $\alpha$  and  $\gamma$  are constants.

**Table 2.1.:** A Probabilistic Prospect Theory Model - Tversky & Kahneman (1992).

sical norms, behavioural finance has moved to accepting many of the same incorrect modelling assumptions while abandoning psychological realism. In models like the SFASM<sup>20</sup> it certainly appears to be the case that the supposedly boundedly rational economic agents are actually unboundedly rational neoclassical utility maximisers action under constraint. Berg & Gigerenzer describe this as 'as-if' behavioural economics.

When the rational representative agent structure of neoclassical models is dropped, applying neoclassical utility maximisation assumptions becomes especially difficult to accept at an individual level. The standard choice axiom of commensurability is a particularly challenging issue in that it requires that for any set of goods or features there are no heuristic cut-offs to limit choice, rather all must be considered. At its most extreme it implies there is always a calculable value for any good to replace another in terms of utility. Berg & Gigerenzer's reductio ad absurdum example highlights this is to take orthogonal, though still utility based choices, of hugs vs ice-cream - according to commensurability there must be a calculable number of hugs which provide the same amount of satisfaction as an ice-cream. Even if this were so the utility calculations required would likely once again be extreme.

<sup>20</sup>See Chapters 3 & 5 for a detailed description & examination of the Santa Fe Artificial Stock Market.

### Fast & Frugal Heuristics

Fast and frugal heuristics, such as lexicographic choice rules, offer better candidates for capturing individual preference behaviours. By reducing choice sets significantly a lexicographic heuristic allows rapid decisions between dissimilar goods under uncertainty which appear to be better than slower, more computationally intensive processes[149].

Gigerenzer[49] provides a number of examples of fast and frugal heuristics in operation which appear to be highly effective in uncertain, information constrained environments and appear to fit well with observed behaviours. Rather than retaining neoclassical optimisation and utility maximisation frameworks, examination of realistic psychological and behavioural processes of choice provides a better, bottom-up approach to behavioural finance with the potential for real explanatory value. Increasingly it appears that it is a misconception to treat cognitive biases simply as errors given that they can outperform rational choice models - examples of this include exploratory inconsistency heuristics[11] and naive portfolio diversification rules[36, 99].

Superficially simple heuristic choice may not only be competitive in terms of computational cost, but under uncertainty may be better adapted than neoclassical, and 'as-if' neoclassical behavioural models. Recognition heuristics; lexicographic choice; saving heuristics; naive portfolio diversification rules; stop-loss; satisficing; and even inconsistent exploratory heuristics may all be adaptations to particular market structures and environments where optimised, rational choice models behave poorly.

**Hybrid Approaches** Although fast and frugal heuristics appear to represent an important, potentially key, component to developing bottom-up models with real explanatory value, it seems unlikely that they will be sufficient in themselves. Many are documented in isolation and identification does not necessarily help with understanding which environments they will or will not work in - i.e. which they are adapted to and whether they need preference modifying structures to function over time. It would be foolish in any case to ignore optimisable, rational algorithmic models where they are appropriate - performing option pricing and hedging without such tools would be *irrational* rather than just boundedly rational. The discussion of risk and uncertainty in Section 2.4.1 supports this. Some combination of models and approaches which captures the strengths of each in a larger architecture is likely to be necessary and beneficial.

An interesting approach to this is Lo's 'Adaptive Markets Hypothesis' (AMH) [88]. This has the merit of recognizing that elements of different schools may coexist and in fact be complementary. At the same time it allows the use of heuristic principles and bounded rationality in expressing preferences. Refreshingly Lo identifies *survival* as a key metric for success and presents the view that the survival of agents or particular trading strategies in an evolving heterogeneous population may be a useful representation of a market.

The AMH allows for the use of heuristic reasoning processes and preference expression - directly supporting both bounded and ecological rationality amongst agents

and strategies. The general direction of this approach fits with the SHaaP architecture Chapter 4, with the caveat that as presented the AMH appears restricted to discussion, lacking practical detail. Incorporation of domain knowledge from situated environments, an experimental framework and appropriate fitness & performance measures are all important elements to deriving operationally meaningful models addressed in the proposed work.

**Situation & Environment Interactions** Shleifer & Vishny provide a powerful example of the importance of considering not only models of preferences but interacting effects with the environment. The EMH requires and begins with the assumption that participants in the market behave as if they are fully rational and have perfect knowledge of all other participants' preferences. In their paper on 'the limits of arbitrage' [121] Shleifer & Vishny present a case for traders operating in an environment *without* perfect knowledge of other participants' preferences, the quality of the information present in the market, or whether it is fully reflected in stock prices - i.e. traders must make decisions in the face of uncertainty. Resources, principally capital to trade, are constrained.

In this system of 'performance-based arbitrage' (PBA) traders gain or lose capital based on performance. Faced with uncertainty and conditional upon past performance a given trader may reject a potential arbitrage opportunity as not being attractive enough, where a fully rational agent would trade. LeBaron refers to such preferences as 'intertemporal' and suggests the added complexity by including them is 'significant' and many rich AFMs avoid it as a problem by reducing the treatment of preferences to abstracted, myopic structures - this is a dramatic simplification and potentially crucial issue in constructing viable market models.

Benink [9] describes the idea of 'imperfect knowledge' as a key characteristic of Neo-Austrian thinking, fitting well with Herbert Simon's definition of 'procedural rationality'. In this context Friedman's self-correcting world [46], where rational traders (arbitrageurs) undo market dislocation caused by less than rational traders (noise traders), can be seen to be over-simplistic: not only does it presuppose a knowable 'fundamental value', it also requires that it be commonly accepted by rational traders. Instead, and more plausibly, Shleifer & Vishny allow the market to consist of heterogeneous traders where putative 'corrective' forces of arbitrageurs do not necessarily operate consistently or even all the time.<sup>21</sup>

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'Arbitrage' opportunities in the restrictive sense employed in financial markets does exist periodically. The common strict functional definition is to simultaneously buy and sell the same asset at different prices to generate a profit. Such arbitrage is relatively rare and generally requires some form of spatial separation to exist caused by the market structure (e.g. on different stock exchanges, or in chaotic markets with fragmented trading networks) or synthetic structures recreating underlying assets, such as swap futures trading. However even with real time data feeds and electronic trade execution, profitable opportunities in well understood, readily trackable arbitrage trades still occur regularly in today's markets. This can be explained by Friedman's logic insofar as the irrationality comes from traders with other motives (preferences), investment horizons or different investment opportunities, so that the arbitrage trade itself is not necessarily important enough to isolate, leaving dedicated arbitrageurs or market repricing to eliminate it, so that it does not conflict with Shleifer & Vishny's paradigm either. The key difference with Friedman's view though is that the arbitrage is relative - the buying and simultaneous selling of an asset. It does not require some known, absolutely understood

Constraints and structures *modifying* individual traders' core preferences, as in performance based arbitrage, need to be considered. It seems unlikely, recalling Simon's distinction between economics and social sciences, that preferences can be usefully investigated by attempts at 'simply' modelling choices - processes must also be considered. The practical issue of preferences in black-box trading model design is a useful illustration of a process based approach, highlighting the kind of structural elements necessary to allow agents to express their preferences (Appendix A.1 provides a detailed example of a trading model). As the models discussed in Section 3.3.4 show, equations expressing core preferences are not enough on their own.

In real world situations it is difficult to avoid or ignore PBA-type effects. These are obvious and much discussed in post-GFC regulatory fora, both in terms of their negative outcomes and normative attempts to explore preference modifying structures which might mitigate them. Shleifer & Vishny's performance related changes in behaviour can be seen in excessive risk taking by individuals and groups under stress, or where bonus incentives & fee structures are asymmetric. Hedge fund and proprietary trading groups provide many examples, Enron and LTCM (see earlier) being isolated but significant cases. Limited liability structures provide important protections for businesses to facilitate and encourage free trade, but when under stress the temptation to increase risk or trade while insolvent can be high given a downside constrained by limited legal recourse. The same temptation exists notably for investment bankers and traders where the limit of their exposure is their employment. Taleb & Sandis's [139] 'skin in the game' heuristic provides an example of proposals to counter such biases.

Directly attempting to model both market structure and trader preference structure is very challenging given the rapid increase in complexity as new features are added. However it makes the case for beginning with situated models as a mode of validation more compelling, working back to isolated systems exploring aspects of agent behaviours identified in this way. Working on stylized isolated systems without this form of validation seems unlikely to produce meaningful results in any reasonable time.

## 2.7. ACE & neo-Austrian Models

ACE research includes work from many different economic schools including neo-classical economists, effectively functioning as an experimental framework. Extensive reviews of the general field, current research directions and issues have been provided by practitioners - see Tesfatsion 2002 [142], LeBaron 2005 [84] and Axelrod & Tesfatsion[5] for good examples<sup>22</sup>. Typically ACE systems are modelled as

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fundamental value. As such the trading profit will persist beyond new market forces which change this 'fundamental' - in market parlance it is 'hedged'.

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It is worth making a distinction between AFM design and automated market design (AMD). Although similar technologies may be employed with software agents playing a central role and although there is considerable overlap of research interests, the emphasis is quite different. Typically AMD research is concerned with efficient market clearing and automated negotiation rather than market behaviours and asset pricing. Although automated agents may be used to

interacting groups of economic agents obeying sets of rules and behaviours defined within the experimental framework providing a controlled environment. Simulations can be run and re-run without the problems or limitations imposed by attempting to work with human subjects.

Recent work on bottom-up models including aspects of Hayek's design [57] has grown rapidly, forming the basis for a neo-Austrian school of economics [8, 146]. A neo-Austrian viewpoint allows the existence of inefficient markets and imperfect knowledge. It also allows the possibility that even if agents are able to generate fully rational behaviours the result will not necessarily imply rational expectations [9]. From a practitioner's and an ACE perspective this sits much more intuitively than neoclassical, unboundedly rational representative agents operating in informationally efficient markets.

Neo-Austrian models may appear to map almost directly to ACE, however they reflect the tenets of an economic school rather than a methodological platform. Neoclassical economists may also use agent-based models as experimental platforms, as may researchers espousing no particular school of economic thinking. For the most part, whatever the school, ACE models are implemented to explore only academic constructs rather than in empirical tests of behaviour in actual market environments.

A common approach in the ACE literature is to build economic systems which are analytically tractable in terms of rational equilibria and to observe the behaviour of individual agents or groups of agents in these environments. Agent behaviour in the context of these equilibria is taken as a key element in validating experimental objectives - specifically where the behaviours are directly compared to human subjects operating in the same economic environment, as in the case of Gode & Sunder's studies of simple auction systems with zero intelligence (ZI) agents [50], or the statistical properties of aggregate behaviours are compared to those of real systems, e.g. in some artificial stock markets such as the Santa Fe Artificial Stock Market (SFASM) [3, 85]. Aside from criticisms specific to particular studies, a constant underlying problem is validation: of assumptions in experimental design; of the computational framework; and interpretation of experimental findings.

'Rich' AFMs, such as the Santa Fe Artificial Stock Market (SFASM) [85, 102] and Chen & Yeh's genetic programming based model [30], have been used to examine underlying agent behaviours in direct comparison with real markets - in fact their design is oriented to produce artificial time series specifically for comparison. The same statistical tools and measures are used to examine the time series produced as are used to examine time series from actual markets. However agents in these

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effect transactions in both AFMs and automated markets they are not necessarily the same - typically agents in automated markets do not 'own' the consequences of their trades, they make the 'most efficient' trade at the time but their remit is restricted to specific trades (this does not preclude learning algorithms and evolution however) - for AFM agents the area of interest is not just specific trades but the behaviour over time.

Makie-Mason & Wellman[93] give a comprehensive review of current issues in automated markets and trading agents, advocating the development of a structured approach to market design. They emphasize the importance of implementation details, problem complexity and context in model design. Even designing automated markets, with the luxury of being able to begin with a blank paper and (reasonably) readily definable objectives analytic theory quickly becomes infeasible as problem complexity grows.

models are cast as unboundedly rational utility maximizers within the constraints of the system, e.g. in the SFASM agents use an evolutionary algorithm to optimize price forecasts, operating consistently in all market conditions but with no knowledge of other agents' forecasts - however, as Gigerenzer [49] is careful to point out, optimisation under constraints is *not* bounded rationality.

Conclusions drawn in such studies rely heavily on this statistical evidence as validation. Although clearly it may be interesting if an AFM generates similar time series to actual markets, there is no obvious direct link between statistical measures at a system level and behaviours at an agent level, so interpretations of agent behaviours based on such measures are tenuous at best.

ACE methodology and infrastructure issues, particularly with rich AFMs, are discussed in detail in Chapter 3, however the use of time series characteristics in supporting inferences about market processes faces a more fundamental theoretical problem. In his original papers [42, 43], Fama identifies the 'joint hypothesis problem': the efficiency of a market is determined only within the context of a given asset pricing model. The use of time series artefacts in systems designed to allow neoclassical efficient equilibria also faces this problem.

In order to avoid the joint hypothesis problem Benink[9] proposes a model independent measure of efficiency using relative returns within and between agent populations. The measure derived is very similar to risk-adjusted measures used in actual trading environments and has the important feature that it is directly derived from agent-level behaviours not the population as a whole. This measure and other situated-measures are discussed and developed in detail Chapter 5.

## 2.8. Summary

This chapter presented a critical analysis of extant economic schools, particularly focusing on weaknesses in experimental methodologies when considered with empirical evidence from actual markets and case studies, providing economic context for the research approach in the rest of the thesis. This is very much a pragmatic, practitioner's viewpoint, separating it from more abstracted academic approaches: here some loss of academic rigour is seen as more than outweighed by gains in operationally meaningful, validated results.

Neoclassical economic models have been increasingly challenged by alternate schools using computational experimental economic models to support this challenge. The global financial crisis has accelerated this process as failings of traditional risk-based approaches to modelling economic systems have been exposed. Such risk-based models fail to deal with uncertainty and non-linear behaviours adequately. However, although alternate models may have better explanatory value, modelling uncertainty effects rather than risk remains a relatively new approach. ACE models provide an avenue for addressing this and current methodologies are discussed in Chapter 3.

Recent work on simple heuristics and policy design described in the chapter suggests a significant role for such heuristics in risk and uncertainty management. With as yet little work in this area reported in the literature, this is an interesting research

topic and is addressed in the experiments in Chapter 6. These experiments use the new SHaaP architecture and population relative performance measures developed in this thesis. This approach allows the use of simple heuristic preference modifying behaviours to be married with both heuristic and computationally sophisticated core preference structures and explored in situated models in a principled manner.

## 3. Agent-Based Computational Models & Economic Preferences

*'In economics, rationality is viewed in terms of the choices it produces; in other social sciences, it is viewed in terms of the processes it employs.'*

Simon 1986

### 3.1. Overview

This chapter discusses methodological issues in designing and using agent-based models (ABMs) of economic preference expression in a principled manner before considering current research in academic agent-based systems and artificial financial markets (AFMs), particularly 'rich' AFMs,<sup>1</sup> in this context. Rich AFMs and the methodologies currently applied to them are described as particular cases of economic systems, although the principles and methodologies are applicable to more general systems.

The approach here is very much that of a practitioner: at all times verification, validation, and replicability are critical concerns. The use of situation and appropriate agent-level fitness & performance measures are proposed as tools to improve these factors. Key choices and considerations in model and methodology design are discussed in some detail, presenting basic guidelines for approaching these issues particularly with respect to operational meaningfulness.

Real-world models and systems are discussed as important, situated starting points for developing viable models, given the complex adaptive nature of economic systems. Such models demonstrate the structural complexity of developing descriptions of even simple economic preference expression, and illustrate the presence of heuristic preference modifying structures. This sets the stage for the development of the SHaaP architecture presented in Chapter 4 - a situated, heuristics based approach to developing operationally meaningful models in a principled manner.

### 3.2. Experimental Platforms & Exploratory Paradigms

LeBaron [84] defines agent-based computational economics (ACE), or as he refers to it 'agent-based computational finance', in terms of its models,

'Models in the realm of agent-based computational finance view financial markets as interacting groups of learning, boundedly-rational agents.'

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<sup>1</sup>AFMs where agents have well specified preferences and operate over extended time periods.

and of its focus,

'In agent-based financial markets the *dynamics of heterogeneity*<sup>2</sup> is critical.'

By this he appears to mean examining the behaviours of individual agents, or small groups of agents within experimental systems. These views appear to be supported by the general thrust and discussion in current published research. However one would also expect it to be fully reflected in the methodologies adopted, which does not necessarily seem to be the case.

Observing the emergence of cheap, powerful computing resource, Tukey's[143] assertion that scientific research needs components which are both exploratory and confirmatory is an important starting point in developing any academic research paradigm, and has particular relevance to ACE research. In some behaviours at least, it appears that experimental economists have adopted Tukey's philosophy and attributed a significant portion of their research efforts to developing exploratory, descriptive models of economic systems adopting many heterogeneous methodologies. More than 30 years on this remains the case, though this seems to be more symptomatic of a lack of common standards, or without (as noted by both Binmore & Shaked[13] and Fagiolo et al[40]) an agreed approach to validating experimental results and methodologies. Lacking such standards it is difficult to readily compare different experimental observations, which is a real impediment to progress within the field overall.

In ACE the systems modelled are frequently combinatorially complex; demonstrate non-ergodicity; software platforms and environments are diverse; and, as a number of researchers, again including Binmore[13] and Fagiolo et al[40], have identified, models presented may be highly parameterised and poorly documented making them difficult or impossible to replicate. The temptation to 'video game' with models, i.e. to play with parameters and see what happens until an interesting result is reached, is high. Although Tukey spoke out for exploratory research, this presumed a continued emphasis on appropriate rigour in determining good research questions in conjunction with follow-up confirmatory experiments and analysis according to reasonable scientific method.

The lack of common methodologies or agreement on validation criteria may reflect a healthy state of flux be observed as a new paradigm is established, as Kuhn[76] proposed[40]. However, although this may be so, after such a protracted gestation period this seems to be a generous conclusion: a more parsimonious inference would be that it reflects the breadth of the problem domain and the ease with which models may be rapidly implemented, revised, and results generated.

Binmore & Shaked in particular have been highly critical of academic practice, identifying problems with replicability; cherry-picking findings; logical failings and over-fitting of models as significant issues. Although some of their arguments might also be indicative of 'normal' critical academic debate, their points are generally well made. In fact an additional criticism they put forward is that such debate is healthy and that there is simply not enough critical analysis in published research -

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<sup>2</sup>

My italics

an observation which review of the literature supports. In terms of models their key proposition is that prediction rather than fitting data should be the gold standard of modelling in experimental economics. While this seems basic, and rather obvious, it has been a concern discussed by practitioners for some time. Gent et al[48] produced their own practical guidelines based on their own experience for experimental protocols with computational models in 1997 highlighting many issues (and providing positive suggestions) with computational protocols. The evidence found in developing and testing performance measures in this thesis using the Santa Fe Artificial Stock Market (SFASM) as a benchmark, as reported in Chapter 5, reflects similar issues. A concerning finding in that work, in reviewing versions of the original SFASM programming code, was the presence of unreported or unexplained structural elements which affected model and agent behaviours and were unrelated to the theoretical model itself.

Equally concerning the premises used in validating ACE models are themselves open to question. A frequently used ACE validation criterion is to replicate stylised statistical artefacts observed in actual financial time series: this appears to be largely a fitting exercise with tenuous causal links to underlying research questions and derived conclusions. Binmore[13] argues that prediction should be the 'gold standard' for validation. This is in theory a clear and precise measure where it is relevant and can be adopted, but the question it raises is 'prediction of what?' and, having decided what, 'how then are those predictions evaluated?'. Throughout this thesis, operational meaningfulness, i.e. predictions and forecasts which have practical use in real world decision processes and provide explanatory value are presented as important component elements to validate experimental inferences and design.

### Scope & Methodologies

Studies of AFMs using agent-based models show great variety both in focus and in the methodologies chosen. Agents and simulations have ranged from deliberately simplistic, highly stylized constructions such as Gode & Sunder's so-called 'zero intelligence' (ZI) agents, which have no learning and zero or minimal rationality[50], to much more complex, 'rich' models like Chen & Yeh's genetic programming business school model [30] and the SFASM [85] incorporating inductive reasoning, agent-learning, evolutionary algorithms and (relatively) large numbers of autonomous, interacting agents.

The majority of studies are isolated from the real world and highly abstracted, using many simplifying assumptions: such systems have been termed *endogenous*[3]. All economic features, rules of behaviour, market structures and price setting mechanisms for agent interaction and learning can be and *must* be specified. All resulting behaviours and time series are generated internally.

Studies working with systems which are not isolated from the real world allow some or all design elements, such as market price setting, to be determined externally. For want of a better nomenclature, these systems will be referred to here as *exogenous*. Where agent survival or viability is wholly determined by its performance in a real world economic environment then not only can it be classified as exogenous but also as 'situated' in Brooks' terms [24, 21]. The two terms may not map directly to each other since situation requires positioning the model in a real environment, whereas

a model could be set up to be exogenous but still not be situated. In contrast endogenous models will always be unsituated.

The choice of endogenous vs. exogenous, and situated vs. unsituated is seen here as critical for any given model. Unsituated, endogenous models have the potential to be strong demonstrations of academic propositions since all elements are explicitly defined and controlled, however the burden of validation may prevent meaningful interpretations being achievable. Even where validation of a model is possible it may not be operationally meaningful - the model may simply not translate to the real world. In contrast, situated exogenous models begin with the practical validation step that they are developed and work (or don't) in the real world. The difficulty here is in isolating and exploring what aspects of such models are important in a principled manner - just 'making money' or surviving are probably not academically useful! Arguments and discussion of Brooks' situation and subsumption architecture in the context of AFMs are set out in detail in Chapter 4.

### 3.3. Methodology & Design Issues

There are three main areas for concern before considering interpretation of model findings:

- experimental & methodological standards, documentation & verification,
- validation, and
- model design choices.

Given the inherent complexity of many agent-based models it is easy to get lost within the design of experimental protocols and confuse (or accept) model results without critical analysis, not just of the protocols, but of the overall computational environment where confounding effects may come from problems with programming, in the form of bugs, or model modifications to establish a stable environment. It is necessary to maintain appropriate standards of verification and documentation of the overall computational environment as well as validation of protocols and experimental design.

#### 3.3.1. Standards, Documentation & Verification

Modern computational platforms make rapid construction of models and simulation runs extremely easy, however even apparently innocuous modelling decisions can have unforeseen implications for experimental observations - unsurprising given models which are, or emulate, complex adaptive systems. These issues have been discussed by Axelrod [4] and Chattoe [27] and are widely acknowledged yet remain problematic. Even deliberately rudimentary structures like Gode & Sunder's [50] minimal intelligence models have proven vulnerable to criticism [32] (see Section 3.4.1).

Taking practical experience from computational experiments for empirical study of algorithms Gent & Walsh[48]<sup>3</sup> presented a pragmatic (and heuristic) set of guidelines for building robust models and protocols, focusing not just on model validation but also on verification, programming issues and platform development. From a practitioner's viewpoint their rules remain highly relevant today and it is somewhat disappointing that in the experimental economic literature the issues they discussed in 1997 appear to have been ongoing since then and remain current today. The evidence presented in Chapter 5 from recreating a version of the SFASM, and subsequent work on it as a model, supports the need for such basic procedural guidelines.

Standards for model designs and protocols in ACE have yet to emerge, however basic elements should include not only descriptions of economic structures, but also documentation of underlying computational elements and parameters. In addition, discussion and description of calibration issues and model stability are important to supporting replicability and as context to interpretation of experimental findings and should be reported. Archival copies of source code versions matched to particular

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The ironic title 'How Not to Do It' speaks well of their approach and how hard earned such rules are.

experimental runs should be maintained. Unfortunately, it appears that such basic steps are often not carried out. The power of the modelling environment and the unpredictability of results makes it easy to 'video-game' with model structure - to tweak, re-run and modify model versions to 'see what happens'.

Despite relatively cheap computing power rich AFM models are still relatively expensive to run, and where many models rely on gross statistical properties for validation, independent replication is a thankless and rather difficult task. When is a model of someone else's model correct? Re-running original code (assuming there is some and assuming it works, and assuming you have the right parameter settings, and version) may give the same results but still often leaves detailed examination of parametrization and computational structures unexplored. Attempts at independent replication and validation seem, not surprisingly, rare. Binmore's[13] cry for more open critical discussion and replication of experimental findings as part of peer review is important, but challenging without sufficient documentation especially as experimenters must pursue their own research interests.

As Fagiolo et al[40] point out a lack of common, 'standard' models makes it difficult to compare experimental results and also to develop models in a systematic, principled manner. Before moving on to validation of experimental results, underlying platform and programming structures should be verified as robust. Beyond basic documentation, such practical verification is critical. In the course of developing the agent-level performance measures reported in Chapter 5 using the SFASM as a proxy for a standard model this unfortunately became extremely obvious: in the published literature; available model code; and supporting materials many elements were missing, incorrectly reported or inconsistent. Ultimately this process highlighted Binmore's criticisms of methodologies with no satisfactory replication of the original SFASM possible.

**Heuristic Guidelines.** Gent & Walsh emphasise strongly that errors in programming and model design are inevitable. Their approach, and the approach advocated here is to critically test models and model outputs. Aside from documentation (which should be a norm anyway), the procedures and data collection align closely with those one would expect for model validation.

- Collect & record all available data
- Do multiple runs with different seeds for random variables
- Do runs with stressed parameter settings
- Explore raw data for anomalies
- Use the same experimental problems or standard test cases across multiple experiments as calibration
- Run experiments on different platforms if possible

To an extent focus on computational elements and problems should be something of a side-issue, but until and unless it is dealt with systematically and to a high standard it cannot be ignored. Once model behaviours can be verified then, as long as the computational environment is consistent (and available), one can simply focus on the economic and functional structure of the agents and the AFM itself.

#### 3.3.2. Validation

Model validation is central to any experimental protocol, requiring justification of model structure, design choices, and inferences drawn from experimental results. The complexity of agent-based systems makes this validation challenging, particularly in the absence of common modelling standards in ACE. To some degree this challenge can be, and often has been, avoided in academic debate by using highly abstracted, unsituated, endogenous models which generate time series properties similar to real world time series and comparing these to neoclassical benchmarks. However, the logical case for this approach seems tenuous at best: if agent-behaviours are the ostensible focus of the research, some agent-level validation of these models is critical. Without this it is difficult to see how such findings can be related to actual markets, either in extending understanding of how markets and economic agents behave, or in practical aspects of risk management, financial forecasting or investment. That said, a successful, situated algorithmic trading system does not necessarily have explanatory power either - at least not without similar analysis of processes and behaviours.

#### Theory, Classifications & Tensions

Fagiolo et al[40] provide an excellent overview of the theoretical and practical issues involved in developing and validating computational models (as opposed to methodological issues, where they also present good critical discussion, in line with Section 3.3.1), illustrating the tensions involved. They highlight four main areas which should be considered in protocol design and validation,

1. Concretization vs. abstraction.

2. Instrumentalism vs. realism.
3. Pluralism vs. apriorism.
4. Underdetermination & identification problems.

1. *Concretization vs. abstraction.* Some level of abstraction is necessary given still limited computational resource and speed, and desirable if a goal of research is models with explanatory value. A fully concretized model - with a one-to-one mapping to the real world - is of limited use. Such a model in a complex adaptive system requires massive parameterisation and in any case presupposes that the researcher knows or can identify all the processes involved. A level of abstraction allows some processes to be isolated for experiment and, in the case of highly abstracted models like neoclassic systems, may still be analytically tractable.

2. *Instrumentalism vs. realism.* An instrumentalist approach is not concerned about whether model processes match those in the real world, rather in whether the model outputs and predictions are accurate. This is very much in keeping with real-world, algorithmic models where the key metric is risk-adjusted return and stability - explanatory value is at best secondary. Validation is simply model accuracy or success. At least to a limited extent it also fits neoclassical models using rational representative agents - there is no claim that these agents' processes actually exist. In contrast realist models require that the processes within the models relate directly to actual processes.

3. *Pluralism vs. apriorism.* Aprioristic models set out and explore a set of declared assumptions *a priori*. This fits with scientific method, but in its strong form apriorism does not allow modification of these assumptions to fit anomalies, so there is a risk of denial and rigidity in approach. Pluralistic models allow more than one explanation and different assumptions to explain observed facts.

4. *Underdetermination & identification problems.* More than one model, at least over some observed results, may give equivalent performance, so the problem is how to establish which is correct or a better representation of the processes involved. Of course a die-hard instrumentalist would not care, except in terms of cost, which leads back to ecological rationality as deciding factor where economic resource is constrained.

#### **Validation Questions**

These classifications highlight some basic questions, important to critically challenging model design and interpretation.

- What is the purpose of the model or research scheme?
  - Exploratory, confirmatory or a practical application?
  - How abstracted and isolated is the model?
- What constraints in the experimental methodology are appropriate?
  - What are the a priori assumptions and how flexible are they?
  - How much is the model a fitting exercise?

- What level of abstraction is present, & how justifiable is it?
- What constitutes an appropriate validation criterion, or criteria?
  - What are the performance measures & are they appropriate?
  - Are there any standard models to compare to, & what does the comparison add?
  - Does the model build directly on an existing framework?
  - Is the model operationally meaningful?

These questions form a critical framework for development of model structures and experimental architectures.

A common ACE approach is to construct the market as a neoclassical two asset system in which agents choose between a risky and a risk free asset and where rational equilibria may be analytically derived. The calculated REE asset price can then be used as a benchmark. Performance relative to this REE benchmark and the presence of 'typical' financial time series properties are presented as key elements in validating experimental findings, any inferences drawn and important to establishing a consistent basis for reference amongst researchers.

Considering the questions above, this is attractive since it presents comparison to a benchmark which is commonly understood. However it does not address the question of whether the comparison to a REE benchmark is appropriate. If the researcher is neo-Austrian then the idea of a rational expectations equilibrium as validation is rather contrary, and presupposes that such equilibria actually exist in real markets. Equally, if the model is designed around agent behaviours as a bottom-up system, judging results on system level behaviour is difficult - a logical link to and validation of agent-level behaviours is necessary. It is intuitively obvious that agent behaviours can be used as a validation of system level observations, it is not so obvious that system behaviours can validate inferences about agents' actions - certainly not without additional evidence. Lastly how do observations relate to the real world? Are they operationally meaningful?

An overriding question, but also the first question on the list, remains 'what is the model actually for?'<sup>4</sup> This should be present throughout model design, implementation, testing and analysis. It can encompass elements of the other questions: if a model is intended to be purely an abstracted demonstration of a theoretical construct then operational meaningfulness is clearly irrelevant - though as a practitioner, there is a question of what purpose that level of isolation actually has. In terms of an exploratory validation strategy operational meaningfulness is again a basic reference point.

Two basic forms validation metric are proposed in this thesis, particularly where explanatory value is important:

- situation, and
- agent-level performance & fitness measures.

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Simon Sinek[130] addresses similar issues in corporate environments dealing with business strategy. Forcing managers to look at what the purpose of their business is, and why they do it is a basis for positive critical analysis of strategy.

#### **Situation & Explanatory Value**

To establish conditions which allow tractable neoclassical benchmark conditions markets and agents necessarily tend to become highly stylized and considerably distant from actual markets. It can prove difficult or impossible in some instances to usefully relate findings to the real world. If the aim of research is to debate academic models in isolation then this is not necessarily an issue, however if operational meaningfulness is desired then some alternative approach is necessary. Situation as set out by Brooks[22, 24] in his work on evolutionary robotics is a potential mechanism to ensure such meaningfulness, although should be noted that this brings with it additional design issues. Brooks' approach is discussed in more detail in Section 4.2.1 since his use of subsumption directly informs the SHaaP architecture development.

Situated models must function in the real world, not an abstracted version of the world. If a model cannot run successfully when situated this raises questions about its levels of abstraction and applicability outside a strictly academic context, whereas if it runs well when situated this provides key support for validation. Situation forces the issue of fitness and performance to the forefront both at the design stage, and throughout experimental runs.

The tensions between abstraction & concretization, instrumentalism & realism may arguably be less for evolutionary roboticists than for ACE researchers - a robot is successful if it performs its particular task successfully or, in a resource constrained environment, better than other robots. The equivalent in ACE terms are so-called 'black-box' models<sup>5</sup>, where performance is the key; underlying processes are not necessarily relevant (depending on the designer's style) ; and no a priori assumption is made that such processes map to any others in the real world.

Even where observed behaviours may be equivalent to other economic agents, as Hayek[57] noted, this does not mean that the underlying processes are the same or can be assumed to be the same. The same can be said however for models replicating time series artefacts, or neoclassical benchmarks in isolated, unsituated environments.

Situation provides a basic validation starting point: however, in itself does not necessarily provide significant explanatory value - for this additional, agent-level exploratory tools are necessary in the experimental design.

#### **Appropriate Performance & Fitness Measures**

The choice of performance & fitness measures within models remains a basic question leading back to model purpose. Forecasting economic statistics or financial time series is different from expressing economic preferences in a competitive, non-linear environment with constrained resources. Fitness and success for an economist can be accuracy of economic statistics, but for an investor those are subsidiary to their own economic performance, in absolute terms and relative to their peers.

The use of neoclassical benchmarks may be insufficient to allow epistemologically valid inferences to be drawn about agent behaviours without some validation of

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<sup>5</sup>See Section 3.3.4 for description of these models

the agent behaviours themselves. Appropriate agent-level measures are vital to the logical connection between market and agent-level behaviours: it simply may not be valid to assume that because at an overall market level certain benchmark conditions are met that this allows any reasonable inference about agent behaviours within the market.

What constitutes an appropriate measure?

1. The measure should operate at the level of the behaviour being measured. So realised profitability for an agent may be a candidate, whereas statistical artefacts in time series or population wealth are not.
2. The measure should relate to the final objective of the agent in the system - its purpose. If the objective of an agent in a population and its survival is determined by forecast accuracy then measures of this are likely to be valid both as fitness measures for the agent and for observing behaviour. However if survival is related to profitability in some form then forecast accuracy is unlikely to be appropriate on its own, whereas return-based measures are obvious candidates.

This does not necessarily require situation, but tests of models and developing models in a situated environment are likely to be useful mechanisms to screen out poor structures and agent designs quickly. In situated applications, models face practical tests of validity - how a model of economic preference expression affects, or is *expected* to affect, the economic state of its users is central to its success.

Some AFMs have presented useful agent-level measures, strikingly similar to those used in real world markets, but this is far from the rule. Frequently market performance and agent fitness are judged in terms of some optimisation tasks rather than population based fitness measures driven by survival. Brabazon & O'Neill[15] are unusual in explicitly stating that appropriateness needs to be considered.

In keeping with Gent & Walsh, work from this thesis would suggest a critical exploratory approach reviewing core validation questions is important where behaviours within the system are examined. This is also suggested as part of Fagiolo et al's review of empirical validation methodologies, where they suggest a 4-step calibration approach to validation: paraphrasing their notes,

1. Identify what set of stylised facts are to be reproduced.
2. Build the model so that microeconomic structure is as close as possible to observed evidence of microeconomic behaviour and interactions.
3. Empirical evidence on stylised facts is used to restrict the parameter space and initial conditions.
4. Explore the causal mechanisms underlying the stylised facts & any new features which emerge.

It can be seen that again these are largely sensible guidelines addressing similar issues to those raised at the start of the section. Step 1 is equivalent to 'what is the purpose of the model', while Step 2 is similar in effect to situation although stops short of this and requires many decisions which may invalidate a model by omission or lead to too much abstraction. Step 3 addresses issues of computational cost and domain knowledge. Step 4 marries closely with the exploratory approach proposed

here - this is an important step in validation. The core of both approaches however is the same, critical examination of behaviours within experimental systems in a principled manner.

Agent-based models permit economic experiments to be run and re-run with the ability to explore interactions at all levels, presuming sufficiently powerful analytical tools and performance measures. The problem is the volume of data which can be captured, but that should not be an excuse for not approaching this in a principled manner. Criteria for evaluating the AFM and agents' behaviours must be established. The design of appropriate agent-level measures for use within agent populations and in evaluation of model validity is the focus of the pilot study set out in Section 5.3.

#### 3.3.3. Model Design Choices

Justification, interpretation and validation: alongside basic model verification, these elements must be considered in detailed, critical examination of detailed economic structures behind the design of any given model. Specific aspects for consideration include decisions regarding objective & utility functions, risk aversion, trading rules, evolutionary protocols and underlying market structure. LeBaron[85] emphasizes the fact that the state space under inspection becomes very complicated very rapidly even with strictly stylized economic models. In general, researchers seem aware of these issues and make some attempt to address them - however acknowledging issues and dealing with them appropriately are not necessarily the same thing.

Design elements may be broadly broken down into five categories: these cover a number of sub-categories and, unfortunately for complexity and experimental rigour, there are often interacting effects across them.

1. *Economic Environment.* The types of securities to be traded and the existence, or not, of definable fundamental values and dividend processes. The available body of work from neoclassical macro and micro-economics has led to a natural tendency to adopt neoclassical frameworks both for expediency and for comparison. LeBaron[84] acknowledges however that it is not clear that such frameworks may be validly transferred to an ACE environment. More concerningly, such an approaches may require some tacit acceptance of a neoclassical rational expectations paradigm.

Representation is also a consideration: how market information is presented to agents; what information they are allowed to see and how it is processed. This includes not only time series return information or prices, but also fundamental variables relating to the economic structure; analytically constructed aspects of this information (such as historic high and low returns, return volatility, yield spreads between different asset classes); cost of information; transparency and so on. Representation choices effectively determine whether the economic environment declared is actually or effectively experienced by the agents.

2. *Preferences & agent structure.* The degree of rationality and information available to agents must be specified. Many bottom-up models cite in one form or another the use of boundedly rational agents acting under uncertainty.

However there is persistent confusion between bounded rationality and optimisation under constraints[49]. Whether coded using readily differentiable functions; explicitly specified in the form of behavioural rules; myopic or inter-temporal; constant or evolving risk aversion, if the rationale for looking at agent based and computational models is to consider markets with interacting economic agents, the choice of preferences is not trivial.

A common choice with rich AFMs is to describe core agent preferences in terms of expected utility maximization with some form of risk aversion, typically constant absolute risk aversion (CARA) or constant relative risk aversion (CRRA).<sup>6</sup> Such agents are usually taken to be single period myopic - their preferences are not affected by recent experience and they do not learn (although in the case of CRRA, it might be argued that there is some effect insofar as changes to agent's wealth are embedded in the CRRA function). Aside from the potential homuncular fallacy of using neoclassical representative agents simply re-invoked at the agent level, there are a number of problematic elements to this approach: an apparently persistent confusion between bounded rationality and optimization under constraints; failure to identify or investigate the use of preference modifiers, and at a general level a confusion between forecasting models, trading rules and preferences.

Actual behaviours expressing preferences must be specified and any preference modifying structures or protocols. These elements are often tacitly ignored. Fitness measures must also be carefully defined in the context of the model's purpose, degree of abstraction & situation.

3. *Market structure.* Central to this is the process by which transactions in economic goods are cleared, trading prices are established and trades are cleared. Since this is one often observable aspect of real markets, at least in a mechanical sense, it is perhaps the most defensible design aspect of model construction, yet it remains subject to many of the problems dogging ACE experimental design in general.

LeBaron[81] identifies four main categories of structure:

- Incremental price changes in response to net demand, so often no established equilibrium in any given trading period. Centralized.
- Forced clearing in each trading period either by simplification or by creation of temporary clearing prices. Centralized.
- Trading book simulation where agents post bids and offers to buy and sell stock, with matching trades crossed. Centralized.
- Where agents interact directly when they encounter each other and trade if preferences dictate. Decentralized.

Each category has problems and benefits: typically trading-off computational and experimental practicality at the cost of realism and vice versa. Except in situated models the choice of clearing mechanism must be directly discussed since different mechanisms can lead to materially different market and agent behaviours.<sup>7</sup>

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<sup>6</sup>See Appendix A.3 for definitions

<sup>7</sup>

4. *Learning & evolution.* By their nature AFMs which are made up of interacting economic agents have aspects which are evolutionary, or at least co-evolutionary in respect of the traded instruments at a population level. A potential source of confusion may occur where evolutionary algorithms are also used at the agent-level as a form of machine learning. In AFMs like the Santa Fe model agents use an evolutionary algorithm (EA) as a learning mechanism, in other models EAs may operate between agents to allow social learning (see Chen & Yeh's genetic programming AFMs [30, 29]). Whether used as a learning mechanism, or applied directly to evolution of agents, or the agent population, care must be taken to define the mechanisms and fitness measures involved.

The use of EAs as learning mechanisms for agents can prove somewhat of a red-herring - particularly in the case of the SFASM the focus jumps to the parametrization of the EA without considering agent-level interactions, no direct measurement or observation of these is attempted.

In contrast, Brock & Hommes' 'adaptive belief system' (ABS)[20, 16] is a good example where agents do not evolve, but the wealth, population size, or influence of a particular agent or type of agent changes as a result of agent interactions (trading). Such systems are the basic starting point for studying system behaviours. With learning or evolution absent or deactivated, they can also provide an important calibration check on experimental paradigms. Lo's Adaptive Markets Hypothesis (AMH) [88] focuses on this aspect as well.

5. *Measurement.* Dependent on market and model structure various performance measures are possible including changes in wealth; realised utility; forecast errors; and higher moments of these as measures of risk. Care is needed both in choosing these measures and their interpretation.

Clear differentiation is required between system-level, agent-level and sub-agent level (e.g. within a learning or evolutionary algorithm) measures; their use and interpretation. Fitness measures for learning mechanisms within agents are not necessarily useful or valid as measures of agent-level or system-level behaviours. As discussed earlier, performance and fitness measures must be appropriate to the system - in actual trading environments forecast accuracy is often less important than profitability, and highly volatile profits less desirable than steady returns. Similarly, as the Austrian school argues, macro-economic measures are not appropriate to describing micro-economic behaviours.<sup>8</sup>

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Pricing and clearing mechanisms are an important field in their own right particularly in automated market design, with enormous potential variations to be studied. Rather than attempt to cover the whole field specific instances will be discussed as they relate to the arguments in this paper and the proposed research.

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See Hartley [54] Chpt. 8 for an extensive presentation of Austrian school arguments against representative agents and the use of appropriate measures in formulating models of systems.

#### Questions, Questions, Questions...

If agents are not imbued with unbounded rationality and utility maximizing preferences (vNM, prospect theory or whatever), then what would be suitable candidates? In fact, what is a *suitable* agent structure? This leads to long lists of questions which usefully direct design towards practical implementation and refocus the modeller on the pragmatic design rather than academic construction, beginning with what a 'trade' actually is. Many of these could (and should) be asked with utility maximizers too, but election to use REE benchmarks makes it convenient to subordinate them.

The list below gives an idea of the types of questions that become relevant. Again taking an AFM economic agent as an illustrative example rather than a more general system, these include,

- Is an agent taken as having one trading strategy, or a collection of competing strategies?
- Do agents in a population compete to provide a single demand outcome modified in each trading period?
- When should a trading recommendation be followed?
- Which recommendation should be chosen from competing strategies?
- What size should be traded?
- When should a trade finish or be closed?
- Can agents or rules die out?

In addressing this list it becomes obvious that trades and trading are not simply an expression of 'core preferences'. A better approach is to construct a collection of heuristic trading elements in which core preferences are not necessarily the most important component. Such collections allow agent preferences to be defined without resorting to complicated homuncular curve-fitting, opening the possibility of subsumption within rule sets and between strategies and behaviours, which fits well with observations of actual trading behaviours.

What properties must a computational agent based on simple heuristics have? For a search heuristic these would be,

1. A mechanism to guide search for information or alternatives.
2. A mechanism to stop search.
3. A decision mechanism based on the outcome of the search.

Similar properties can be envisaged for economic agent behaviours both for single strategies and for a population of candidate strategies competing for risk capital, either discretely or in aggregate, within an agent and expressing its preferences.

Defining a strategy, this will consist of

1. a mechanism to activate the strategy - e.g. search, rule matching, a core preference - generates a buy/sell instruction given specific conditions (a search element)

2. a mechanism to initiate a trade given specific structural rules, including some core preference rules expressing trading size
3. a mechanism to realise profits or losses - i.e. to decide when a trade must finish
4. a mechanism to record and apportion strategy success

An economic agent would consist of a collection of competing strategies and require additional supervisory rules for choice between strategies and apportionment of risk capital. Described in these terms, a functional economic agent is a collection of interacting layers of rules which when broken down can be heuristic in nature. This is the approach taken in developing the SHaaP architecture presented in Chapter 4.

Agent-based models have been used by supporters and detractors alike to explore what types of agents and rationality can produce 'real' market behaviours, however lack of situation and a frequent absence of appropriate validation metrics generally reduces such research to demonstrations of possibilities rather than operationally meaningful or applicable conclusions. The significant potential pitfalls in current approaches, particularly where neoclassical assumptions are used to simplify analytical frameworks and model infrastructures, are a consistent theme revisited throughout this thesis.

The approach suggested here involves both systematic review and justification of model decisions, standardisation of methodological protocols (whether within a particular research scheme or in general agent-based models), and exploratory critical analysis of model behaviours at various levels within the modelled system, particularly within agent populations. Real world models - whether algorithmic, 'black-box' models or business cases derived from observing trading and investment practices serve to illustrate these types of structures and processes.

#### **3.3.4. The Real World - 'Black-Box' Models**

Given natural evolutionary pressure from traders and banks, it is not surprising that some of the most detailed empirical work on financial markets is sponsored, or directly carried out, by financial practitioners themselves. Indeed it is reasonable to assume that much work which might contribute significantly to academic debate is deliberately unpublished - where such work has obvious economic value its effect seen only in the markets themselves by attempts to crystallize this value.

Typically within institutions many trading operations and identifiable trading models (trading strategies) compete for capital. Differentiating sources of revenue and correctly attributing profits and losses across specific models is problematic in itself since business functions are often blurred, but at a basic level these models are *situated* and face *realistic* measure of success independent of the model itself. Whether core mechanisms generating expectations or expressing preferences are top-down or bottom-up, the final test of the model is independent of the model itself.

Models survive while they, or the institutions using them, are competitively superior. Situation forces practitioners to realistically address all aspects of implementation and it becomes clear that models are never simply expectation generators or utility maximizers in isolation. In all cases preference modifiers are implemented which

reflect regulatory, structural and pragmatic aspects of model design informed by domain knowledge. Typically such preference modifiers are not coherently addressed in academic treatments, yet their effect in the real world may be profound. As an example, in the UK stock market bear market ending in 2003, life insurance companies were forced to liquidate assets to maintain key regulatory capital ratios radically distorting core investment models contributing at least in part to the severity of the falls experienced in equity markets at that time. The fears of contagion from Long-Term Capital's collapse (see Section 2.4.2) and larger impacts from the GFC all serve to illustrate how preferences are subject to modification by external and internal processes in the face of uncertainty.

#### **Technical Analysis and Mechanistic Trading**

A number of academic studies have been published investigating the use of so-called 'technical', or mechanistic, trading rules across a variety of markets, and provided useful evidence in the ongoing debate over market efficiency (see Boswijk et al [14] for an example and overview of the literature). In some cases they have also informed the construction of AFMs attempting to demonstrate agent behaviours. This direction for model development, starting with systems situated in the real world and examining them in artificial settings, seems more reasonable than the more common 'unsituated' approach favoured in the literature. Whether or not they yield consistent profits, the behaviours associated with 'technical analysis' and mechanistic trading are some of the few which we definitely do know to be present in actual markets, as such they are an important example of operationally meaningful model structure. High frequency models are presented here in some detail since they directly relate to situated design and performance measures.

There is a huge body of literature on technical analysis and an industry devoted to developing and selling trading systems employing it.<sup>9</sup> Of greater interest is the growth in so-called 'black-box' trading systems. Increasing numbers of funds run by both hedge funds and the traditional fund management community are marketed on the basis that all trading decisions are made by autonomous models without human intervention (Bluecrest Asset Management discussed in Section 2.5 is an example of these). Although it is debatable just how autonomous these models and trading decisions are, it is clear that significant sums of money are now managed in this way and that this trend is increasing. This has significant effects in terms of market behaviours and adaptation by extant participants and regulators alike, as in the case of the Flash Crash described in Section 2.4.3.

These developments allow some useful insights into behaviours necessary to profitable trading models and design (and inevitably therefore viability) but inevitably raise a variety of new questions: why are specific features required and what elements of these are functionally relevant; what relation does this bear to 'normal trading' behaviours; and in the long term what effect black-box funds will have on behaviours and time series characteristics? An excellent, detailed view of one sector of this market is provided in 'An Introduction to High-Frequency Finance' (Dacorogna et al [34] - hereafter Dacorogna).

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<sup>9</sup>See Murphy [96] for a general overview and the finance section of almost any bookshop for endless texts and self-help guides.

Dacorogna's approach is applied to models working and making trading decisions with real market data and attempts to identify and categorize a variety of statistical elements to model design - there can be little doubt of its situatedness. The authors describe in detail the process by which they identify empirical regularities in particular markets (using multiple regression analyses for different moments of time series returns) and, having done so, design and test models to trade on these regularities (Chpt.11, pages 295-347).

Although the sophistication and provenance of their approach is undoubtedly enormously higher than simple 'technical analysis', in essence, the overall protocols are basically similar:

1. Identify a market regularity or pattern, enable and calibrate (optimize) a trading decision rule based around some testing.
2. Allow the model to trade until the rule is invalidated and recalibrate or discard the trading rule.

In fact elements of the underlying infrastructure presented bear similarities to the subsumptive architecture of interacting heuristics proposed and developed in Chapter 4. A key element in Dacorogna's approach is the use of preference modifiers - these are applied to limit trading frequency, monitor performance, allow trades to be opened and profits or losses taken. All models employ such modifiers and all employ realistic return based performance measures. Forecast performance is seen as relevant but not on its own. Dacorogna emphasizes that '*a good forecasting rule on its own is of little use*', stating

'in the assumption of a heterogeneous market, no particular trading strategy is systematically better than all the others' (Pg.323).

For a practitioner dedicated to identifying trading strategies this is a clear, strong statement potentially crucial to how modelling is approached. It points to preference modifiers as critical differentiating factors for success, i.e. consistent, resilient profitability in the face of uncertainty - the key fitness measure for any situated model.

#### **High Frequency Black-box Models**

The 'high frequency' models dealt with by Dacorogna focus (unsurprisingly) on high frequency time series data, using various intra-day data periods down to single minute intervals, tick by tick and individual trades.<sup>10</sup> Anecdotally, high frequency models are the most common type used in 'black-box' systems - periods greater than a single day for trading models are not generally reported to be favoured

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In financial markets terms a *tick* is generally regarded as the change in either the mid-market price of a quoted commodity, share, index or future. In some cases it is applied to changes specific to the bid or offer of a price quote. This is the finest possible granularity for financial time series and care needs to be taken in handling such time series: price quotes are not necessarily updated regularly, quoted bids and offers are not necessarily fresh or tradable. While practitioners have to deal directly with the problems associated with this, academics often make simplifying assumptions both about the regularity of quotes and of trading liquidity.

by participants. There are obvious reasons why this may be a natural direction for traded models to take - in particular the volume of data available is vastly greater when considering high frequency time series, allowing greater confidence in observed features than for lower frequency time series. However, with smaller time periods single period absolute returns diminish while frictional costs remain largely unchanged and become a significant consideration in trading models. Even if a model generates highly accurate, profitable trading signals, if it produces too many, too frequently it may not be profitable after allowing for brokerage, taxes and bid-offered spread.

In terms of structure, it is important to make clear that each of Dacorogna's models consists of only *one* signal-generating rule. Although a trader/manager will often have many such models active at any time competing for capital - a given model may only generate a trading signal infrequently, but if the black-box approach is applied when a signal is generated and approved by the model it should be executed subject to a supporting preference modifying infrastructure overlaying each model. It should also be noted that operators of such systems report a constant need to recalibrate and re-optimize models in their portfolios. Dacorogna comments that 'construction of successful trading strategies is not an easy task and many possible pitfalls must be avoided'.

Examining a detailed example of a simple model set out by Dacorogna, elements of the model and the design process appear somewhat ad hoc, and certainly good candidates for designation as heuristic in character. The process set out below points to the preference modifying heuristics surrounding the trade signal generating elements are at least as important as signal generation itself. This observation is important to the SHaaP architecture presented in Chapter 4 and examples of such structures are investigated in Chapter 6.

#### 3.3.5. Sample Model Structure

Dacorogna's overview of model design gives some idea of the heuristic elements above and beyond trading rule generation, calibration, and optimisation. He is careful to note that a clear distinction must be made between price or return forecasts and trading recommendations. A trading recommendation necessarily requires some form of triggering signal which may take the form of a forecast, but must also account for the risk profile of the trader/manager and must 'take into account the past history of the model'. The recommendation will also be modified by current risk exposure. Of course some elements of this are captured by utility functions and preferences within systems like the SFASM, however note the inter-temporal requirement - here performance measurement is specifically tied to historic model returns, *not* historic forecast accuracy. Note also that forecast and trading are separated as behaviours, with forecasting subsumed by trading.

For overall real-time model design Dacorogna highlights the following features. Models should,

1. Give reasonable warning in advance of a deal.
2. Not change recommendations too rapidly.
3. Not give recommendations outside market hours and allow for holidays.

#### 4. Support stop-loss at all times.

Of these, 1 & 3 seem rather obvious, but certainly can't be argued with on grounds of practicality. Points 2 & 4 are more interesting. The concept of a stop-loss as a separate heuristic element to trading behaviour seems generally absent from most academic discussions (Brabazon & O'Neill[15], Lo[88] and Kaminsky & Lo[65]<sup>11</sup> are notable exceptions), though it is necessarily a part of 'finishing' a trade as set out in Section 3.3.3. From a practical point of view in real environments it is often regarded as the most important of any trade.

The trade recommendation constraint might initially seem slightly odd, but is reasonable once one accepts that there are frictional costs to trading and, despite increased liquidity in many financial markets, a large part of the time bids and offers differ by a meaningful amount. Generating trade recommendations too frequently rapidly increases frictional costs and creates execution problems.

These components all have Simon's procedural rationality - they can be seen to function to increase success in model profitability and therefore survival. As features they are incorporated in the structural elements necessary to support the model described below. It can be seen that as with a subsumption architecture, the whole is made up of layers of interacting elements. It can also be seen that a large element of operator interaction is required. Basic accounting elements, such as average cost and P&L calculators, are also necessary, but the key structural elements set out are,

- *A 'Gearing' Calculator.* The gearing calculator generates recommendations for changes to exposure. This incorporates a number of rule elements. Firstly it monitors the model's risk exposure in the face of changing market prices. Secondly, it contains all the market indicators monitored, trading rules and their performance history. These rules are evaluated in the context of current exposure and trading returns as well as rules for frequency of trading and trade entry.

In essence it appears to represent the basic, core preferences for the model incorporating heuristic rules alongside optimisation procedures. A detailed example provided by Dacorogna is however set out in Appendix A.1. It is clear from these that some elements are quite ad hoc. The model components are relatively simple, but the set of interactions they represent will still be complex.

Interestingly, given the preceding 300 pages of statistical discussion of market indicators and characteristics in his book,, Dacorogna states that '*the gearing calculator is the real model...*' and '*the gearing calculator ... provides the model with its particular properties.*' Taken together with his earlier statement about trading strategies, this is an important assertion.

- *A Recommendation Maker.* Working with the output of the gearing calculator, this set of rules validates trade recommendations. It acts as a control point checking a variety of factors: prohibiting rapid successive trades; checking the

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<sup>11</sup>Kaminsky & Lo's work is also notable insofar as it refers to stop-loss in terms of an overlay strategy, although it does not explain this further. This seems likely to be similar to the ideas of preference modifiers and subsumption of behaviours in preference expression introduced in the work here, discussed particularly in Chapters 4 & 6 dealing with the SHaaP architecture and experiments.

market is open; the 'quality' of market pricing - here filters are necessary to check that trades are possible and that erroneous price feeds are not triggering false trades. Trades have to pass through the recommendation maker to be executed, otherwise they are rejected and the model waits for the next recommendation and so on.

- *Stop-loss Detection*. As well as passing through the gearing calculator, all prices are monitored (after filtering for erroneous feeds) by the stop-loss detector. For any trade that is initiated a specific maximum acceptable loss threshold is set at initiation. If this threshold is reached or exceeded a stop-loss trade instruction is issued and executed. Other than price feed filtering to check that a reported market move is real, stop-loss trades are not subject to being overruled by the recommendation maker.
- *Stop-profit Control*<sup>12</sup>. Similar to the stop-loss, but designed to preserve profits within a trade, this has strong similarities to *satisficing* heuristics<sup>13</sup> except that it requires a decrease in unrealized profits rather than simply reaching a threshold profit objective before being triggered. Typically this can be set up to follow market prices upwards only once threshold price (and profit) targets for a particular trade have been reached. As a simple heuristic it is an elegant extension to satisficing, as it allows for the possibility of profits on individual trades significantly greater than threshold satisficing levels while at the same time remaining conservative about realising gains on a consistent basis. Not very surprisingly, it is a common heuristic throughout the trading community alongside stop-loss limits.

### 3.3.5.1. Performance Measures

Dacorogna emphasizes the importance of robust performance measures in model design and evaluation. In keeping with typical market practice, and given that these models are intended to be situated, the measures adopted are adjusted for market risk and the volatility of experienced profit a loss on trades. Identifying the instability of Sharpe Ratios at low variances, Dacorogna proposes two alternative measures, a symmetric effective returns measure,  $X_{eff}$ , and an asymmetric effective returns measure,  $R_{eff}$ . These measures share the common feature of incorporating a utility function:  $X_{eff}$  fits inside a standard continuous utility function framework, while  $R_{eff}$  attempts to include asymmetric risk aversion<sup>14</sup>. Both are notable in that for a given strategy they work on the cumulative total return,  $R_T$ , realised from all previously completed transactions and unrealised from a current trade.

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<sup>12</sup>This is Dacorogna's term, in the case study experiments in Chapter 6 it is referred to as a 'trailing stop', in common with my own market experience.

<sup>13</sup>

*Satisficing*. Coined by Simon in 1956 [126] - the behaviour of attempting to achieve at least a minimum level of success for a particular variable, while not concerned with achieving or searching for the maximum.

<sup>14</sup>

See Dacorogna [34] Pgs.306-309 and Appendix A.2 for a full description.

This structure is clearly quite different from performance measures in AFMs like the SFASM. Performance and survival are directly related and affect all the elements that make up a strategy, not simply forecasting or demand.

#### **3.3.5.2. Black-box Model Design & Evolutionary Algorithms**

The design issues in creating trading models are formidable. Many degrees of freedom are present at all stages of model development. The models presented are not simply a combination of some form of pattern recognition and demand functions, but have a detailed structure involving preference modifiers. Separate elements may be represented as heuristic rules arranged in layers to form a subsumptive structure in which some equivalent to Dacorogna's gearing calculator appears to be the ultimate level of integration for behaviours. This is a potentially powerful structure: instead of attempting to curve-fit complex demand functions: individual components may be broken out and may be modified independently using performance measures appropriate to the component.

Interestingly however, the majority of Dacorogna's presentation deals with pattern recognition and identifying forecasting rules, while his performance measures relate to the model behaviour as a whole expressed by the gearing calculator. Given his comment that no trading strategy is systematically better than another, this is an odd emphasis. An potentially useful research direction, and the one proposed here, is to focus investigation on preference modifying superstructural elements and their function.

Dacorogna also proposes the use of evolutionary algorithms as a means to search for and generate new models as well as for calibration (optimisation) of model parameters. There is some alignment with the AFM literature's use of EAs - as a search and calibration mechanism to identify potentially interesting market indicators in forecasting. A number of researchers, including Allen & Karjalainen [1]; Neely et al [98] and Bhattacharyya et al [12], have published work in this area.

## **3.4. Academic Models - Poor to Rich, Unsituated to Situated**

'Rich' AFMs are generally presented in terms of the overall model complexity, principally in terms of the scale of potential interactions rather than agent sophistication. Within such markets agent preferences may be described in terms of simple biases in behaviour - e.g. a tendency to buy rather than sell as in Brock & Hommes' ABS [16] - or unboundedly rational utility maximizers constrained by available market information - as in the Santa Fe Artificial Stock Market. In a rich model the combinatorial complexity of the system makes analysis problematic. Meaningful discussion of agent behaviours in terms of their levels of rationality or separation of these from other computational structures is difficult.

As has been discussed, the typical academic solution is to make the environment less complex, allowing the construction of neoclassical equilibrium conditions - ease

of tractability may yield meaningful results in comparison to time series from real markets.

An alternative solution is situation. As discussed in Section 3.3.2, by forcing models to perform in realistic environments with realistic agent-level performance measures many problems of validation are avoided. Further if an agent is not viable in a situated environment in the first instance some theoretical musings become moot. Naturally situation does not preclude working with simplistic models, though the design requirements of real world models are likely to make such models as combinatorially complex as rich academic models.

Once situation is adopted as a design ethic, the questions posed in developing such models tend to naturally address realistic (operationally meaningful) issues. A basic question quickly becomes apparent - 'how sophisticated does a model have to be to be successful?'<sup>15</sup>

Such questions do not naturally arise in unsituated AFMs and if posed have no obvious answer emphasizing their distance from operationally meaningful analysis.

A natural way to address this question is, in keeping with ACE descriptions, from the bottom up. What then is the minimal level of sophistication that an *economic agent* must have to be successful? This necessarily requires clear definition of 'success', with a natural bias towards situated measures (survival, profitability, stability of returns, relative outperformance) rather than unrealistic or potentially inappropriate measures like simple forecast accuracy or total return.

#### 3.4.1. Minimal Intelligence Models. Zero Intelligence Traders and Market Micro-structure

Separate from rich AFM work there are many relatively 'poor' AFMs (though very good in terms of quality). A useful example by Dave Cliff [32, 33] considers earlier findings by researchers working with so-called Zero-Intelligence (ZI) agents in simple auction markets.<sup>16</sup> For a particular market structure as, presented by Gode & Sunder [50], it appears that agents do not require *any* rationality at all. From a methodological standpoint a major benefit is that the reduction in required assumptions and complexity that results from using ZI agents can make analysis of market structure and market mechanisms easier and increase explanatory power. However, Cliff's analysis highlights severe pitfalls even in apparently rudimentary unsituated systems.

Gode & Sunder presented a simple market auction process in which ZI agents appeared capable of establishing an efficient market equilibrium despite having no ability to learn. This builds on an experimental framework presented by Smith in 1962 [131], considering human trading behaviours in limited run, repeated auctions<sup>17</sup>. Smith's work showed that over short trading runs with minimal information

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<sup>15</sup>

This might be recast as how 'intelligent' does an agent have to be, but given the discussion of *intelligence* in 4.2.1 that is not necessarily useful.

<sup>16</sup>See Ladley[77] for a recent review of current work with this class of model.

<sup>17</sup>

and simple rules governing their activity, human traders quickly reached theoretical equilibria,  $E_0$ , suggested by relative supply and demand of the goods they were instructed to trade.

Gode & Sunder's work replicated Smith's using software agents in place of people. The results were surprising and dramatic - it appeared that the auction mechanism and the rules for trading were sufficient to generate 'efficient' markets<sup>18</sup>. Although the experimental setting is deliberately artificial and, while not ruling out some elements of utility maximization in actual traded markets, Gode & Sunder's conclusions are rather stark.

*'The primary cause of high allocative efficiency of double auctions is the market discipline imposed on traders; learning, intelligence or profit motivation is not necessary.'*

In contrast to REH models where utility maximizing agents are required to derive market equilibria, if 'market discipline' is a sufficiently powerful normative force they suggest such maximization is no longer a strict requirement. In effect, this would suggest that the market imposes rationality!

Unfortunately, despite such interesting conclusions, work by Cliff and others [32, 33] casts considerable doubt on this interpretation and suggests that the underlying infrastructure in the computational system is driving the apparent equilibria rather than the market structure. Cliff points out that as ZI agents in Gode & Sunder's study generate bid and offer prices randomly subject to upper and lower limits, if prices vary continuously it is possible to generate probability density functions (PDFs) for these prices in the market and from these, with the transaction rules in the system, PDFs for transactions. Using these PDFs to generate expected values,  $E(P)$ , for transactions with particular supply and demand curves, Cliff is able to consider analytically the performance of ZI agents under a variety of cases including those presented by Gode & Sunder. Cliff's analysis suggests that only in a limited set of special cases, including Gode & Sunder's, will  $E(P)$  be equal to  $E_0$ . Cliff tested this empirically by constructing cases where  $E(P) = E_0$  and  $E(P) \neq E_0$ . The results of these experiments strongly suggested that ZI agents were actually converging on the expected value,  $E(P)$ , rather than  $E_0$  as human traders did.

Cliff's analysis and experimental observation leads to the conclusion that the ZI agents form a stochastic system of bids and offers and that the mean transaction

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The auction process used was a Continuous Double Auction (CDA). Typically in a CDA buyers and sellers can publicly post prices at which they are willing to buy or sell at together with the amount they are willing to trade. Other participants can choose to trade on those prices. This is a widely used mechanism on commodity, futures and stock exchanges. In a CDA there is no obligation for a trader to 'make a market' (to show a bid and offer at the same time) or even to publish his trading interest - traders can simply observe and trade when they see a price which suits them assuming that no other trader has already acted on it.

In Gode & Sunder's protocol necessary modifications included setting profit targets for the agents for buying or selling, setting the supply and demand, and constraining agents to only profitable trades.

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The definition of efficiency here is similar to that of REH proponents - an *efficient* market here is one where demand and supply are equally matched.

price is simply the intersection of the PDF for sellers and for buyers. This was supported by constructing a variety of supply and demand curves for selling and buying agents with different theoretical equilibria and PDF intersections. In each case over the trading runs the price converged to the expected value given by the PDF intersection rather than the theoretical supply-demand equilibrium.

However, by modifying the ZI agents by the introduction of a simple machine learning mechanism where agents adjust their profit targets adaptively in response to observed trading prices in each trading period, Cliff created agents (he called them ZI-plus or ZIP agents) which under the same experimental markets as were tested on the original ZI agents performed much closer to theoretical forecasts in some cases and in general better than the ZI traders. The ZIP agents were not tuned to specific test markets, rather the aim was to demonstrate that the effect of minimal modification to the ZI agents.

The learning mechanism employed was effectively a small collection of ad hoc trading rules with a target profit adjustment mechanism to incrementally adapt agents' bids and offers in response to the market - no complex utility function was required. The learning mechanism is rudimentary yet yields surprisingly good results. Clearly the CDA market is equally basic, however as an approach, this is attractive and fits well with the subsumptive architecture of heuristic layers described earlier. It allows the possibility of exploring the interaction of trading components and the trading/economic environment while at the same time giving some comfort over unforeseen or ignored aspects in the experimental protocol.

#### **A Situated Test**

The ZIP approach has another attractive aspect. Although the market explored in Cliff's paper and Gode & Sunder's is not situated at all, implementations of the ZIP agent protocol have been tested against other trading algorithms in the 'Trading Agent Competition' (TAC)<sup>19</sup> meeting with some reported success. This contest is run annually with a variety of trading environments and acts to promote exploration of agent-based technologies in a competitive environment - in effect it creates a quasi-situated benchmarking arena. More recent studies extending the use of ZIP agents to express preferences have shown some success in developing models of systems which remain simple while at the same time highlighting interesting technical and structural features of markets. Ladley & Bullock[78] investigate market structural effects and degree of trader connectivity on performance vs other traders who are more or less informed, while Straatman et al[134] propose a generic framework.

The direction here is unsituated to situated. It would have been interesting to see how quickly the problem Cliff identified might have become apparent if Gode & Sunder's original work had been carried out in the opposite direction - *situated to unsituated*, where minimalistic agents in such a protocol would first have to pass practical, situated success measures of survival. The explanatory benefits of such deliberately simple models of agent behaviours are significant, but without some form of adaptation of learning, as in the case of ZIP agents, limited. It remains the

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<sup>19</sup>

<http://www.sics.se/tac/page.php?id=1>

case that behaviours in situated environments are more complex and competition will drive out poorly adapted behaviours. More appropriate is to look at such models as demonstrations and tests of simple behaviours in isolation, allowing their properties to be explored before incorporation or subsumption into larger structures, as is the case for the SHaaP architecture.

#### 3.4.2. 'Rich' Artificial Financial Markets

A large range of rich AFMs are explored in the literature (see Axelrod & Tesfatsion [5] and LeBaron [84, 80] for recent reviews). Typically the models presented are,

- unsituated
- have a relatively large number of interacting economic agents
- agents range from minimally intelligent to unboundedly rational utility maximizers
- populations may be heterogeneous in beliefs, but homogeneous in preference structure. Few AFMs work with heterogeneous preferences.
- some form of learning is often present - either at a population level or at an agent level - in some cases social learning may be allowed
- financial time series are the major experimental output and are tested for 'validity'
- agent behaviours are observed in the context of the time series analysis
- agent performance measures do not map clearly to measures observed in actual economic agents

Without examining a specific example of an AFM in detail it is difficult to appreciate the methodological challenges presented in their construction or the problems in fairly interpreting results. The SFASM is a useful working case: one of the more commonly studied architectures, its gross features are reasonably representative of the general approach in the AFM literature. It is presented here in considerable detail as it also forms part of the case study developing the agent-level performance measures presented in Chapter 5.

This section covers the basic market structure, the main experimental findings, a brief look at some subsequent studies extending and reassessing the market and some overall criticisms. The whole is then re-assessed in Chapter 5 where a version of the SFASM forms the underlying agent-based market used to explore realistic situated, agent-level measures designed for use in these markets. It should be emphasized however that it is not implied that specific design issues raised here are necessarily present in other AFMs.

After covering the SFASM some alternate structures are discussed including some more situated models.

##### 3.4.2.1. SFASM Structure

Described by Arthur et al in 1997 [3] and LeBaron et al [85], the SFASM was an extension of earlier work at the Santa Fe Institute [102]. The stated aim is that a

simple, 'standard' model structure allows for ease of replication and allows experimenters to focus on market related rather than protocol implementation problems. While the structure is simplistic, it is considerably richer in complexity than many AFMs.

The SFASM is designed around a two asset, rational expectations equilibrium (REE) framework made up of a population of interacting economic agents. The resulting market is wholly endogenous: it is not simply isolated from real markets, rather it has no direct connection with real markets at all. The authors cast the AFM as a structure in which the 'aim' of the agents is to learn the REE underlying the market<sup>20</sup>. This allows an analytical benchmark to be derived using the REE price for risky assets traded by agents in the AFM - a key feature used by the authors in assessment and validation of the time series produced by the market, in conjunction with comparison of the statistical properties of the time series to those observed in actual financial markets.

The market consists of  $N$  autonomous, one-period myopic, utility maximizing agents with constant absolute risk aversion (CARA)<sup>21</sup>.  $N$  has generally been set to 25 - qualitatively more complex than in simpler AFMs, while still making extended simulation runs feasible. Agents evolve trading strategies over a large number of generations using a learning classifier system (LCS).

Only two assets are available to agents to trade. In each period,  $t$ , agents must choose what proportion of their wealth to invest in a risk-free asset in unlimited supply paying a rate of interest,  $r$ , and in a risky asset in limited supply paying a dividend,  $d$ , which follows an auto-regressive (AR) process,

$$d = \bar{d} + \rho(d_{t-1} - \bar{d}) + \varepsilon_t. \quad (3.1)$$

The AR coefficient,  $\rho = .95$ ,  $\bar{d} = 10$  and  $\varepsilon_t$  is Gaussian and IID such that  $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ . Agents are heterogeneous in expectations, however they share a common

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<sup>20</sup>

ACE literature is riddled with anthropomorphic statements as to agents' intent, thinking and behaviour. While researchers may be rigorous in their definition of the problem space so that in those terms reasoning may be strictly defined, such usage is still dangerous insofar as mapping to actual processes observed in the real world is either limited or ill-defined. Sometimes optimisation is just optimisation.

<sup>21</sup>

CARA functions do not allow for changes in agent's preferences as their wealth changes. Other models have favoured constant relative risk aversion (CRRA) allow changes in proportion to an agent's wealth (see Appendix A.3.2 for examples). There has been considerable debate as to the validity of both types, not only from researchers in behavioural finance and cognitive psychology, but also on the basis of some of the underlying mathematical implications of these functions in particular cases. Rabin[109] identifies circumstances under expected utility theory where it appears that a utility maximizer who has turned down a small bet for small stakes, would not undertake a larger bet no matter how large the pay-off. Rubenstein [110] commenting on Rabin's findings, suggests that this is a result of problems in applying expected utility theory rather than the theory itself, deriving from a common use of *final* wealth rather than *changes* in wealth. He highlights two issues. Firstly this misapplication is commonplace amongst economists. Secondly, the broad brush application of expected utility ignores changes both in risk aversion at different levels of wealth and at different times. This impacts on the issue of inter-temporal preferences similar to that referred to by LeBaron [80].

CARA utility function,

$$U^i(W) = -exp(-\lambda W^i) \quad (3.2)$$

for the  $i^{th}$  agent, where  $W^i$  is an agent's wealth and the coefficient of absolute risk aversion,  $\lambda = .5$  also constant across all agents and subject to

$$W_{i,t+1} = x_{i,t}(p_{t+1} + d_{t+1}) + (1 + r)(W_{i,t} - p_t x_{i,t}). \quad (3.3)$$

as a budget constraint.

### 3.4.2.2. Trading Process

An agent,  $i$ , uses a set of trading rules to formulate its expectation for the stock price,  $p$ , in the next trading period, choosing the fittest activated rule in any period to produce a forecast (see the following section for the learning process and rule evaluation). Using the expected price for the next trading period and the utility maximization constraint, demand for stock by an agent,  $x_i$ , is given by,

$$x_i = \frac{\hat{E}_{i,t}(p_{t+1} + d_{t+1}) - (1 + r)p_t}{\lambda \hat{\sigma}_{i,t,p+d}^2}, \quad (3.4)$$

where  $\hat{E}_{i,t}(p_{t+1} + d_{t+1})$  is agent  $i$ 's best forecast for the next period and  $\hat{\sigma}_{i,t,p+d}^2$  is the agent's forecast of the conditional variance of  $p + d$ . This relationship holds if stock prices and dividends are normally distributed under *linear* rational expectations, such that,

$$p_t = f d_t + g \quad (3.5)$$

From this it is possible to solve for a homogeneous REE as all agents share the same degree of risk aversion. This provides the REE benchmark within the system<sup>22</sup>.

By setting the total number of shares equal to  $N$  and forcing each agent to optimally hold 1 share, i.e.

$$\sum_{i=1}^N x_{i,t} = N \quad (3.6)$$

optimal forecasts are given in the form,

$$\hat{E}_{i,t}(p_{t+1} + d_{t+1}) = a(p_t + d_t) + b. \quad (3.7)$$

A Walrasian auctioneer is used to solve for an equilibrium settlement price,  $p_t$ , in the current period matching demands for all  $N$  agents using their forecasts from

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By using the demand Equation 3.4,  $f = \frac{\rho}{1+r_f-\rho}$  and  $g = \frac{\bar{d}(f+1)(1-\rho) - \lambda\sigma_{p+d}^2}{r_f}$ . The same relationship does not necessarily hold for non-linear REE.

Eqtn 3.7 & Eqtn 3.4. The fittest rule which provides values for  $a$  and  $b$  is chosen according to the protocols in Section 3.4.2.3.

At the start of a trading period each agent provides the auctioneer with its demand,  $x_{i,t}(p_t)$ , given its expectation for the closing price and dividend. Thus for an activated rule  $j$  for an agent  $i$ ,

$$x_{i,t}(p_t) = \frac{a_{j,i}(p_t + d_t) + b_{j,i} - (1 + r)p_t}{\lambda \hat{\sigma}_{j,i}^2}, \quad (3.8)$$

The auctioneer calculates an instantaneous equilibrium price which satisfies these demand functions subject to the supply constraint Eqtn 3.4 and communicates this to the agents<sup>23</sup>. Using this price agents recalculate their demands and execute their trades. The new dividend for the period is posted according to the AR process, Eqtn 3.1, allowing final wealth and rule forecast accuracies to be calculated for the period. Traders were subject to the budget constraint Eqtn 3.3 and some trading constraints.<sup>24</sup>

Although the demand function derived under the utility maximization constraint incorporates agents' final wealth, it can be seen that changes in wealth do not affect risk aversion or demand, except where budget constraints are reached in any given period. In terms of the experimental structure, this auction mechanism has the inherent problem that, once chosen, an activated rule is not changed: trading execution is taken to occur at the clearing price even though the auction process might result in a significantly different settlement price to the one at which the rule is activated. In effect trading and forecasting are temporally separated.<sup>25</sup> Insofar as the performance measure used in the system is forecast accuracy, not wealth, this is consistent, but does highlight the separation of the model framework from real world experience.

### 3.4.2.3. Agent Rules

Each agent has a pool of 100 forecast rules. Agents learn by updating the forecast accuracy of their active candidate rules in each period and an EA is implemented periodically to eliminate poorly performing rules and generate new candidate rules from currently fitter rules for each agent. In this case, the EA uses a genetic algorithm (GA) as the basis for a learning classifier system (LCS) of the form proposed

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<sup>23</sup>In fact, it appears that a number of different computational approaches were used but whether all produced similar findings is not stated. See Chapter 5.3.2.1 for a discussion of this.

<sup>24</sup>

These trading constraints were somewhat arbitrary. Agents could not trade more than 10 shares per trading period and could not go short more than 5 shares at any one time. LeBaron does mention that trading limits were rarely invoked after early training. There was no mention of how often budget constraints came into effect however in implementing a working version of the model it was found that they were important to the stability of the system - Chapter 5.

<sup>25</sup>

LeBaron notes that the auction process is not perfect since it is not conditioned on the current price information ([85], pg. 1495), stating that if it were the demand function would be a 'complicated non-linear function for which the equilibrium would be hard to find'.

by Holland et al [58]. Rules in such classifier systems consist of a condition-action pair of elements. The condition element of each rule contains a set of condition 'bits'. When environmental conditions match the condition bits of a particular rule it is activated and becomes available for choice by the agent in that period. If chosen its action element is implemented - if more than one rule is activated the rule with the best (lowest) forecast error score as calculated by Eqtn 3.9 is chosen.

In the SFASM this action is to set the  $a$  and  $b$  forecast parameters of Eqtn 3.7 - trading is not an action in itself, rather this is a result of the preferences expressed in Eqtns 3.2 & 3.4 - in fact, Arthur et al refer to these as 'condition-forecast rules' emphasizing the difference.

As a structural choice this is important since it introduces the confusion between forecasting and trading. This is exacerbated by the fact that all trades are myopic adjustments to a notional portfolio rather than identifiable independent actions. Some studies, particularly those where prices are supplied exogenously [116, 67, 69], avoid the problem of choosing a particular utility function by allowing the condition part of the rule to specify the resulting trading position. Demand functions are no longer specified except in so far as the parameters available to the EA to set them. This sidesteps potential debate about choice of utility and demand functions, but means that deriving an explicit REE benchmark case is no longer possible.

For each SFASM rule the condition bit string is 12 bits long, and each bit can have a value of 0, 1 or #<sup>26</sup>: 0 or 1 values correspond to the truth of the environmental state, while # is a wildcard 'don't care' symbol passing a particular bit whatever its state when deciding which rules are active or not. Condition bits are of two types:

1. 'fundamental' bits carrying information about the risky stock dividend to price ratios
2. 'technical' bits giving simple historic price moving average data.

Truth states for these bits are chosen by the experimenter - this choice is arbitrary, although the authors refer to some tests for 'robustness' in their selection. The values taken are shown in Table 3.1. These are simplistic in the extreme and bear little resemblance to trading signals used in realistic trading environments.

The forecast element of each rule is a real valued vector of length 3, corresponding to the linear forecast parameters,  $a$  and  $b$ , and a conditional variance estimate used to supply the value of  $\hat{\sigma}_{i,t,p+d}^2$  in Eqtn 3.4. For each rule values of  $a$  and  $b$  are set randomly to values uniformly distributed about the REE and remain fixed for the life of a given rule, new values are set only as part of the EA. The conditional variance value is initially set to the REE value and in each evolutionary round is updated across each rule to the then active variance of forecast accuracy. The authors highlight the fact that the mechanisms chosen for setting new parameters and updating variance estimates were relatively arbitrary, based on what seemed reasonable.

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Strictly 'trits' given each can hold 3 values. Real valued conditioning bits have been used in other studies, either directly such as in Tay & Linn's [140] where fuzzy inference is also applied, or indirectly with a conversion mechanism into binary bitstrings.

Bit	Condition
1	Price*interest/dividend > 1/4
2	Price*interest/dividend > 1/2
3	Price*interest/dividend > 3/4
4	Price*interest/dividend > 7/8
5	Price*interest/dividend > 1
6	Price*interest/dividend > 1/8
7	Price > 5-period MA
8	Price > 10-period MA
9	Price > 100-period MA
10	Price > 500-period MA
11	On: 1
12	Off: 0

**Table 3.1.:** Condition Bits

Potential criticisms have been raised by both experimenters themselves [84] and many others (including but not limited to [29, 30, 67]). The major criticism presented is that the experimenter chooses both the type of conditioning information and the types of actions that can evolve, constraining the novelty of agent behaviours which may emerge.

#### 3.4.2.4. Agent Learning & Forecast Accuracy.

Learning progresses via two mechanisms: (1) Updating the forecast accuracy of activated rules and (2) using the LCS to eliminate poorly performing rules and generate new rules.

At the end of each trading period, the accuracy of all active forecasting rules is updated using the exponentially weighted moving average of the squared forecast error,  $e_{t,i,j}^2$ , such that

$$e_{t,i,j}^2 = (1 - \theta)e_{t-1,i,j}^2 + \theta[(p_t + d_t) - \hat{E}_{t-1,i,j}(p_t + d_t)]^2, \quad (3.9)$$

for the  $j^{\text{th}}$  rule of an agent,  $i$ , where  $\theta$  is a weighting constant for the exponential moving average. The use of an exponentially weighted accuracy measure ensures that all of a rule's historical performance is available to the agent, while weighting its impact. Low values of  $\theta$  place higher emphasis on historic performance, while a value of 1 for  $\theta$  would mean that only the last period's error would be used. While the authors identify this parameter as important within the model, in their original papers they leave it fixed with a value of 1/75 - again citing trials to establish 'robustness'.

In describing its relevance they heavily anthropomorphise their interpretation - for values of  $\theta$  approaching 1 agents are described as *believing* that they are in a rapidly changing environment where the asset return process is non-stationary, while for small values that they believe they are in a relatively static environment where historic performance is more relevant. While a plausible description of agent beliefs if such anthropomorphic descriptions are accepted, if instead the model is considered

as a distributed collection of interacting heuristic search optimizers operating in a noisy environment, then varying  $\theta$  seems closer to simply degrading population performance.

The LCS allows agents to eliminate poorly performing rules and explore the rule space. Agents invoke the EA asynchronously. Agents are selected randomly so that, on average, the genetic algorithm is invoked every  $k$  periods for each agent in the population. The authors report that the GA used is fairly standard, however, they provide little discussion about how its final structure was developed or its stability.

In each instance where the GA is implemented for an agent,  $i$ , each rule,  $j$ , is ranked in terms of its strength,  $s_{i,j}$ , according to,

$$s_{i,j} = -(e_{i,i,j}^2 + cB_j), \quad (3.10)$$

where  $B_j$  is the number of bits not equal to  $\#$  in a rule and  $c$  is a constant representing the 'cost' per bit - this effectively penalizes rules which are highly specific, pushing rule evolution to more generalized cases (i.e. with a high proportion of  $\#$ 's).

Each agent's rule set is generated at random on initializing the system. At each iteration of the GA the worst performing 20 rules are discarded and 20 new rules are generated by crossover and mutation. These processes are described briefly here with a detailed description and discussion in Chapter 5. Mutation occurs with probability 0.9 and crossover with probability 0.1. In mutation a single parent is chosen by tournament selection and bits are flipped at random.<sup>27</sup>

For crossover two parent rules are chosen by tournament selection from the population. The bitstring element is recombined by uniform crossover while the real valued component is recombined by an ad hoc randomized procedure. Child rules inherit the average forecast accuracy of their parents - this is again an arbitrary decision within the protocol and like some of the other instances is open to criticism since in this case there is no reason to assume accuracy of a child rule will be anything like those of the parents particularly given the lack of granularity of the conditioning bits. The protocol also contains routines to deal with rules which are never matched, whether because they are logically invalid or simply never encounter activation conditions - they are periodically randomized using the  $\#$  symbol to replace a portion of their bits.

### 3.4.2.5. Experimental Runs & Discussion

Repeated simulation runs were carried out with  $k$ , the average number of trading periods between GA operation, set to either 250 or 1000, described by the authors as *fast learning* and *slow learning* respectively. The results of the simulation were described in terms of REE conditions and the statistical properties of the resulting

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<sup>27</sup>

Bits are flipped with probability 0.03 according to the following rules.  $0 \rightarrow \#$ ,  $P(2/3)$ ;  $1 \rightarrow \#$ ,  $P(2/3)$ ;  $\# \rightarrow 0$ ,  $P(1/3)$ ;  $1 \rightarrow \#$ ,  $P(2/3)$  and  $0 \rightarrow 1$ ,  $P(1/3)$ . Real valued elements are reset or incrementally adjusted according to the rules,  $P(0.2) \rightarrow \text{reset}$  to a uniformly random value within the allowable range and  $P(0.2) \rightarrow \text{an incremental adjustment of } \pm 5\%$  of the allowable range.

time series. Differences in rule complexity both in terms of the total condition bit usage and use of 'technical' vs 'fundamental' bits formed a key elements in their analysis and discussion.<sup>28</sup>

An experimental run consisted of an initial 250,000 trading periods to allow 'sufficient learning and transients to die out', followed by recording trading in the next 10,000 trading periods. This was done for 25 separate runs with different random seeds generating the initial agent populations in each run. A time series of 10,000 trading periods gives the equivalent of over 40 years trading data if compared to daily closing exchange prices (assuming approximately 250 trading days in any given year). This does beg questions about the 'learning' period, equivalent to over 1,000 years of daily transactions. Clearly, concerns over realism in this learning period might be addressed by considering trading periods as 'intra-day', however a whole new set of assumptions and decisions would then have to be made about the structure of the market and the trading day.

The principal reported observation from experimental runs [3, 85] was that varying  $k$  was crucially important to the rate at which the rational expectations equilibrium was approached and whether it was reached at all. 'Slow' learning ( $k = 1000$ ) the results were consistent with a linear rational expectations equilibrium, matching expected values for kurtosis, returns and volatility persistence. The experimenters' conclusion was that agents in this market structure were capable of learning the REE. However, in the 'fast' learning case ( $k = 250$ ) no recognisable equilibrium was achieved. It is reported that the statistical features observed are similar to common features observed in real markets: exhibiting volatility persistence, higher than theoretical returns, weak forecastability and some cross-correlation between volatility and trading volume. (See Section 5.3.2.1, Table 5.3 for summary statistics).

Time series generated were unstable - subject to bubbles and crashes at least superficially like those in actual markets. Examining the rule structure of the agent populations under slow and fast learning it was found that in slow learning condition bit usage was low (i.e. a higher number of #-type bits) relative to fast learning and that the fraction of 'technical' bits set to 'fundamental' bits was higher in fast learning than slow learning.

This difference in bit usage with a change only in the rate of GA implementation is an interesting finding. Drawing on the time series properties as validation, the authors interpret the increase in technical bit usage as GA learning increases as evidence of co-evolution of rules between agents with 'technical trading' as an emergent phenomenon of the system. They propose this as a link between their system and agents' behaviours to behavioural finance work on inductive reasoning. They were careful to highlight that agents were homogeneous in risk aversion and utility functions.

'It is important that the time series results were obtained without having to resort to any type of ad hoc differences across our agents. The results were made much stronger since they were subject to this constraint.' [85]

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There is no measure of market efficiency independent of the underlying models and assumptions of the REH. Without such a measure experimental conclusions are codependent on the underlying model - the so-called 'joint hypothesis' problem identified by Fama [42, 43].

The only elements varied across experimental runs are the value of  $k$  and the initial rule make up of the population.

Although from a procedural standpoint LeBaron presents this is a strong aspect of the methodology, as with Cliff's criticism of ZI traders covered in Section 3.4.1, questions of how much the computational structure may shape the equilibrium are not addressed.

#### **Structural Issues & Performance Measures**

There is little published work examining structural issues. Ehrentreich [39, 38] provides one example - his work appeared to highlight a fault in the mutation operator used in the SFASM. His concern was that the operator had an upward bias and was responsible for the set bit level being higher in agents with faster learning (lower  $k$ ) rates. On re-examination using analytical techniques from population genetics he found the bias was in fact absent and the original operator was valid. In fact, after allowing for genetic drift effects, he found that the upward bias he had calculated was *necessary* to counter the push from genetic drift to all 'don't care' (no bits set). Although his final conclusion is that the operator is functioning as originally described, the actual processes are very complex and required extensive work to verify the result.

Ehrentreich analysis is very detailed, however, bearing in mind that it is agent behaviours that are the point of the model, the question becomes why is his work necessary or relevant: if the model can be shown to validly relate to behaviours in actual markets, if agents are successful in trading then discussions of bias in the mutation operator are not relevant except potentially at a later stage of investigating *why* the agents are successful.

The problem again is that as the markets are not situated and without agent-level performance measures there is no way to directly link agent behaviours to actual markets. There is still no answer to why bits are set and whether this has any relevance in agent behaviour - whether indeed agents are deriving 'useful information' or acting in a meaningful way. LeBaron [85] carried out a regression analysis of his time series and found that the series had potentially exploitable information - i.e. the market was not efficient according to REE theory. However this still does not mean that agents are acting or able to act on it, a fact revisited in the pilot study.

Agent wealth is a natural candidate for an agent-level measure<sup>29</sup> and SFASM agents do accumulate wealth. Unfortunately it does not fit well with the design of the system - in all runs although there are differences between agents wealth is seen to rise monotonically across the population as a whole. Joshi & Bedau [64] attempted to analyse their version of the ASM by applying game theory to identify theoretical equilibria within the system. They found that wealth increased monotonically for all agents and that agents with classifier rules increased faster than agents with non-classifier rules.

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<sup>29</sup>

See Section 3.4.2.7 for discussion of why it actually is not a good measure.

Ehrentreich presents a useful analysis demonstrating a base level of wealth accumulation within a population as a function of the risk premium in the form of the discount in traded prices to the REE price. He also shows that differences in wealth are highly correlated to average stock holdings and provides some qualitative evidence that this may be linked to the GA frequency rather than 'technical trading' in the form of some kind of pattern recognition.<sup>30</sup>

#### **SFASM Conclusions**

A basic problem within the system is that trading and forecasting are not the same thing. The LCS used in the model acts within agents to improve the quality of their pool of forecasting rules. Although there is a link to trading via the demand function and the specialist agents do not optimise their trading performance. Although agents are designed as utility maximizers they are not *profit* maximizers - a rule which yields a more profitable trade than expected may be penalized more than a rule with a highly accurate forecast from a rule resulting in a loss. It is difficult to see why or how agents can usefully exploit inefficiencies in the market.

Arthur et al [3] identified this as an issue but stated that their intuition was that profits from rules based on forecast accuracy would be higher for more accurate rules than less accurate rules. In an unpublished PhD dissertation Wilpert [148] apparently found evidence supporting this conjecture, however the modification presented, which bases rule fitness off wealth, actually reduces to another form of forecast based fitness similar to Eqtn 3.9, albeit non-linear, so it remains an open question. The agent-level measures presented in the pilot study in Chapter 5 go some way to addressing these issues, however the fact that the model is unsituated remains a major problem and although testing it in a situated environment might be of interest given the basis of the overall design this seems unlikely to yield operationally meaningful results.

#### **3.4.2.6. Other 'Rich', Unsituated AFMs.**

There are many alternate endogenous AFMs with interesting feature sets, though relatively few published studies of exogenous AFMs. This section will briefly discuss aspects of two further endogenous models, since their design aspects have relevance to the overall themes of this paper, before considering two of the very limited set of published exogenous models.

#### **An Adaptive Belief System**

In a series of papers, Brock and Hommes (hereafter BH) developed a model structure they refer to as an Adaptive Belief System (ABS) [18, 19, 20, 59]. The economic system follows the familiar neoclassical two asset investment structure - agents can

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Strangely in a later part of his dissertation he concludes that there is technical trading, but bases this on system level measures which are different from those applied here.

either invest in a risky asset in limited supply or a risk-free asset in unlimited supply. Agents are again myopic, utility maximizers sharing the same utility function as set out in the SFASM. The ABS differs from the SFASM in 3 distinct ways: expectation formation; learning & population evolution; and agent performance measures.

- *Expectation formation.* Having established a special case equivalent to the SFASM where agents are homogeneous in expectation formation, yielding a theoretical REE for benchmarking, BH introduce a truly heterogeneous agent population (at least in expectation formation). All agents have the same beliefs about future dividends incorporating these in conditional expectation of stock prices. However BH generate agent subpopulations with simple biases to these expectations, allowing the creation of simple trend followers and positively or negatively biased agents. The price dynamics of time series generated by various mixes of interacting heterogeneous populations are then investigated. This represents a level of complexity significantly greater than in the SFASM.
- *Learning.* In the ABS, the proportions of different agents in the system are updated over time according to their relative fitness - analogous to population evolution in A-Life studies. As with Lo's AMH this fits well with observations of actual trading floors and behaviours, while avoiding complications of EA parameterisation at the sub-agent level.
- *Performance measures.* This is perhaps the most significant difference from the SFASM. Agents differ only in their belief bias. BH propose that a natural fitness measure might be realised profits and that excess profits compared to the REE price over time would be useful. However they reject this in favour of a measure which incorporates *risk-adjusted* profits. This is a significant change - as a measure it approaches measures used in the real world much more closely than measuring forecast errors. In fact their measure,  $\pi_{ht}$  is structurally similar to the  $X_{eff}$  described in Section 3.3.5.1 and Appendix A.2 for black-box models. BH define the measure as follows

$$\pi_{ht} = R_t z_{h,t-1} - \frac{a}{2} \sigma^2 z_{h,t-1}^2 \quad (3.11)$$

where  $z_{h,t-1}$  is the demand by a trader of 'type'  $h$  and  $R_t$  is the realised excess return ( $a$  is a risk aversion parameter and  $\sigma^2$  is the conditional variance of expected returns). As with  $X_{eff}$  this can be formally tied to an expected utility framework and since BH's excess return measure is constructed with respect to the REE it can also be related to a neoclassical benchmark.

In their experimental runs, BH demonstrate that the various statistical artefacts common to real time series can be reproduced in the ABS. Given reservations about using these artefacts as validation for examining behaviours, the runs themselves will not be discussed here.

This AFM has several aspects to recommend it.

- By introducing agents which are heterogeneous in beliefs it moves the AFM significantly closer to considering real markets - although highly stylized, this allows the preferences expressed to be easily understood (Dacorogna's black-box model in Appendix A.1 shows a counter example, where simple heuristic rules combine to give highly complex preferences!).

- Learning is population rather than rule-based - this has much to recommend it since it avoids the apparent potential pitfall confusing learning or adaptation with optimisation when using GA's.
- The performance measure is more situated than in the SFASM and similar AFMs - it directly addresses agent performance & the use of risk-adjusted performance is a significant improvement.

**A Hybrid Multi-Agent Model** A second, more recent, hybrid multi-agent model has been proposed by Chen et al[28]. This model extends a mathematically defined multi-agent model presented by Lux & Marchesi[91, 92] in which agents adopt heterogeneous aggregate preferences according to a set of equations developed by the research to simulate different forms of trading behaviour. Of itself this is not particularly interesting insofar as it is subject to the same validation issues as other endogenous academic models relying on the recreation of time series artefacts as a success criterion. It also suffers from the fact of reintroducing de facto representative agents since its mathematical structure determines the overall model performance rather than direct agent interaction.

However, Chen et al extend the model by first implementing a true agent-based version and then by introducing a class of agents using a copy-cat heuristic and observing the resulting system dynamics. As a step towards exploring preference structures in an agent-based system this has merit and relates to the work presented in this thesis. Beyond extending Lux & Marchesi's model however there is no attempt to specify a clear architecture or to explore agent-level behaviours, so validation criticisms remain and its actual explanatory value is limited.

#### 3.4.2.7. Exogenous models.

Two (slightly) contrasting protocols are presented here.

#### **A Learning Classifier System Model**

Schulenburg & Ross [115, 116] (hereafter SR) present a LCS model where agents make trading decisions between a risky asset and a risk free asset (cash). In this case the risky asset is a real stock and in each trading period agents are provided with market information about the daily changes in stock price and trading volume in the stock.

The LCS has three traders, each with its own set of classifier rules evolved over the trial runs. Initial rule sets are randomly generated with protocols to capture logically invalid rules and a number of runs with different random number seeds are performed. The condition elements are of the same form as in the SFASM (see Table 3.1), although the variables represented by each bit are different. The three traders differed in length of conditioning-bit string and in the types of bits used. The lengths and types were held constant in each trader type - as with the SFASM, bits could either be used or ignored.

In contrast to the SFASM, the rules here are condition-*action* rules rather than condition-*forecast* rules. The action element is 4 bits long, with 1 bit representing a

buy or sell instruction and the other bits capturing how much of an agent's current holding or maximum potential holding to buy or sell. Traders were not allowed to go short<sup>31</sup> and were subject to budget and trading volume constraints in each period. Each trader was allocated a specific starting position in both stock and cash. Over trading runs LCS rules within each trader competed for activation and were evolved using a GA. Rule performance was updated using a bucket-brigade mechanism based on changes in net wealth (realised and unrealised) in each period, with all active rules receiving credit based on their level of specificity whether used in trading or not. The stock used for sample time series data was IBM.

It should be noted that different trader types do not interact at all - in the SFASM trader interaction determines the stock price - here that is not necessary and no specialist clearing mechanism is required. Neither is there any form of 'social learning' (learning between agents [145]). As part of the study after an initial training period, the GA was turned off and the agent performance when they could not update their rule performance metrics was monitored.

SR present extensive results for different trial runs comparing performance for groups of agents of each type and showing these relative to simple 'buy&hold' and 'all cash' strategies (in the later study [116] they also compared these results to a 'random walk trader' which changes its position randomly in each trading period and to a simple trend-following trader). They look at the behaviours of specific agents and their trading decisions.

They conclude that agents are able to learn to trade and to outperform simple strategies and that this was evident both in the training period and in the non-adaptive period when the GA was not active. They also conclude that 'a good set of indicators (for conditioning bits) is crucial' to the process of developing new traders.

Unfortunately it is not clear from the information presented just how useful their results are. The final performance measure of success for agents presented was total returns with average total returns by trader type and 'best' total return at the end of the relevant periods. Comparing these results to the benchmark simple cases, specifically 'buy&hold' it is clear that the average trader performance in each case is markedly better than buy&hold and it seems likely that this may be significant at a statistical level, but no analysis of this type is provided nor even a total return for the entire population.

More problematically, the use of total return as a result is a poor and potentially misleading performance measure - both risk-adjusted and internal rate of return (IRR) measures are important. Highly volatile trading performance is not accepted in the real situated world, either by risk managers or regulators, nor is performance judged only at the end of a trading period, it is inevitably an ongoing, quasi-real time metric.

From the graphical data presented even for the 'best' traders, it would appear that in the later phases of the trial runs most traders' total profit declined markedly and much more rapidly than the 'buy&hold' strategy - there appears to be rapid

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<sup>31</sup>

i.e. Sell assets they did not own.

convergence. Obviously without access to underlying experimental data and more detailed protocols it is not possible to comment on the reasons for this. A plausible explanation might be that given a strong bull market over a number of years of training data the SR agents evolved to a particular kind of trending market, as conditions changed they were unable to adapt back - it may well be that there was not enough diversity in the rule base to allow the trader to adapt successfully.

#### **A Neural Network Model**

A second model, presented by Kendall and Su (hereafter KS) [69, 67, 68], is grossly similar both in the problem addressed and in the experimental findings, although the underlying agent structure is quite different.

Again working with time series from real stocks, initially British Petroleum, they take groups of agents and allow them to learn and trade over an extended period (though still very short in SFASM terms). The principal difference to SR is that the EA used is an artificial neural network(ANN) constructed to allow evolution and learning via mutation as well as social learning via the replacement of indicators with alternatives taken from a public pool with published performance figures. The social learning element may help to encourage speciation, avoiding problems with reduced rule diversity (as may have been an issue in SR's work). KS cite Vriend's paper [145] and Chen & Yeh's study [30] in support of this approach. The use of a genetic ANN is proposed to avoid the problem of the experimenter explicitly choosing and constraining the market variables on which to condition agents.

While some interesting design issues are addressed in this approach, these are somewhat secondary to problems in the results. As in SR's model, total returns are presented alongside population averages vs. simple buy&hold and cash benchmarks. These are then used to support examining particular strategies. As before total returns on their own are not a useful performance measure and without detailed analysis or the use of a more appropriate agent-level performance measure it is not possible to comment on the usefulness of the work at length.

Neither study takes into account previous trades and extant positions. As a result, although their trading structures avoid the problem of mapping forecasts to demand, it is not at all clear that there is any meaning to considering individual strategies. Both studies focus on the best performing traders in the population, ignoring survival bias issues and the performance of the population as a whole.

In both cases performance measures need to be appropriate to the market and agent structure with clear differentiation between agent-level, intra-agent and system-level measurement.

#### **3.4.3. Bio-inspired & Other Approaches**

Beyond forecasting, learning and optimisation using common evolutionary and machine learning techniques, other biologically inspired algorithms provide potentially encouraging exploratory approaches to modelling economic systems. Brabazon &

O'Neill[15](BoN) provide an extensive review of these types of models, with discussion of supporting evidence from experimental protocols demonstrating them in application.

The algorithms assessed include versions of particle swarm optimisation (PSO), ant colony optimisation (ACO), ANNs, artificial immune systems (AIS) and grammatical evolution (GE). The case studies are similarly broad, ranging from asset allocation & trading systems to corporate failure prediction & bond rating. BoN report some success across all of these, although the overall approach taken is still one of optimisation rather than resilience, which as discussed in Chapter 2 is a better approach in the face of uncertainty.

That said their basic methodology and experimental design process are noteworthy and much stronger than most in this area: not only do they work primarily with exogenous models, so there is some degree of situation, but they specify a clear working process. The structure they adopt is very similar to those used by practitioners like Dacorogna and is clearly influenced by practical experience. Key elements they set out are,

- Choice of appropriate performance & fitness measures. Rather than looking at forecast accuracy where profit is a success factor, risk-adjusted return measures are used.
- Validation. Models are tested against historic and out of sample market information. This brings a high degree of situation, so although BoN according to their results are generally seeking to optimise performance, there is some intrinsic validation here.
- Processing model output. Although, not very clearly expressed, BoN discuss converting model outputs into economic preference behaviours. For trading models, BoN recognise a need for trade entry and exit strategies based on model signals and also on gearing (the strength of the preference behaviour).

In particular, although they do not identify them as distinct components that need to be explored as part of a larger model structure, they explicitly talk about risk limiting behaviours, such as stop-loss and profit-taking triggers. This is highly unusual in academic analyses but basic to practical implementation as set out in Sections 3.3.3 & 3.3.4.

- Implementation & Maintenance. Again this is interesting and strongly similar to the practical high-frequency model structures Dacorogna describes and sets out what are effectively preference modifying superstructures. BoN describe the need for mechanisms to decide when to use particular models and when to reject them, however the details they provide are quite limited and indeed heuristic.

Of the algorithms they review, grammatical evolution[15, 100] is intuitively particularly interesting as an algorithm for developing sophisticated, flexible behavioural rule populations which can capture domain knowledge. A particularly attractive feature of GE being the potential explanatory value of such rules when experimental results are explored with agent-level performance measures and worth considering in future work.

Unfortunately, despite the strengths in their work there are some problems in BoN's approach. The first is specific to their reported findings and is a serious limitation.

In all of their evolutionary models they appear to report only the results for the best evolved candidate solutions without providing data for all simulation runs and all solutions (although they do provide averages for the best solutions). Without full reports on all population distribution statistics these results are of quite limited use and should only be regarded as exploratory (while of course, accepting that exploratory models are no bad thing in Tukey's exploratory-confirmatory framework[143]).

The second issue is somewhat more philosophical. The use of nature and biologically inspired algorithms is widely accepted in agent-based computational economics in optimisation, learning and classification - indeed it is rare to find models which do not use them in some form. However, given the uniqueness of economic systems where agents are self- and other-regarding, and where the systems are complex and adaptive, exhibiting non-linear, recursive behaviours, it seems fair question whether biologically inspired algorithms are necessarily the best, or at least sufficient, solution when modelling these systems.

In the presence of uncertainty in economic exposures it seems more reasonable to look to financial markets themselves for inspiration (alongside nature). Neth & Gigerenzer[99] suggest that efficient heuristic structures observed in existing financial markets are specifically adapted to deal with uncertainty where optimised algorithms fail - effectively they encapsulate domain knowledge in behavioural responses. Their viewpoint is quite purist: for practitioners (and ex-practitioners) the view is less clear or more pragmatic - use the best optimisers and classifiers where they work best and use domain knowledge and uncertainty adapted, heuristic structures with them to build resilient, well adapted models.

## 3.5. Summary

This chapter followed Chapter 2's presentation of economic context, giving a critical discussion of experimental methodologies in agent-based models of economic preference expression. Beyond reviews of current approaches, the main issues highlighted here centre on experimental protocols and how agent-based models of economic systems can be developed in a principled manner. In Chapter 1, lack of operational relevance was cited as a motivation for this research. The proposals and guidelines in this chapter, illustrated by existing models, put this in context and address these concerns.

A number of suggestions for improving current approaches are presented in the chapter. From a practitioner's point of view these may appear somewhat basic, yet often seem to be missing in academic methodologies. Those elements surrounding model verification and documentation amount to good practice, but despite that it appears that minimal standards still need to be accepted and processes followed.

Validation of experimental protocols and inferences is a more challenging issue. Some simple guidelines have been proposed here. The overall approach, recognising the inherent complexity present in agent-based models and economic systems, requires both exploratory and confirmatory behaviours. Other than a critical approach recognising a model's purpose, key components identified for this are situation as a basic validation metric; the use of appropriate agent-level performance measures

when exploring experimental results; and clearly defined model structures within a recognisable exploratory framework.

Candidate performance measures are introduced and developed in Chapter 5. Model structure and implementation are core elements in the experimental process: the SHaaP architecture is presented as an appropriate open & extensible experimental framework for modelling economic systems. Subsumption and population-based design are core meta-heuristic structural elements to this architecture and are discussed Chapter 4 and Chapter 6, where case studies using the architecture are described.

# 4. SHaaPa - An Experimental Platform & Architecture

*'We need exploratory and confirmatory.'*

Tukey, 1980

## 4.1. Overview

This chapter sets out the structure and design of the Subsumptive Heuristic Adaptive Agent-based Preference Architecture (hereafter either 'SHaaPa' or 'SHaaP architecture'<sup>1</sup>), a novel, agent-based experimental platform building on domain knowledge-based insights from situated environments, and the discussions presented in the thesis as a whole.

The central focus of the work here is economic preference expression and behaviours in economic systems: the structure of the architecture reflects this. Subsumption as a structural meta-heuristic is at the core of the architecture, facilitating a design which emphasises the processes behind expression of agent preferences and their effects, rather than simply choices. As discussed in Chapter 2, while in social sciences this focus on processes is not a particularly unusual approach[129], in economics it is uncommon and processes appear only to be explored only as a secondary topic - the SHaaP architecture is designed to address that shortcoming.

The second key component of the architecture is its population-based structure. Any agent-based model is inherently population-based, however in the architecture developed here each agent itself is made up of sub-populations of behaviours competing for expression - again following a subsumptive approach. This design reflects the stochastic nature of outcomes in a non-linear system: the same behaviour at different times in apparently similar conditions does not guarantee equivalent outcomes, so this exploratory component is important. Versions of such structures are frequently observed in situated economic systems. Their frequency and apparent operational significance raises the question of whether, similar to Knight's attempts

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In the context of case study applications reported in Chapter 6 SHaaPa acronym might also reasonably well be taken to be *Simple Heuristic* Adaptive Agent-based Preference Architecture, since simple heuristics as identified by Gigerenzer and others are an important topic for research in themselves as mechanisms for cognitive choice in realistic, uncertainty prone environments. However this is separate from the architectural structure itself. In both cases however the emphasis, driven by subsumption is on easily decipherable, testable processes in keeping with Simon's description of social science investigation.

to identify and categorise uncertainty mitigating structures[74], they serve a similar meta-heuristic function and forms one of the research questions explored in Chapter 6.

The SHaaP design explicitly recognises that in real economic systems, rather than highly abstracted academic models, preferences are rarely if ever consistently the result of a single demand function, be it simple heuristic or the unboundedly rational expression of a von Neumann-Morgenstern utility maximisation function, and certainly not ever if viewed over multiple time periods. Rather, choices are the result of multiple layers of subsumed and, potentially, subsuming behaviours. Economic systems are not isolated and preferences are constantly modified either by new information, by structural constraints or, more significantly, by internal or external regulation. By encapsulating rational or other choice algorithms within subsuming preference structures this leads to concentration not just on these choices, but also on the processes expressing and modifying those choices.

Encapsulation and explicit subsumption allow functional behaviours to be more readily isolated, explored and tested in a principled manner. The subsumptive structure allows complex behaviours to emerge from subordinate systems severally or together. The ability to probe and explore system and agent behaviours is critical in this regard, otherwise validation and principled experimental analysis and development is compromised. The measures & protocols discussed in Chapter 3 and developed in Chapter 5 and Chapter 6 are integral to the overall approach.

The chapter presents the rationale behind the use of subsumption and a population-based modelling architecture before moving on to consider specific agent-based modelling issues, choice of modelling platform, and implementation. The overall structure of the architecture and its main components are presented in some detail together with discussion of issues and practical considerations around the design.

## 4.2. SHaaP Architecture - Design & Principles

The SHaaP architecture addresses methodological issues discussed in Chapter 3. Whereas in earlier agent-based models preference behaviours are often simply bundled together in a single, neoclassically influenced demand function or documented as programming structure rather than integral to overall expression, here functional behaviours are explicitly mapped and modelled as part of a subsumptive, population-based structure. Similarly, structural constraints on behaviour which can directly affect preference expression are recognised in the design process.

In the SHaaP architecture component behaviours may be isolated before aggregation and subsumption into larger more complex, and potentially more sophisticated or ecologically rational, entities. Economic agents express their preferences as the result of subsuming the outputs of subsidiary systems. This aligns closely to real world solutions dealing with complex financial computational problems: although sophisticated mathematical models may represent specific aspects of a trading system, they form only subcomponents and are subsumed within a larger risk control and trading infrastructure which reacts to market changes, makes recommendations, modifies preferences and delivers trading instructions - see Sections 3.3.4 and A.2 for further discussion.

The key design objectives behind the SHaaPa design philosophy beyond exploring particular economic systems are

- replicability,
- verification, &
- validation.

The experimental design approach and modelling environment are both critical to replicability and verification. The structural subsumption meta-heuristic represents an interesting research area in itself, however methodologically it also yields the ability isolate, identify and investigate specific programming structures and functions. Models can be developed piecewise adding complexity and sophistication in a principled manner so that they can be critically examined and explored. In agent-based models, as in economic systems as noted in Chapter 2 and in Haldane[53], the variable and parameter space become extremely large extremely quickly in economic systems. Clear documentation and reporting of parameter sets, model source code and computing environments are important starting points. Without these elements the first two objectives are at best difficult, at worst infeasible. The SHaaP architecture lends itself to a systematic approach to these problems.

Validation is more complex and requires an appropriate set of investigative tools and measures to explore the inner workings of the model and its agents. Situated performance measures, i.e. measures tested and drawn from real-world observation, form a basic initial validation measure, while other exploratory agent-level tools allow the behaviours within the system to be critically examined with some level of confidence. Basic validation principles and protocols are discussed and set out in Chapter 3. These tools also enable verification that what is being measured is actually what was intended. All these are supported and facilitated by the choice of a stable, generalised modelling platform - in this case Repast Symphony as set out in Section 4.4.

The potential pitfalls of poor programming decisions and documentation are discussed extensively in the following chapter, Chapter 5, while reviewing the Santa Fe Artificial Stock Market (SFASM) as part of the development of risk-adjusted relative efficiency measures and exploratory processes. It is worth reiterating however that, even without the particular methodological issues identified there, in their original research and in subsequent versions of the SFASM the authors gave little emphasis to exploratory data analysis as Tukey proposed[143]. What agent-level behaviour was examined was not linked to model validation: rather the model validation criteria were primarily based on statistical artefacts in market time series and only summary statistics pointing towards potential agent behaviours were presented - no actual agent behaviours were examined.

The exploratory data analysis and, indeed, forensic process proposed in Chapter 3 and demonstrated in Chapters 5 & 6 are important techniques in design and development of experimental protocols. The features examined will inevitably be context and domain specific: in the case of economic systems the agent behaviours & interactions in the context of their environment as it changes are a basic level of information which should be directly observed. Given the volume of data this can produce however visualisation techniques are vital exploratory tools. There is little evidence in the literature of significant efforts to use agent-level exploratory data

analysis, indeed the tendency is to provide relatively little information on programming structures within the models and with little standardisation across platforms.

### 4.2.1. Subsumption - A Structural Meta-Heuristic

Subsumption is central to the SHaaPa design. Its validation and design benefits as a structural meta-heuristic are described by Brooks[21, 23, 25] and are discussed in the following section. However, in common with the abundant use of biologically inspired computational algorithms and algorithms taking metaphors from physics in computational methodologies, subsumption as a structural choice is perhaps the ultimate nature inspired architectural choice and is evident throughout the evolution of simple and complex organisms. It represents both a biologically inspired algorithm and a form of domain knowledge. Importantly, as a meta-heuristic it does not preclude the use of other well researched algorithms and methods, rather it provides a mechanism to incorporate these in a coherent manner.

#### Evolutionary, Adaptive Context

A common popular misconception of evolutionary processes is they are inherently progressive, have intent and direction. In this viewpoint evolutionary progress is towards 'higher' orders of more sophisticated organisms with greater complexity. The problems in this interpretation are now reasonably widely recognised and discussed eloquently by Henry Gee[47] as a scientist reporting on evolutionary biology. Philosophical arguments aside, it is at odds with natural selection as a process: rather than a progressive process, natural selection (and evolution) simply allows success with some stochastic element by organisms suited to the environment they are reproducing in, particularly where there is competition for constrained resource.

Evidence of evolutionary subsumption appears to be abundant. As Gee has pointed out, subsumptive structures in nature present both evolving complexity in multicellular organisms, but equally as a path towards simplification. A classic example would be the presence of mitochondria within eukaryotic cells: mitochondria carry out a specialised function in cell metabolism but at the same time, if current theories of their incorporation and co-evolution are correct, have lost or no longer require their own control mechanisms. Their metabolic function is subsumed by and part of the original host cell.

In evolution there is no direction and no guarantee of increased sophistication or indeed complexity. Gee provides a number of counter examples to the idea of evolution as progressive, where competitive pressures favour resilience and ecological costs are a determining force. Under ecological pressures complexity may arise not as a process of sophisticated cooperation, but one where cooperation and simplification within a subsuming structure yield competitive efficiency advantages - the equivalent of ecological rationality in Simon's terms. Mitochondrial subsumption in eukaryotic cells is a particularly clear biological example.

Subsumptive structures are readily observed in modern corporate and economic entities: examples of economic simplification and subsumption in broking and banking functions directly mirror biological subsumption and support its place in ecologically

rational structures. There are clear analogies with economic uncertainty and risk mitigating structures discussed in Section 2.4.4.

Specialisation as an uncertainty mitigating structure is an interesting case for subsumption and evolutionary pressure. In an unsubsumed form it yields benefits in exploiting economic resource, but under competitive pressure is particularly exposed where specific changes in market structure could eradicate overspecialised entities. Where large profits are available in new unstructured markets or where new trading opportunities are created, as for instance when tax regimes are reviewed or international trade barriers removed (or imposed) pure, individual specialist entities may be sufficiently economically resilient to survive uncertainty. As inefficiencies are eliminated, either by other entities competing in the same area, or by policy makers adjusting the playing field, profit margins and resilience will fall - in that case a subsumption structure may yield benefits for both subsumer and subsumed, further pressuring isolated entities.

Subsumptive architectures present an opportunity to explore heuristic processes already well established in nature and in economic systems, while at the same time potentially increasing substantially methodological rigour and explanatory value.

### **Evolutionary Robotics, Subsumption & AFM Design**

In evolutionary robotics Rodney Brooks' subsumption architecture [21, 22, 23, 24] presents an interesting example of applying subsumptive principles similar to those presented in the SHaaP architecture. In Brooks' architecture agent design is based on layered collections of interacting simple heuristics and situated agent-level measures. Complex behaviours are built from the bottom up. Layers of simple behaviours are subsumed by more complex levels: the overall behaviour results from the interaction of component simpler behaviours.

In contrast to traditional AI (or at least AI prior to Brooks' papers), subsumption allows models to distribute representation functionally throughout a system instead of requiring centralised abstract, manipulable, symbolic representation of goals and the environment.

Brooks proposed four ideas as central to developing AI models beyond what he identifies as 'traditional' AI, which are worth discussing in the context of economic preferences and ABM design.

- *Situatedness*. He identifies traditional models as 'problem solvers working in a symbolic abstract domain' where agents are not situated in the real world at all. While there is no claim that the agents in the ABMs discussed here are attempts at producing financial AI the same criticism can be levelled. In some sense the ABMs are static model constructions where although the agents co-evolve the world or the problem does not. If attempts are made to port such models to the real world, a fresh new set of representation issues arises.

Brooks sums up this aspect as 'the world is its own best model'. This is a viewpoint that is largely subscribed to here - if ABMs are situated questions over the validity of the economic environment fall away. Discussions about the behaviours of the agents become worth having, not because they necessarily

have any semblance to economic agents external to the ABM, but in responding to such agents.

There is a question of the 'degree of situatedness' and this relates to the level of abstractedness. As with traditional experimental economics the impact of a fully situated model will have consequences not just for the experimenters and subjects but potentially also for the world at large - some degree of separation is desirable not just to limit this impact but to allow repetition. In this thesis traditional approaches have been criticised for their high degree of isolation and abstractedness. The approach taken here and in the case studies is to partially situate the models - historic financial data and time series are used in test environments; performance measures are based on realistic situated measures; and behaviours are based on observed behaviours.

- *Embodiment.* Bearing in mind Brooks' work is primarily in robotics, this is presented as the extent to which an agent physically deals with the real world, i.e. in terms of effectors and sensors. The arguments supporting this again focus on validation of the agent - no secondary testing or conversion is necessary to enable the agent to perform - this is part of the development process.

For ABMs and economic agents or real economic agents embodiment may not be strictly applicable. At a basic level economic transactions and settlement systems are highly organised with recognisable protocols for 'back office' functions. The environmental challenges relevant to evolutionary robotics are not present in financial transactions. However it is still pertinent to ask even for a fully situated ABM, how embodied it is. A trading system may work perfectly well on paper but fall down on execution protocols - a simple example being that market prices quoted often do not reflect the tradable prices, frictional costs, or market structures and some elements of these need to be captured in the form of structural modifiers.

At higher structural levels embodiment may have some relevance, where subsumptive economic entities interact with each other, as in a regulatory environment, though again this will still be at a somewhat abstract level relating to forms of economic capital and modes of exchange.

- *Intelligence.* Brooks argues that intelligence should be looked at from the bottom up, so that 'reasoning' is only one small facet of intelligence with a larger aspect being environmental interaction. This again supports the idea of reasoning as not simply 'core preferences' and behaviours bundled up in a homunculus.

Although this makes a clear definition of 'intelligence' problematic, in an A-life context rather than AI such a definition is not necessarily important: the 'reason' or intent behind a behaviour is secondary to the behaviour itself and is in keeping with Herbert Simon's ecological and bounded rationality arguments. Simon[128] suggests that an important aspect of rationality and the development of intelligence is the complexity of the environment.

Arguments over definitions aside, in an economic context if an agent behaves as if it is intelligent then arguably it is - certainly in terms of performance metrics and also in terms of its impact on other agents and its environment.

- *Emergence*. Having allowed a definition of 'intelligence' to include 'environmental interaction', Brooks suggests that 'intelligence' is an emergent phenomenon resulting from the dynamic interaction of components within a system or agent rather than located in an identifiable component opening up discussion of how structural and procedural elements combine in agent behaviours.

As is acknowledged in the literature [84, 5], rich ABMs of financial systems are often neither *situated* nor *embodied*, yet many studies spend time considering both the time series characteristics and the behaviours of individual agents with eventual inferences about how these relate to behaviours in the real world. Agents preferences are defined 'simply' as utility maximizers so discussion of intelligence becomes irrelevant.

The analytical tractability gained in ABM studies which begin by considering markets which are entirely endogenous, where only isolated risky assets are traded, unrealistic co-evolutionary simulation periods and where agents are homogeneous in utility and risk aversion functions, is at the expense of plausibility and testability. These models are again based on choices not processes. Unlike such models, real world systems are not closed; agents are not homogeneous in utility and risk aversion functions. While co-evolution clearly occurs it is in an open framework with agents, or agent behaviours, entering and leaving trading arenas, with differing investment horizons, risk and return profiles and tax structures. If models are to have viable practical application and credibility alternative approaches are required.

By adopting an approach which begins with unsituated design, all aspects and assumptions underlying a model are open to question and this includes validation tests themselves. Design issues and problems in isolating causal effects of these assumptions and combinatorial complexity are potentially overwhelming issues for agent-based approaches. The luxury of restricting design is not available to real-world trading models and such models are instructive when considering ABM design.

In the methodologies presented here subsumption in the SHaaP architecture allows for principled investigation of situated behaviours and experimental protocols with realistic validation metrics.

### 4.2.2. Population Structural Heuristics

Population-based heuristic structures are common in biologically inspired optimisation and local search algorithms - particularly evolutionary algorithms. While the specific mechanisms of these algorithms vary widely, a common feature is the application of machine learning and generational selection techniques to population of candidate solutions, aiming to improve the quality of these solutions. Analogous to evolutionary processes in natural selection, the fittest solutions are selected and weaker solutions eliminated. A problem in simple versions of such algorithms is speciation, where solution populations become too well adapted to particular environments and cannot adapt to changes. This is a significant issue in situated environments where underlying asset distributions are unknown, subject to change over time, and to uncertainty.

Population-based structures are also commonly observed in situated economic systems, where economic agents in the form of corporate entities, regulatory bodies or individuals interact and compete for expression. In contrast to evolutionary algorithms however, the function here is not necessarily solely focussed on optimal performance. While poorly performing agents may be constrained or sidelined they are not necessarily eliminated from a population so their knowledge and behaviours remain available for use as systems change over time and where they may become successful. Essentially compared to optimisation algorithms there is no certain generational death or total replacement.<sup>2</sup>

The SHaaP architecture uses this second structure, maintaining populations of economic agents subsuming key components and behaviours for expression which may themselves be population-based. This design allows successful behaviours to be retained and reused even after periods of poor performance without specifically requiring sophisticated algorithms to recognise suitable conditions or changes in economic system states. The design does not preclude optimisation or machine learning by agents or interaction between agents, but generational replacement is also not required.

An effect of this structure extends beyond homogeneous preferences, where it could be viewed as simply supporting adaptability via a distributed population of candidate solutions. For heterogeneous preferences and agents it allows the possibility of maintaining multiple, possibly conflicting, beliefs and behaviours simultaneously. This is analogous to real world experience, where heterogeneous investors have different investment horizons, degrees of risk aversion and investment criteria but may co-exist successfully exploiting apparently different opportunities within the same systems.

In the form presented here this population-based approach bears some resemblance to the initiation stage of a simple estimation of distribution algorithm as described by Pelikan et al[106] without progressing to an optimisation stage. Rather, after generating candidate solutions, performance of each candidate is monitored and the best performing selected in each decision period.

The overall SHaaP structure is described in detail in the following sections and explored in the experiments presented in Chapter 6.

## 4.3. SHaaP Structure & Agent Representation

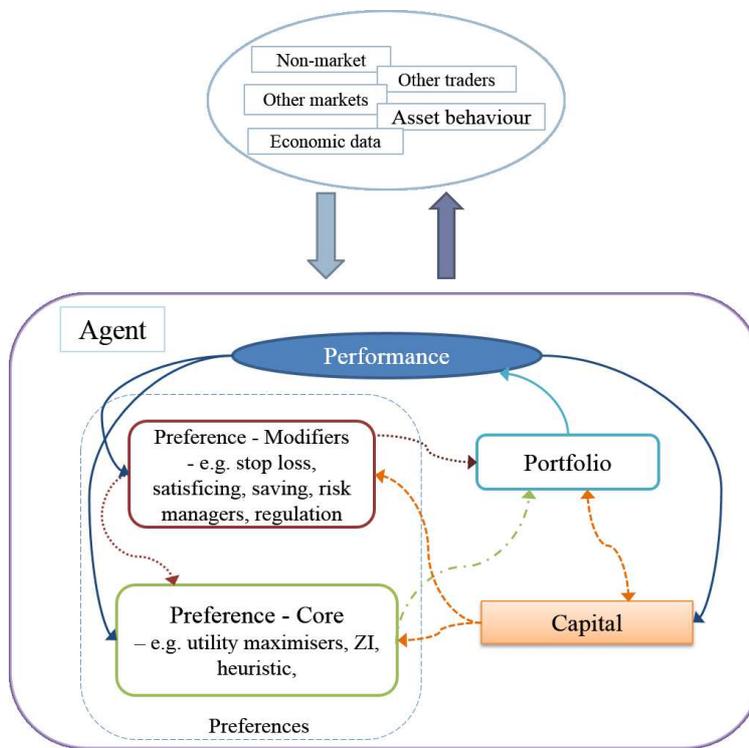
### 4.3.1. Economic Agents

A schematic structure for an economic agent in terms of key preference structures, modifiers, assets and market interaction is shown in Fig 4.1. This form identifies basic functional structures, behaviours and intra-agent interactions for such agents.

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<sup>2</sup>Anecdotally this is a highly criticised feature of banking post the GFC where many involved in arguably creating the circumstances for the crisis remained fully employed after it broke and were highly paid to deal with its effects. However in such circumstances their knowledge was valuable and may have, in terms of uncertainty mitigation, forestalled even more serious effects. The question of appropriate salary however is a moot point.

While core preferences are inevitably present, in this schematic their relationship to non-core processes is highlighted, as is the dynamic interaction between changes in behaviour, performance and feedback into modification of core preference expression as an agent interacts with the external economic environment.



**Figure 4.1.:** Economic Agent Structure

There are obvious simplifications in this representation: for example, no market clearing or auction processes are shown; allocation and changes in available capital other than by market performance are not dealt with; nor are learning or adaptation mechanisms shown either at agent or higher levels. However these elements are subordinate, indeed subsumed, component behaviours of the main structures.

The schematic identifies the main functional components,

- Preferences - Core. The central economic preferences of an economic agent. In a typical neoclassical agent, these would take the form of an unboundedly rational risk averse utility maximisation function. Here they simply represent the behaviours an economic agent would carry out if not constrained or subject to modification by non-central behaviours.
- Preferences - Modifiers.
  - Risk & Uncertainty Modifiers. These may be generated internally or from external sources and may be explicit or structural. Hence stop-loss behaviours, regulatory constraints, or simple panic when confidence in a core preference model falls all constitute preference modifiers.
  - Structural. Such as trading frequency and settlement constraints. While not directly affecting risk and uncertainty exposures, again there may be interaction effects.

- **Portfolio & Capital.** The accounting and organisational structures. An agent's investment portfolio and its capital. Decisions moderated by core preferences and preference modifiers effect changes in the portfolio, while changes in market state and transactions from the portfolio affect economic capital.
- **Performance.** The ongoing measurement of the results of an agent's behaviours. Realistic measures are obviously important - P&L over time, risk-adjusted measures and total wealth all fall into this category.

The key structural feature of this representation is subsumption. Subordinate structures are subsumed within a larger whole. These structures may be simple, fast and frugal heuristics or sophisticated, highly specialised quantitative functions. This diagram also helps to highlight a frequent basic misconception over preferences and beliefs: forecast models, which are prime candidates for optimisation, help form beliefs about future market states, but are in themselves not preferences - forecast models inform agent beliefs and in turn preferences (and preference modifiers). So, again, forecast models and belief functions, are subsumed by preference structures.

### 4.3.2. Recursive Structures & Agent Representation

An appealing aspect to a functional representation of an economic agent as shown in Fig 4.1 is that the same structures can be identified in larger subsuming economic actors. Figure 4.2 shows this repetition across entities in a market setting, emphasising the recursive nature of interactions between the main components as performance from preference expression and changes in market state feed back into ongoing behaviour and expression.

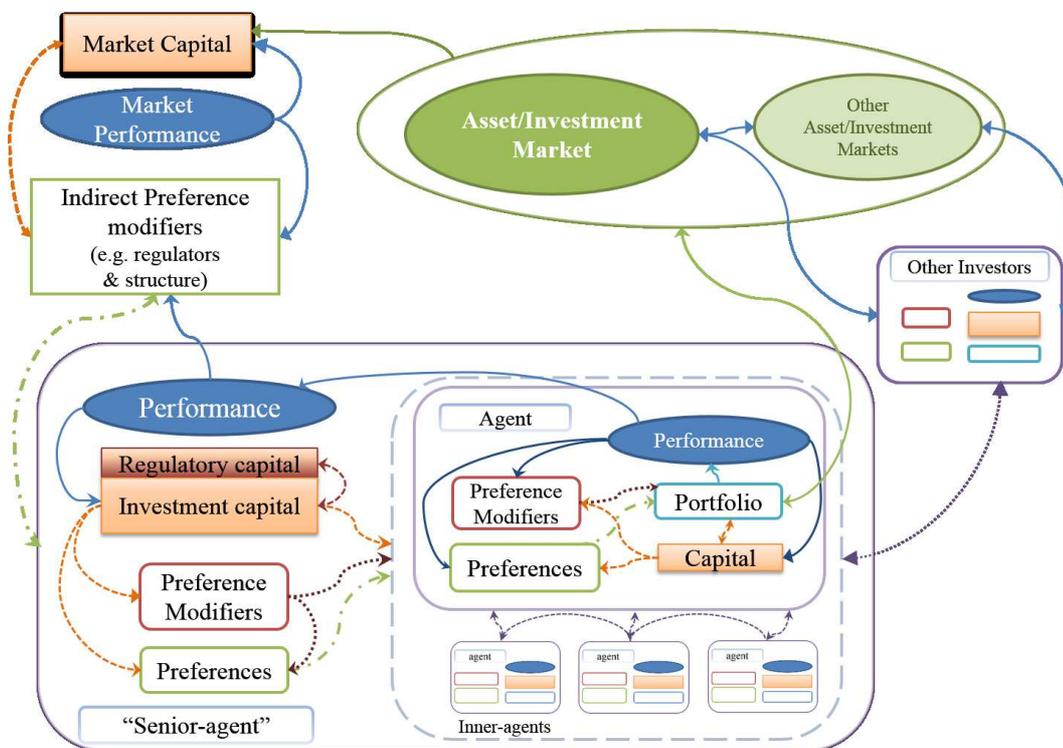
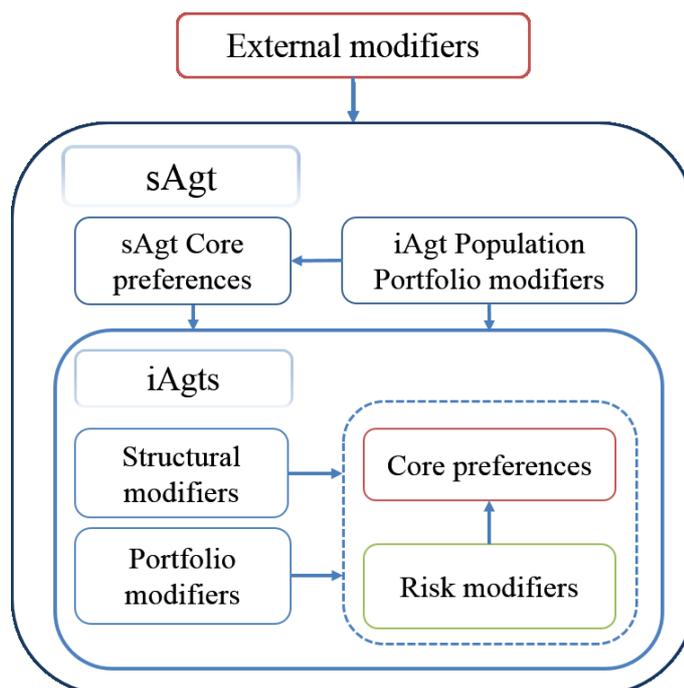


Figure 4.2.: Recursive Structure in a Financial System

In this schematic a super, or 'senior' agent subsumes a population of agents as described in Fig 4.1. Each of these 'inner agents' (iAgt) has its own personal preferences and preference modifiers - for an entity like an investment bank these would represent traders or trading departments for different markets or functions. iAgt may compete for capital or may cooperate, but must also respond to the economic status of their population and entity as a whole. The subsuming senior agent (sAgt) representing the entity as a whole has its own preferences and modifiers which it uses to manage and allocate capital to inner agents. Their performance using this capital affects the overall capital available for investment throughout the institution.

In the larger context a senior agent's behaviours are also affected by regulatory bodies, legal and market structures - providing a set of preference modifiers which feed into its own preferences and are then passed on to inner agents. Similarly, while regulators do not generally invest capital themselves, so that the idea of core preferences is perhaps inapplicable<sup>3</sup>, they do respond to market behaviours imposing regulations and guidelines which affect and modify economic activities in the system.

Of course, taking subsumption and recursion further, the sAgt in Fig 4.2 could itself be subsumed in the same way or otherwise form part of a population of heterogeneous competing, cooperating or otherwise co-evolving economic entities.



**Figure 4.3.:** Modifier Cascade

Stripping away the feedback effects and structures from the results of trading behaviours in the form of performance to focus only on preferences and preference

<sup>3</sup>

However as has been seen in the response to the GFC, even this is subject to change in extremis as governments and regulators stepped in both to acquire distressed assets or entities, and to inject liquidity directly into markets by effectively printing money.

modifiers, it is possible to see a modifier cascade in a subsumptive hierarchy. Developing the concept of preference modifying behaviours in Fig 4.3 risk modifiers are differentiated from portfolio and structural modifiers, while modifiers from subsuming agent structures and regulators act on agents as a whole.

In this arrangement core preferences and risk modifiers form the central cognitive decision process for individual preference expression. These decisions are modified within an agent by structural and portfolio modifiers which operate across an agent's entire exposure. Above this any subsuming senior agent preferences (both core and modifier) or overall external structural or regulatory modifiers operate.

When the different modification structures are not activated, either severally or together, an agent's core preferences are free to operate unimpeded, but when activated a much more complex, non-linear set of behaviours is likely to result. Whether or not this is intelligent is open to question, whether it is 'fit' is a function of the market. In keeping with Brooks' case for situation as a validation metric, successful aggregate behaviours over time will at least give the appearance of intelligence and exploring these behaviours is likely to be a source of usefully applicable domain knowledge in the design and understanding of economic policy and agency.

### **4.4. Experimental Platforms - Programming Environments & Design Choices**

This section revisits some practical issues in ABM design for financial systems informing the choice of Java and Repast as the environment for the current SHaaP implementation, prior to presenting an actual architecture in the remaining sections of the chapter.

A large number of modelling choices are required in developing ABMs for any system - not least of these is modelling environment, infrastructure flexibility and robustness, transparency and speed. Although a number of standards exist for agent-based platform development, such as those developed by FIPA (the Foundation for Intelligent Agents - an IEEE body), these focus principally on programming interoperability standards for software operating in commercial environments - see [108, 45] for a detailed discussion of these standards and critical comparison. In contrast there does not appear as yet to be a clear description of equivalent standards for experimental platform design.

As discussed in Chapter 3 such standards are a necessary part of principled methodology design, experiments and validation. Although there is some discussion of the need for clear, robust experimental protocols in the literature there is little evidence of cohesive work in this direction. A specific problem is the absence in agent-based computational economics of a clearly described architecture in which both structural and economic elements are represented. The SHaaP architecture sets out to address this shortcoming, combining domain knowledge of economic systems and structures with an established Java, agent-based modelling platform. The scope of the problem is large, reflecting the problem space, but the SHaaP architecture, with exploratory tools and performance measures represents a starting point to addressing it.

### 4.4.1. Programming Languages & Modelling Platforms

A number of agent-based modelling environments and programming languages are regularly used in ACE research.<sup>4</sup> These are typically generalised agent-based resources which require some adaptation by the user. Higher level packages such as SWARM (supported by the Santa Fe Institute and used in later versions of the SFASM), JADE and Repast (the platform adopted for SHaaPa) have GIS and visualisation capabilities, although it is up to the user to structure these according their model and problem domain.

The process for choosing a platform and programming environment for SHaaPa was itself somewhat heuristic. Based on the somewhat challenging experience of building a working model of the SFASM in MatLab documented in Chapter 5 several factors were obvious. As a development environment MatLab benefits from an enormous, widely used suite of robust, well documented and supported analytical packages and programming tools. Its scripting language is simple and powerful, however it is very slow to run for even relatively small models and more importantly difficult to debug with rich ABMs.

From the available agent-based software platforms Repast Symphony in Java was chosen, with additional analysis carried out in MatLab: it satisfies a number of criteria,

- systematic development and releases, with implementations in a variety of languages to suit ease of implementation vs computing power & platform requirements, including C++, C#, Java, Python and ReLogo
- an established core set of classes for creating, manipulating and organising agents and agent populations during simulations
- an extensive, well audited library of analytical tools and classes
- batch processing and parameter specification
- strong visualisation tools with GIS integration available within models and reporting of model parameters during simulations
- ready export of large volumes of simulation data for audit and external analysis
- relative to MatLab
  - Repast simulations in Java run significantly faster - up to 10x for similar models - satisfying a 'good-enough' heuristic criterion in terms of a trade off between computing speed and programming flexibility
  - Java offers object oriented features for developing agent classes which are attractive in terms of robust model implementation and development
  - Use of MatLab for post-simulation analysis provides flexibility in presenting and access to powerful off-the-shelf statistical and financial toolkits

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See Leigh Tesfatsion's Iowa State University resource, <http://www2.econ.iastate.edu/tesfatsi/acecode.htm>, for an extensive list of these.

A final key criterion in the selection process was evidence of a reasonable set of examples of agent-based models developed and available in Repast, including at least some models of economic systems. There exists a large published library of sample models developed and implemented on Repast demonstrating its capabilities. This includes one of the best documented derivative implementations and investigation of the SFASM[38, 39].

### 4.4.2. Repast's History & SHaaPa Implementation

The Recursive Porous Agent Simulation Toolkit (Repast), originally developed jointly by researchers at the University of Chicago and the Argonne National Laboratory in the late 90's, is a free, open-source agent-based modelling simulation toolkit, with good track record of stable systematic cross-platform releases<sup>5</sup>. The version used to implement the SHaaP architecture is Repast Symphony (RepastS) 2.2. This is a rich, interactive Java-based simulation environment, building on earlier versions of Repast in Java (RepastJ).<sup>6</sup>

RepastS allows graphical model development for rapid prototyping of simple models, however in SHaaPa this functionality is not used, preferring direct, explicit control of all the different agent and other classes. Similarly graphical probes and charting of intra-agent variables and performance measures are used only sparingly since they radically slow down simulation speed - in any case these can be set up ad hoc by users since they form part of the core RepastS functionality. Detailed analysis is done post simulation by recording all simulation data and porting this to MatLab. Repast provides standardised data recording tools and processes for this purpose.

### Documentation & Support

The Repast project has reasonably extensive documentation based largely around exemplars of Repast models. Although the reference manuals and API are quite detailed these exemplars are perhaps the most useful support element in getting models up and running and to understanding Repast modules and programming styles. To some extent, as usual, given the complexity of establishing agent-based models this is not a surprise.

The detailed programming relating to setting up the SHaaP architecture on the platform is not presented here.<sup>7</sup> Instead basic platform concepts and structures are

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5

Full documentation for the Repast project can be found at <http://repast.sourceforge.net/>.

6

RepastS was a very significant upgrade and improvement to RepastJ, automating and hiding much of the programming previously required to set up scheduling, reporting and graphical interface structures in RepastJ. Although the conversion process for existing models is fairly extensive, it was highly warranted as it reduces programming error risks which could feed through into simulations.

7

For that refer to the Repast User Guide; Repast Quickstart Guide and Repast API online.

set out together with an overall description of the class structure, individual classes and relevant methods. Specific additional detail relevant to the architecture is to be found in Appendix A.4.

As Repast was developed as a generic agent-based platform many of the available structures are not particularly relevant to economic agents, although some which facilitate modelling network interactions, such as projections, may prove useful in future extensions of the SHaaP architecture. It is safe to say that the platform is itself extensive and the feature set has elements to support a wide variety of programming styles and tastes. Overall it has proved, and continues to prove, to be a stable, reasonably friendly environment to work in.

### 4.4.3. Java, Object-Oriented Design & SHaaPa

The modelling approach using RepastS follows an object-oriented design path using Java. This sits well with agent-based simulations, improving robustness by encapsulating component elements & behaviours, while at the same time allowing flexibility & extensibility through the use of generic types developed in the architecture.

Subsumptive structures are used throughout SHaaP models: in agent populations, subsuming agents and within economic agents themselves. Other than speed benefits relative to MatLab, RepastS using Java offers a strongly-typed, class based, object-oriented environment. Model development in RepastS comes bundled in a full Eclipse IDE package allowing normal Java development and programming processes to be followed. The SHaaP architecture implementation on RepastS makes use of abstract classes, interfaces and inner classes to maintain robust encapsulation of agents and behaviours<sup>8</sup>.

The general approach has been to attempt to standardise and encapsulate all the main processes and behaviours. Given the potential richness and complexity of economic agents this is a somewhat iterative procedure, as particular processes are identified and their interactions with other components developed in the architectural design are created and integrated. Once again subsumption as a development paradigm was beneficial here.

Extensive use of abstract classes is made to create generic types of the main agent and architecture structures to allow polymorphism with type compatibility. Passing and manipulating instances of preference and modifier classes in a generic manner prevents problems in creating multiple, hard to maintain extensions of parent classes, particularly given an objective to have agent populations with truly heterogeneous preferences. Interfaces and inner classes are used to achieve this: interfaces allow additional behaviours and type compatibility to other preference classes, while inner classes allow multiple instances of particular standard class types, such as portfolio modifiers, within agent classes.

Generic array-based structures and classes are used to hold and manipulate agent populations, strategy rules expressing preferences, and portfolios.

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There is no claim here however to be an expert Java programmer or that the program structures are optimised - the key implementation concern was robustness.

Overall, particularly given its widespread use, relative robustness and availability of extensive libraries, Java and RepastS is a good working platform and suits SHaaP development well with only minor issues<sup>9</sup>.

### 4.5. SHaaP Architecture Implementation

In this and the remaining sections of the chapter the actual SHaaP implementation developed on the RepastS platform for use in the case study experiments and future work is set out schematically and described.<sup>10</sup> The design follows the economic agent representation shown in Figures 4.1 & 4.2 creating a population of Inner Agents (iAgt) encapsulated and subsumed by Senior Agents (sAgt). This establishes the cascading subsumption structure for preferences and preference modifiers shown in Fig 4.3.

Given uncertainty, risk and non-linear behaviours in economic systems the success or failure of an individual agent in particular economic conditions is unlikely to be particularly informative: survival and mortality effects for single instantiations are not readily interpreted. Rather the performance of heterogeneous agent types within larger populations over time and in different market conditions is important. To capture this populations of iAgt instances are grouped together and subsumed within sAgt in the SHaaP implementation.

#### Main Population Structure

Figure 4.4 shows the practical population structure for an sAgt subsuming iAgt populations.

Here sAgt are the top level of economic agents, though of course additional subsuming agent layers could be introduced, with sAgt, or super-sAgt, subsuming other sAgt. For the moment however this is sufficient, both in terms of the complexity of the population structure, and for the experimental questions to be addressed.

As can be seen, each sAgt has a number of iAgt groups which are then sub-divided into sub-populations. This creates opportunities for partitioning by agent type and for networking capabilities allowing social learning or direct interaction within sub-populations. Networking and inter-iAgt social learning capabilities have not yet

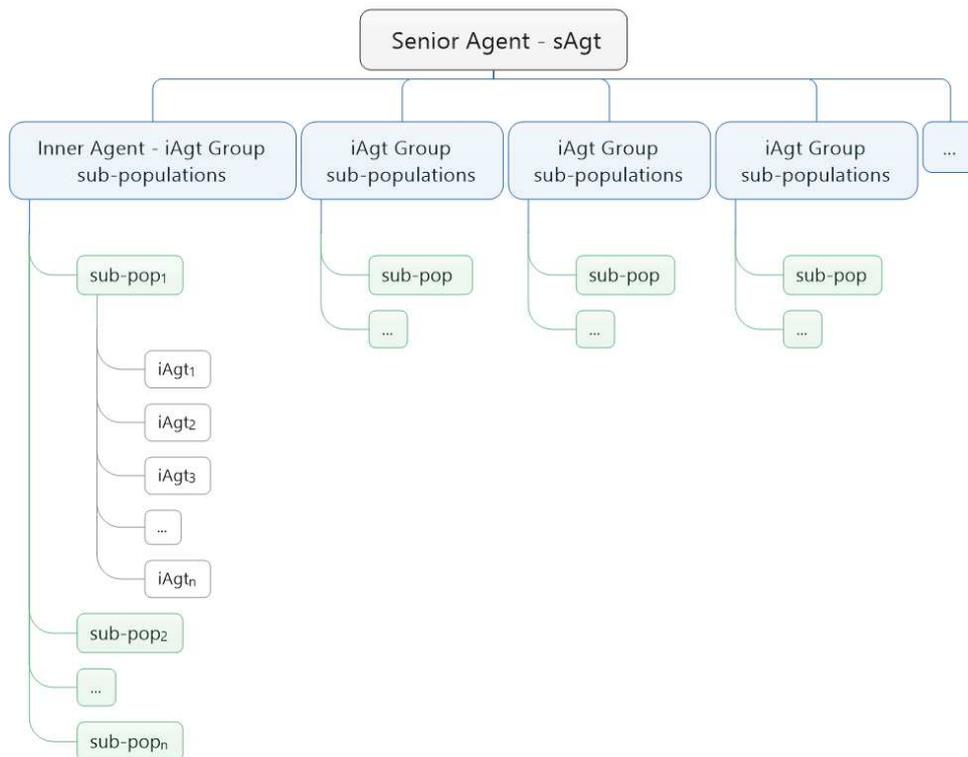
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These are mainly due to Java conventions and language structure - the principal drawback, or potential pitfall, being exposure to 'copy-by-reference' errors when passing instances of components like core preferences around. The solution for this, though not ideal and somewhat convoluted, was to develop extended parameter collection routines, where specific new instances of objects are created and populated with parameters of referenced objects to give true, isolated copies.

<sup>10</sup>Except for examples of pseudocode to illustrate programming structures or algorithms no Java source code is presented here, since this in itself would not be particularly informative and is in any case several thousand lines long. Rather, full archive copies of the SHaaP implementation are submitted separately as part of the thesis documentation.

Schematics & tables showing model structure and parameter settings, interfaces etc. are set out in the following sections and in the Appendices.



**Figure 4.4.:** sAgt Population Structure

been implemented, but combined with RepastS’s functionality this is potential future work. Presently the group and sub-population structure provide flexibility and the direct capability to explore differential, path dependent behaviour in financial systems.

### Inner Agent Population Structure

Inner agents are themselves population-based. In addition to its embodying structures, each iAgt subsumes a population of investment rules (rulepackets) competing for expression. The structure of these rules is explained in detail in Section 4.5.2, however Fig 4.5 shows their population structure within an inner agent.

#### 4.5.1. SHaaP Agent RepastS Implementation

The following sections set out agent structures and their Java implementation in RepastS following the ideas outlined in Section 4.4.3. Simplified schematic diagrams for Java classes and interfaces described are provided here and in Appendix Section A.5, which also has detailed tables for these components, setting out their methods and other model structures.

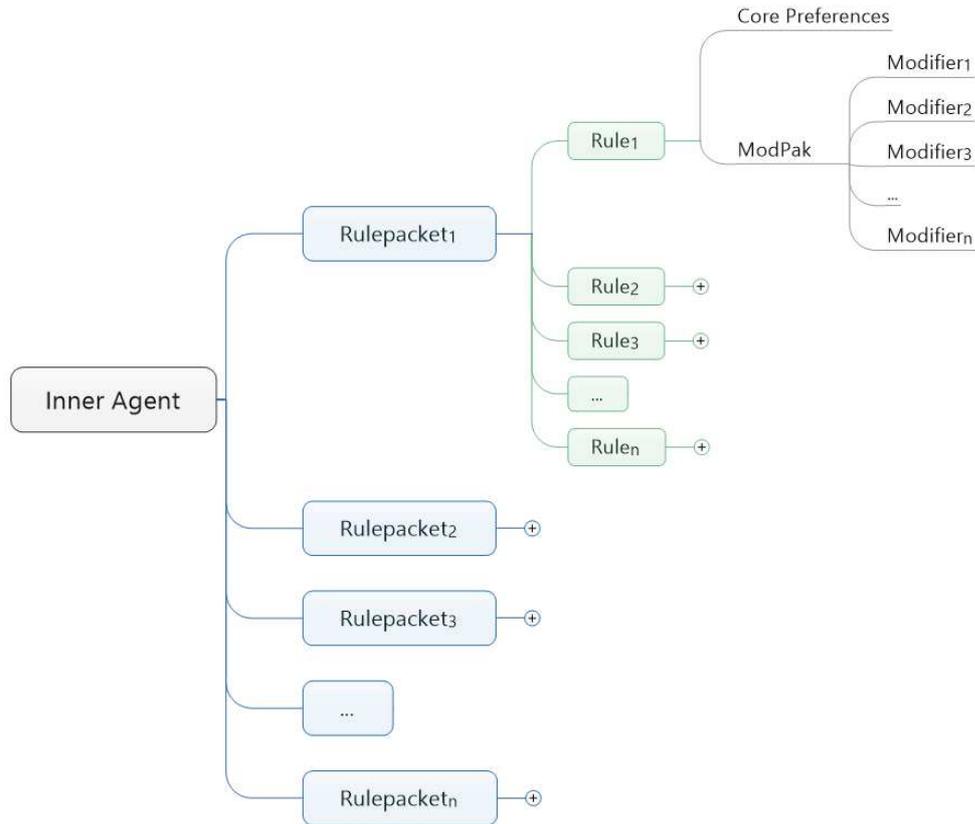


Figure 4.5.: iAgt Rule Population Schematic

### Inner Agents - iAgt

Inner agents form the basic functional unit in the architecture, being the first subsuming level which is capable of carrying out a full range of economic activities, having not only preference structures, but also necessary other accounting structures, they are sufficiently embodied to interact with their environment.

Inner Agents in the SHaaP architecture are extensions of the abstract economic agent class `Agent`<sup>11</sup> - these child classes (with a naming format of `Tn_iAgt`) form concrete types with particular behaviours and characteristics. Instances of `Agent` are effectively complete economic agents with their own preferences, modifiers and other structural behaviours.

A simplified schematic for the `Agent` class is shown in Fig 4.6 - while not a full UML diagram<sup>12</sup>, this sets out the main structures associated with the class and its child

<sup>11</sup>

As a convention, in this chapter and the thesis as a whole, where model structures are being described capitalisation indicates a specific object class, interface or implementation of a particular class. Hence `Agents` and `SeniorAgents` are abstract classes of agent types, `T1_iAgt` is an example of an inner agent implementation extending `Agent`, while `PrefModifier` is a preference modifier interface used in implementing preference modifier classes.

<sup>12</sup>

classes.

As they are instantiated concrete instances of iAgt are responsible for creating the preference objects, both core and risk modifiers, and passing these to a rulepacket population which they also generate. At the same time they create sets of structural and portfolio modifiers, such as budget and total portfolio size constraints, which act across the inner agent's entire portfolio rather than directly. This follows the organisation shown in Fig 4.3: the sequence of events for this and the top level algorithm for the model is set out in Section 4.5.4.

Inner agents maintain ArrayList based data structures recording investment and portfolio performance. Each iAgt has a population of rulepackets (as Section 4.5.2 below explains, each rulepacket, as an instance of the RulePacket class, represents a single set of core and risk modifying preferences), choosing from these based on performance to make portfolio decisions. An iAgt's portfolio rules take copies of the best performing rulepacket's preferences and use these, subject to portfolio and budget constraints in economic preference expression.

### **Senior Agents - sAgt**

The simplified class diagram for the Senior Agent class in Fig 4.7 follows the same design pattern as for the iAgt Agent class. Unsurprisingly, given a subsumption structure and recursive economic agent behaviours, many of the functional elements are repeated. In Senior Agent however rulepacket populations are replaced by iAgt populations.

In both the Agent and Senior Agent classes learning, where activated, is currently carried out only at the rulepacket population level: adaptation at agent levels however still occurs as competition for expression and differential individual performance allows particular behaviours to be chosen over others. Additional learning and adaptation algorithms can be added as the architecture is developed for specific experimental studies: at this stage, given an already high level of combinatorial complexity, it is a conscious choice not to permanently introduce it as yet. However a simple PSO structure is described and tested in the experiments in Chapter 6, which may form the basis of a learning mechanism.

### **4.5.2. Rules & Rulepackets, Preferences & Preference Modifiers**

Abstract classes are used for preferences, preference modifiers and rules, allowing specific concrete types of each class to be built extending common behaviours to concrete types.

Core preferences and risk modifiers for specific investment decisions are bundled together into rules for expression. Preferences and preference modifiers are added to these rules as preference objects using Java interface classes: these interfaces require basic embodying methods to be implemented in concrete instances helping to enforce a level of standardisation in model development.

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A full UML diagram would have limited explanatory value here given the number of variables, particularly in the form of inner classes and instances of classes in the Agent structure.

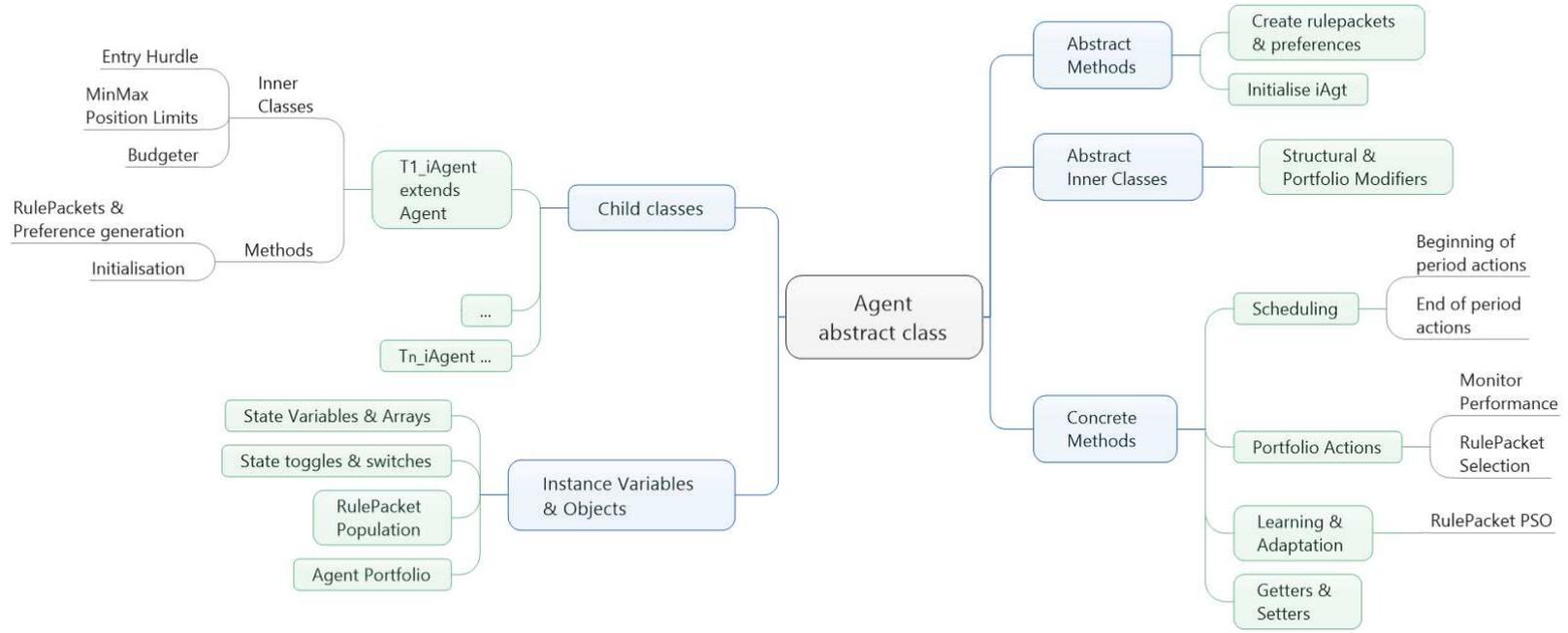


Figure 4.6.: Inner Agent Class Schematic Structure

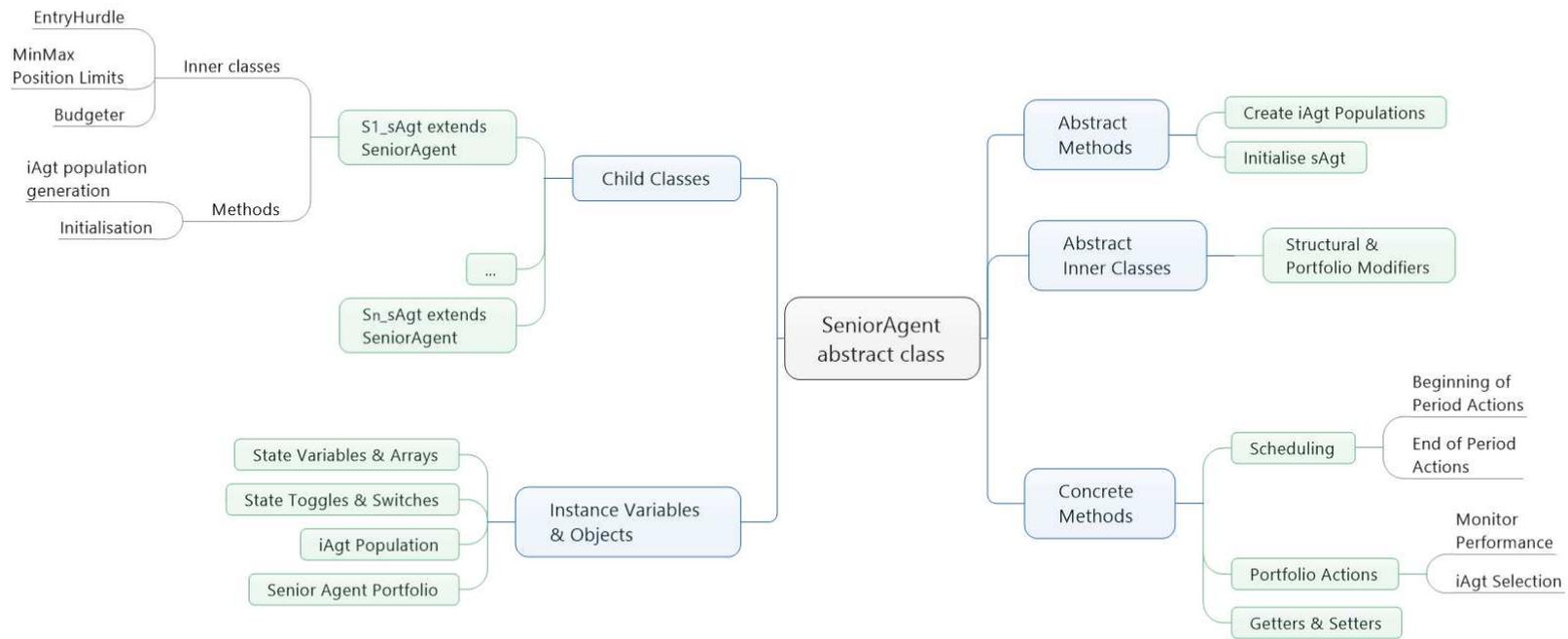
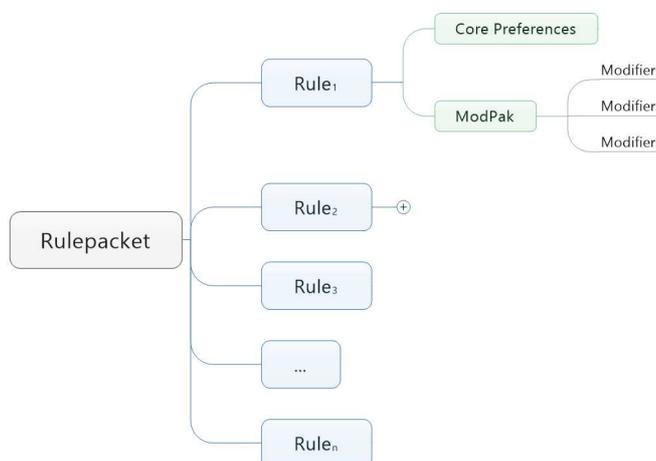


Figure 4.7.: Senior Agent Class Schematic Structure

Portfolio modifiers & structural constraints, since they act on whole portfolios rather than single rule behaviours, are instantiated as inner classes within agents, however, again Java interfaces are used for each portfolio modifier type to require standard embodying methods to be implemented. These classes and interfaces are set out below.

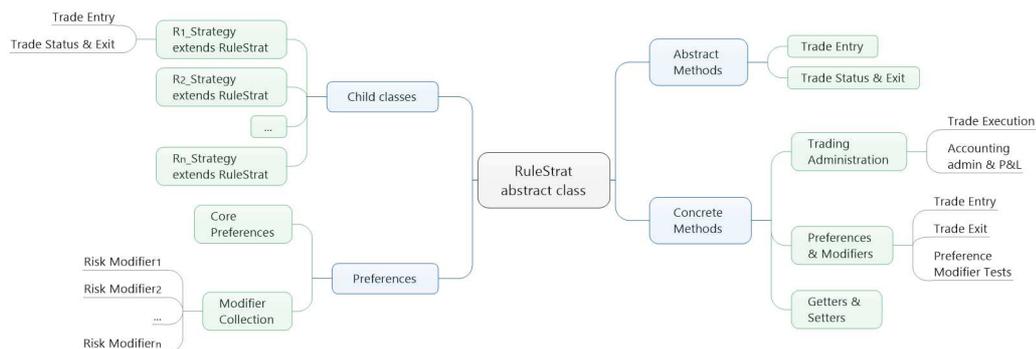
### Rules & RulePackets

The RulePacket design reflects the stochastic component to both trade entry and exit decisions vs. actual performance in any given period. Apparently equivalent trading decisions in similar circumstances can give quite different realised returns. Each RulePacket holds multiple copies of the same core preference and modifier structures so that these may be active at different times giving increased granularity to performance metrics.



**Figure 4.8.:** Rulepacket Schematic

RulePacket rules copies are bundled together as shown in Fig 4.8. Aggregate performance metrics for all the rule copies in a RulePacket are maintained and updated in the RulePacket during model runs. Each rule in a rulepacket is a concrete instance of the RuleStrat class - Fig 4.9 shows a simplified class diagram for this class.



**Figure 4.9.:** RuleStrat Class

Rulepacket populations engage in 'paper trading': these are theoretical trades, based on their preferences and market conditions, where no cumulative accounting effect is recorded, only the effective performance based on entry and exit. Figure 4.1 shows pseudocode for these operations for a single period. By aggregating data across all rules in a packet this establishes performance data for each rulepacket instance and hence for the combination of core preferences and modifiers in use.

---

**Algorithm 4.1** Rulepacket Paper Trading Pseudocode

---

```

1 Beginning of period
2   For each rule in packet
3     For current trades
4       Check core preference exit criteria
5       Exit if met
6       Check preference modifier criteria
7       Exit if triggered
8     If there is no current trade
9       Check core preference entry criteria
10      Register new trade if satisfied
11    Update rule instance performance
12  End
13 Update whole rulepacket performance
14 End period

```

---

**Core Preferences** Concrete core preference classes implement the CorePreferences interface extending the abstract class Core, and are used by rules and agents using those rules for core preference expression and behaviours. Beyond preference criteria, be they heuristic or quantitative, core preference expression shares some common behaviours. The CorePreferences interface requires methods common to all core preferences to be implemented.

These methods essentially

- set out the core preference criteria

- set out the behaviour, i.e. the preference choice, given the economic environment
  - to make an investment & how much investment to make
  - to exit an investment & how much to exit

The algorithms behind these choices are encapsulated in Java methods. Other methods may be added to, again, subsume more complex, or simple, structures. However the overall call on CorePreferences object types is the same throughout the architecture, which ensures some level of robustness and facilitates ongoing investigation of subsumptive structures and behaviours.

### Preference Modifiers

Preference modifiers are functionally divided into risk modifiers, portfolio modifiers and structural modifiers. Risk modifier classes implement the PrefModifier interface extending the abstract class RiskMod. Operating in conjunction with core preferences at individual rule levels, they may be thought of as part of the overall cognitive process expressing agent preferences.

Portfolio and structural modifiers operate across entire agent or portfolios of agent exposures. Again they follow the same design pattern as risk modifiers, concrete classes are instantiations of abstract types implementing interfaces with associated behaviours.

The distinction between structural and portfolio modifiers is somewhat blurred, with structural modifiers forming a subset of portfolio modifiers. Restrictions on trading frequency are an example, these may be imposed by markets or external regulators, but may also, as in the case of Dacorogna's model discussion in Section 3.3, be internally imposed since models which trade too frequently or are too sensitive to noise may perform poorly.

As with core preferences, preference modifier interfaces share some common behaviours. Setting out,

- modifier criteria
- modifier behaviours
  - when to override or augment core preferences
  - and by how much

Again, the use of interfaces ensures some level of robustness since it gives a standard set of method calls for the particular modifier type, as well as allowing polymorphism in an object-oriented framework.

### 4.5.3. Agent Choice & Performance Metrics

Following the subsumption hierarchy shown in Fig 4.5, iAgts choose which rules to use in any given period and hence their core preferences and modifiers based on historic performance of individual packets in the agent's rulepacket population. As

set out in the previous section, each iAgt maintains a set of portfolio rules. These rules form the behavioural centre of an iAgt, choosing preferences for expression from the iAgt's rulepacket population based on historic performance.

Similarly sAgts choose from their iAgt populations based on iAgt performance. In the current implementation sAgts adopt the preferences of the most successful iAgt individuals and use them to make investment decisions. An equally valid mode of expression would be to differentially allocate more, or less, capital to successful, or unsuccessful, iAgts, with the result that their investment capacity and size of investments would change. Both modes are themselves forms of meta-heuristic preference operators: the current mode being a form of copycat heuristic for proprietary change; the latter a form of portfolio preference modifier where iAgts represent the portfolio elements.

**Performance Metrics in SHaaP** Performance metrics are a critical experimental design choice in terms of validating experimental results and inferences. This has been discussed both in this chapter and in Chapter 3. For economic agents evaluating rule and preference performance similar criteria apply: realistic, situated return measures are important where profitability and resilience of profitability are survival and success criteria.

Agents evaluate rulepacket performance and are in turn evaluated by subsuming agents using the risk-adjusted performance when selecting preferences for expression. As with other components in the architecture, this can be modified relatively easily to suit the form of model, or to evaluate different measures. In the current implementation the symmetric risk-adjusted measure,  $X_{eff}$ , described in Section 6.3.4 and Appendix A.2 is used, although an asymmetric form  $R_{eff}$  has now also been implemented for use in future experiments.

### 4.5.4. Scheduling & Timing - The Top Level Algorithm

Scheduling within the architecture is naturally a critical simulation component. In the SHaaP architecture this is carried out by the ShBuilder class which is called by RepastS at runtime. ShBuilder sets up all the state variables, model parameters, contexts and simulation environment, calling the simulation GUI into existence before creating agent populations.

As the simulation progresses ShBuilder has control of scheduling: this is a hierarchical process iterating over all agents within populations and sub-populations as shown in Fig 4.2.

**Algorithm 4.2** SHaaP Scheduling - Top Level Algorithm

---

```
1 Call ShBuilder
2   Set up common world; distribution;
3     & agent parameter object instances
4   Create main 'grid' context
5     Initialise parameters
6       Read batch settings & GUI changes
7     Add common objects
8     Create sAgt instances
9       Create all iAgt; parameters;
10      preferences & rules
11     Add sAgt to context
12
13 Begin main schedule method & run simulation
14 For each timeseries period
15   Update asset price & information
16   For each sAgt
17     Carry out BOP(beginning of period) actions
18     For each iAgt Group
19       For each Group iAgt Subpopulation
20         For each iAgt
21           Carry out iAgt BOP actions
22             Update Rulepackets
23             Check core prefs & pref. modifiers
24             Do paper trades
25             Update Rulepacket performance
26             Carry out Rulepacket learning
27           End iAgt BOP actions
28
29           Carry out iAgt EOP(End of period) actions
30             Update iAgt portfolio
31             Check core prefs & pref. modifiers
32             Close old trades
33             Choose new rules & do new trades
34           Update iAgt performance
35           End iAgt EOP actions
36         End
37       Identify top iAgt in each Subpopulation & Group
38     End
39   End sAgt BOP actions
40
41   Carry out sAgt EOP actions
42     Update sAgt portfolio
43     Check core prefs & pref. modifiers
44     Close old trades
45     Choose new rules & do new trades
46   Update sAgt performance
47   End sAgt EOP actions
48 End
49 End
50
51 Output simulation data & records to file
```

---

### 4.5.5. Learning, Evolution & Adaptation

Learning and evolution are important areas for ongoing development and study in the SHaaP architecture. Even without specific memory capabilities beyond performance metrics which capture historic information, subsuming agents can be adaptive and apparently successful - this is an important finding of the work in Chapter 6 .

Learning in the form of optimisation and parameter tuning may fit well with particular subsumed functional elements such as forecasting algorithms, however these are independent of the top-level agents. Where success metrics for subsumed components are orthogonal to those of an agent as a whole the problem is relatively simple. However where there is only one performance metric, such as risk-adjusted returns, across multiple components attribution of contributing factors becomes problematic.

Following the subsumption heuristic ethos then, the preferred route in this thesis is to develop modular components individually. Then to assess and test elements before incorporating them in larger structures where testing will need to be re-done to understand their interaction in a larger system. There is no assurance that a component successful when tested in isolation will be successful in a larger structure. Again validation processes are important to assessing situated performance and explanatory value.

As such learning mechanisms will form a significant area for future work and investigation. In Chapter 6 a preliminary implementation of a basic PSO is set out forming the basis of a set of experiments discussed there. This maintains the modular development approach.

### 4.5.6. Other Architecture Structures

Global variables and commonly used structures are set up as standard classes within the architecture. The aim here remains to standardise and ensure robustness in model development.

In its Java implementation SHaaP source code is segmented into various categories under an overall package name sHaaP\_v1. The main class called to set up and run simulations, ShBuilder, is held in SHaaP in the top level package, sHaaP\_v1. Other components are held in sub-packages according to function. These are set out in Table 4.1.

Individual class lists and descriptions are set out in Appendix A.5. This appendix also contains practical implementation notes on SHaaPa and Repast specific structures.

## 4.6. Ongoing Development & Conclusions

This chapter presented the SHaaP architecture - an original, domain knowledge driven, experimental architecture for developing and exploring economic preference expression. The architecture's design addresses the concerns raised around methodological robustness & transparency in existing approaches. Using a well established

Package	Description
sHaaP_v1	Top level algorithm - ShBuilder. Sets up simulation.
sHaaP_v1.agents	All agent abstract classes, concrete classes and agent population classes.
sHaaP_v1.assets	Classes relating to economic assets. Abstract classes & concrete classes defining particular assets; time series; asset specifications; and asset states.
sHaaP_v1.common	Global and common parameter classes; utilities and distribution classes. Common parameters and random distributions used throughout SHaaP models are collected here so that they can be readily identified and, when modified, changes are not required throughout classes using them.
sHaaP_v1.prefs	Preference classes. Concrete instances of particular core preferences & preference modifiers; interfaces for preference class implementation; and collection classes for multiple preferences or modifiers.
sHaaP_v1.strat	Strategy and rule classes. Concrete instances of rules encapsulating core preference and preference modifier instances; interfaces; and rulepacket collection types.

**Table 4.1.:** SHaaP Project Package Structure

and supported Java-based, agent-based modelling platform, Repast Symphony, to develop a clear, extensible architecture for exploring economic preference expression. Repast's object-oriented programming environment, with well documented libraries of mathematical and analytical tools, was an important choice in this process, adding to the overall strength of the modelling environment.

The design incorporates novel aspects in the form of subsumption and population-based structural meta-heuristics, which allow functional decomposition of economic preferences reflecting observed behaviours and processes in situated environments.

As discussion in Chapter 6 emphasises, while the architecture itself, with its core subsumption and population-based structural elements, is in place, its development as an experimental platform is still a work in progress and remains exploratory, forming a research area in itself. The work presented here provides a fully functional experimental platform while limiting the scope to a reasonable extent within the context of the thesis as a whole.

Key areas for future development are additional performance measures for exploratory data analysis tools, and as fitness measures in population adaptation. Domain driven structures will be important: most significant of these seems likely to be network and social learning mechanisms. These are functional components in developing realistic, situated heuristic structures. Their design and implementation will once again be driven by the specific systems under investigation.

The main architecture itself however is at a stage where these developments are feasible and can be carried out in a principled manner, while experiments using the architecture, such as those in the case study, offer potential for operationally mean-

ingful contributions to the literature on agent-based models of economic preference expression.

# 5. Performance & Fitness Measures: Verification & Validation

*'Buy in the valleys, sell in the rallies - if only I had a brain.'*

Trading aphorism, unattrib.

## 5.1. Overview

The choice of appropriate performance measures and validation criteria are important to any methodology and agent-based models are no exception. This chapter builds on the review in Chapter 2 and Chapter 3, where these and other methodological issues were discussed. New agent-level performance measures are proposed and developed here - these are model independent measures which measure relative risk-adjusted performance in populations of economic agents. These population relative performance (PRP) measures are potentially important exploratory tools and are used in conjunction with the SHaaP architecture case studies in Chapter 6.

A case study, implementing a MatLab version of the Santa Fe Artificial Stock Market (SFASM), is described in which these measures are demonstrated. This case study also provides an opportunity to consider and critically examine methodological issues in such models with the benefit of seeing how they impact on an actual model in practice, providing direction for the development of the SHaaP architecture.

While noting the importance of highlighting methodological issues and emphasising the need for principled model development, the main contribution of the work in this chapter is to demonstrate the effectiveness of the PRP measures, and how they can be applied in exploring agent and population behaviours.

## 5.2. Performance Measures

Appropriate, realistic performance measures are vitally important in model validation. In the SHaaP structure described in the previous chapter they are also essential to agent performance in selecting preference behaviours. This section sets out the measures developed for testing in this chapter.

### 5.2.1. Model Independent Measures

Section 2.7 raised the question of how to measure efficiency in artificial financial markets (AFMs) given the 'joint hypothesis' problem. Benink et al [9] (hereafter

'Benink'), working on neo-Austrian AFMs, propose a model independent approach and develop an empirical measure using excess profits. This is of interest not only because it addresses the joint hypothesis issue, but also because it incorporates a relatively realistic risk-adjusted performance measure. Their approach is uncommon in the artificial financial markets (AFM) literature, where statistical feature sets and REE benchmarking are generally preferred measures. As has been pointed out in earlier chapters, statistical properties are not a direct test of agent behaviours and REE benchmarks are reliant on the underlying asset pricing model: when looking at equilibria it is not possible to differentiate between poor AFM structure and poor asset pricing models.

In Benink's approach fitness and performance are measured in terms of *excess profit* relative to a passive, moving benchmark. This passive benchmark is defined as an agent's position if the action generated by a rule had not been effected in a given time period,  $t$ , i.e. no trade occurred. The intent is to isolate *market* returns (the returns from a movement simply because the market moved anyway) from *trading* returns, where an active trading decision occurs. Differentiation between passive, market returns and excess return due to trading performance is a common one in finance where they are referred to respectively as 'beta' and 'alpha' - so this demonstrates some element of situation in the construction of this measure.

However as a measure of performance Benink's definition is in itself slightly awkward and potentially problematic. Firstly, it requires each strategy to have the same measurement period. Where traders' strategies are heterogeneous in investment horizons it may be inappropriate. That said, it remains a useful generalized starting point which may be addressed separately in model structure by careful definition of strategy investment periods.

Secondly, and more fundamentally, as the following section sets out, it is presented as an 'efficiency' measure insofar as it attempts to capture non-market returns. In the context of Benink's models this may be appropriate since his markets are by definition closed systems where total market returns in any single period across the whole population inevitably sum to zero. However in situated models which are open systems, and where agents trade over multiple time periods, it is difficult to see that what it captures is actually some form of market inefficiency - more literally it represents a form of population-based relative, risk-adjusted performance for excess returns between agents.

As such, while acknowledging Benink et al's contribution, for open systems such as those in the situated case study in Chapter 6, and more generally, it will be referred to as *population risk-adjusted relative performance* or simply *population relative performance* (PRP), since, as set out below, all performance measures used in analysis here have a risk-adjustment component.

In Benink's work, and in the SFASM described in Section 5.3 the systems are closed so the excess return measures set out below are applied, however for the open, situated systems described in Chapter 6 gross returns are used to construct the relative performance measure. This is functionally equivalent to the excess return measure except that gross returns are substituted, so also falls into the class of PRP measures, while requiring no assumptions about agent strategy types.

### 5.2.2. A Population Risk-Adjusted Relative Performance Measure Definition

This section sets out Benink's definition of a population risk-adjusted relative performance measure.

Following Benink's approach, for a group of  $J$  traders with  $I$  strategies, for a trader  $j$  using trading strategy  $i$  for the period from  $t - 1$  to  $t$ , excess profit  $e_{i,j}$  is defined as,

$$e_{i,j}(t, t - 1) = (\delta V_{active}(i, j, t) - \delta V_{passive}(i, j, t)), \quad (5.1)$$

where  $\delta V_{active}(i, j, t)$  increase in portfolio value between  $t$  and  $t - 1$  for  $j$  using strategy  $i$  and  $\delta V_{passive}(i, j, t)$  is the increase in portfolio value from simply retaining the portfolio position from the previous period. This can also be written,

$$e_{i,j}(t, t - 1) = \delta n_{i,j}(t) \delta p(t), \quad (5.2)$$

where  $\delta n_{i,j}$  and  $\delta p$  are the change in portfolio position and the change in stock price respectively. The excess profit between any two periods  $t$  and  $t'$  is given by,

$$E_{i,j}(t, t') = \sum_{q=t'+1}^{q=t} e_{i,j}(q, q - 1), \quad (5.3)$$

which allows summary measures to be calculated, such as average profit for a given period,  $(t, t')$ , for a given strategy across all traders employing that strategy,

$$E_i(t, t') = \frac{1}{N_i} \sum_j E(t, t'). \quad (5.4)$$

Given an assumption that noise in a market affects prices as well as information, there is a non-zero probability that purely by chance a trader will make or lose money in any given trading period. So in measuring the effectiveness of a strategy or set of strategies it becomes necessary to report on performance in terms of the magnitude of excess profits *and* the variance of these excess profits.

Benink makes the case that reporting absolute profits is not as useful as relative profits. Notwithstanding the concerns expressed in the previous section, this has intuitive appeal as much of the real market is organised on this principle - fund managers commonly report on and trade to beat benchmark indices which are not static; traders are judged against their peers. In economic systems, where there is competition for constrained resources and evidence of ecological rationality which also drives evolution and adaptation, relative performance is an appropriate measure.

The result is a measure which is simple to derive, situated in structure, and may also may provide the basis for a suitable population-based fitness measure within groups of strategies or agents.

The excess profitability measure,  $E$ , defined in Eqtn 5.4 is used to define a relative measure,  $I$ , between two strategies,  $i$  and  $j$ , such that

$$I_{i,j} = \frac{(E_i(t, t') - E_j(t, t'))}{\left( \frac{\sigma_i^2(t, t')}{N_i} + \frac{\sigma_j^2(t, t')}{N_j} \right)^{\frac{1}{2}}} \quad (5.5)$$

where  $\sigma^2(t, t')$  is the variance of excess profits for a given strategy. The *relative excess return*,  $I_{i,j}$ , is defined in terms of the difference in excess returns between  $i$  and  $j$  divided by the standard error to give a statistical confidence measure in the difference.

Using Eqtn 5.5 the structure of relative excess returns across an entire market or population of strategies can be generated. For a strategy population,  $m$ , an Excess Return Matrix,  $I^m$ , is generated for a given period,  $(t, t')$ , with matrix elements  $I_{ij}(t, t')$ . Where the strategy population is complete and the market a closed system, this matrix represents the entire market and gives a complete description of all the relative inefficiencies between strategies in the market. This allows a single excess return measure to be constructed,

$$\mathcal{I}^m(t, t') = \frac{1}{N'} \left( \frac{1}{2} \text{Tr}(-(I^m)^2) \right)^{\frac{1}{2}} \quad (5.6)$$

where  $N'$  is a normalisation factor, such that  $N' = N(N-1)/2$  and  $N$  is the number of trading strategies in the population.<sup>1</sup> The approach has considerable intuitive merit insofar as it allows *specific* strategies to be considered over time against a dynamic benchmark - this is analogous to situated measures of performance.

### 5.2.3. Risk-adjusted Performance

'Risk adjusted performance' measures are commonplace in applied situations, where large volatility in profits is detrimental to survival either as a trader, bank, hedge fund, or indeed any economic entity. One of the most commonly recognized such measures is the so-called 'Sharpe Ratio'.<sup>2</sup>

First described by Sharpe in 1966 [117], the Sharpe Ratio,  $S$ , gives a single measure to compare performance of portfolios, assets, traders or trading strategies,

$$S = \frac{R_{ast} - R_{ref}}{\sigma} \quad (5.7)$$

where  $R_{ast}$  is the expected return of the asset,  $R_{ref}$  the expected return of a reference asset such as the risk free interest rate and  $\sigma$  is the standard deviation of differential return,  $R_{ast} - R_{ref}$ , over the period. Sharpe [118] is careful to stress that it is *excess*, or differential, returns that are important: simply taking the ratio of mean return to the standard deviation of return of a single investment loses information that is important in real environments.

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<sup>1</sup>

$I^m$  is antisymmetric, since by definition across any elements measurement of profit is relative, thus  $I_{i,j}(t, t') = -I_{j,i}(t, t')$ .

<sup>2</sup>

There are many other similarly inspired, or derivative, risk-adjusted measures, including Modigliani risk-adjusted performance, a dimensionless derivative of the Sharpe Ratio, and Sterling, Calmar, and Sortino Ratios which are asymmetric and look at returns vs maximum cumulative trading losses or vs targeted returns. Each has its own flavour and proponents. The Sharpe Ratio remains the most commonly used despite its instability, simplicity and transparency of assumptions being a key strength.

Examining Eqtns 5.5 & 5.7, it can be seen that their general forms are strikingly similar - Eqtn 5.7 differs insofar as it uses an expected variance of differential return instead of a standard error, but otherwise as performance measures they have the same general structure.

In applying this measure two structural issues should be noted.

- Firstly, as with the Sharpe Ratio, the relative excess returns measure becomes unstable as the variance of excess returns across strategies approaches zero. Dacorogna's risk-adjusted return measure,  $X_{eff}$ , described in Section 3.3.5.1 and detailed in Appendix A.2 does not suffer from this effect. In the pilot study which follows a relative excess returns measure based on  $X_{eff}$  is constructed for comparison.
- Secondly, it applies only to definably different *strategies* or *agents*. As an agent-level measure this is appropriate, however it means that where there no clear distinction between different rule recommendations and trades it will not be suitable. In the SFASM this means that it cannot be applied to specific rules within agents. However this serves to highlight a structural issue in the model rather than the measure: in the SFASM agent performance is not a function of a situated measure, rather forecast accuracy at a rule level determines behaviour. The question this leads to is what the SFASM is actually modelling.

### Model Independent Measures In Use

Benink [9] set out a series of experiments with a highly stylized AFM in which agents choose between a risky asset and a risk free asset, and are described in terms of a bias with respect to expected values for the risky asset. In each trading period agents adopt a position in the risky asset according to a defined probability distribution and the market clears via continuous double auction mechanism. Key experimental variables were the degree of bias for a trader type, the number of different trader types in a population, and the relative proportions of different trader types. Although clearly not situated and using agents which are minimalist in terms of preferences, this market structure proved a useful mechanism for exploring concepts of market efficiency, and demonstrating the use of the relative excess return measures.

Agent performance was recorded in the form of excess returns Eqtn 5.2. Homogeneous populations were clearly characterized by unimodal distributions of excess return with total excess return for the population summing (by definition since the system is closed) to zero - this unimodal distribution is taken as demonstrating market efficiency. In a market with heterogeneous traders at the end of the experimental runs distinct bimodal distributions were observed. The population structure does not satisfy the REE requirements and the multi-modal structure is taken as symptomatic of inefficiency.

Extending these simulations Benink demonstrated the evolution of excess returns within agent populations over time. This is a potentially powerful mechanism for directly linking agent behaviours and learning to performance. In the context of the SFASM, which is also a closed system, this provides the opportunity to critically

examine if 'technical trading' emerges and where. Rather than relying on general population statistics reporting gross changes to overall levels of bit usage in rules, directly identifying agents responsible. More generally it addresses problems with total return measures of the type used by Schulenburg & Ross [115, 116] and Kendall & Su [67, 69, 68].

Summarizing from Benink's work on a relative efficiency measure and observations of problems in other AFMs,

- Observed success (or failure) of specific agents or subsets of agents in a population in isolation is not in itself particularly useful - as with survival bias described by Taleb, such observations only become meaningful if reported in the context of return information from the *entire* population.
- Summary measures like total return for agents or strategies are not a good performance measures in isolation. Arthur et al [3] observed that agent populations in the SFASM are co-evolutionary - successful agents removed from the population and reinserted later in a simulation run were observed to be unsuccessful. By the same token, in exogenous models successful agents, or particular strategies may not be consistently successful, or indeed may only be successful at a particular period in a market's history.

Similarly, taken on their own benchmark 'buy&hold' agents, or comparisons to 'cash only' investment strategies add little value if not presented in the context of situated measures.

- Risk-adjusted and ongoing rate of return measures are important as situated performance measures. Traders or trading strategies which may be profitable in particular periods, but show too much volatility in ongoing performance, or become unprofitable over an extended period will simply not survive in a real market. Agent performance measures need to reflect these features if the models under investigation are to preserve operational meaningfulness.

The excess return and population-based risk-adjusted performance measures presented here directly address these issues: summary measures are built from agent-level, strategy specific, situated performance metrics and are independent of an underlying economic model. Using such measures the statistical properties of time series can be considered in an informed manner, i.e. based on the behaviours of the agents rather than in the opposite direction.

Relative risk-adjusted performance measures have immediate relevance for rich AFMs - developing and exploring these measures is an important stage contribution of the research and is the main focus of the case study presented in the rest of this chapter.

### **5.3. Performance Measure Case Study - The Santa Fe Artificial Stock Market**

This section covers in detail the reconstruction of the SFASM using MatLab. The principal aim here is to use the example of a well known, 'rich' AFM as a basis to develop and evaluate realistic return-based measures suggested by Benink et al

[9], Dacorogna et al [34], and practitioners in actual markets in the context of more traditional economic measures commonly used in reporting on agent-based systems.

The SFASM has been widely cited in the academic community as a useful example of a rich AFM in overviews of the area and as background for new research into rich AFMs. Performance of agents in this system, and its success as a model for financial markets has been judged on the ability of agents to learn a rational expectations equilibrium present in the system. Success in this task has been used to discuss implications for learning and behaviour of economic agents in AFMs and in the real world. However analysis of agent performance in terms of trading behaviour and wealth generation has been more problematic. The reconstruction of the SFASM with agent-level situated performance measures presented an opportunity to explore this aspect of the SFASM while also testing the measures themselves.

A secondary, but important, outcome of the pilot study is to critically examine the model as a whole as it was presented in the literature. This is in keeping with Binmore's[13] criticism that there is not enough critical review of published research. As became clear from this reconstruction - without detailed documentation and archival material such reviews are in reality difficult and costly - though this serves to further support Binmore's criticisms in terms of verification and replicability.

The results of this analysis given the model's history in the literature are quite concerning and reflect directly on the issues raised in Chapter 3 regarding the quality of ACE models in research, verification, documentation, and validation. For the research presented in this thesis the result of this process was to highlight the need again for a clear robust architecture and principled process for agent-based models of economic systems. This provided the impetus for, and informed the development of, the SHaaP architecture described in Chapter 4.

### 5.3.1. Model Reconstruction

In initial implementations the SFASM was set up exactly as described as in the original journal papers.<sup>3</sup> Agent populations and dividend series were generated using different random number seeds for each experimental run and while in runs with longer  $k$  set to 1000 the results were more stable than  $k$  at 250, the market was in no way as stable as reported by LeBaron et al[85, 82] and highly dependent on the initial population drawn using the random seed. Further periods of instability were frequently observed even late in simulation runs. This was again at odds with the reported market behaviour and prompted an examination of the published code and available literature.

Notes on the reconstruction of the model, issues and observations on problems encountered along the way are presented here: these are limited to elements directly affecting the processes of the market and programming details directly relevant to these processes. Other details of the reconstruction effort and its forensic aspects are recorded in the Appendix A.7 reflecting the need to fully document the implementation process and the rationale behind the various design choices required. The

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<sup>3</sup>

Refer to Section 3.4.2 for detailed notes of this model structure.

appendix also gives a detailed list of parameter settings and variables used in the Matlab model with equivalents from Badegruber's SFASM v2.4[6].

### 5.3.1.1. Model History & Overview

As set out in 3.4, the SFASM is a deliberately simplistic economic environment where the existence of known rational equilibria can be established - these equilibria are then used as key benchmark criteria. The AFM consists of a population of constant absolute risk averse, utility maximizing, myopic economic agents. The effect of modifying agent learning parameters on evolving individual and aggregate behaviours is examined. Learning is driven by an error minimization process within an evolutionary algorithm in the form of a learning classifier system (LCS).

Although forecasting error statistics are reported, these relate only to rational equilibrium benchmarks: together with statistical artefacts in the financial time series produced by the AFM, they are the only quantitative form of validation presented prior to subsequent discussion of individual agent behaviours. The case was made earlier that if behaviours of agents as individuals are going to form part of the analysis and description of a system, relevant measures of such behaviours at an agent level are required. Even with such measures any conclusions drawn must be strongly caveated.

A number of studies have extended the original SFASM structure, variously modifying agent learning mechanisms and reasoning processes - a chronology of the main lines of work referred to in this paper is set out in Table 5.1 below. No doubt there has been other exploratory work, but, despite the apparent status of the model in terms of citation and discussion in reviews of agent-based models in finance, little appears to have reached publication. A possible explanation for this became apparent only during the process of reconstructing the model and then only after having access to the PhD thesis of Thomas Badegruber [6]. This details his contribution to creating Version 2.4 of the SFASM, the last published version of the SFASM source code claiming a direct connection to the original authors. It became obvious that a number of programming details and design choices were not fully documented in the available literature. A number of relevant programming parameters were omitted and full parameterisation settings were not recorded in the journal papers. Without these, models developed using only the original journal articles were frequently highly unstable and the time series produced failed to replicate the earlier findings.

According to Badegruber *no* version of the source code publicly released up until Version 2.4 was capable of producing the results obtained in the journal papers. At some point, or points, during the process of translating the model from its original ObjectC on a NeXTSTEP platform and porting it to the SWARM<sup>4</sup> platform, it appears that a number of changes and/or errors were introduced. Examining

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An open-source agent-based modelling platform pioneered at the Santa Fe Institute, designed to promote a standard computational environment speeding up model development and replicability.

	Chronology	
1994	Palmer et al. [102]	Early version of SFASM. Rules mapping directly to actions.
1996	Arthur et al. [3]	First full version. Preliminary results.
1998	Joshi & Bedau [64]	Extension of original using 59bit agents and considering wealth effects.
1999	LeBaron et al. [85]	Further results for the original model - including bitwise reporting of behaviours.
1998 -2000	Santa Fe Institute	Releases of source code for Objective C version 7.1.2 & Swarm versions 1.3 - 2.0 of SFASM.
2001	Tay & Linn. [140]	Fuzzy inference based reasoning agents extending original model.
2002	Paul Johnson	Version 2.2 of SFASM released.
2003	Badegruber [6]	Detailed revision of released code, bug fixing leading to Version 2.4
2002 & 2006	Ehrentreich [38, 39]	Various papers analysing mutation operator in detail & examining operator design.
2005	Polhill et al. [107]	Demonstration of rounding error artefacts for computationally equivalent code.

**Table 5.1.:** SFASM Chronology

Badegruber's code and an earlier release, v1.0<sup>5</sup>, some sense of the development and translation process is evident: there are numerous redundant or undocumented sections of code; unreported model parameters are present; a number of elements also appear to conflict with the model structure reported in the original studies. This may reflect a natural process of exploration and tuning in model development, however it does little to support the case for replicability in agent-based research. In order to maintain credibility models should support reliably reproducible results. A basic standard in this process is to preserve properly documented archive copies of the models employed including original source code. Clearly this failed in this instance.

Badegruber's solution was, with the help of LeBaron, to hunt down an old copy of a version of the original code located on an old personal computer in LeBaron's lab. According to the version nomenclature reported by Badegruber, this version (numbered '6.21.xx') pre-dated all publicly released versions. Badegruber states that

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I had mistakenly assumed the earlier version would be truer to the original code by dint of having had less time to diverge.

only versions numbered 6.21.xx were 'able to come close to the desired behaviour' and even then some parameters had to be hard-wired to avoid permanent market crashes.

Unfortunately, although Badegruber reports that his released version, v2.4, reproduces the original findings, he did not present any time series analysis to verify this. He relied on the changes in bit usage within the LCS to determine that the system was functionally the same, so there is no way of knowing if in fact the time series, and therefore the model, actually exhibited the same properties.

In contrast, Ehrentreich [38, 39] devoted considerable effort to analysing both time series properties, the mutation operator and agent wealth, however his model is definitively different from the original implementation. Aside from being ported to a Java environment, Ehrentreich's agents use 64 technical and fundamental bits and assigns default and new rule variances in a radically different manner.<sup>6</sup> So although his system 'works' it is not '*The SFASM*', though it is very useful for comparison and his methodology appears relatively robust and internally consistent.

Other than the papers by the original authors [3, 85], the papers in the chronology can be split into 2 groups.

- Those which take the original model structure and extend it, essentially accepting its findings as a platform to explore the underlying premise of a model of interacting agents with wholly endogenous pricing further. Tay & Linn's[140] and Joshi & Bedau's[64] fall into this category.
- Those which critically examine the underlying model and computational structures before attempting to extend the scope of research. Ehrentreich's and Polhill et al's follow this route[38, 39, 107]. Given the complexity of the model itself and the large number of parameters required to make it work, this is important to addressing the underlying model dynamics. If, as Ehrentreich had supposed, there is a possibility that a structural computational component (the mutation operator) was responsible for a key element underlying agent behaviour rather than an emergent property of agent interaction then that needs to be tested.

Unfortunately, a common issue for both types of approach is that they have very clearly not started from the point of replicating the original model and testing that before progressing - although this has proved to be very challenging in itself. Neither do they systematically employ agent-level performance measures.

### 5.3.1.2. Development Environment

The model reconstructed here was developed in Matlab, an array-based mathematical programming environment. As such the programming structures are significantly different from either the original versions of the SFASM developed in Object C

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The effect of his variance assignment in the code implemented here was to create very stable time series early in all simulation runs which tallies with his account. A question therefore is whether this design choice was introduced to overcome market crashes and instabilities observed elsewhere.

or later releases in the SWARM environment, however the computational economic structures should not have been affected. Matlab has advantages in that the array structures are quite transparent. It also benefits from availability of robust packages developed for financial modelling and time series analysis. A major drawback however is that it is relatively slow - for normal financial modelling this is not usually a problem, however for the SFASM simulations running to 260,000 trading periods per single simulation, each run took between 4 and 5 hours compared to less than one hour reported by Badegruber.

### 5.3.1.3. Design & Parameterisation Choices

Table 5.2 sets out the undocumented programming structures and parameters identified during implementation of the SFASM.<sup>7</sup> Some are rather prosaic, such as constraining the market to non-negative stock prices, however others directly affect the economic processes and purported rationality of agents in the market - all can at some level affect the overall stability of the market. They are roughly segregated here into two functional categories according to whether or not they generate dynamic, computationally derived parameters within the market: recall that in the original experimental design the only explicitly defined parameter which was changed between simulation runs was the frequency of the GA operation and hence that form of learning.

1. *Static Structures.* Parameters for these structures are defined before simulation and remain fixed throughout. This group is subdivided into those with direct impact on the economic structures and processes of the market, and those dealing with background infrastructure and general parameterisation. The latter are dealt with in the appendix: although varying parameters or their computational structure clearly may affect the market since they relate to general reconstruction rather than the operational meaning of the model.

Elements which directly affect economic behaviour and learning, such as the value of  $\theta$ , the exponential moving average smoothing constant used in the forecast accuracy calculation are relevant to the design and assumptions behind the model. However, combinatorial complexity within the model make attempts to delineate the scope of their effects extremely difficult if not, for practical purposes, impossible. It is easy to become bogged down in the attempt and does not necessarily significantly progress debate over the value of the model as a whole. To do that we still need appropriate agent-level performance measures.

Others identified include,

- a) Minimum rule activation counts. Rules in the LCS for each agent must have been activated a minimum number of times before they can be used to generate a forecast for trading (MinCount in v2.4). This is incompletely specified in Arthur's 1997 paper, and not at all in the later, main paper, yet is an important structural constraint. A low count threshold means the rule forecast error has had few trials and may be inaccurate - this will directly affect price volatility in the market as it feeds into the

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<sup>7</sup>Unless otherwise specified v2.4 is the code version referred to.

demand function. High threshold values lead to small eligible rule pools - a result can be overuse of the 'default' rule when no rules qualify.

Within the GA the code appears to specify that a rule must have been activated at least once to be eligible for use as a parent in the reproductive process. Again this can lead to problems in eligible pool size. The code also contains a workaround forcing a rule choice after 50 attempts.

It is not clear how significant this activation count is, however it does fall into the category of a preference modifying heuristic.

- b) Maximum forecast error constraint calculation. This is incompletely specified in the journals, where the authors simply state that the squared forecast error,  $e_{t,i,j}^2$ , is subject to a limit of 500 - as seen in Eqtn 3.9 this is an exponentially smoothed moving average. In fact, the implemented version limits the squared error from each forecast trial to 500 *before* using it to calculate  $e_{t,i,j}^2$  - considerably reducing its volatility and the possibility of being locked to the upper limit. As  $e_{t,i,j}^2$  is used to update rule variance and this feeds into demand via Eqtn 3.4 there is a substantial potential knock-on effect within the model. Having identified this difference, it is the one implemented in the Matlab code and appeared to reduce brittleness in the market operation.

These parameters and their settings may come from tuning the LCS, but in any case some record should have been present in the experimental documentation.

2. *Dynamic Structures*. Structures which directly generate or indirectly affect computationally derived variables used in the model.

- a) The specialist auctioneer.

Although the auctioneer type remains fixed throughout groups of simulations, the output of the auction process - the clearing price for stock transactions - is clearly dynamic.

In the published studies it is unclear which type of specialist agent was used to clear the market. Arthur reports that two types were available but not which was applied. The 'Slope specialist' type uses the first differential of the demand function to solve for a clearing price where buying and selling demands are matched. Typically it does this in one iteration. The ETA specialist iterates towards a clearing price using the imbalance in demands to adjust successive trial prices and recalculate new demands in each case. This type of clearing rarely reaches an equilibrium price within the limit on iterations specified by the system resulting in reduced overall the stock price volatility in the model as a whole. The parameters constraining the number of iterations and the minimum excess allowed in searching for a solution are important in this process.

Both types of auctioneer incorporate budget and trading constraints set out in the literature, however they also use additional apparently arbitrary coding constraints, which may have some effect in the system. Overall the effect of the auctioneer is to introduce discontinuities into the demand process for the agent population, which necessarily affect the time series produced.

- b) The default rule. This generates a forecast whatever the market conditions and when no other rule is matched and eligible then this forecast is used by the agent. Two forecast bit values must be maintained. A 'default' forecast variance is also required in order to generate a demand for the agent. The protocols for generating these values are incompletely specified in the journal papers - and there appears to be some difference between the mechanisms in the 1997 and 1999 paper. The v2.4 code seems to be an amalgamation of elements from both - the Matlab implementation uses the same basic protocol since there is no easy way to determine what was originally implemented.

Forecast bit values are generated in a two stage process. If any rules pass the minimum activation hurdle, a variance weighted value<sup>8</sup> of their forecast bits is used. If no rules qualify a 'global mean' is used: in the v2.4 code this is an exponentially smoothed moving average of the clearing price in the time series.

Default rule variance and forecast accuracy are updated in each period using the forecast bits generated above. Given the mechanism for generating the default forecast bits however, it can be seen that this is not a simple mechanism.

Presented as a workaround mechanism to deal with possible gaps in available rule forecasts it is conceivable that the role of the default rule in the market may be much more significant. If invoked frequently, agent trading decisions cease to be a direct function of learning based on forecast accuracy. It becomes difficult then to discuss rationality without some information and analysis of default rule use. This aspect is not addressed in the original work on the SFASM, although Ehrentreich[38] does discuss it briefly.

- c) New rule variances and activation counts. Again there appear to be conflicting descriptions of this process between journal papers. The code in v2.4 appears to take elements from both original papers, although there still appears to be a conflict where a new variance is overwritten to a median value, although this could be argued as an interpretation issue.

The protocol for setting new rule variances is a major structural element in the agent population learning process and was found to be critical to the overall stability of the market in conjunction with the minimum activation hurdle. Given a GA where, particularly in crossover, offspring may bear little resemblance to parents a plausible case can be seen for ensuring the 'inherited' variance is not far from the general population level since this directly affects demand on activation.

No protocol for setting the activation count for new rules is specified in the journals. In v2.4 the code appears rather confused: in crossover the

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This is the rule variance,  $\hat{\sigma}_{j,i}^2$ , updated only each time the GA is performed, *not* the forecast accuracy measure  $e_{i,i,j}^2$ . I explored using the latter but found the market to be very unstable over repeated runs.

general case is that the count is set to zero, but in some cases and more frequently in mutation new rules inherit parent activation levels. The interaction of new rule variance and activation levels appears to have a large impact on stability.

In his Java version, Ehrentreich sets new rule variance and the default rule variance to a simple average of the whole population's variances. Using this setting in the Matlab version the time series were observed to become very stable very quickly (a feature of Ehrentreich's reported results). When not set, i.e. using the computational structure in v2.4 time series were much less stable, particularly for 'fast' runs with  $k$  set to 250. In the interests of the reconstruction, the protocol used in v2.4 is followed as closely as possible.

**Table 5.2.:** Undocumented or Incompletely Specified Structures

Description	Documentation	Pilot Study	Value - Journal	Value – published code	Impact
Minimum rule activation count	incomplete	2	Unspecified	2, 5 & 10	Learning, trading & stability - stops new rules from use in trading and in the GA.
Default rule variance	conflicted	Initial value 4, follows journals	Journal protocol ambiguous or conflicted	Calculated value	Directly impacts trading demand and stability
Default rule forecast bit values	incomplete	calculated	As above	As above	As above.
Forecast mutation operator	Conflicted	0.05%	5.00%	5.00%	Unclear whether this is a typographical error or not.
Maximum rule error	incomplete	as in v2.4	Ambiguous	Implemented code differs from basic journal description	Use of exponential smoothing increases stability.
Auction specialist agent	yes	Slope	No clear statement of auction process – two types outlined	'ETA' & 'Slope' code implemented	Impacts clearing prices. Choice of protocol is important in the auction process
Maximum auction iterations	Yes	10	Unspecified	10, 20	Only appears to impact where 'ETA' specialist is used.
Maximum excess	Yes	0.001	Unspecified	0	As above.
ETA value for auction process	Yes	0.0005	Unspecified	0.005, 0.0005	As above.
Maximum stock price	No	200	Unspecified	200, 500 & 9999	Stability
Minimum stock price	No	0.001	Unspecified	0	Stability. No non-negative prices
GA Start period	No	501	Unspecified	100	Learning & stability
Technical moving averages	No	Ema	Unspecified	Simple & ema	Choice of simple (arithmetic) or exponential (ema) moving averages affects the agents' available information.

### 5.3.2. Experimental Runs & Protocol

Following Arthur et al [3] and LeBaron et al [85] the basic experimental design was retained.

- Using different paired random number seeds, 25 sets of sample runs were carried out. Two random number seeds were used in Matlab since it has the facility to run independent random number processes for uniform and Gaussian distributions. This allowed the dividend process to be isolated from other elements in the model such as the GA where the evolution of the population could impact on the random sequence used in calling other elements of the system.

- In each set one run was performed with the GA frequency,  $k$ , set to 250, 'fast learning' and one with  $k$  set to 1,000, 'slow learning'.
- The resulting time series were analysed for time series artefacts in the context of the known theoretical REE within the system. The behaviour of the LCS at different learning speeds was examined using the same measures as Arthur and LeBaron, the frequency of condition bits being 'set', i.e. to 1 or 0 as opposed to # ('don't care').
- Additional information was recorded as part of each experimental run, including some elements not reported in the original studies. This included,
  - Forecast accuracy data by agent
  - Agent wealth and trading decisions
  - The proportion of agents using the default rule generate forecasts in each trading period.
  - Which rules were used by each agent and how frequently a given rule was used
  - Bit usage in rules used, including the proportions of technical and fundamental bits set as well as 'always on' bits.
- Having repeated the original analysis, the data sets were then analysed using the population relative risk-adjusted performance measures set out in Section 5.2.1. Two sets of measures were used,
  1. Benink's relative performance measure, hereafter referred to as  $R_{REF}$  giving an aggregate risk-adjusted performance score,  $I_R$ , as in Eqtn 5.6.
  2. A computationally equivalent measure using the symmetric effective return measure,  $X_{eff}$ , suggested by Dacorogna, hereafter referred to as  $X_{REF}$ . In this measure the same matrices were generated as for  $R_{REF}$ , except that the excess return measure is the risk-adjusted value represented by  $X_{eff}$  so that the population relative risk-adjusted performance,  $I_X$ , is given by,

$$I_{X,ij}(t, t') = \left( \bar{X}_i(t, t') - \frac{\alpha \sigma_i^2(t, t')}{2} \right) - \left( \bar{X}_j(t, t') - \frac{\lambda \sigma_j^2(t, t')}{2} \right) \quad (5.8)$$

Where  $\alpha$  and  $\lambda$  are risk aversion coefficients for agents  $i$  &  $j$ , and  $\bar{X}_i(t, t')$  is the mean total excess return between  $t$  and  $t'$ . This measure avoids the instability at low return variances Sharpe based measures are potentially prone to.

The results of these analyses are set out in the following section.

### 5.3.2.1. Experimental Results

In the original studies the model is presented as a learning task, where agents seek to learn the underlying linear process set out in Eqtn 3.5. As with these studies,

observation of the output time series shows periods of apparent stability punctuated by volatility, which the authors compare to actual market behaviours. In order to analyse this, the market price and dividend is regressed on a lag and a constant,

$$p_{t+1} + d_{t+1} = a + b(p_t + d_t) + \varepsilon_t \quad (5.9)$$

and the estimated residual series,  $\hat{\varepsilon}_t$ , analysed for structure. The results of this analysis are contained in Table 5.3.

**Table 5.3.:** Residual Summary Statistics

Description	LeBaron k=250 fast learning	Matlab k=250 fast learning	LeBaron k=1000 slow learning	Matlab k=1000 slow earning
Std Deviation of residuals	2.147 (.017)	2.336 (.068)	2.135 (.008)	2.289 (.009)
Excess kurtosis	0.320 (.020)	0.194 (.081)	0.072 (.012)	0.102 (.071)
autocorrelation, $\rho_1$	0.007 (.004)	-0.012 (.004)	0.036 (0.002)	-.007 (.0016)
ARCH(1)	36.98 [1.00]	115.76 [0.960]	3.159 [0.44]	31.211 [0.48]
$\rho_1^2$	.064 (.004)	0.088 (.012)	0.017 (.002)	0.031 (.009)
Excess return	3.062		2.891	
Trading volume	0.706 (.047)	1.58 (.126)	0.355 (.021)	1.026 (.089)
Mean difference to REE price	not reported	2.833 (.1.06)	not reported	7.874 (1.46)
Total Relative Performance, $I_R$	n/a	40.45 (4.83)	n/a	37.90 (4.94)
Mean, rolling Relative Performance, $I_R$ - 256 period	n/a	1.818 (.147)	n/a	1.590 (.109)
- 40 period	n/a	0.481 (.028)	n/a	0.413 (.018)

Means over 25 sample runs. Numbers in parentheses are standard errors estimated for these runs. Numbers in brackets are the fraction of tests rejecting a 'no-ARCH' hypothesis.

**Time Series Structural Analysis** In an homogeneous REE as constructed in the SFASM, the residual series should be independent and identically distributed (IID),  $N(0,4)$ . It can quickly be seen that there is a clear difference in agent success in learning the REE between the original runs and in the Matlab implementation. In all cases there is higher than theoretical volatility, but in Matlab learning this is much higher, as is also reflected in the ARCH statistics, reflecting a considerably

higher degree of variance in the residuals. The squared autocorrelation statistic is also higher in both fast and slow learning. In all cases however the direction is the same as the original studies - fast learning shows higher variance and more evidence of volatility persistence.

Interestingly however the results for excess kurtosis are similar to the original studies and the serial autocorrelation statistic shows no evidence of linear structure. In addition, the number of runs rejecting a 'no-ARCH' null hypothesis at 95% confidence is similar in Matlab slow learning to the originals. So although there are clear differences in the time series produced, there are also striking points of similarity.

It is worth noting that the standard errors across all of the sample measures are higher in the Matlab runs: given higher overall volatility in sample time series this is perhaps not surprising. Given the problems in reconstructing the market set out in Section 5.3.1.3, it is also hardly surprising that the results are different, but the mixture of elements which agree and differing leads to interesting questions about what the market is actually doing. This is also reflected in the bit level analysis of agent 'behaviour'.

**Bit-Level Agent Structure** LeBaron & Arthur extend their analysis of SFASM time series properties by examining the structure of agent rules in terms of their forecast and conditioning bits. Having shown using further regression analysis that there is evidence of exploitable market structure for particular indicators<sup>9</sup> in fast learning but not in slow learning, they examine changes in the numbers and types of condition bits set in agent rules over the course of their sample runs. If there is exploitable information, is there evidence that it is being exploited? Changes in bit usage are taken as potential sources of evidence for this.<sup>10</sup> Retaining set bits has a cost within the GA used in the SFASM, so that if they had no value to an agent then numbers of bits set would be expected to decline to some base level reflecting an equilibrium between mutation introducing new bits and cost eliminating them.

Their broad finding was that in slow learning the proportion of bits set falls steadily from the original 10% set in the initial model parameters to a stable level below 5%. In fast learning, after an initial decline, set bit levels recovered and appeared to stabilize between 10 and 15%. Similar findings were produced in the Matlab implementation and are set out in Figures 5.1 and 5.2 below.

An interesting difference becomes apparent when the *types* of bits set are examined. In the Matlab implementation during fast learning fundamental bits, using dividend ratio information, on average reached high stable levels while technical bits, using simple moving average indicators, decreased to relatively low levels - directly opposite to the original findings.

As a behaviour within the system this could be symptomatic of some use of technical indicators in trading. A possible explanation is that in the Matlab runs, where the

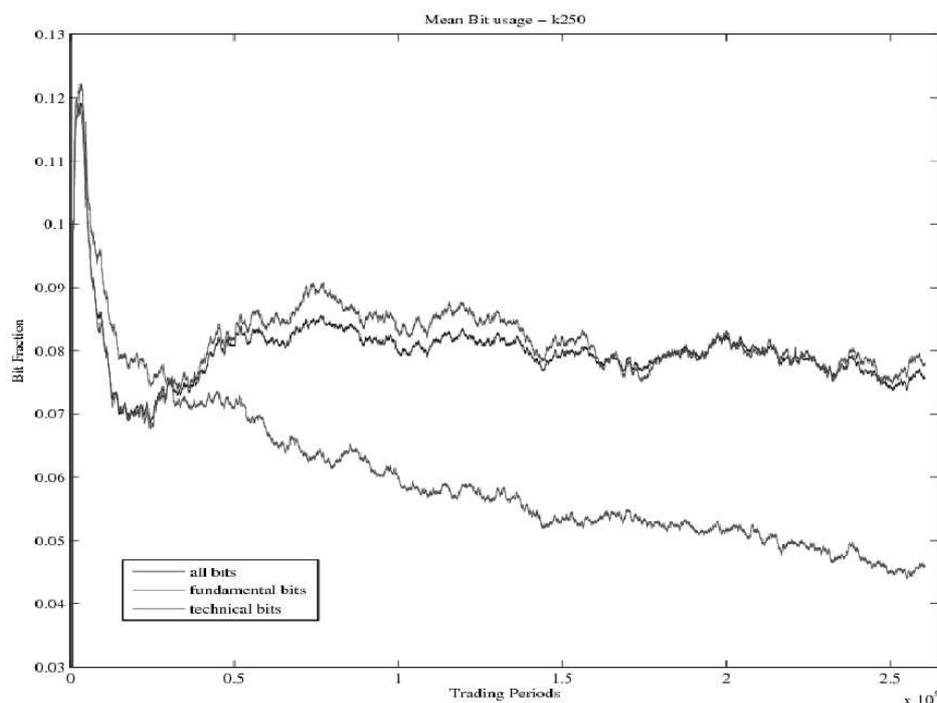
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Recall the REH definitions of efficiency.

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It should be emphasised that it is the average bit usage across all agents that is examined, no attempt is made to look at individual agent behaviours, rules or performance.



**Figure 5.1.:** Bit Usage - 'fast' learning

time series are already observed to be relatively more volatile in comparison to the original SFASM, fundamental indicators which are not changed very often in the original SFASM runs change value more frequently here, and so have more information content, while technical bits change so rapidly that noise and information are indistinguishable. To some extent this intuition is borne out in slow learning where technical bit usage remains higher than for fundamental bits: in a more stable, less volatile market technical indicators are less noisy whereas fundamental bits are activated less often so their cost tends to remove them from rules in the GA. It should be emphasized however that, in the 12 bit classifier rules of the SFASM, for two bits to be set this implies a bit usage of just 0.167 - higher than the maximum aggregate state achieved in fast or slow learning. So it remains difficult to see how rules of any sophistication can exist in either scenario and anthropomorphic descriptions of the rationale behind trading activities remains problematic.

Nevertheless in both the Matlab and original SFASM implementation there does appear to be some differential behaviour involving bit usage. However this still does not clarify what causal relationships exist, if any, between bit usage, agent trading and the presence of exploitable market structure and this issue is ignored in the original studies. To investigate trading behaviour it is necessary to explore trading decisions and wealth effects, i.e. to examine the system at an agent level. Earlier arguments have made the case that *relative* changes in wealth are key elements in judging trading performance. Given the effect of market level on all agents' wealth identified by Ehrentreich[39] this is particularly relevant in the SFASM.

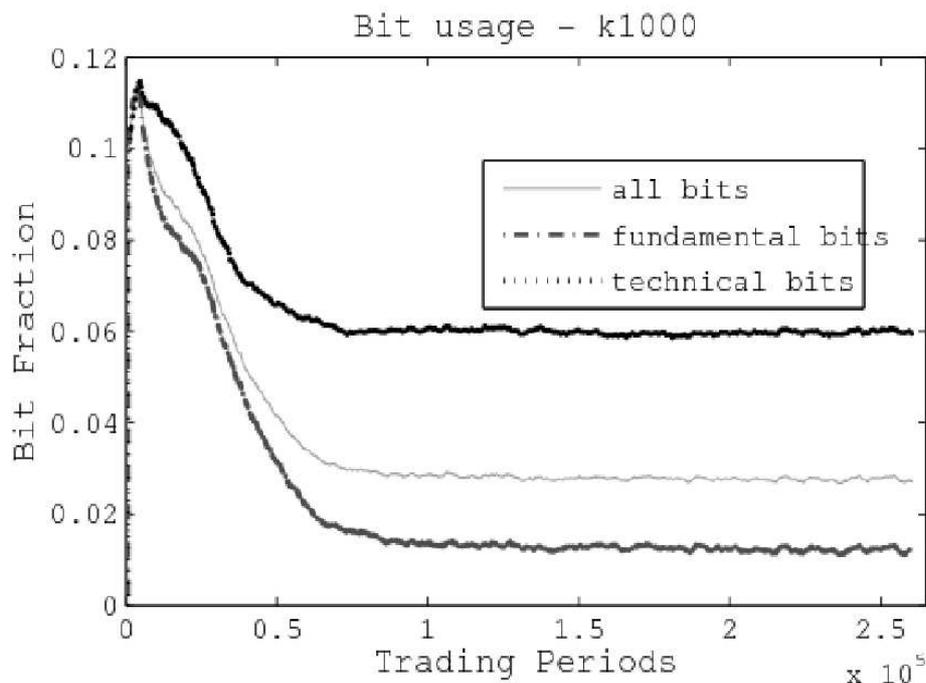


Figure 5.2.: Bit Usage - 'slow' learning

### 5.3.2.2. Risk-Adjusted Relative Returns & Agent Trading Performance

This section applies and evaluates several candidate measures based on the definitions set out in Section 5.3.1.3.

Two considerations are important in setting up these measures for the SFASM.

1. Identifying what constitutes a unique strategy. Where specific trades can be identified and classified these may be used, however if this is not possible higher level groupings may still be used. Given the structure of the LCS and the demand and trading functions, the finest granularity that can be established in the SFASM is at the agent level. Each agent with its pool of forecasting rules forms a unique 'strategy'. This flexibility in granularity is an important strength of the measures.
2. A trading window,  $w$ , representing  $(t, t')$ , must be defined. This is an important design parameter and could be set to a single trading period or to the length of the entire sample. Very low values of  $w$  were found to have limited use as noise inherent in financial markets obscures performance. At high values, the extreme being the whole sample giving an overview of agent performance, much information about the dynamics of the system is lost. Although an aggregate score for  $w$  equal to whole sample was recorded, after exploring values of  $w$  between 256 and 20 a final value of  $w = 40$  was adopted for use in analysing the SFASM constructed here. While somewhat arbitrary, this value appeared to give a good balance between noise and sensitivity.

An immediate observation is that the relative performance scores appear to rank markets with fast learning as less efficient (following Benink's definition) than with slow learning, since higher scores imply greater extracted excess returns. The relative performance scores also appear more volatile in fast learning than slow learning.

**Table 5.4.:** Relative Risk-adjusted Return Scores

	Fast - k250	Slow - k1000
$I_R$ - mean, rolling 40 period window	0.48 (.028)	0.41 (.018)
$I_R$ - aggregate	40.4 (4.83)	37.9 (4.94)
$I_X$ - mean, rolling 40 period window	0.150 (.030)	0.071 (.017)
$I_X$ - aggregate	920.6 (229.1)	576.6 (154.6)

Values in parentheses are standard errors. Summary statistics for both  $I_R$  and  $I_X$ : the measure for the 40 period window is a simple average over the sample period.

Statistical analysis appears to confirm this for the rolling scores at least. A null hypothesis that the mean relative performance scores for fast and slow learning are equal is rejected for only the rolling relative performance measures at a 95% confidence level (  $I_X$  p-value = 0.028,  $I_R$  p-value = 0.0468 ) and in neither case for the aggregate measures (  $I_X$  p-value = 0.229,  $I_R$  p-value = 0.719). Given the loss of granularity in aggregate score with  $w$  covering the whole sample period the outcome for these measures is not particularly surprising and suggests that they may have limited usefulness in evaluating model behaviours.

Correlation analysis of time series statistics shows a somewhat mixed picture - mean values for fast and slow learning are set out in Table 5.5. There is reasonable correlation of relative performance scores with historic volatility for both learning speeds and low correlation between  $I_R$  or  $I_X$  and bit usage; risk premium; and rule accuracy.

However there are also moderately high scores for correlation between active rule variance (used in rule choice) vs. volatility and vs. total bit use. In other words, high error rates seem correlated with volatility and with bits used in the LCS, which is difficult to reconcile with agents using technical information to extract high returns.

Overall however the findings do give some apparent support for the validity of the SFASM as a useful simulation of actual markets - the intuition being that more volatile markets have more exploitable structures - inefficiencies - in keeping with EMH definitions. Plots of the relative performance measures vs. historic volatility emphasize this support - showing increases in relative performance coincident with increases in volatility (see Figures 5.3 & 5.4 in the following section). However there is no means to directly compare them to an external benchmark, nor, as with the regression analysis carried out by LeBaron, can direct causal links be demonstrated between these summary statistics and actual behaviour in the markets. Such inferences require examine underlying structure of the behaviours to be examined.

### 5.3.2.3. Exploring Trading Behaviours & Performance Using Agent-Level Measures

To explore and understand the correlation findings it is necessary to look at specific examples within experimental runs. Without agent-level measures this would be at

**Table 5.5.:** Fast & Slow Learning Correlation Coefficients,  $w = 40$ 

	$I_R$	$I_X$	Historic volatility	REE price differential	Passive rule variance	Active rule variance	Total Bit Use	Open Interest
$I_R$	1	.47/.42	.54/.36	.09/.15	.15/.19	.28/.30	.13/.16	.44/.33
$I_X$	.47/.42	1	.66/.58	.03/.27	.15/.26	.31/.62	.08/.18	.50/.57
Historic volatility	.54/.36	.66/.58	1	.19/.44	.25/.49	.40/.66	.30/.48	.46/.40
REE price differential	.09/.15	.03/.27	.19/.44	1	.15/.68	-.04/.36	.25/.40	.52/.20
Passive rule variance	.15/.19	.15/.26	.25/.49	.15/.68	1	.40/.44		
Active rule variance	.28/.30	.31/.62	.40/.66	-.04/.36	.40/.44	1	.22/.36	.24/.45
Total Bit Use	.13/.16	.08/.18	.30/.48	.25/.40	.25/.46	.22/.36	1	.09/.16
Open Interest	.44/.33	.50/.57	.46/.40	.52/.20	.12/.40	.24/.45	.09/.16	1

Key: fast learning / slow learning.

best very challenging, however the relative performance scores and their component measures allow the detailed dynamics of trading models to be examined. The examples here emphasize the flexibility of the measures, and also the pitfalls of applying only system-level measures to agent-based models. Specific to the SFASM it emphasizes once again that forecasting and trading are *not* equivalent behaviours.

Analysis of trading behaviour at the agent level is a somewhat forensic process - it is certainly exploratory. The complex nature of the interactions makes the use of summary statistics difficult if not hazardous, while looking at individual trades in isolation serves little purpose besides being simply infeasible. Relative performance measures give a mechanism to potentially identify interesting periods in the market and work through the underlying dynamics of inter-agent behaviours.

Two samples are used here as reasonably representative illustrations of the model behaviour. A 'fast learning' case (seed 80222) with a high ARCH score of 309.2, giving a p-value of effectively zero which rejects 'no ARCH' with a very high degree of confidence and  $\rho_1^2 = .176$  indicating significant serial autocorrelation of volatility, and a slow learning case (seed 19567) with a moderately low ARCH score of 4.64, giving a p-value of 0.0312, rejecting 'no ARCH' at the 95% certainty level, and  $\rho_1^2 = . - 21$  confirming low serial autocorrelation of volatility.

The exploratory process applied refers back to the validation criteria questions set in Section 3.3.2. Despite reservations about the neoclassical structure of the market and its agents, the original SFASM was presented as a demonstration that a market made up of boundedly rational (sic) economic agents could demonstrate similar properties to actual markets, and that the agents engaged in a mixture of technical and fundamental trading. Given that time series volatility is a key feature discussed in the experimental analysis and that agents competitively determine this time series by expressing their demand, it seems reasonable to consider the relationship between volatility and relative agent performance. The steps in the analysis based on this are,

1. Identify sample runs for detailed examination using the summary statistics as a filter - in this case examples which satisfy presence of ARCH in the time series. Obviously samples rejecting ARCH could also be used but since this is an exploratory process and the aim is to see what typical agent behaviours are, it makes sense to begin with samples where these behaviours are likely to be interesting.
2. Plot rolling relative performance measures vs. volatility for the time series in question.
3. Identify regions of increased volatility and increased relative performance in the time series as determined by  $I_R$  and  $I_X$ .
4. Explore the relative performance surface of the agent population to see which agents are outperforming others.
5. Explore the dynamics of the outperformance or underperformance in terms of trading behaviours during the inefficient periods.

This process does not look at the logic of particular trading rules or attempt to evaluate them, rather it is an initial step towards validation of the causal links between agent behaviours and the time series artefacts, which can then be a basis for further, confirmatory experiments.

$I_R$  vs  $I_X$  Plots of rolling population relative performance scores for particular sample runs reveal interesting features. Fig. 5.3 shows a plot of an agent population under fast learning with a high ARCH score, while Fig. 5.4 shows an equivalent plot for a population under slow learning with a moderately low ARCH score. In both there are obvious periods of correlation between volatility in stock price and relative performance, consistent with an interpretation of inefficiency and trading opportunities associated with increased volatility.

Looking at the two relative performance measures however, it is immediately apparent that  $I_R$  is considerably noisier than  $I_X$ . The Dacorogna-based measure demonstrates clear sensitivity to changes in market volatility, whereas  $I_X$  generates relatively high values even when market volatility is relatively stable particularly in the slow learning case. This is consistent with the construction of  $I_R$  and the problems with instability of Sharp-type measures at low variances identified in Section 5.2.2.

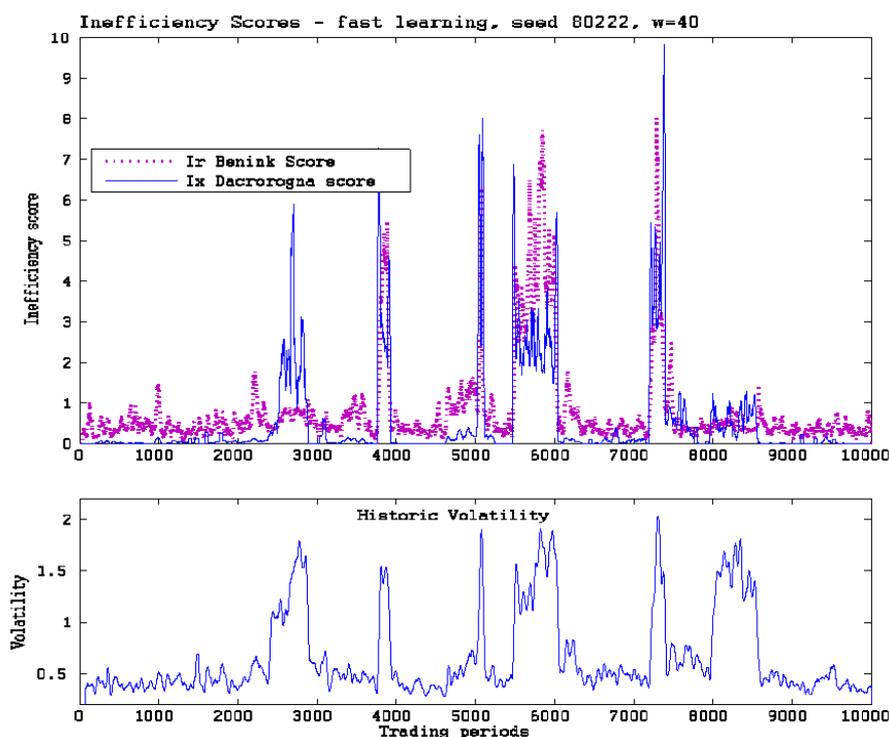
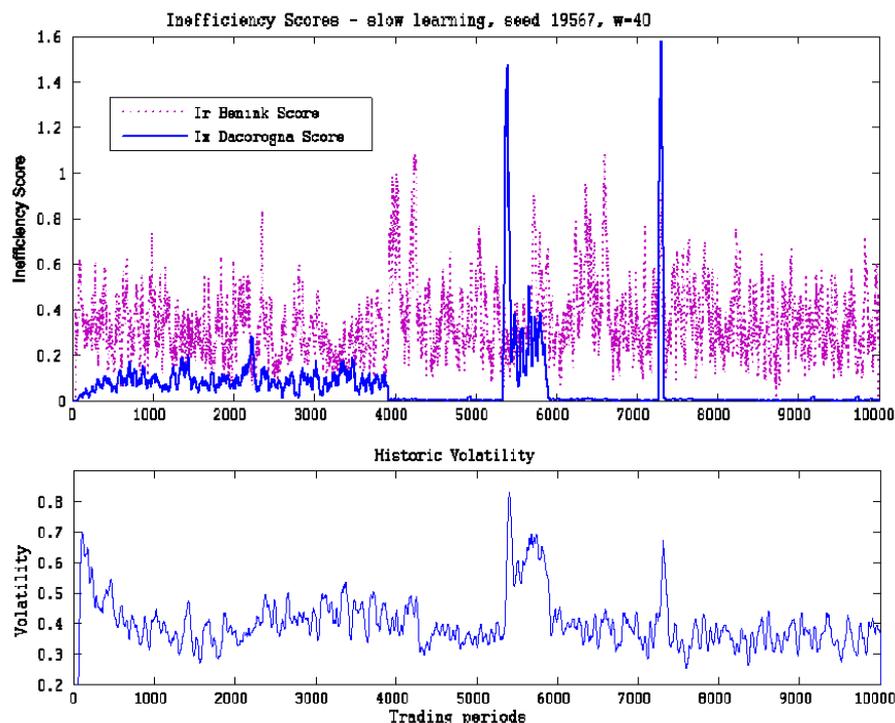


Figure 5.3.: Fast Learning Relative Performance Scores



**Figure 5.4.:** Slow Learning Relative Performance Scores

However, using the relative performance scores to focus on periods of volatility, the proposed interpretations of the causal relationships involved presented in the literature become rather problematic. Examining the main periods of volatility both in fast and slow learning, the same basic pattern is repeated. Inter-agent relative performance score matrices underlying the  $I_X$  scores from Eqtn 5.5 are used to generate a relative performance surface for the agent population for the sample period. This reveals that, in general, single agents appear to dominate, generating very large negative returns relative to the rest of the population. Examining wealth changes in the period does not pick up this effect - all agents increase wealth monotonically in the period as Ehrentreich [39] describes.

In the fast learning example, Agt 20 massively underperforms its peers in the window from 5,400 - 6,100, Fig. 5.5. Similarly in the slow learning case, during the major period of volatility in the sample a single agent, Agt 25, dominates, Fig. 5.6. The same type of behaviours were found in high volatility periods across all the samples in the study although in some cases agents massively *outperformed* their peers.

In contrast the Benink-derived inter-agent relative performance scores give little useful insight. During the periods of volatility no single agent appears to stand out in fast or slow learning, Figures 5.7 and 5.8. Due to the instability noted earlier, as a measure  $I_R$  appears to be over-sensitive, obscuring relative performance amongst agents, where low return, low variance scores are magnified in importance. Both as a performance and a fitness measure this effect seems likely to make it unsuitable for use in a situated environment, although it may still have some use as a crude measure in preliminary runs. Given these limitations, the rest of the analysis focuses on the Dacorogna derived  $I_X$  measure and its components.

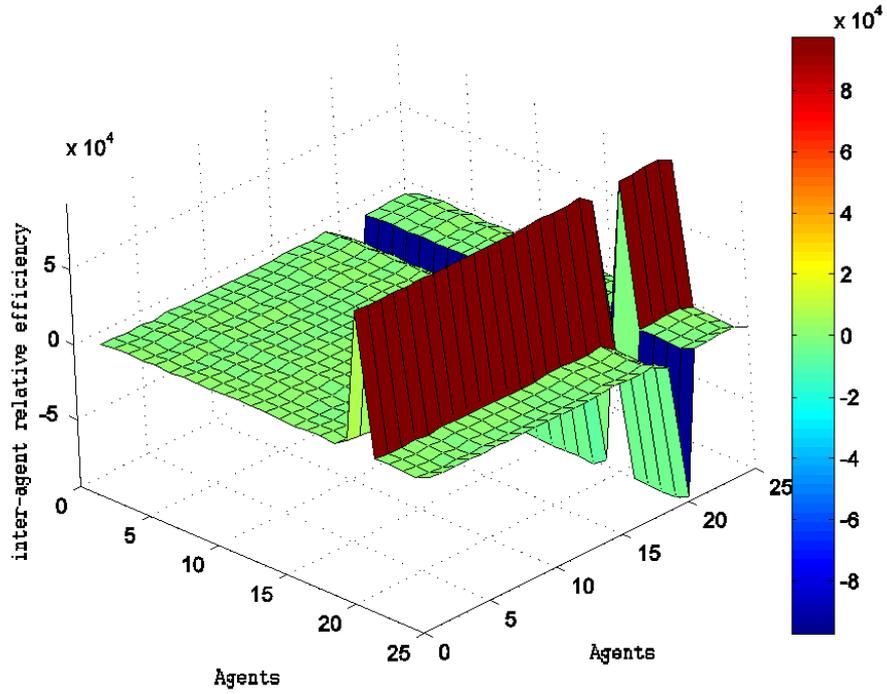


Figure 5.5.: Inter-agent relative performance,  $I_X$ - seed 80222, fast learning, window 5,400 to 6,100

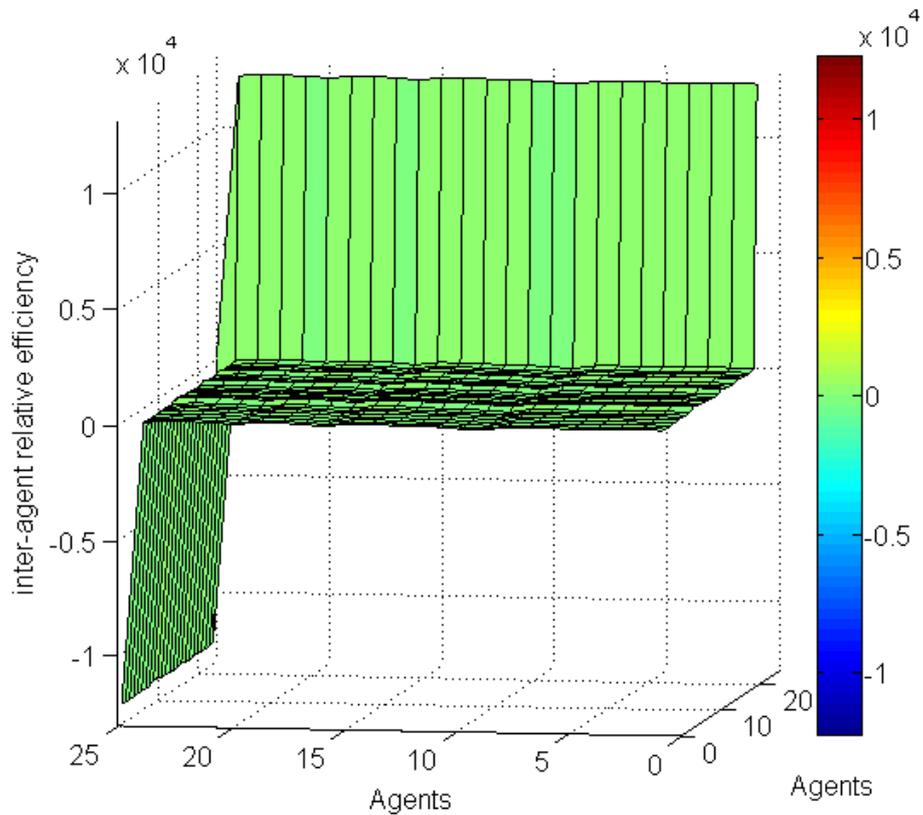
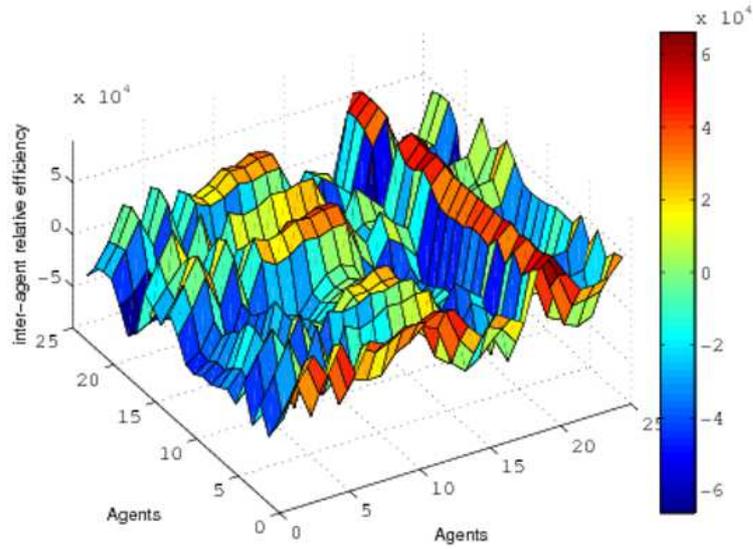
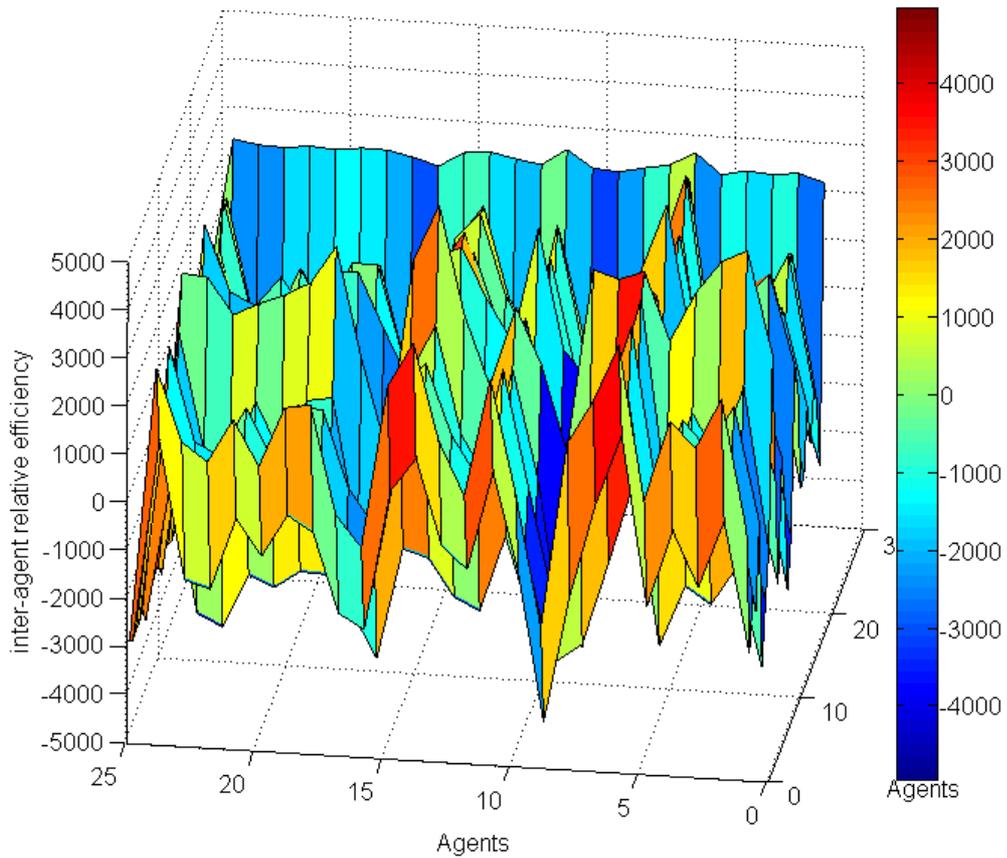


Figure 5.6.: Inter-agent relative performance,  $I_X$  - seed 19567, slow learning, window 5,350 to 5,910

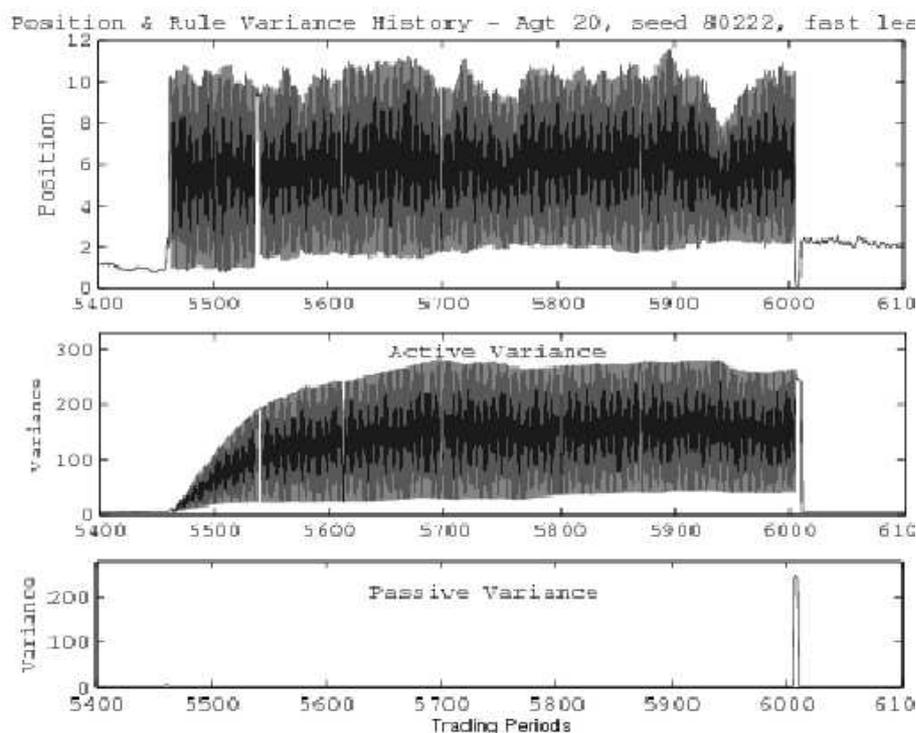


**Figure 5.7.:** Inter-agent relative performance,  $I_R$  - seed 80222, fast learning, window 5,400 to 6,100



**Figure 5.8.:** Benink-based Inter-agent relative performance,  $I_R$  - seed 19567, slow learning, window 5,350 to 5,900

**Agent-level Trading Behaviours** Given such striking differentiation of performance by individual traders the next step is to look at their trading activity in periods of increased volatility. Figs. 5.9 & 5.10 show examples of the dominant agents' stock holdings as well as the active variances of the rules employed and the passive variances for active trading rules. In both cases the agents are making very large position changes relative to the size of the market and these changes are extremely frequent. Given total tradable stock of 25 units, the dominant fast and slow learning agents respectively are regularly making trades in over 40% and 20% of the market's total size. Average trading volume for the dominant agent in the fast learning case is 8.12 units vs an average for the rest of the population of 0.49 units; in the slow learning case these figures are 2.3 units and 0.14 units respectively.

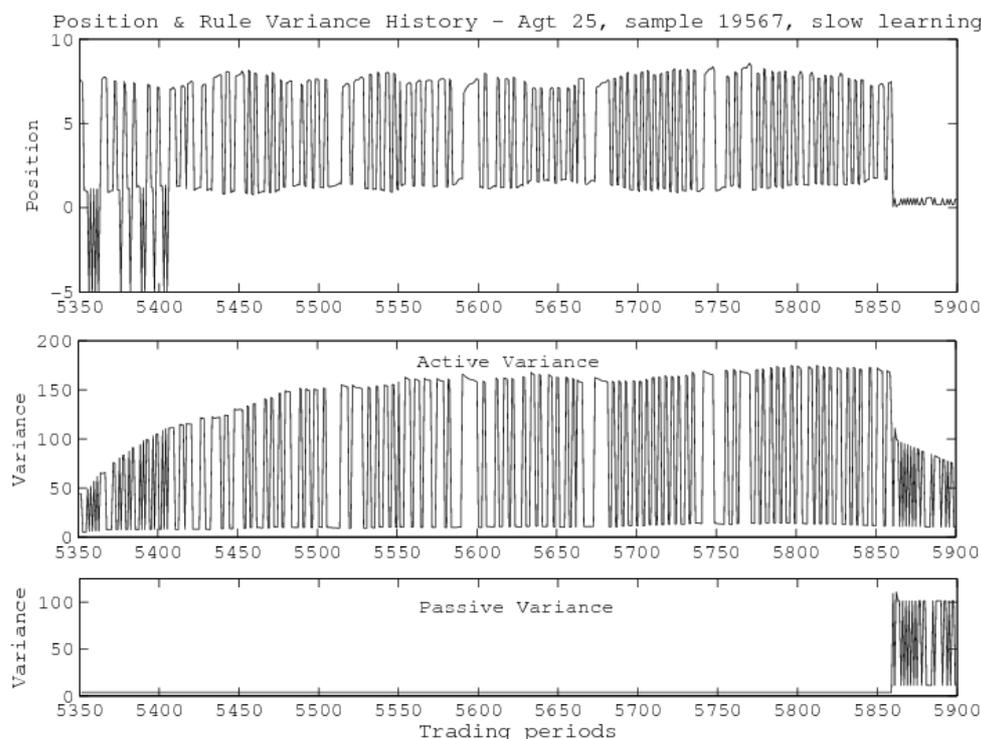


**Figure 5.9.:** Agt20, seed 80222, window 5,400-6100

The dominant agents may be creating excess volatility by forcing the market to move in response to large trades, while the rest of the agent population benefits. As a behaviour excessively large trades are known to happen in actual markets, though not for extended, successive periods or on a systematic basis. As a self-limiting process, excessive rapid, under-performance tends to eliminate highly unsuccessful market participants relatively quickly, while superior performance disappears as strategies are copied. In contrast, in the SFASM constructed here once the behaviour begins it carries on for extended periods - there is no market sanction for irrational, inefficient or unprofitable behaviour. This, again, calls into question the appropriateness of the fitness function used by agents in their learning classifier system, which is driven by forecast accuracy not profitability (risk-adjusted or otherwise).

This suggests the agents are not exploiting market inefficiency, rather they are *creating* it. Of itself this does not mean that the SFASM model is poor, however it

leads to questions about what is causing the agent to trade in this manner. It is difficult to make some causal connection between exploiting volatility and technical trading, when only one agent seems to be actively trading and losing cash relative to the rest of the population.



**Figure 5.10.:** Agt25, seed 19567, window 5,350-5,900

The evidence presented in the original papers pointed towards the learning classifier system and bit usage as a source of technical behaviour. However, given the nature of the evidence (See Section 3.4.2.5) no causal link could be proved and no direct examination of agent level behaviours was attempted. Without some ability to identify periods of excess returns in the agent population this would have been difficult or impossible. In this version of the SFASM we have identified such periods and specific agents which appear to generate it. However although average bit usage by the agents is higher than their peers (Agt 20, fast learning bit use = 0.58 vs a mean of 0.48; Agt 25, slow learning bit use = 1.45 vs a mean of 0.55) the evidence below suggests that rule use does not appear to be causal in itself.

LeBaron et al demonstrated that learning within the system does occur and that error rates approach REE norms. Agents optimise their forecasts and use their best active rules to generate trading demands. Given the dominant agent performances, if they are using optimized rules to exploit inefficiency in the market one would expect this to be reflected in the rules used and their forecast accuracy. In fact this is simply not so in this reconstruction. Average forecast accuracy of rules used as calculated in Eqtn 3.9 is much poorer in the dominant agents than in the rest of the

population: in the fast learning example, Agt 20 mean active variance = 127.3 vs. a mean for the rest of the population of 26.9 and in slow learning, Agt 25 mean active variance = 78.7 vs a mean for the other agents in the population of 9.11. So these agents are using rules which are on average less accurate but still outperforming their peers.

### Forecast Accuracy vs. Trading Profitability

Revisiting Figures 5.9 & 5.10 an explanation becomes clear. Agents switch rapidly between low variance, accurate rules and high variance, inaccurate rules. It is interesting that in these examples the passive variances of the rules used are not materially different, however recall that the passive variance is only updated when the genetic algorithm is invoked for a given agent.

The learning classifier system chooses amongst activated rules based on their *active* variance, but agent demand is calculated using the rules' *passive* variances - Eqtn 3.4. Thus when a rule loses accuracy but is the 'best' available, the conditions exist which can allow high demands even with poor forecasts. When these are combined with market conditions which trigger alternate activation of good and bad rules, the agent makes large switching trades in response. Figure 5.11 illustrates this effect as a poor rule comes into action: forecasts are plotted against actual market settlement prices in the period running up to the anomalous behaviour and as the behaviour continues. Large changes in the agent's forecasts combined with low passive variance lead to large position changes until the behaviour is eventually switched off either by rule replacement or updating the passive variance.

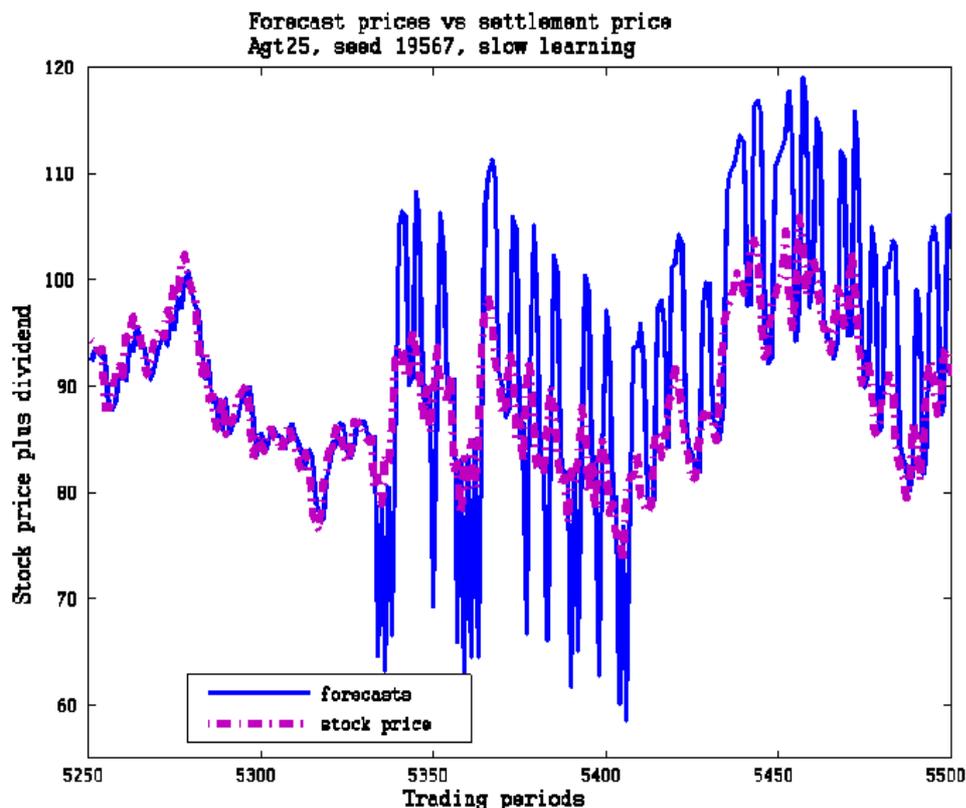


Figure 5.11.: Agent Forecasts vs. Market Settlement

This emphasizes two problems in the AFM design identified earlier.

1. Forecast accuracy is not equivalent to trading profitability as a fitness measure within a trading system.
2. Agent preferences within the model are not simply an expression of a core utility maximizing function.

The trading behaviour observed in the AFM is the combined result not only of the rule forecast, accuracy and trade clearing mechanisms, but also the trading protocols defined to support the learning classifier system. By making new demand in each period myopic, i.e. a function solely of the 'best' rule in that period, portfolio switching can be dominated by variance & forecast effects in the demand function as different rules become activated in successive periods. Switching between very accurate and very inaccurate rules can cause large swings in demand.

In the examples here, it is difficult to see how the agent behaviour could be interpreted as exploiting market inefficiencies to the benefit of the agent. The presence of clustered volatility observed in the time series does not appear to be related to the use of technical information about the market. Exploitable information may be present in the market, but agents do not in fact appear to use it. It appears that outperformance is a piggybacking effect, the general agent population benefiting from anomalous trading behaviour and poor performance in individual agents whose dominant trading behaviour is correlated with increased volatility. Although some examples were observed where dominating agents *outperformed* their peers, the same switching activity prevailed. In these cases excessive profit does not appear to be linked to the use of good rules, rather an effect of switching activity driven by very poor rules.

### 5.3.3. Reconstruction Discussion

The work presented here provides an important insight into the application of agent-level performance measures to rich AFMs. The population relative performance measures developed during the course of this study appear to be effective, novel analytical tools for analysing agent-based AFMs as part of an exploratory validation process.

The SFASM reconstruction process uncovered worrying problems in published accounts of the AFM, highlighting the importance of basic documentation in establishing standard protocols for rich agent-based models of markets. Such documentation should include availability of archived model source code matched to sample data and record complete parameter settings. As work on these models progresses it is inevitable that new model versions are investigated - again this needs to be rigorously reported and controlled.

The use of MatLab as an ACE modelling platform may also have contributed to some of the issues in the reconstruction. Although it allows rapid prototyping for simulation runs it was very slow, with a single run taking up to 5 hours. This seriously affects the development and testing process. Although the scripting language allows some function encapsulation the overall structure is vulnerable to errors in agent construction and referencing. Given the complexity inherent in ACE simulations

this is not acceptable as a weakness as it imposes a heavy verification cost, particularly given the relative slowness when compared to compiled languages. The major strength of MatLab for agent-based models is its financial analytical toolboxes. The approach taken therefore following this study is to develop an experimental architecture (the SHaaP architecture presented in Chapter 4) on a specific agent-based modelling platform, Repast Symphony, while using MatLab, where appropriate, for analytical support.

It is clear from the problems encountered in replicating reported model performance that the SFASM developed here is not the same as the original model reported in 1997 and 1999. This limits any inference which may be drawn about agent-behaviour in the original studies. However the findings still raise interesting questions about the interpretation of the results presented by the authors and may merit further investigation. It has proved to be a very useful testing ground for population risk-adjusted relative performance measures. The ability these give to break down agent-level behaviours has proved invaluable in understanding the dynamics of the model, and is very encouraging for their use elsewhere.

In the case of the SFASM version implemented here, the evidence uncovered by the relative performance scores makes it clear that traditional analysis of time series features is not sufficient to draw valid, operationally meaningful conclusions about market structure or agent behaviours. This study strongly suggests that no link between learning and technical trading is present at least in this SFASM reconstruction. Although some bit-level behaviours are similar to those reported in the original studies there is no apparent connection between bit usage and trading behaviours.

However since no agent-level analysis was carried out in the original SFASM studies it is impossible to say whether the same was true of their results. Previous studies have frequently confused forecast accuracy with trading success and the absence of appropriate, agent-level performance measures made this confusion difficult to address. In general unrealistically large sample trading periods, summary market measures have been used to direct inference about agent behaviour. Population risk-adjusted relative performance measures allow agent trading behaviours to be directly examined with greater confidence in a principled manner.

## 5.4. Summary & Conclusions

The agent-level measures introduced and developed in this chapter have proved to be effective in the exploratory analysis of a rich artificial financial market, yielding valuable insights into agent behaviours in this experimental system. The relative performance measures,  $I_X$  and  $I_R$ , are potentially important tools for both exploratory and confirmatory analysis when used in combination with inter-agent, relative effective return matrices and relative return surfaces. The comparative study here showed that  $X_{eff}$  based measure  $I_X$  was more stable and acted as a better filter, so this is used in the experiments in Chapter 6 - similarly, gross returns rather than Benink's 'excess returns' are used in constructing  $X_{eff}$  as a more realistic, situated measure in constructing  $I_X$  and its components.

Unsatisfying and challenging though parts of the work on the SFASM were, overall it was a highly beneficial exercise. It provided a useful working example highlighting

problems with existing agent-based modelling approaches both in terms of verification and validation, supporting the arguments presented in earlier chapters of this thesis.

Reconstructions such as this are seldom reported in the literature, a fact which Binmore[13] identifies as a failing in experimental economics. This may be due to a number of reasons. One factor, amply evidenced here, is that reconstruction is an expensive undertaking in terms of time and effort, often with little obvious reward, especially given that researchers have their own research interests to pursue, and particularly if suitable agent-level measures are not available. The measures developed here go some way to addressing at least this latter issue, making forensic, agent-level, exploratory analysis possible during primary research as well as in later reconstructions.

A second, arguably more interesting, factor may relate to validation and the operational relevance of many agent-based models of economic systems. It has been argued in this thesis that situation is a basic validation criterion linking directly to operational meaningfulness. Models which are used and applied in situated environments are inevitably subjected to close scrutiny, repeated replication and testing - particularly in performance driven, highly regulated economic systems. Absence of reconstruction efforts may be symptomatic of a lack of operational relevance in many models, and a consequent lack of pressure to test and validate their results.

This is not to say that abstracted academic models are not useful or relevant in themselves, however the need for adequate validation and verification metrics and techniques remains. The new measures presented in this chapter appear to be useful tools for this objective.

# 6. Simple Heuristics vs. Risk & Uncertainty - A Comparative Case Study

*'A good trader is someone who knows when to panic.'*

Trading aphorism, unattrib.

## 6.1. Overview

'Fast & frugal' heuristics have been presented as effective candidates for cognitive decision processes and preference expression based on observations of such heuristics applied in situated environments[49, 10, 99]. Preference modifying heuristics are also commonly observed in economic preference expression, however their role remains relatively uninvestigated. This chapter presents an investigation of heuristic and subsumptive preference modifiers using the SHaaP architecture described in Chapter 4, applying the performance measures and exploratory data analysis tools introduced and developed in Chapter 5.

The experiments presented here are primarily exploratory, reflecting the view from earlier chapters that this is both a necessary precursor step, and complementary element to validating and verifying confirmatory work.

Two main objectives are addressed.

- To investigate the role of heuristic preference modifiers as part of larger cognitive processes and economic preference expression and in doing so to demonstrate subsumption as a functional structural meta-heuristic in this expression.
- To critically examine subsumptive and population-based structures as implemented in the SHaaP architecture; and to demonstrate and review performance metrics and exploratory tools developed in this research.

The potential range of application for the SHaaP architecture in exploring heuristic preference modifiers is rather large, as is the number of methodological issues raised in earlier chapters. Necessarily there are limitations to the scope of research which can be reasonably addressed in one PhD, so in the case studies presented here the focus is on heuristic preference modifiers as part of overall individual economic agent and agent population preference expression, and particularly their role in risk management and uncertainty mitigation.

The structure of this chapter is as follows,

- After presenting the background context for simple heuristic preference modifiers, simple heuristics in economic decision making are revisited, presenting exemplars for use in the experimental work.
- The experimental protocols are set out detailing the main model structures and parameterisation, together with trial & calibration experiments.
- Three sets of experiments investigating simple heuristic economic preference modifiers in the SHaaP architecture are presented exploring and analysing,
  - Economic agent behaviours, where 'minimal core' preferences and heterogeneous cognitive bias interaction with heuristic modifiers are the focus.
  - Adaptivity and learning using a particle swarm optimisation (PSO) implementation.
  - Higher levels of subsumption, 'agent-subsuming' agents, and decision making in situated environments.

The overall results and their implications for the architecture as an experimental platform and future work on performance measures and tools developed thus far are critically examined.

- Finally, the experimental findings are discussed in terms of risk and uncertainty mitigation in situated environments.

## Background

Nature inspired heuristic optimisation, local search, and classification algorithms have been, and remain, a major computational intelligence research area, yielding significant successes for otherwise computationally prohibitively expensive problems and cases where speed is an important criterion. There are many studies in the literature providing systematic comparisons of the relative merits and efficiency of different algorithms, having, in Tukey's terms, moved beyond simply exploratory investigation to confirmatory work.

In financial applications practical solutions to problems are frequently cast in terms of optimisation, where accuracy becomes a key performance metric, and issues with speed being addressed by increasing computational resource. This is neither surprising nor a major issue in itself: indeed in the same way as experimental economists frequently address questions with tractable answers, optimisation problems are attractive in that they have identifiable and measurable success criteria.

However resilience in model performance when transferred to, or tested in situated environments is seldom formally discussed. Where models are recognised to be unstable or brittle, supplementary structures or behaviours designed to stabilise them are generally presented as secondary aspects to model management rather than significant components of a larger overall model. Dacorogna et al[34] describe such structures in practical trading model design, while Kaminsky & Lo[65] refer to 'overlay strategies'. In this thesis such structures are set within a subsumptive framework, forming part of overall preference expression rather than ignored or downplayed.

As discussed in Chapter 3 typical academic models of economic preference expression and investment have similarly centred on optimisation. Investment models based around risk rather than uncertainty typically fall into this category, as do forecasting models where predictive accuracy is the core performance metric. Modern portfolio theory (MPT) focuses on creating an optimal portfolio structure - maximising return, while minimising volatility of returns. MPT requires a large number of assumptions as part of a neoclassical framework (see Section 2.5), many of which are known to be incorrect, or, at best, poor abstractions of actual experience. As a modelling approach this would be acceptable except that, when applied to even moderately complex real world scenarios, some assumptions that MPT requires can break down catastrophically under uncertainty, or are simply intractable. At the same time some studies have shown simple heuristic investment rules to outperform MPT itself[36] - if ecological rationality is adhered to, where performance is better, or even equivalent, such simple heuristic rules should be preferable then since their cost and speed are distinct advantages.

Whether core preferences take the form of simple heuristics or sophisticated mathematical models, such as MPT, they are seldom if ever used in isolation in real situations. Minimally they require supporting administrative, subsumptive structures for expression, and generally also are observed to be subsumed with, or by, preference modifying heuristic structures. Such structures are regularly observed both in individual economic actors, and in larger commercial, economic entities, and environments, where operational functions and specialisations are incorporated into a larger operational whole. Within such entities, in industry and in dedicated financial institutions, gross heuristic preference modifying structures are frequently observed, working both alongside, and overarching quantitative risk management analytics and tools. Corporate governance, audit, and legal structures act on commercial behaviours modifying core corporate preferences, whether implicit or explicitly recognised.

A functionally useful, though rough categorisation of modifiers would be to split them into two groups: those which interact directly with core preferences and form part of an economic entity's individual preference expression & cognition, and those which are applied externally as part of the overall economic environment imposed on and affecting whole populations of independent economic agents. This is a broad distinction and it will be clear that it is somewhat artificial: the distinction between external and internal becomes blurred as behaviours are incorporated into day to day activity, while in subsuming agent entities the portfolio modifiers described in Section 4.5 could be regarded as both internal and external. However it is a useful starting point.

The experiments in this chapter, while recognising and catering for structural and portfolio modifying heuristics, focus primarily on direct modifiers in the SHaaP architecture. External modifiers will form part of the future work discussed in Chapter 7 in the context of external regulatory structures and policy formulation. In describing the experiments and protocols here, reasonable familiarity is assumed with the functional structure of the SHaaP architecture and component elements set out in Chapter 4.

## 6.2. Simple Heuristic Preference Modifiers & Economic Cognitive Processes

Economic systems, i.e. systems in which economic preference expression is a key component, are differentiated from other naturally occurring systems by their complex adaptive, self-regarding nature giving rise to non-linear behaviours and uncertainty<sup>1</sup>. Economic agents in such systems, such as regulators, policy makers, investors (who may just be the apocryphal 'man in the street'), or corporations, exposed to such uncertainty have typically focused on optimisation based methodologies for managing risk, rather than overall economic exposure, particularly uncertainty. As presented in the preceding chapters, this emphasis ignores potentially useful and informative heuristic uncertainty mitigating structures, which appear to be present in practical application within real economic systems.

Simon's[129] concepts of procedural, bounded and ecological rationality were discussed in Section 2.6. The central ideas here being not just that individuals are boundedly rational, having limited computational power, but that they must make decisions with only limited information in uncertain environments subject to change. In situations where computationally efficient and unboundedly rational tools are available and appropriate, such as arbitrage trading, it would clearly be a cognitive error to ignore these. However, it is also a cognitive error to ignore uncertainty exposures, and similarly ignore structures which may deal with these - this appears largely to have been the case until relatively recently.

The SHaaP architecture set out in Chapter 4 appears well suited to investigating heuristic preference uncertainty mitigating modifiers since it offers a clear experimental platform on which individual processes can be isolated and explored. Following the economic agent representation described in Section 4.3 it also gives the opportunity to investigate subsumption as a meta-heuristic structural mechanism for developing sophisticated combinations of core preferences, preference modifying behaviours, and models of economic systems.

In ABMs investigating heuristic rules it is important to recognise that combinatorial complexity grows extremely rapidly as agent populations grow and new rules are added, as does the overall parameter space. It is necessary then to begin with simple systems, adding new components as behaviours within systems become clearer. To that end, in the experiments that follow the economic agents are imbued with only minimal core (MC) or near-minimal core (near MC) preferences - these are akin to Gode & Sunder's 'zero intelligence' (ZI) traders[50] and Cliff's near-ZI traders[32] although in those cases demand functions were explicitly defined and the system structures highly simplified and abstracted.

The aim here is to examine preference modifiers in relative isolation rather than to build or tune better investment rules - modifier performance and their effect on agent behaviours are the focus. In itself it is an interesting question to see how important a modifier can be to economic performance with deliberately weak, or non-existent, core preferences. If modifiers are sufficient on their own to generate profitable investment behaviour that would be quite significant if for no other reason than to challenge assumptions about investment performance and survival bias. In later

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<sup>1</sup>(see Chapter 2 for definitions of risk and uncertainty used here)

work more sophisticated, or at least alternate, core preferences can be introduced and explored with modifier components.

A problem with this approach is that it potentially ignores the interaction of different modifiers when combined with different core preferences and as such is, ironically, an oversimplification. This is a valid criticism, however the same is true of any studies of economic preference expression: to date studies of core preference expression have effectively ignored preference modifying behaviours and structures. The work here is the first step in setting out a principled approach to systematically exploring and understanding these interactions, so in this case the oversimplification is a necessary precursor step to building larger, more realistic models.

A second issue, and one which affects much ABM research, is the ease with which models can be developed and simulations run. It would be easy to become bogged down in confirmatory behaviours of particular model effects, without a good understanding of the modifiers themselves. The aim of the work at this stage is to develop an understanding of modifier characteristics in situated environments, while at the same time developing and validating the experimental framework and investigative tools.

Finally, it is also important to recognise that although advocates of simple heuristics in cognition argue strongly for their power, it would be incorrect to claim that the heuristics have evolved to solve particular problems. This would suffer from the same flaws as 'evolution as progress' advocates<sup>2</sup>, or ACE researchers imposing anthropomorphic intent on optimisation algorithms. It seems more likely that empirically identified uncertainty mitigation heuristics share common properties which are beneficial in uncertain competitive economic environments. These properties are key areas of interest, as will be developing tools to identify them.

In the next section some common simple heuristic behaviours are identified before moving on to set out the main experimental protocols.

### 6.2.1. Situated Simple Heuristic Preference Modifier Exemplars

Three preference modifying heuristic structures are identified for investigation in the following experiments. These are specifically heuristics which work directly with core economic preference structures to form an agent's overall preference behaviour. The main experiments concentrate on a detailed investigation of stop-loss as a preference modifier and its characteristics, illustrating the exploratory process and analysis involved using the architecture. Experiments with trailing stop and satisficing heuristics allow critical comparison of modifier characteristic and behaviours, while serving as a reminder of the sheer scope of the domain as component elements in preference expression are combined.

1. Stop-loss. For an investment where the return hits a pre-determined loss level the investment holding is exited immediately, or as soon as is practicable.
2. Trailing stop (or stop-profit). Similar to stop-loss except that as an investment moves into profit the investor maintains a stop relative to its highest profit (a

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<sup>2</sup>As discussed in Chapter 4, Henry Gee[47] provides elegant examples of why this is flawed thinking.

high-water mark) so that if it starts to fall the investment will be closed and profits realised rather than risk moving back to a loss.

3. Satisficing. For a given investment, as Gigerenzer[49] following Simon[125, 126] describes, there is a point where the investor may decide they are satisfied with the return for an investment, rather than having a specific investment target even if they believe they could make greater returns.

Obviously other preference modifying heuristics can be identified, for example 'saving heuristics' where a fixed proportion of available investment capital is always kept in cash and added to from profits, or 'copycat heuristics' where core preferences are overridden by observed successful behaviours. However, the three listed above are common and, particularly with stop-loss, cited at all levels of institutional and professional trading as important in basic operational good practice. They provide a reasonable set of test cases for the approach developed here, and as has been noted do not appear to have been systematically investigated previously using agent-based models.

These heuristics can be applied to individual investments, at portfolio, and at overall corporate activity levels, separately, or in combination to create compound behaviours. A number of other portfolio and population-based modifying and structural heuristics are also identified, including absolute position limits, trade frequency and budgeting constraints. Since they have direct operational relevancy to actual trading behaviours they have been incorporated in the experimental design although they are not systematically investigated at this stage.

### 6.2.2. Preference Modifier Heuristic Payoff Profiles

The three preference modifiers listed affect the payoff profile for an investment strategy. The critical effects are to make a profit and loss profile for an investment asymmetric and path dependent. Figure 6.1 shows simplified payoff profiles for the stop-loss and the satisficing heuristic with trigger thresholds of -0.005 and 0.008 respectively<sup>3</sup>.

These profiles are similar to those for a simple financial Call option.<sup>4</sup> However they differ from basic financial options in that they exhibit some path dependency (if the trigger value for the heuristic is reached while the rule is active before normal exit it is activated immediately) and no premium is paid or received as for an option.

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<sup>3</sup>The simplification here is effectively a view at the limit when the investment is exited. Various quantitative approaches have been suggested to estimating the value of stop-loss rules, such as Shen[119] and Kaminsky & Lo[65], where expected return profiles are generated probabilistically. While these have merit they immediately fall prey to the problem of presuming underlying asset distributions are known or knowable, a fundamental issue highlighted in the discussion of risk vs. uncertainty in Chapter 1 and Chapter 2. Where they may be useful however is in parameter setting and this is discussed in Section 6.3.2.

<sup>4</sup>

Financial options give a buyer of the option on an underlying asset the right, but not the obligation to sell, in the case of a put, or buy, for a call, an amount of the asset at a fixed price for an agreed period. There are many more exotic derivatives with option-like return profiles, some of which also exhibit path dependency.

The stop-loss payoff graph in Fig 6.1 shows an equivalent long Call option profile for comparison, the difference being the effect of a premium cost on the Call payoff (for simplicity taken to be equal to the stop-loss trigger level - in reality of course time value and market conditions determine option premia and these are unlikely to match off to make graphs easy to compare). The satisficing profile is similar in shape to a short (sold) Call position. The trailing-stop profile would look similar to the stop-loss profile, but would have to dynamically change and would be difficult to show in a simple chart.

The important characteristic of these heuristics is their ability to overlay an asymmetric payoff structure on core preferences. Such structures may be useful (as with option trading) in tailoring desired return and volatility of return characteristics. It can also be seen that if a stop-loss behaviour was combined with satisficing the profile limits loss but caps profitability. In the experiments later in this chapter compound modifier experiments are discussed. If this process were continued and more components added, potentially including extra core preference behaviours, a very complicated profile could result.

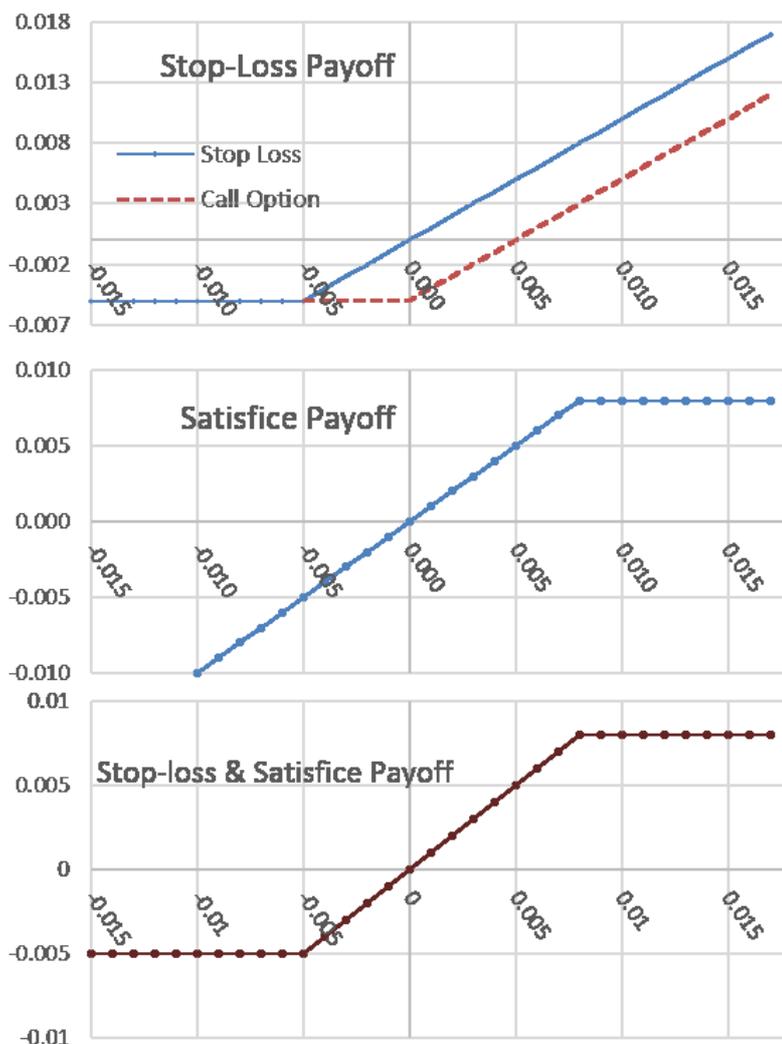


Figure 6.1.: Option-like Preference Modifier Payoffs

## 6.3. Experimental Protocols & Structures

This section sets out the main protocols and structures used in the experiments presented in Section 6.4, describing the core preferences, heuristic modifiers, and operational market structures for agents as they make investment decisions, addressing the design questions and considerations set out in Section 3.3.

The experiments consider preference modifying heuristics as they affect minimal core (MC) preference and near-MC traders situated in equity and foreign exchange (forex) markets.<sup>5</sup> Traders must choose between investing in a single risky asset (either equity or forex as determined at the beginning of each simulation) and a riskless asset in the form of cash.

Such deliberate simplification of core preferences can obviously be criticised as a form of abstraction, however this is a necessary initial step to understanding at least the gross characteristics of preference modifiers. Given risk-adjusted economic performance metrics as common measures of both core preference and modifier performance, separating the contributions of core preferences from modifier is challenging. The use of minimal core preferences here allows modifier behaviours to be systematically examined in some degree of isolation. It remains a valid concern however and will be considered in the discussion of experimental results.

### 6.3.1. Minimal Core Preferences

The core preference structure used by simple MC economic agents<sup>6</sup> is straightforward. Each trader maintains a set of 20 portfolio rules each of which can have open positions at any time. Each portfolio rule takes its core preferences and modifiers from the best available rulepacket at the time of trading (see Section 4.5.3 for a description of this process in the architecture).

Each position represents a separate investment governed by core preferences and modified by any active heuristic structural or preference modifiers. Open positions are aggregated across the portfolio and can net out, i.e. some can be long while others short, while returns for each position are monitored separately.

For each portfolio position traders have three basic core decisions in any period: trade entry; trade exit; and trade size ('gearing'). For MC traders these are based on probabilistic parameters drawn when each agent and its rule population is instantiated.

- Trade Entry. The decision to initiate a trade, opening a new position for any portfolio rule that does not have an open position already. MC traders may do nothing, buy, or sell according to probability weights set at the start of each simulation, as shown in Eqtns 6.1, 6.2, & 6.3. Here the 'do nothing' weight,  $dn$ ,

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<sup>5</sup>

It should be emphasised that these are not the same type of traders as those used by Gode & Sunder[50] or Cliff[32, 33] which were designed around demand function experiments. The MC preferences here are simple bias driven investment decisions and are closer to those used in Benink et al[8, 9].

<sup>6</sup>Hereafter referred to variously as agents, traders or MC traders.

is set to 0.6 in all experiments so that 60% of the time traders will not trade even when they have an opportunity - this is a somewhat arbitrary choice to ensure that not all traders of a given type commit to a trade at the same time.

The *bias* parameter is a fixed value in the range 0 to 1, determining how bullish (positive) or bearish (negative) the agent is about the market and affects how likely they are to buy or sell. A neutral bias is 0.5, so that when a trader decides to trade they would be equally likely to buy or sell.

$$P(\text{doNothing}) = dn \tag{6.1}$$

$$P(\text{buy}) = (1 - dn) * bias \tag{6.2}$$

$$P(\text{sell}) = (1 - P(\text{buy}) - dn) \tag{6.3}$$

- Trade Exit. The decision to exit an existing trade, closing an open position. As each trader's population of rules, with associated core preferences, is initialised each core is assigned an exit (investment) horizon either fixed or drawn from a Poisson distribution. This sets the maximum length for any individual trade. The default fixed value is set to a maximum of 50 trading periods, while the mode of the Poisson distribution is set to 5.

At the same time there is a fixed probability of early exit in each trading period, creating a Bernoulli process. Anthropomorphically this would represent a difference in the traders 'view' and investment horizon. In simulations this has typically been set to 0.07 giving a median expected trade length of 10 periods and a mean expected length of around 14 periods. Obviously there is an interaction here with the maximum trade length, be it fixed or drawn from a Poisson distribution, although the default maximum length is sufficiently long as generally not to be a factor.

- Gearing & trade size. In these experiments gearing is set to a fixed base currency amount for the risky asset. At trade entry traders can only buy or sell units of the risky asset based on this fixed amount. The default value for this is 3,000 currency units: thus for GBPUSD at a rate of 1.5 would equate to 2,000 asset units, while for the FTSE 100 index at a price of 2,500 this would equate to 1.2 asset units. Since we are interested in returns rather than specific currency amount the underlying currency amounts or notional unit sizes are not seen as particularly important in the current experiment set<sup>7</sup>.

At this stage there is no additional gearing component, i.e. no confidence element, which would increase or decrease trading size dependent on the strength of a trading signal or confidence, although there is provision for it in the RepastS SHaaP architecture implementation.

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In all the experiments the assumed base currency symbol will be '\$' to simplify reporting - clearly FTSE prices are reported in British Pounds and GBPUSD is US\$ per British Pound, but since agents are only trading between cash and the risky asset this is not significant.

### Structural & Portfolio Protocols

Heuristic rules govern transactions at the portfolio level constraining activity and preference expression according to budget, frequency, and size limits. These fall into the category of preference modifiers: as such, although they are not investigated here and have fixed parameters, they may be directly controlled in the SHaaP implementation allowing for future work in this area.<sup>8</sup>

- Latency. After each portfolio transaction is closed out the rule governing that position becomes latent for a specific period before it can initiate a new position. This prevents over-trading. Here it is arbitrarily set to a single period.
- Minimum & maximum net positions. Although a fixed limit of the number of portfolio rules acts as a hard constraint, smaller net position limits can be enforced.
- Budget. Accounting functions operate across the portfolio. For buy transactions this means that if initial allocated capital is exhausted no new long positions can be initiated.

### Transactions & Operational Mechanisms

- Friction. Trades are assumed to be frictionless, i.e. there is no bid-offer spread for securities and no brokerage charge. This is clearly not the case in actual markets, but simpler to choose zero as a value than an arbitrary number for exploratory work. Costs in many markets have collapsed in recent years and for even medium sized transactions become relatively insignificant.<sup>9</sup>
- Entry price, exit price, and transactions. All transactions are carried out at the closing price of the asset in any given period. Although trades could be carried out throughout any trading day, given a MC premise this is not particularly controversial mirroring many investment fund execution protocols. It is also assumed implicitly that an agent's trade does not itself affect the market - to all intents and purposes and certainly for index trading and foreign exchange trading this is fair. In smaller, less liquid markets for individual stocks or commodities this assumption, and the friction assumption would need to be defended or changed.

In each case the SHaaP architecture implementation allows these mechanisms to be directly controlled and modified, so that friction and pricing rules can be introduced if required.

#### 6.3.2. Preference Modifier Structure & Parameterisation

In their basic forms the three preference modifier heuristics examined in the experiments here are relatively simple to implement and can be set up with only a single

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<sup>8</sup> In the Java SHaaP version here these are set up as inner classes implementing generic interfaces to allow for robust future model design.

<sup>9</sup>see Section 2.4.3 where this is discussed in terms of high-frequency trading

parameter representing a trigger threshold which invokes the modifying behaviour. As with any other parameterisation decision, some coherent (or at least clearly specified) rationale is necessary, although ideally it would be preferable to have agents acquire or learn good settings from their experience in the markets themselves. To establish a parameterisation rationale it is worth revisiting Fig 6.1 while considering the perceived operational purpose of the modifying heuristic in dealing with uncertain exposures.

### Stop-loss

Stop-losses are widely used in trading and investment management as uncertainty and risk limiting structures. A stop-loss serves to force an investment or trade to be closed out as new information becomes available, to update investment models, or as uncertainty increases and estimates of potential loss become unacceptable.

A stop-loss can operate on specific open positions or total exposure to reduce risk in a targeted or more general way. In these SHaaP experiments it operates on individual open positions (long or short) in the risky asset which are closed out if a trigger negative return,  $r_{exit}$ , is reached or exceeded.

Setting initial values for  $r_{exit}$  is to some extent an arbitrary process, though there is the obvious practical constraint that  $r_{exit}$  should never be positive or all trades would immediately be stopped. Various computational approaches have been described to establishing good parameter settings, including Shen[119] and Kaminsky & Lo[65], however these typically require assumptions about underlying asset distributions, which is potentially problematic for uncertainty prone environments.

Pragmatically and intuitively,  $r_{exit}$  should not be too large or the stop-loss will never operate and at some level it should relate to an expected return from the investment, whether as individual transactions, or where a portfolio is being managed the targeted or expected return on the whole portfolio.

It should also have some relationship to expected market volatility, or, given uncertain exposure, to estimates of unacceptably extreme movements. A low value for  $r_{exit}$  in high volatility conditions could lead to loss making exits too frequently and too early in the life of a trade, when recovery and profitability are still reasonable expectations. A high setting in low volatility would render the stop-loss behaviour ineffective - losses would be realised only at high levels and infrequently relative to expected returns - as an uncertainty mitigation measure the stop would only be triggered late in a market shift.

A heuristic (and pragmatic) domain driven approach was taken here to establish reasonable starting values for  $r_{exit}$  in trial runs by relating it to mean absolute returns for the underlying market. For both FTSE100 and GBPUSD time series absolute daily returns for the sample periods in these experiments appear to lognormally distributed with the properties set out in Table 6.1.

Given the practical non-positive  $r_{exit}$  constraint, the protocol adopted here is to draw a value from a lognormal distribution,  $X$ ,<sup>10</sup> reflected on the ordinate axis with mode and scalar to produce a wide range of candidate stop-loss values. A uniform distribution could be used but this would then result in a relatively high number of tight (low value) stops compared to what might intuitively seem reasonable.

Timeseries	Mode returns	Median	70% <	Mean[ln(X)]	StdDev[ln(X)]
FTSE100	0.120%	0.476%	0.882%	-5.348	1.176
GBPUSD	0.060%	0.248%	0.464%	-5.998	1.193

**Table 6.1.:** FTSE and GBPUSD Continuously Compounded Daily Returns

The choice of mode and scalar determines the modifier parameters for iAgt's' rule-packet populations and their dispersion. For the initial trials modes were set in a range around the median of the underlying timeseries daily absolute returns - for FTSE100 the range tested was 0.25 to 1%, while for GBPUSD the range tested was 0.10 to 0.5%. Scalars in a range from 0.5 to 1.0 were also tested.

Figure 6.2 illustrates probability distribution functions approximating the FTSE100 timeseries and sample distributions with mode 0.5% and scalar values of 0.5 and 0.75. As can be seen with a scalar of 0.5 extreme values are rare relative to the underlying distribution, whereas with 0.75 this generates a much broader range and allows a greater proportion of tight stop values.

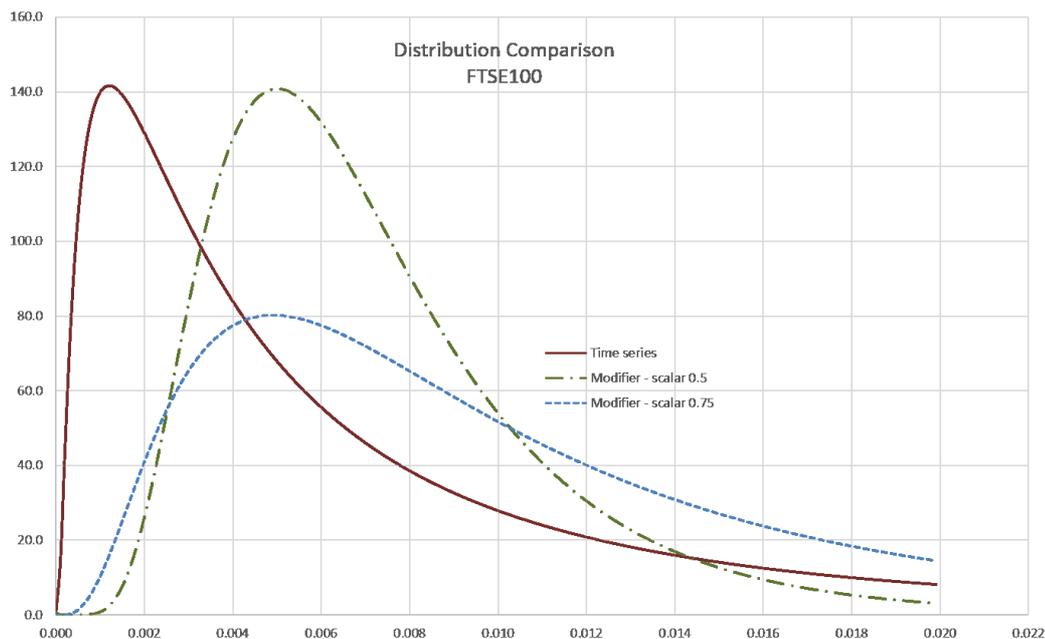
The results of these trials are shown in Section 6.4.2 The overall question of how sensitive preference modifiers are to parameter settings is addressed in later experiments applying a PSO to allow traders to learn over time.

### Satisficing

Described by Simon[125, 126] satisficing takes the form of a general exit choice heuristic when searching for a particular outcome, but where it is not possible to know what the optimal outcome is - the user is satisfied at some point when the current outcome is judged 'good enough'. When applied to investment decisions, specifically for existing investments it becomes a form of core preference modifier: instead of following a core preference behaviour to its conclusion, when a given level of return is reached the investment is closed. What that given level is, or how it is established appears to be a subjective process.

This behaviour could, as with other heuristics have added components, but in the simple form implemented here it is based on a single target threshold. Recalling that the MC preferences here have no target return for any trade, a satisficing heuristic gives a mechanism to capture and realise profits on unusually large, unexpected favourable market moves. This can function well as such moves are often subject to some retracement so a mechanism to capture sharp spikes up or down can be advantageous.

<sup>10</sup>Where  $X = e^{\mu + \sigma Z}$ ,  $\mu$  &  $\sigma$  are the mean and standard deviation of  $\ln[X]$  and  $Z$  is a standard normal random variable.  $\sigma$  is also referred to the scalar for the lognormal distribution and affects its overall dispersion.



**Figure 6.2.:** Lognormal Distribution Comparison

In setting a trigger threshold  $r_{exit}$ , when the payoff profile in Fig 6.1 is considered, it is clear that this should not be too low otherwise small profits would be realised. Although this could form a conservative profit realisation strategy, potential losses would be unconstrained. It can also be seen from this profile that whereas stop loss heuristics modify risk preferences and address negative uncertain outcomes, satisficing heuristics leave negative exposure open. As such they can be viewed as augmenting rather than modifying core preferences in terms of uncertainty at least.

The parameterisation approach used is as for the stop-loss heuristic, except that the mode trial range for the distribution is taken to be generally higher, and obviously the threshold chosen is strictly positive. Based on the results in Table 6.1 mode range was tested around a 70% cut off value for each data series.

### Trailing Stop-loss

The basic payoff profile for a trailing stop-loss is the same as for a simple stop-loss, the difference being that it is adjusted to follow the high-water return level for an investment, so it has a path dependent temporal component. As with the simple stop loss heuristic, trailing stops act to limit exposure to both established risk distributions and uncertainty.

Typically, given an objective to capture profits while still allowing further gains, one might expect a trailing stop threshold to be closer to the high-water mark than for a stop loss threshold. However, given that this might be too sensitive for the initial portion of a trade it can make sense to incorporate different trigger thresholds for negative returns and the trailing stop itself. This is the form examined here: until a trade has a positive mark to market return a wider threshold is used (in effect a basic stop-loss), but after a positive return is recorded setting a non-zero high-water mark the trailing stop return is activated. This is in fact a form of compound modifier as

described below.

As with stop loss a range of  $r_{exit}$  values were explored in the trials reported in Section 6.4.2 before default values for the remaining experiments were established. The trial range was the same as for the stop loss trials since this encompassed some relatively low values, intuitively suitable for trailing stop heuristics.

### Compound Preference Modifiers

As with the trailing stop-loss it is possible to combine modifiers, or simply to overlay them within a subsumptive structure to give more complex profiles which may suit particular environments. An example payoff profile for a combination of stop-loss with satisficing is shown in Fig 6.1. The advantage of this layering, subsumptive approach is that the processes are simple to observe and understand, when added to simple core preferences the results have potential to both capture real world behaviours and to give insights in to both risk management and policy formulation. While not explored in the experiments here compound behavioural structures are an important natural extension of preference expression structures available in the SHaaP architecture.

#### 6.3.3. Learning, Adaptation & Population Structures

Parameterisation and parameter choice within any agent-based model is an important issue: in SHaaP models potential combinatorial complexity as components are subsumed makes it highly relevant and a research question in its own right. For a stop-loss heuristic, what threshold should trigger a trade or investment to be exited? For a satisficing heuristic, what is the appropriate return to be satisfied? For a saving heuristic, what proportion of capital and profit from investment should be set aside from each transaction?<sup>11</sup>

The domain knowledge driven approach for initial modifier settings described in the previous section seems reasonable, or at least plausible, but some systematic approach to exploring the parameter space is desirable. With so many degrees of freedom this is not a trivial issue and a computational method would be preferred. In addition, uncertainty and heteroskedasticity in the problem domain mean that a single parameter setting for a modifier is unlikely to be sufficiently responsive over time. Mechanisms or structures which allow agents to learn good parameter settings or to adapt over time may be important model features.

The population-based structure for the SHaaP architecture described in Chapter 4 provides individual economic agents (iAgt) with a group of candidate solutions in the form of rulepackets to select from. In this structure each iAgt and its rules have only one core preference type and one set of preference modifiers (or none at all). Agents choose from amongst their rulepacket populations based on recent historic

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<sup>11</sup>

There are of course many additional model specific structural parameters to be chosen. These are set out in the following tables and in the appendices, but as they are not varied during or between experiments, are not the focus of this section.

performance when deciding on what rules to use for preference expression. Success for an agent relies on having a large enough population of preference modifying rules to select from which should match and be identifiable in a large range of conditions. This is historic information, but as performance is weighted using exponentially smoothed averages it gives a measure of what is currently working in the market.

Even without learning or evolution, given a reasonably large number of candidate rules, this population structure allows the possibility of iAgt adaptation to the current market state. The extent of this adaptation and whether this reaches as far as the senior 'sAgts' in the SHaaP structure, which choose strategies from successful iAgts in the same way as iAgts choose from rulepackets, are experimental questions explored here.

A further question, which relates to both parameterisation and ongoing adaptation, is what value learning has when applied to preference modifiers? Given that they are presented here as uncertainty mitigation behaviours, beyond gross tuning, is there a value or benefit to further attempting to optimise their settings? By the nature of uncertainty prone systems, their underlying distributions are not known and potentially not knowable, so optimal parameters may also be unknowable. To address this question a basic learning algorithm, particle swarm optimisation, is described here and its effect on agent behaviour investigated in the experiments in Section 6.4.4.

### 6.3.3.1. Particle Swarm Optimisation In SHaaP Models

Remembering that the subsumption architecture is not primarily an optimising structure, successful agents and agent populations should demonstrate resilient behaviours and performance over time. In traditional approaches the trade-off between resilient behaviour and performance may have been cast as a multi-objective optimisation problem and many algorithms have been developed which could be applied, however this ignores the nature of economic systems demonstrating uncertainty where underlying distributions are unknown. PSO is an interesting candidate method being relatively straightforward to implement, and not reliant on assumptions about the underlying distributions of candidate solutions. As a flocking algorithm it also opens the possibility of adaptation rather than optimisation.

A basic PSO was adopted following Kennedy & Eberhart's PSO formulation[70, 71]. This was then modified based on observations of experimental runs to address rapid convergence and lock-in problems. The modifications are relatively straightforward, reflecting the fact that adaptation not optimisation is the objective: parameters should be tuned rather than optimised, maintaining some dispersion in the swarm cloud so that exploratory behaviour persists over time. These are set out in the following sections.

Rulepackets (see Section 4.5.2 for more detail) form the particles within the swarm with their modifier parameters representing location. As with rule selection by agents, rulepacket performance and fitness for the PSO uses the effective return measure  $X_{eff,i}$ . Given that core preference performance is also measured off the same metric there is the same potential confounding issue as with packet selection.

**Algorithm 6.1** SHaaP - Rulepacket Learning Pseudocode

---

```
1 iAgt RulePacket Population PSO
2
3     get Cognitive Factor parameter
4     get Social Factor parameter
5     get Inertia Weight
6
7 For each Rulepacket
8     Check pBest & current position for each Pref. Modifier
9
10    If (new pBest == true){
11        update pBest performance score and position}
12    ElseIf (no new pBest == true){
13        If (pBest decay == true){
14            apply exponential decay factor to old pBest score
15        }
16    End
17
18 For each Rulepacket
19     Check gBest & current position for each Pref. Modifier
20     If (new gBest == true){
21         update gBest performance score and position
22     }
23
24 If (no new gBest == true){
25     If (gBest decay == true){
26         apply exponential decay factor to old gBest score
27     }
28
29 Update Rulepacket population positions
30     For each RulePacket
31         For each Pref. Modifier
32             Get current pBest, gBest, position & velocity
33             Generate new position & velocity using
34                 cognitive factors ,social factors & inertia weight
35             If (velocity < threshold){
36                 Add stochastic noise to velocity if true
37             }
38             Pass new position & velocity back to preference modifier
39         End
40     End
41 End
```

---

**The Basic PSO**

Velocity,  $v_i$ , in any time period,  $t$ , is given by Eqtn 6.4 where the personal best position (pBest) is  $y_i$ ; the global best position (gBest) is  $\hat{y}$ ;  $\omega$  is a momentum weight factor;  $c_{cog}$  &  $c_{soc}$  are cognitive and social weights respectively; and  $r_1$  &  $r_2$  are random coefficients taken from a uniform distribution. Initially gBest is set randomly to one particle's position and pBest is set to the starting location for each particle in the swarm. The starting value for  $v_i$  is chosen from a uniform random distribution set between minimum and maximum values,  $v_{min}$  and  $v_{max}$ , which are

generated from minimum and maximum position values,  $x_{min}$  &  $x_{max}$ .

$$v_i(t+1) = \omega v_i(t) + c_{cog} r_1 (y_i - x_i(t)) + c_{soc} r_2 (\hat{y} - x_i(t)) \quad (6.4)$$

Position,  $x_i$ , is given by Eqtn 6.5.

$$x_i(t+1) = x_i(t) + v(t+1) \quad (6.5)$$

In application the PSO operates over each preference modifier in the rulepacket population sequentially, updating pBest and gBest values before updating particle velocities and positions.

Momentum, cognitive and social weights are fixed in each experimental run. A set of exploratory calibration runs were used to establish reasonable starting values for these weights: the PSO appeared relatively insensitive to these, though of course this should be revisited at some later date. The cognitive weight,  $c_{cog}$ , was fixed at 1.5 and the social weight,  $c_{soc}$ , was set at 1.5, following general recommendations for PSO settings in biologically inspired financial models by Brabazon & O'Neill[15].

The momentum weight,  $\omega$ , was set at 0.9 to limit the rate of particle velocity decay, while  $v_{min}$  and  $v_{max}$  were limited to a fraction of the overall potential particle range, counterbalancing the momentum setting and at the same time limiting potential convergence rates.

As the results reported in the following sections show, in its basic form convergence and lock-in occurred relatively quickly compared to the length of the experimental run. To address this, and explore whether this is in fact a bad thing for resilience or not, some SHaaP specific modifications to the basic PSO were explored.

### 6.3.3.2. PSO Modifications

There are many variants of the basic PSO presented in the literature addressing issues of rapid convergence and trapping around local optima. These include adaptations to use multiple swarms, social network topologies and adaptive variants ([150, 61, 62] give examples of these), as well as predator-prey structures, in which the swarm is subject to disruption by a chasing predator particle[124, 123]. Many attributes of these variants are appealing, however as other work on deliberately simplifying PSO models seems to suggest [105, 104], it is not clear that adding extra layers of sophistication is necessarily better than taking them away.

In the end the work presented here is not about evaluating new or novel PSO variants: simplicity is favoured over adding combinatorial complexity in the form of PSO variants designed for optimisation rather than resilience. With that in mind the modifications presented here are deliberately simplistic and domain inspired (and may well themselves be a new PSO variant).

Two mechanisms are introduced to moderate premature convergence & lock-in effects, and to maintain an exploratory behaviour.

- Particle velocity noise
- pBest & gBest decay

*Particle velocity noise.* Noise is added to a particle's velocity when it falls below an absolute threshold level,  $v_{thresh}$  - effectively when it has stopped moving. If this noise function is switched on in the PSO process then, after velocity and position updates in Eqtns 6.4 & 6.5, a noise factor is added according to Eqtn 6.6, where  $r_{noise}$  is drawn from a uniform distribution between 0 & 1 and  $v_w$  is a velocity weighting factor set to 0.4 in the current experiments<sup>12</sup>.

$$\begin{aligned} \text{If } Abs(v_i(t+1)) < v_{thresh}, \\ v_i(t+1) = v_w(v_{min} + r_{noise}(v_{max} - v_{min})) \end{aligned} \quad (6.6)$$

*pBest & gBest decay.* In each PSO update period for each particle, if no new pBest is found the performance score for a particle,  $i$ , its pBest score  $p_{pBest}$  decays according to Eqtn 6.7 to give a new pBest value,  $p_{pBest*}$ , using the mean for all particles, where  $\bar{p}$  is the mean performance score.  $\alpha_d$  &  $\beta_d$  are exponential smoothing factors where  $\beta_d = \left(1 - \frac{1}{\alpha_d}\right)$ . This function pulls the old pBest score value back towards the mean for the swarm.

$$p_{pBest*} = \frac{1}{\alpha_d}\bar{p} + \beta_d p_{pBest} \quad (6.7)$$

After all particle pBest's have been updated, gBest for the swarm is updated following the same protocol: if no new value for gBest is found in the update period, then a new gBest score  $p_{gBest*}$  is produced using Eqtn 6.8. Figure 6.1 shows the order of events for the PSO, including conditions where these modifications are invoked.

$$p_{gBest*} = \frac{1}{\alpha}\bar{p} + \beta p_{gBest} \quad (6.8)$$

The intended effect is to encourage continued exploration as over time - if not updated then pBest and gBest values decay, reflecting the fact that market conditions may change and historically successful particle positions may no longer be valid. A typical value for  $\alpha$  in the experiments was 20: given PSO invocation rates of 1 in 10 to 1 in 20 days, the potential decay rate is actually quite slow relative to the time frames involved in trading.

The combined effect of adding noise to velocity when swarm movement has collapsed, and to decay historic pBest and gBest values is intended to address the fact

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<sup>12</sup>

As with many of the parameters this is somewhat arbitrary, chosen with the view that there is likely some value to the location even if decaying, so that a random move too far away would be badly disruptive.

that the aim is to use PSO not to optimise, but to exploit its original flocking behaviours. Whether or not this is successful, or even necessary, is another matter and is discussed in the experiments in Section 6.4.4.

### 6.3.4. Risk-adjusted Performance Measures

Realistic risk-adjusted relative performance measures are a significant component in model validation and exploratory analysis (see Chapters 3 and 5 for discussion). The main measure used here, building on the work in Chapter 5, is the symmetric effective return measure,  $X_{eff}$ , which is also the basis for population-based relative performance measure,  $I_X$  developed in combination with inter-agent relative return matrices as part of the analysis of the Santa Fe Artificial Stock Market case study in Chapter 5. In the population-based SHaaP structure  $X_{eff}$  is used by rule and agent populations as a form of fitness measure and to analyse agent-level performance, its stability being preferred to alternate risk-adjusted measures such as the Sharpe ratio. The full derivation  $X_{eff}$  can be found in Appendix A.2, however it is worth looking at its core structure here, given its position in the experiments.

#### Risk-adjusted, symmetric effective return - $X_{eff}$

$X_{eff}$  is a symmetric measure, derived from Keeney & Raiffa's utility function framework[66]. In the end it is relatively straightforward to apply, as shown in Eqtn 6.9. Here profits and losses are treated equally as part of the total return measure  $\bar{X}_{\Delta t}$ , which is the mean total return in each period,  $\Delta t$ ,  $\sigma_{\Delta t}^2$  is the variance of these returns and  $\alpha$  is a risk aversion coefficient.

$$X_{eff} = \bar{X}_{\Delta t} - \frac{\alpha\sigma_{\Delta t}^2}{2} \quad (6.9)$$

The second term in Eqtn 6.9,  $\frac{\alpha\sigma_{\Delta t}^2}{2}$ , acts to reduce the effective return of highly volatile strategies. As noted earlier, this measure is attractive because at low variances, unlike the Sharpe ratio, it remains stable.

#### Return Windows

For  $X_{eff}$  as a return measure, as Dacorogna notes, using overlapping rolling return periods where these are relatively long reduces the potential loss of information inherent in using extended periods,<sup>13</sup> and in the implementation here overlapping rolling periods are used. The length of this return period, or  $X_{eff}$  window, is an important parameter choice in exploring fine detail of model simulations as discussed

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<sup>13</sup>

If the value of a portfolio is the same at the beginning and end of a set of periods but varies wildly in between that information would be lost. However this is a common problem to any periodic measure, and intra-period risk evaluation is also practically required by regulators and risk managers to trap for this.

in Chapter 5, and also when applied as a fitness measure since it relates to the trade horizon for investments. It should be noted that the windows used in the population relative performance measures and summary measures are independent of those used by agents themselves in the SHaaP models.

For the population relative performance measures (PRP) developed in Chapter 5 the window size was set to 40 periods following the experience from replicating the SFASM. For agents themselves using  $X_{eff}$  in rule selection, the window was initially set to 40 periods for sAgt observing iAgt performance and to 15 periods for iAgt observing rulepackets. However, as part of the general exploration sAgt performance and decision processes in later experiments both were set to 15 periods. This adjustment affects only sAgt performance and its effects are discussed in the experiments of Section 6.4.5.

Reflecting the concerns expressed in Section 5.2.1 regarding Benink's excess return definition[9] in situated environments and open systems, gross returns rather than excess returns are used in calculating  $I_X$  the aggregate PRP measure set out in Eqtn 5.8. This choice was supported by trial runs in which the excess return measure was found to be insensitive, picking up only very extreme volatility.

### 6.3.5. Data Series

Two main time series are used in the experimental runs documented in these case studies. Daily closing price data from the UK FTSE100 equity index (FTSE100) and foreign exchange closes for the British Pound vs. the US Dollar (GBPUSD). The time series used cover an extended period - over 35 years and 30 years respectively.

Daily closing data is used rather than intra-day information. This simplifies time period normalisation for equity markets, since this would introduce an additional structural difference between intra-day periods, beginning and end of day periods. In terms of information however it reduces the amount of data and ability of agents to capture intraday movements. Daily trading is arguably a more challenging environment for agents, while empirical evidence from HFT algorithmic funds shows that they focus on intra-day activity.

The FTSE100 time series runs from 1984 to the present. Figure 6.3 shows historic index closing prices and daily returns plotted against historic volatility<sup>14</sup>. Typical financial time series characteristics are evident in this chart - return stationarity and volatility clustering with some autoregressive component. Figure 6.5 plots historic volatilities for the FTSE100 against a volatility of volatility measure<sup>15</sup>, emphasising volatility clustering and volatility as an ARCH process.

A chart for the US Dollar to British Pound exchange rate, Fig 6.4, exhibits the same features, albeit without observable long-term trend in prices.

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<sup>14</sup>

Where volatility is the annualised standard deviation of continuously compounded returns.

<sup>15</sup>

Here simply the annualised standard deviation of the FTSE100 volatility time series.

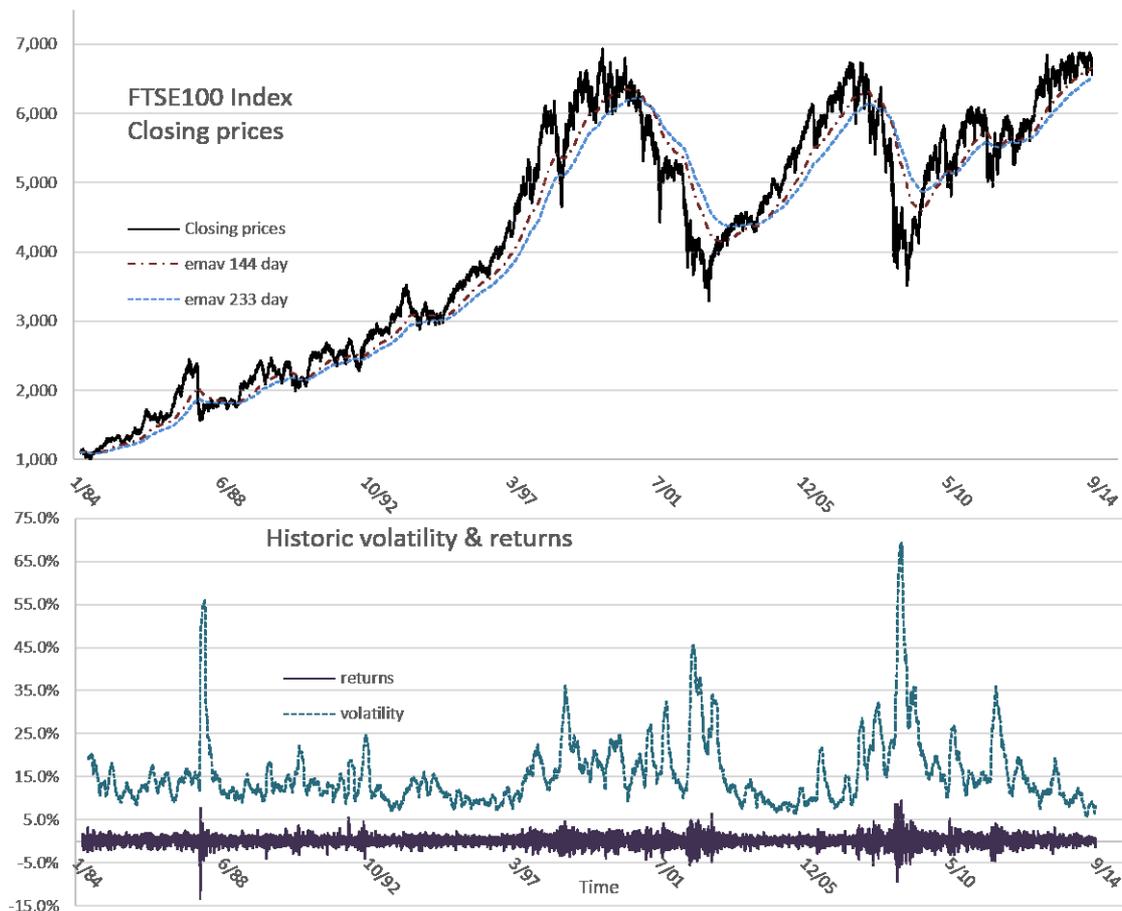


Figure 6.3.: FTSE100 1984-2014 - Closing Prices & Volatility

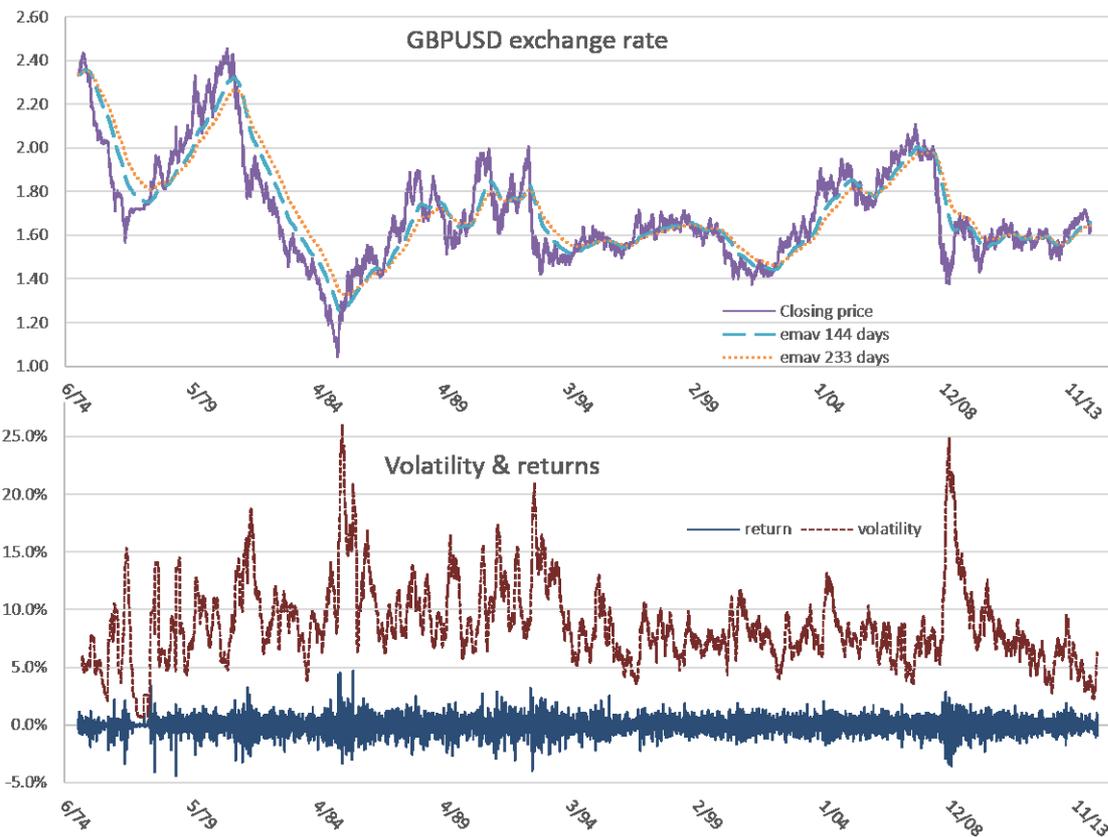


Figure 6.4.: GBPUSD Exchange Rate 1975-2014

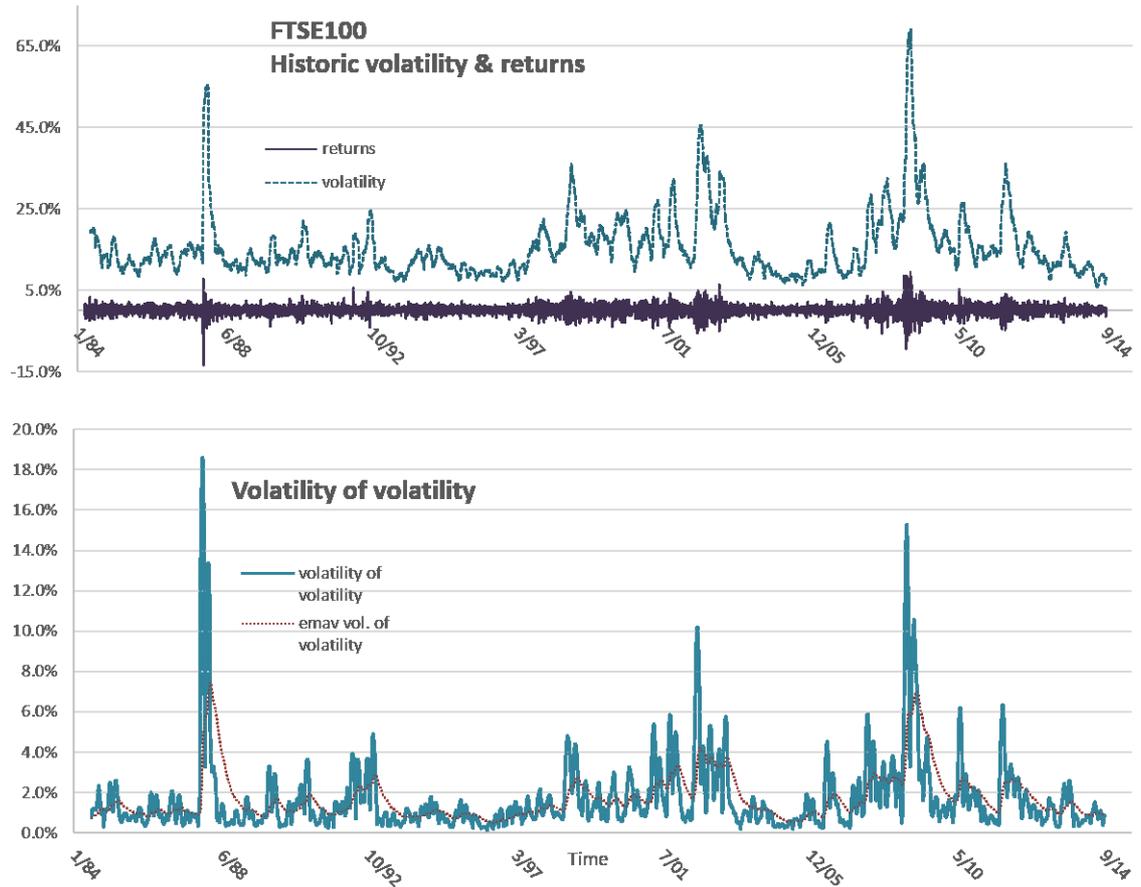


Figure 6.5.: FTSE100 Volatilities

## 6.4. Main Experiments

After preliminary trial calibration sample runs, the series of experiments presented here begins by looking at examples of iAgtS with neutral-bias minimal core preferences and no modifiers compared to iAgtS with the same core preferences and a preference modifier. The experiments then progress to comparing iAgtS, again with MC preferences and the same modifiers, but with different cognitive biases. iAgtS with different modifiers are then compared.

In all these experiments an agent-subsuming sAgt attempts to use its iAgt populations to make investment decisions. This is a form of copycat heuristic, and allows further examination of the subsumptive structure and of performance measures in operation, particularly with heterogeneous iAgt populations.

Extending the use of risk modifier heuristics allows compound behaviours to be examined and the effect on overall agent preference expression to be explored.

### 6.4.1. Model Structure & Parameters

Table 6.2 shows the default model parameters for the main experiments these are held constant in each case.

Parameters	Value	Notes
sAgt	1	Total number of sAgt
sAgt & iAgt portfolio trades	20	Maximum number of open positions at any time
Inner Agent population size	25	iAgt in each sub-population within sAgt
Rulepackets per iAgt	60	Rulepackets available to each iAgt
Rules per rulepacket	10	Number of rules in each packet, identical by preferences
Agent $X_{eff}$ period	15 - 40	Performance window for iAgt, monitored by sAgt
Agent $X_{eff}$ eMav	10	Exponential moving average period for $X_{eff}$ score
Rules $X_{eff}$ period	15	Performance window for rules, monitored by iAgt
Rules $X_{eff}$ eMav	10	Exponential moving average period for $X_{eff}$ score
Poison mean	10	Mean of Poisson distribution for maximum trade lengths
Learning period	20	Mean period between learning algorithm runs

Table 6.2.: Main Model Parameters

### 6.4.2. SHaap Trial Runs & Calibration

Initial trial runs were carried out against the main preference modifier under test, stop loss, using both GBPUSD and FTSE100 timeseries to explore the parameter setting assumptions. The main parameter for each modifier representing its trigger threshold for the modifying behaviour,  $r_{exit}$ , was drawn from a lognormal distribution following the protocol set out in Section 6.3.2. Fig 6.6 shows the net profit for 2 populations of iAgt subsumed by a single sAgt with neutral bias MC preferences trading the GBPUSD foreign exchange rate for a period from 1975 to 2014. In this trial the mode was set to 0.25% and the scalar for the lognormal distribution to 0.5.

Initially at least the results are rather striking. The mean wealth of MC traders with only a stop loss modifier is markedly different from iAgt with the same core preference but no modifier. The sAgt shows a similar profile, albeit somewhat less striking - recall that the sAgt chooses its preferences from the best performing iAgt whether they use modifiers or not. Before exploring alternate mode and scalar settings, several points are worth noting,

- Type 1 (T1) iAgt - agents with no modifier - show no material P&L variation throughout the trial. This would be expected given a random choice to buy or sell whatever the market conditions and to exit according to the MC preference protocol set out in the previous sections.
- Type 2 (T2) iAgt - agents which have a stop loss modifier - are as likely to

go short (sell) as to go long (buy), so profits accrue in all stages of a market where a trade is not stopped out.

- While T1 and T2 agents are in populations of 25 and the plotted figures are mean net profit & loss, the sAgt is a single instance and as a sample may not be representative. As this is only an initial set of experiments while exploring the parameters settings this limitation is not followed further at this point, but in the experiments in Section 6.4.5 larger sAgt populations are generated and examined.
- T2 iAgts and the sAgt perform well till around period 4,400 which equates to 1994 historically, after which although the iAgts do not lose all their gains, neither do they continue to perform well, while the sAgt gradually loses money. Recalling criticisms of agent-based models which work only off cumulative returns the same applies here. Given that there is no learning mechanism in this model the question obviously is, what changed at that point? Certainly many more electronic data feeds were available from early 1990, so it may reflect Sullivan's observation of a change in market behaviours[136].<sup>16</sup>

Recall also that while the experiments are set up to see how significant an effect preference modifiers can have on core preferences, the second important measure is how they affect risk adjusted performance, which will be explored in the main experiments.

A second trial run for the FTSE100 timeseries using the same parameters and protocol exhibited the same general features, including a similar decay in performance later in the simulation, albeit a much more significant decline - this is also shown in Fig 6.7. That agents in both markets suffer decaying performance in the 1990's again raises the question of 'what changed?'. Given an equivalent model & agent structure to the trial shown in Fig 6.6 this more severe decline is an interesting finding, though arguably not surprising given the quite different gross time series characteristics of the two markets.

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<sup>16</sup>As a practitioner at this time I spent considerable time building and using real-time spreadsheets for basic arbitrage trading. The change in day-to-day activity was profound and very profitable for a period of around 3 years till this technology became commonplace.

## 6.4 Main Experiments

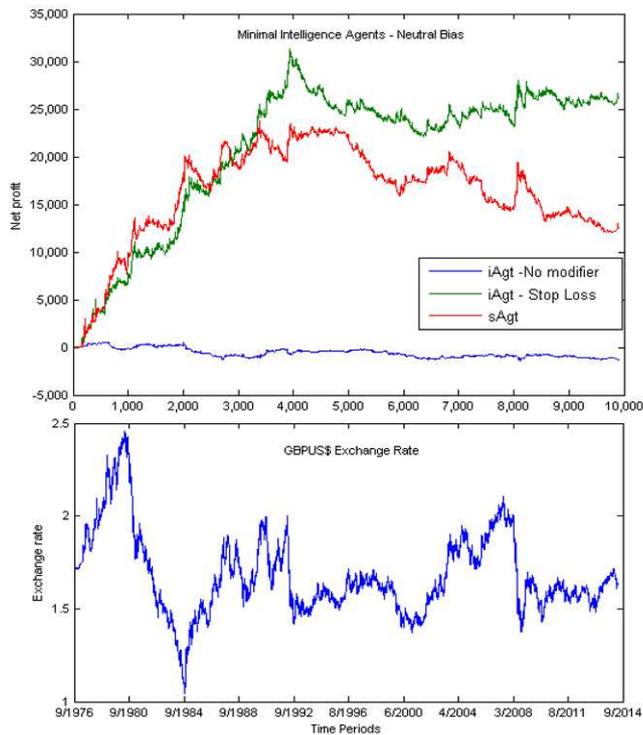


Figure 6.6.: MC Trader Comparison - GBPUSD

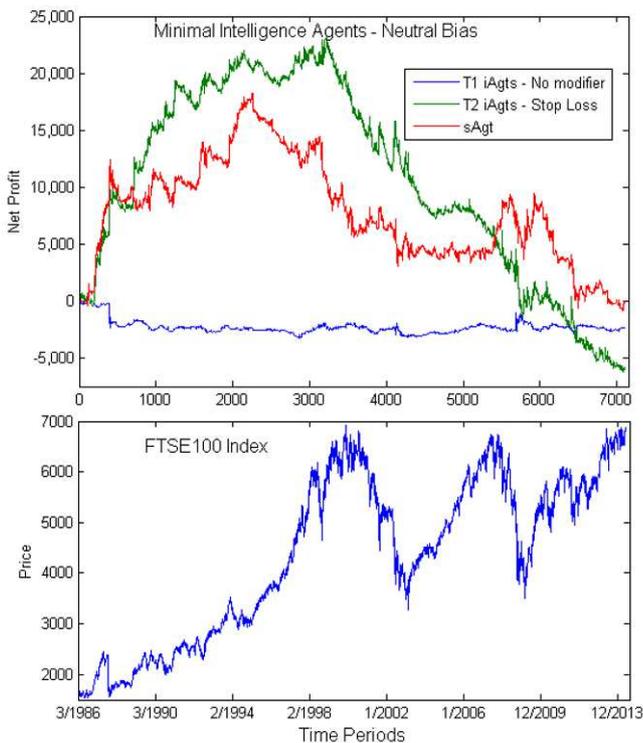


Figure 6.7.: MC Trader Comparison - FTSE100

### Mode & Scalar Settings.

Having completed initial, 'proof of concept' trial runs, a set of calibration trial runs were carried out in which the mode and scalar settings for the stop loss modifier trigger parameter were varied to explore the iAgt's rulepacket population sensitivity to these values during initialisation. Figures 6.8 & 6.9 show the effect on cumulative profitability over time on iAgt's for different mode and scalar settings for GBPUSD and FTSE100 respectively. The results support the general protocol outlined in Section 6.3.2, however there is no particularly clear result for either mode or scalar values. Similar results were found for equivalent trials on both trailing stop and satisficing heuristics. The general tendency being towards wider settings on both modes and scalars giving slightly more consistent, higher cumulative P&L profiles. In the end since the work is not concerned with optimising parameters to particular underlying markets a range of settings was used.

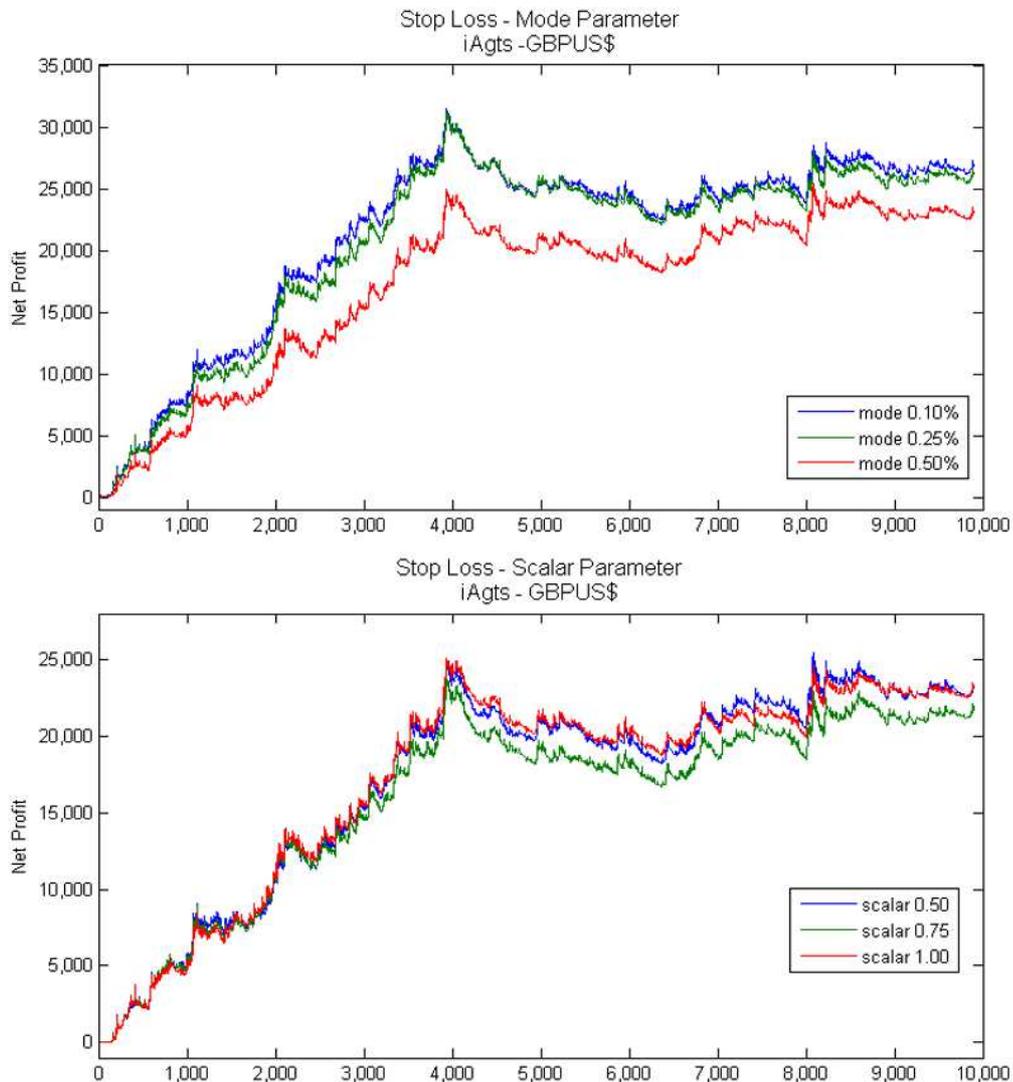
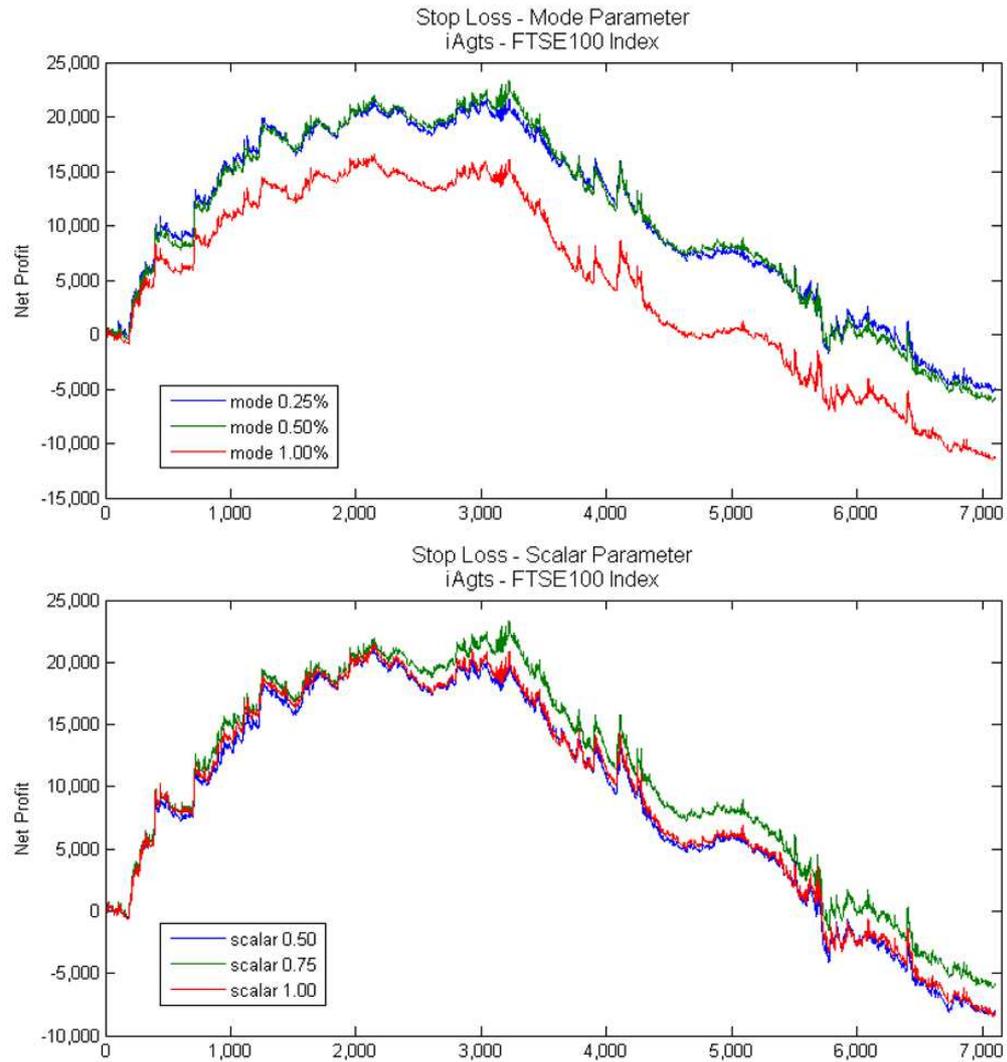


Figure 6.8.: Mode & Scalar Parameter Comparison GBPUSD



**Figure 6.9.:** Mode & Scalar Parameter Comparison - FTSE100

	Default value	Range
MC 'do nothing weight', $dn$	0.60	
MC neutral core, $bias$	0.50	
MC positive core, $bias$	0.90	
MC weakly positive core, $bias$	0.70	
MC negative core, $bias$	0.10	
Stop Loss mode range	0.50%	0.25 - 0.50%
Stop Loss scalar	1.00	0.75 - 1.00
Satisfice mode range	1.00%	0.5% - 1.00%
Satisfice scalar	0.75	0.75 - 1.00
Trailing Stop mode range	0.25%	0.10 - 0.25%
Trailing Stop scalar	1.00	0.75 - 1.00

**Table 6.3.:** Preference Parameters

The default settings for the following experiments are set out in Table 6.3 - except

where stated otherwise these default settings are used in the remaining experiments. Typically modes were set in the range 0.25 - 0.50% for stop loss; 0.10 - 0.25% for trailing stops; and 1.0 - 2.0% for satisficing heuristics. For all the preference modifiers the scalars were set in the range 0.75 - 1.0, although the trials for satisficing tended to support lower settings for that scalar value which is reflected in the default setting.

### 6.4.3. Preference Modifier Behaviours & Cognitive Bias

In this section stop loss, satisficing and trailing stop modifier performance is explored for traders with heterogeneous MC preferences. Neutral, positive (*+ve*) and negative (*-ve*) biased iAgt are compared to iAgt with equivalent core preferences and no modifiers: here the analysis provides a series of detailed investigations of risk-adjusted returns for economic agents, highlighting the importance of these measures. Two groups of experiments were run, systematically exploring the behaviours and characteristics of economic preference expression in agents with heterogeneous core preference and modifiers.

**Group 1 - Heterogeneous Cognitive Bias Experiments.** Four iAgt types with heterogeneous MC biases were initialised with one sAgt. These iAgt were each then used to demonstrate the three preference modifiers under consideration in both FTSE100 and GBPUSD markets. The four iAgt types for each experiment were,

	Core Bias	Modifier
T1	MC Neutral	none
T2	MC Neutral	yes
T3	MC <i>+ve</i>	yes
T4	MC <i>-ve</i>	yes

**Table 6.4.:** Group 1 iAgt Types - Heterogeneous Cognitive Bias

Runs using both the FTSE100 and GBPUSD timeseries were repeated using each of the preference modifiers, giving six runs in total. The sAgt in these simulations chooses between the best performing agents in the population of iAgt. The experimental design allows direct observation of differential performance of heterogeneous cognitive biases and preference modifiers amongst iAgt and preliminary information on sAgt decisions in a subsumptive hierarchy, providing support for the more detailed analysis in Group 2.

These experiments do not allow direct comparison of all agent types with modifiers to the same agent types without modifiers, that comparison is carried out in the second group of experiments. The Group 1 experiments give high level information about the interaction of cognitive biases with preference modifiers used to direct subsequent experimental focus.

**Group 2 - Matched Cognitive Bias Experiments.** Two iAgt types were initialised for each experiment in this group. Each agent type had the same MC core preference structure, i.e. Neutral, *+ve*, or *-ve* as shown below. T1 iAgt had no preference modifier, while T2 iAgt were allocated a preference modifier at

the start of each experimental run. Twenty-five iAgt of each type were used in each run. As with the Group 1 experiments, each run was repeated using both timeseries for each of the 3 preference modifiers, giving a total of 18 runs in this group.

	Core Bias	Modifier
T1	MC Neutral/+ve/-ve	none
T2	MC Neutral/+ve/-ve	yes

**Table 6.5.:** Group 2 iAgt Types - Matched Cognitive Bias

The objective of these experiments was to directly compare iAgt with the same core preference biases with and without preference modifiers, so that the effect of their trading biases modifier performance could be observed and analysed. Although an sAgt was created by default as part of the SHaaP model, its performance is not analysed in this set of experiments - that forms part of the later experiments in the chapter.

**Parameter Settings & Experimental Run Protocols** In these experiments the default parameters for agent structure and preferences used are as set out in Tables 6.2 & 6.3, with the exception that in the Group 1 experiments with 4 iAgt types, due to memory constraints there were only 20 iAgt per type, giving a total population of 80.

Risk-adjusted return measure window - the  $X_{eff}$  window for analysis and summary statistics was 40 periods in all cases.

### Group 1 Experiments - Results & Discussion

Each sample run allows comparison of iAgt with the same preference modifier but different cognitive bias, and also to the behaviour of a subsuming sAgt for each population of iAgt. Agents show markedly different performance depending on their combination of MC preference bias and preference modifier. Figures 6.10 & 6.11 show the net profitability for the various trader types with each modifier trading GBPUSD and FTSE100 respectively. Although cumulative P&L statistics and charts have limited use, they do allow high level path dependent behaviours to be observed - information which is lost in summary statistics.

Some characteristics stand out and are discussed in more detail below. The Group 1 experiments continue the exploratory approach begun in the calibration experiments in Section 6.4.2. The main areas of interest are,

- **PREFERENCE MODIFIER RELATIVE PERFORMANCE & COGNITIVE BIAS EFFECTS.** Following the trial run results which showed significant changes in neutral bias MC iAgt performance, how do cognitively biased MC iAgt with modifiers perform compared to iAgt with no bias?

The earlier trials showed marked differences in iAgt performance between GBPUSD and FTSE100 timeseries. The markets themselves exhibit quite different gross asset price properties, with the stock market index showing long

term positive trends. One might expect differential performance for iAgt with bias given these conditions. The presence or absence of cognitive bias effects may be important in informing the design of later experiments.

- **SAGT PERFORMANCE & CHOICE.** Given a population of heterogeneously biased iAgt with preference modifiers, how does a subsuming sAgt with a simple choice heuristic perform? Is such an agent able to successfully choose between competing iAgt?

**PREFERENCE MODIFIER PERFORMANCE & COGNITIVE BIAS EFFECTS.** There are clear differences in iAgt performance across agent types between GBPUSD and FTSE100 sample runs. For the latter *+ve* bias iAgt stand out from all other agents, including the sAgt, reflecting the long term price trend observable in the FTSE100 timeseries. In terms of P&L and risk-adjusted returns, as will be seen from the results of the Group 2 experiments, if an iAgt has a highly successful core preference structure then this appears to dominate.

This should not in itself be surprising: to take an extreme illustration, if my core preferences are 100% accurate in forecasting future prices then I should never have to take on a loss making investment so that preference modifiers for risk and uncertainty will never have to come into play.<sup>17</sup> At the opposite extreme, for very poor core preference, *-ve* biased iAgt in the FTSE100 simulation, profits are seldom realised and modifiers are triggered very frequently. Although a modifier may limit losses, overall performance is dominated by poor core preferences.

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<sup>17</sup>This presupposes of course a) that I can legally trade and it is not inside information, i.e. I am not subject to regulatory preference modifiers, b) that I can actually trade on the market prices and c) that I am consistently rational as an investor.

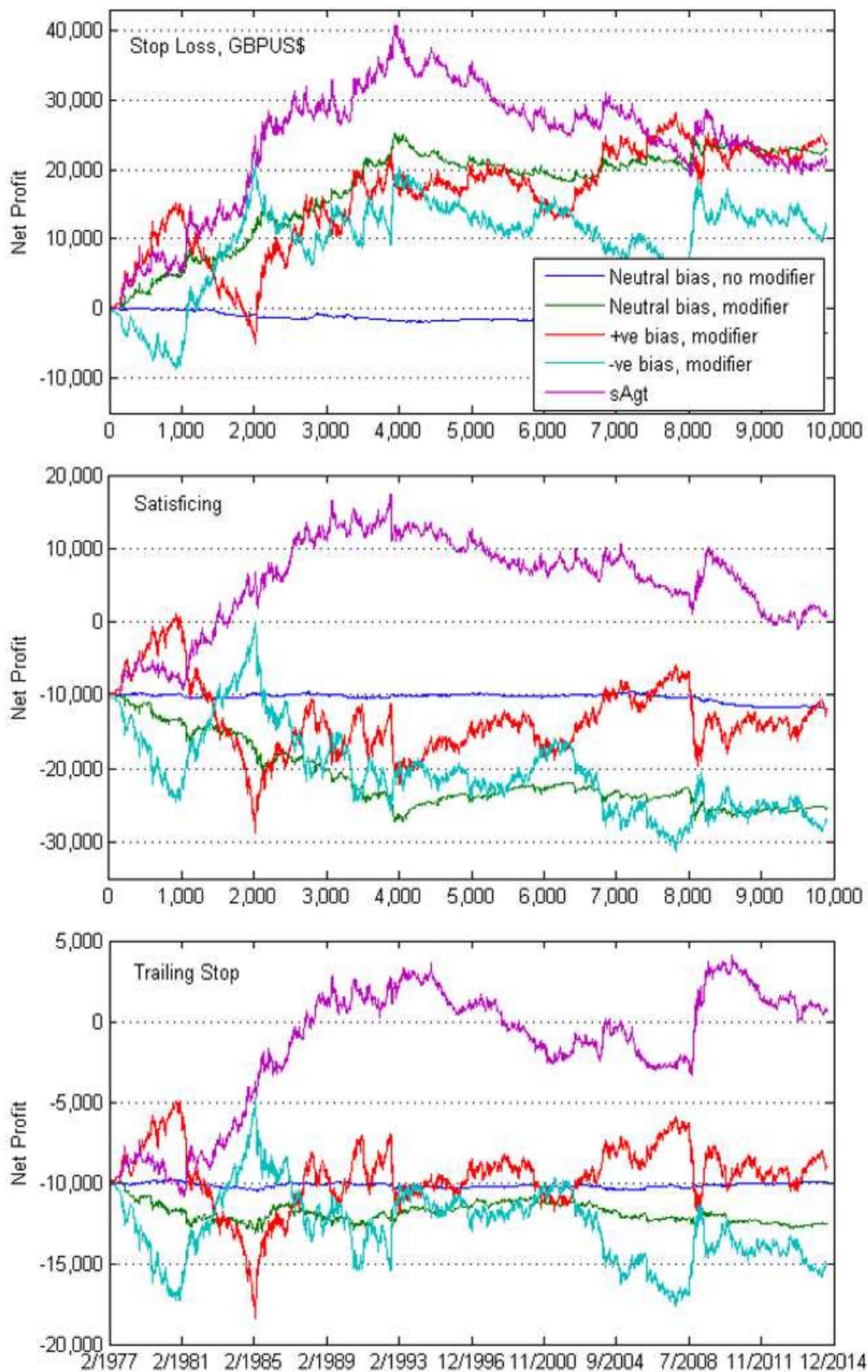


Figure 6.10.: 4 Trader Types & 3 Modifiers - GBPUSD

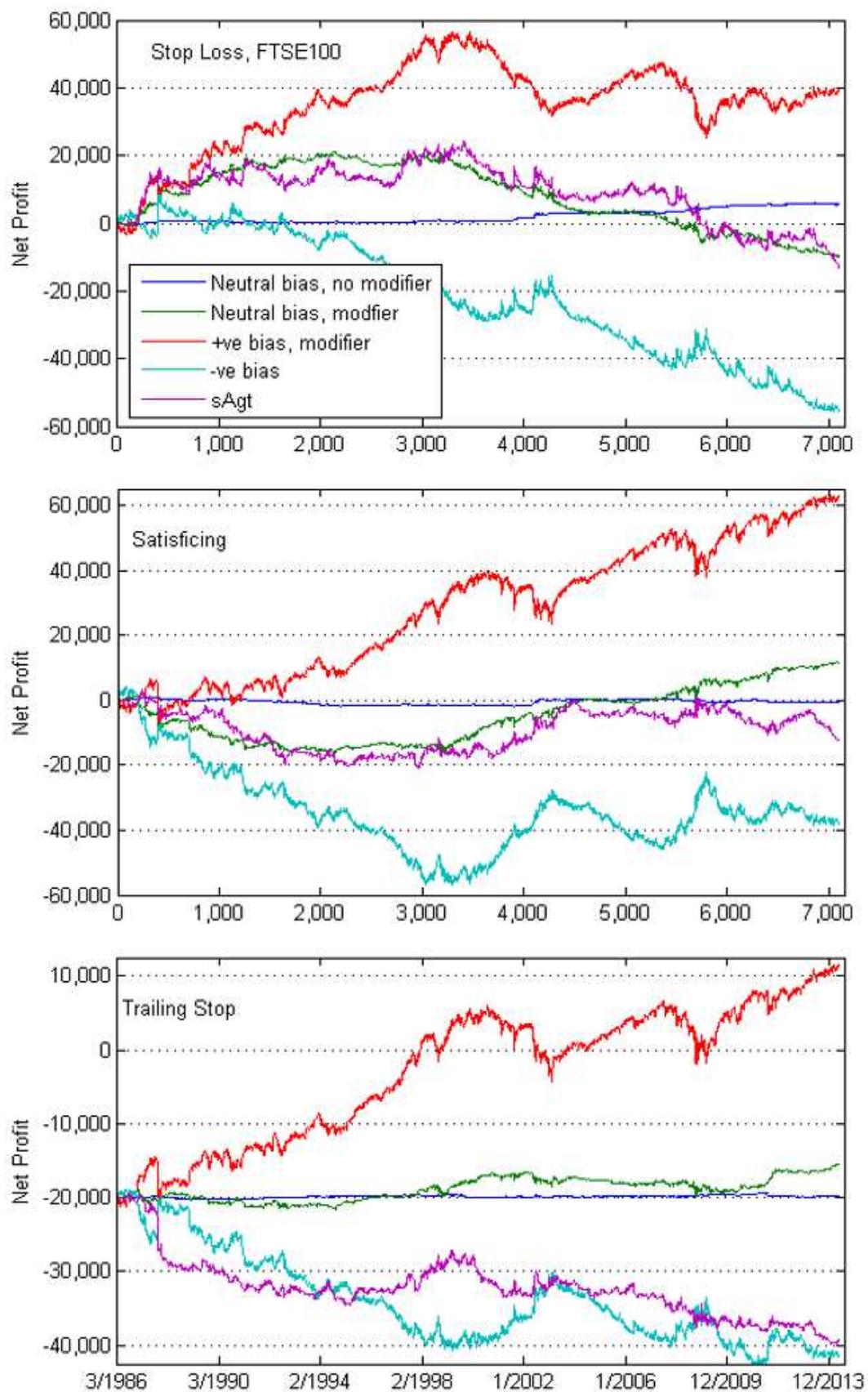


Figure 6.11.: 4 Trader Types &amp; 3 Modifiers - FTSE100

From P&L alone it is difficult to extract much useful information on modifier behaviours, however iAgt performance in GBPUSD is interesting: this is not dominated by the underlying market direction allowing some qualitative observation of modifier effects on iAgt with different biases. As shown in Fig 6.4, although there are large swings in the forex rate over the timeseries, there is no obvious long term trend - at the end of the series the exchange rate is still within its main trading range. Here it is possible to see iAgt with different biases succeeding and failing in different market conditions. In contrast to FTSE100 runs, although *+ve* bias agents still appear to outperform, the effect is much less clear and at different points in each run there is little obvious difference in performance for agents with the same modifier.

The finding that neutral bias iAgt perform relatively competitively overall is interesting in itself, however as the results from the FTSE100 runs emphasise, the fitness of core preferences themselves is important. The minimal core preferences used here are a deliberate abstraction intended to allow modifier behaviour to be explored in a principled manner. Despite the striking results for MC preferences with modifiers like stop loss, behaviour with more realistic, 'situated' core preferences will inevitably be different - exploring such preferences will form part of future work.

In terms of the work here, a flaw in these Group 1 experiments is that, other than for the neutral biased iAgt, the results here do not compare like with like. A better comparison is to look at iAgt with the same core preferences with and without modifiers. This allows the effect of modifier behaviour to be examined directly having stripped out market performance effects and is the focus of the next set of experiments.

SAGT PERFORMANCE & CHOICE.<sup>18</sup> sAgt in the GBPUSD simulations are markedly more successful than those in FTSE100, performing at least as well as their iAgt, and in the case of satisficing and trailing stop modifiers outperforming all subsumed iAgt whether using modifiers or not. In contrast for FTSE100, where cognitive bias dominates iAgt performance, sAgt performance is much poorer - performing at best in the middle of the iAgt population, and in the case of trailing stop modifier iAgt populations not much better than the worst, *-ve* bias iAgt.

The reason for this is likely to be the structure of the sAgt choice rules, which perform like a copycat heuristic.<sup>19</sup> sAgt choose preference combinations from the best performing iAgt in their populations to use in their own trading portfolio. The success of this selection process will determine the overall profitability and risk-adjusted performance of this behaviour.

For GBPUSD, where the effect of cognitive bias is less strong, sAgt seem able to successfully choose between iAgt types over time (although with the same caveat as before), which may indicate that sAgt are able to extract useful information

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<sup>18</sup>Again it must be emphasised that the sAgt shown in this set of experiments are individuals - to properly assess sAgt performance multiple runs need to be performed to collect a reasonable sample size for analysis. Memory constraints on a standard PC configuration make it impossible to run simulations with multiple sAgt currently - each sAgt here has 25 iAgt under its control and each iAgt has 60 rulepackets with 10 rules in each rulepacket, all of which are active and require arrays to monitor performance.

<sup>19</sup>Such behaviours are common in actual trading institutions, where proprietary and senior traders observe retail and institutional traders for information about successful behaviours and strategies.

from their iAgt population even without learning mechanisms. If nothing else this certainly challenges neoclassical efficiency assumptions and may point to a structural population heuristic effect. However, in a situation where a single iAgt core preference is dominant, as in the FTSE100 example, if the sAgt ever chooses another agent type (when for instance that type outperforms for short periods) then over time the sAgt will always underperform its best iAgt. However, if volatility of returns is more resilient than that is not necessarily an issue in itself.

**CONCLUSIONS.** Of the three modifier types only stop loss seems able to generate a stable positive return, and notably this is for all iAgt bias types. In contrast no iAgt with satisficing or trailing stop modifiers is consistently profitable. For satisficing agents this may relate to the asymmetry discussion in Section 6.2.2 where it was noted that this heuristic can cap positive returns while leaving losses open - a serious potential exposure in periods of uncertainty. For trailing stops, although losses are limited in volatile environments this heuristic will tend to trigger early exits from trades except in periods of strong trends and this may be the case here.

It is also worth noting that the timeframe for the sample runs is long relative to absolute returns: on a continuously compounded basis although returns are significant they are quite small - around 0.5% per annum for positively biased iAgt using the satisficing heuristic trading FTSE100. As a caveat to this caveat, in professional fund management a 0.5% differential between funds trading similar asset classes would be often regarded as a highly significant performance differential.

Finally, while exploring overall P&L performance particularly using these types of charts showing temporal path dependent performance is interesting, and obviously has some value, it also serves to emphasise the risk of becoming lost in attempting to make useful inferences from such relatively high-level information. It does not give much insight into any risk or uncertainty mitigation effects from preference modifying heuristics, which after all is one of the key experimental questions under investigation for preference modifiers. Agent-level analysis is necessary to identify areas of interest and explore these systematically.

The second group of experiments using trader types with matched cognitive biases, but different modifiers addresses this issue. In particular the agent-level analysis for satisficing in Section 6.4.3 shows its weaknesses and raises questions about the value of such heuristics applied in isolation.

### **Group 2 - Results & Discussion**

Using T1 & T2 iAgt pairs described in Table 6.5, risk-adjusted effective returns and P&L statistics were examined for each MC preference case, with and without a preference modifier. The results for these experiments are set out in Table 6.6. These tables show summary statistics, mean  $X_{eff}$  and  $X_{eff}$  variance means for iAgt populations in each sample run together with cumulative mean iAgt P&L. The data captured is also roughly partitioned to observe the effect noted in Section 6.4.2, where cumulative P&L performance decays after a period of initial positive returns - in the FTSE100 runs this effect is very clear, but less so in GBPUSD.

Several points are worth noting from these tables.

- P&L vs. risk-adjusted return measures. In the trial runs in Section 6.4.2 (and in the methodology discussion in Chapter 3) the weakness of cumulative P&L as a performance measure in analysing experimental findings was discussed. Here the summary risk-adjusted return measures expose this weakness further by clarifying performance over time. Taking the case of neutral bias iAgtS with stop loss trading GBPUSD as an example, these agents actually record losses in the period from 1993 to 2014 - their mean risk-adjusted return,  $X_{eff}$ , is negative and lower than neutral bias agents without modifiers.
- For biased MC agents, runs with iAgt pairs with and without modifiers make performance comparison feasible and meaningful. Relative performance between iAgtS with the same core preferences is important, particularly where these preferences dominate as in FTSE100.
- Variance of risk-adjusted measures. Beyond gross return performance, the effect of modifier behaviours on risk-adjusted performance is a central question for this case study. The tables here show a consistent pattern across all modifiers in which  $X_{eff}$  is lower for all iAgtS with modifiers in each pair, except for neutral MC bias, where the variance relationship was inverted. The explanation for this is discussed below.

NEUTRAL BIAS MC PREFERENCE  $X_{eff}$  VARIANCE. As the results from both GBPUSD and FTSE100 time series show, risk-adjusted return variances for neutral bias preferences with no modifier are very low compared to neutral bias cores using modifier behaviours. This is a structural effect from the underlying core preferences as the P&L and risk-adjusted return are for an agent's portfolio rather than individual trade. Remembering that for neutral bias MC preferences an agent is equally likely to go long or short for any given trade, then for a portfolio of such trades the expected position will be zero. This is reflected in the P&L profile for these traders and in the return statistics. In contrast, the asymmetry created by preference modifiers in this study means that for neutral MC iAgtS the expected position will not be zero, and so increase  $X_{eff}$  variance compared to neutral bias unmodified core preferences. This effect is shown in Fig 6.12 which also shows histories for *+ve* bias and *-ve* bias agents with and without stop loss.

(a) GBPUSD - Summary Statistics

	Bias	Mean $X_{eff}$ (e-06)			$X_{eff}$ Variances Mean (e-06)			Mean iAgt P&L		
		1975-14	1975-93	1993-14	1975-14	1975-93	1993-14	1975-14	1975-93	1993-14
No modifier	Neutral	-2.43	-3.79	-1.52	0.014	0.020	0.010	-2,167.7	-1,687.5	-480.2
	Positive	4.68	-2.98	9.71	0.142	0.221	0.089	5,335.8	-2,997.8	8,333.6
	Negative	-6.24	-1.01	-9.63	0.158	0.235	0.105	-5,271.5	2,067.1	-7,338.6
Stop Loss	Neutral	20.74	53.99	-1.80	0.021	0.030	0.013	23,148.5	24,747.4	-1,598.9
	Positive	21.80	41.34	8.48	0.081	0.129	0.048	24,779.2	17,316.0	7,463.2
	Negative	11.09	43.89	-11.06	0.090	0.135	0.058	11,596.4	21,005.5	-9,409.1
Satisfice	Neutral	-15.22	-43.06	3.63	0.031	0.040	0.023	-13,724.4	-16,253.2	2,528.8
	Positive	-5.76	-29.93	10.48	0.140	0.200	0.098	-4,977.9	-12,934.3	7,956.4
	Negative	-19.74	-27.68	-14.24	0.145	0.201	0.106	-17,108.3	-9,063.2	-8,045.1
Trailing Stop	Neutral	-6.69	-14.04	-1.69	0.021	0.032	0.013	-6,237.3	-5,346.0	-891.3
	Positive	0.42	-8.79	6.55	0.116	0.178	0.073	760.8	-4,723.1	5,483.9
	Negative	-10.00	-5.03	-13.25	0.128	0.187	0.088	-8,783.4	-265.6	-8,517.8

(b) FTSE100 - Summary Statistics

	Bias	Mean $X_{eff}$ (e-06)			$X_{eff}$ Variances Mean (e-06)			Mean iAgt P&L		
		1986-14	1986-98	1998-14	1986-14	1986-98	1998-14	1986-14	1986-98	1998-14
No modifier	Neutral	2.04	-2.23	5.51	0.036	0.036	0.036	1,871.1	-644.9	2,516.0
	Positive	69.15	125.95	22.76	0.216	0.292	0.418	63,376.3	49,307.98	14,068.3
	Negative	-149.82	-213.18	-98.43	1.055	0.641	1.340	-63,519.8	-49,286.4	-14,233.4
Stop Loss	Neutral	-13.35	53.39	-68.08	0.065	0.058	0.063	-8,261.0	19,340.0	-27,601.0
	Positive	49.19	136.18	-22.03	0.146	0.174	0.111	41,606.7	54,052.9	-12,446.2
	Negative	-115.32	-79.37	-145.05	0.440	0.269	0.577	-54,576.2	-21,759.1	-32,817.1
Satisfice	Neutral	20.59	-46.35	75.51	0.078	0.082	0.067	16,178.7	-13,745.6	29,924.3
	Positive	47.15	131.72	-22.08	0.151	0.179	0.118	39,559.2	51,995.9	-12,436.7
	Negative	-124.36	-76.19	-164.11	0.441	0.263	0.584	-57,699.6	-21,068.6	-36,631.0
Trailing Stop	Neutral	6.63	1.21	11.04	0.051	0.049	0.052	5,508.2	451.7	5,056.5
	Positive	67.46	106.47	35.53	0.166	0.218	0.120	61,383.5	39,694.6	21,688.9
	Negative	-84.58	-144.71	-35.68	0.483	0.415	0.533	-43,805.4	-37,174.4	-6,631.0

Table 6.6.: iAgt Modifier Summary Statistics

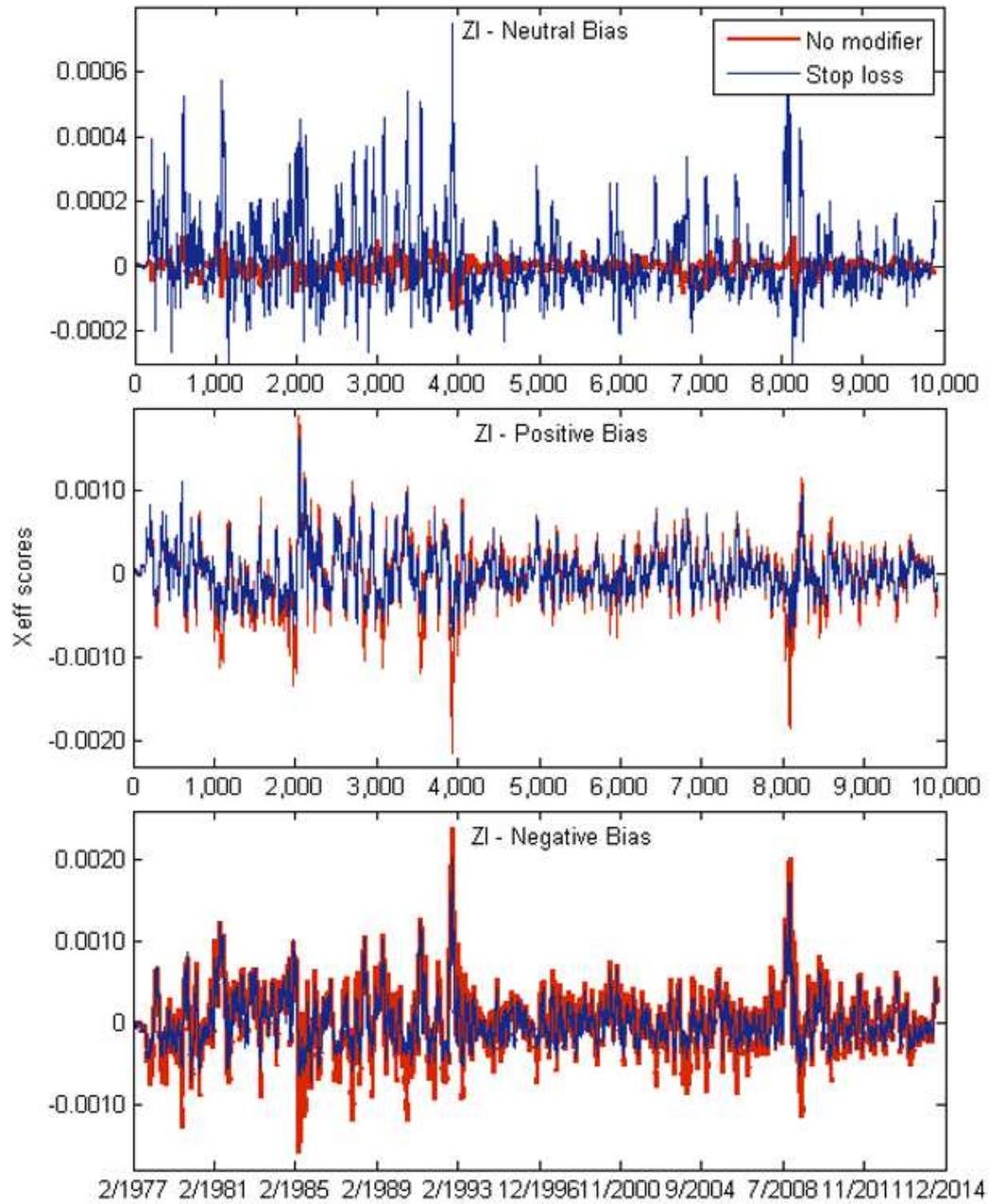


Figure 6.12.: iAgt  $X_{eff}$  History - Stop Loss vs. Unmodified MC Preferences

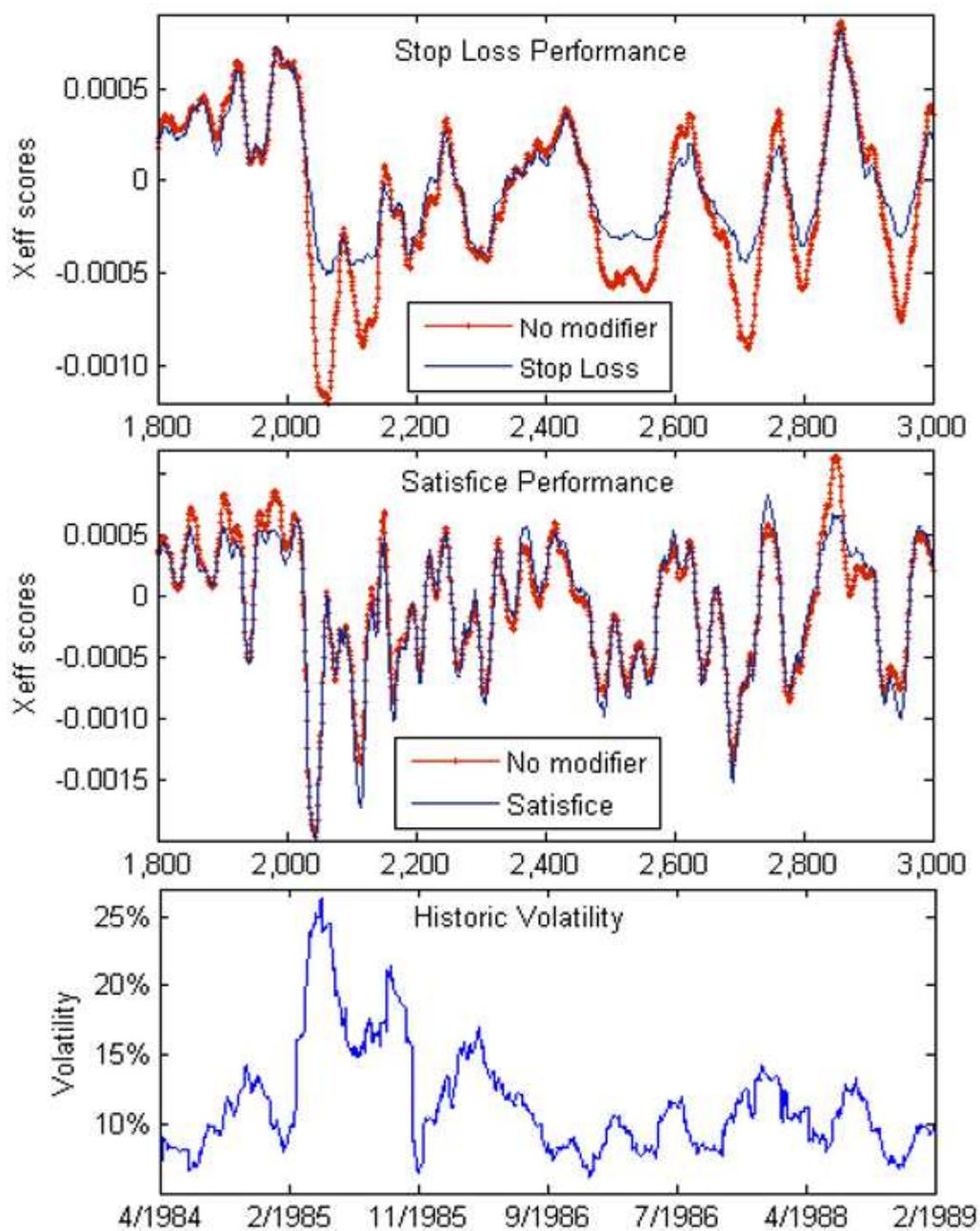
NON-NEUTRAL BIAS MC PREFERENCE VARIANCE. Where the cognitive bias is not neutral variance increases to reflect the change in expected position, whether for agents with modifiers or without. In the results here the main question is, how preference modifier behaviours affect risk-adjusted return and variance of these returns? Although not terribly clear in these overlaid histories, worth noting is the apparent asymmetry in  $X_{eff}$  scores for stop loss agents in each case. In particular for non-neutral bias iAgt's stop loss agents do not appear to experience the same extremes for negative scores as agents without modifiers. These observations support the summary return statistics in Table 6.6, illustrating the dynamics of modifier behaviours.

Plotting shorter  $X_{eff}$  histories allows a better understanding of modifier behaviour micro-structure. Figures 6.13 & 6.14 show smaller sections from the experimental runs including periods of both high and moderate volatility, plotting stop loss and satisficing iAgt's  $X_{eff}$  scores against agents without modifiers for GBPUSD and FTSE100.

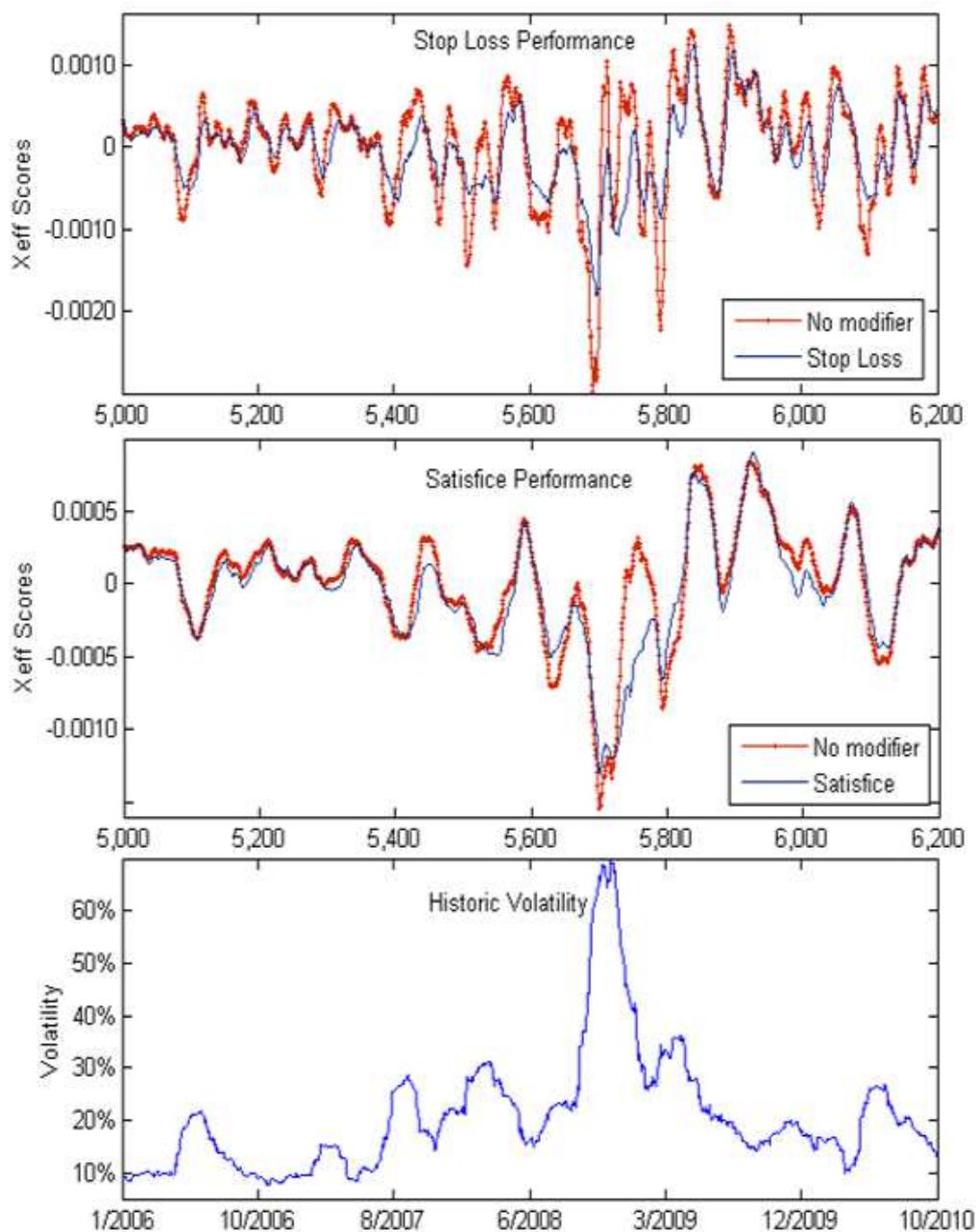
From these charts differences in modifier performance become obvious. In Fig 6.13, where *+*ve bias MC iAgt's trade GBPUSD, the beneficial effects of stop loss modifiers is apparent. In periods of extreme volatility, such as early 1985, iAgt's using stop loss significantly outperform and do not experience the same sharp losses as agents without modifying behaviours. It is also notable that stop loss seems to be operating effectively even in periods of moderate volatility. In this case it might be interpreted as functioning in a 'risk' rather than 'uncertainty prone' market condition.

In contrast, satisficing iAgt's experience the full negative returns while missing out on the peak positive return periods. This effect is even more striking in Fig 6.14 which contains the main period of the GFC - certainly a period when markets were prone to uncertainty, and risk models highly stressed. Satisficers underperformed in almost all periods particularly after the most volatile zone in early 2009.

For stop loss, although significantly outperforming in periods of negative return iAgt's underperformed in positive periods when some trades will have exited early as stop loss thresholds were breached. This pattern is borne out in the results in Tables 6.6 for all bias types of stop loss iAgt.



**Figure 6.13.:** Stop Loss & Satisficing  $X_{eff}$  Micro-structure,  $+ve$  Bias iAgt, GBPUSD



**Figure 6.14.:** Stop Loss & Satisficing  $X_{eff}$  Micro-structure,  $-ve$  Bias iAgt, FTSE100

For stop loss it seems clear that the trade-off is between lower relative returns and significantly lower volatility of returns, this translates into the quality of returns, which is what risk-adjusted measures try to capture. For satisficing heuristics there is a more negative interpretation, which is that not all reductions in return variance are positive. The asymmetric return profile for satisficing heuristics leaves agents using it exposed to uncertainty and short term market noise.

### Modifier Performance - Statistical Tests

Taking the results from the Group 2 sample runs, iAgt types in each sample population were compared to test for significance between iAgts with and without modifier behaviours. iAgts' mean risk-adjusted performance (as opposed to the mean for each agent type as reported above) and iAgt variance of risk-adjusted performance are tested for each iAgt population, giving  $n = 25$  (the number of each iAgt type in the sample run).

The results for these tests are shown in Table 6.7. A two-tailed, two sample t-test was used for  $X_{eff}$  means - the null hypothesis being that there was no difference in risk-adjusted returns between iAgt populations with and without modifiers. For  $X_{eff}$  variances a one-tailed, two sample t-test was used<sup>20</sup>, with  $H_0$  being that  $X_{eff}$  variances for iAgts with modifiers is not less than for iAgts with no modifiers. Tests for neutral MC preferences were not carried out given the structural variance effect noted in the previous section.

**Results & Discussion** Somewhat unsurprisingly, given the size of the overall effects observed in the earlier experiments, and borne out in the breakdown of risk-adjusted returns, many of the tests proved to be significant at the 1% level. Indeed, as can be seen in Table 6.7 above, the P values reported are, in many cases, extreme. Such values raised some initial concern about the tests, however given the meaningful absolute effect sizes<sup>21</sup> in each case where there is statistical significance and reasonable but not excessive sample sizes ( $n=25$ ), the results appear to be valid with a statistical power approaching 1.

Charts showing scatter-plots of iAgt  $X_{eff}$  means against  $X_{eff}$  variances serve to illustrate the degree of separation between population types & effect size, as well as giving a useful means to better understand and interpret the data. Figs 6.15 & 6.16 show two examples from the results in Table 6.6a and Table 6.6b respectively.

Figure 6.15 shows the results for *+ve* bias iAgts with stop loss vs unmodified iAgt preferences in GBPUSD. Here the data for different agent types samples under test is quite distinct in the plot, however the risk-adjusted return means separation largely disappears in the second phase of the sample run while the return variances remain spatially distinct, reflecting the results of the statistical tests where no significance was found for  $X_{eff}$  mean returns across the 2 agent types. This helps illustrate the basic rule of thumb for choosing between investments with similar returns - i.e. one should generally choose the candidate with lower variance of returns.

<sup>20</sup>Note that here the mean  $X_{eff}$  variance for iAgts is the test statistic of interest, we are not testing the variance of mean risk-adjusted return across agents in the samples where a  $\chi^2$  test would have been more appropriate.

<sup>21</sup>See Sullivan & Feinn[135] for a useful discussion of the importance of effect size over P values.

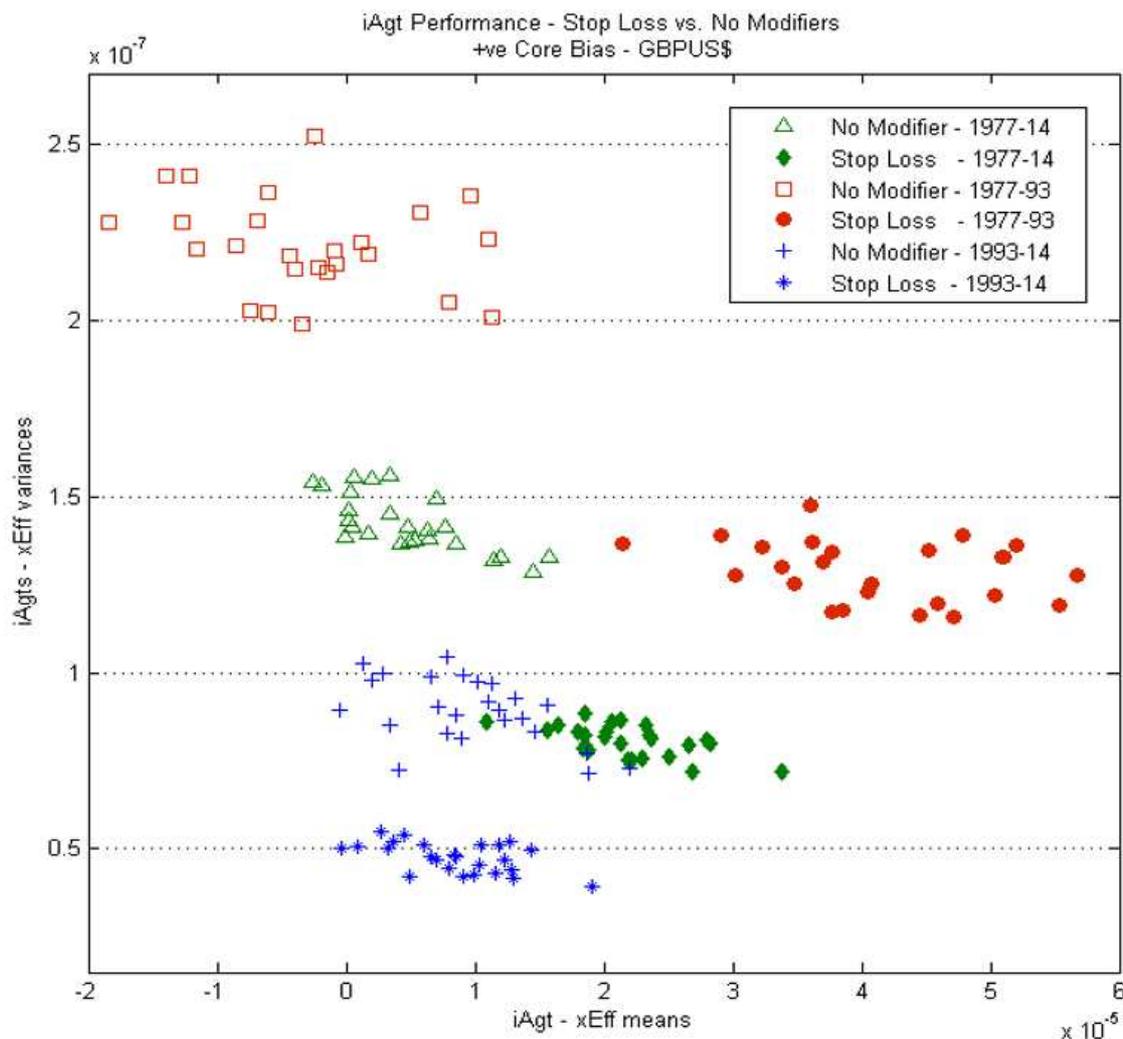
(a) t-tests - GBPUSD  $X_{eff}$  Mean & Variance Comparisons

Sample Periods		Whole Series				0 - 4,000				4,000 - end			
		$X_{eff}$ means		$X_{eff}$ variances		$X_{eff}$ means		$X_{eff}$ variances		$X_{eff}$ means		$X_{eff}$ variances	
		$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value
Stop Loss	+ve bias	1	1.1e-16	1	3.9e-35	1	1.3e-23	1	3.4e-32	0	0.409	1	9.9e-7
	-ve bias	1	1.5e-15	1	5.0e-30	1	1.5e-15	1	2.8e-26	0	0.294	1	2.2e-24
Satisfice	+ve bias	1	1.8e-8	0	0.390	1	4.1e-12	1	1.8e-4	0	0.642	0	0.999
	-ve bias	1	1.7e-9	0	0.015	1	2.6e-11	1	3.9e-8	0	0.051	0	0.950
Trailing Stop	+ve bias	1	3.6e-4	1	3.1e-13	1	0.004	1	3.4e-11	0	0.71	1	1.7e9
	-ve bias	0	0.419	1	2.7e-9	0	0.198	1	2.4e-9	0	0.871	1	5.2e-5

(b) t-tests - FTSE100  $X_{eff}$  Mean & Variance Comparisons

Sample Periods		Whole Series				0 - 3,000				3,000 - end			
		$X_{eff}$ means		$X_{eff}$ variances		$X_{eff}$ means		$X_{eff}$ variances		$X_{eff}$ means		$X_{eff}$ variances	
		$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value
Stop Loss	+ve bias	1	1.9e-11	1	2.6e-18	1	1.9e-6	1	5.1e-21	1	2.1e-19	1	6.4e-10
	-ve bias	1	2.3e-4	1	2.3e-18	1	3.1e-22	1	1.9e-24	1	6.4e-4	1	8.7e-16
Satisfice	+ve bias	1	1.4e-12	1	1.0e-21	1	2.1e-4	1	7.6e-23	1	7.8e-24	1	2.4e-9
	-ve bias	1	0.003	1	1.0e-7	1	2.1e-25	1	8.4e-28	1	9.2e-7	1	2.2e-15
Trailing Stop	+ve bias	0	0.099	1	5.4e-11	1	2.9e-7	1	8.2e-9	1	0.003	1	1.1e-9
	-ve bias	1	1.3e-9	1	2.3e-13	1	1.6e9	1	7.9e-17	1	7.2e-7	1	4.6e-12

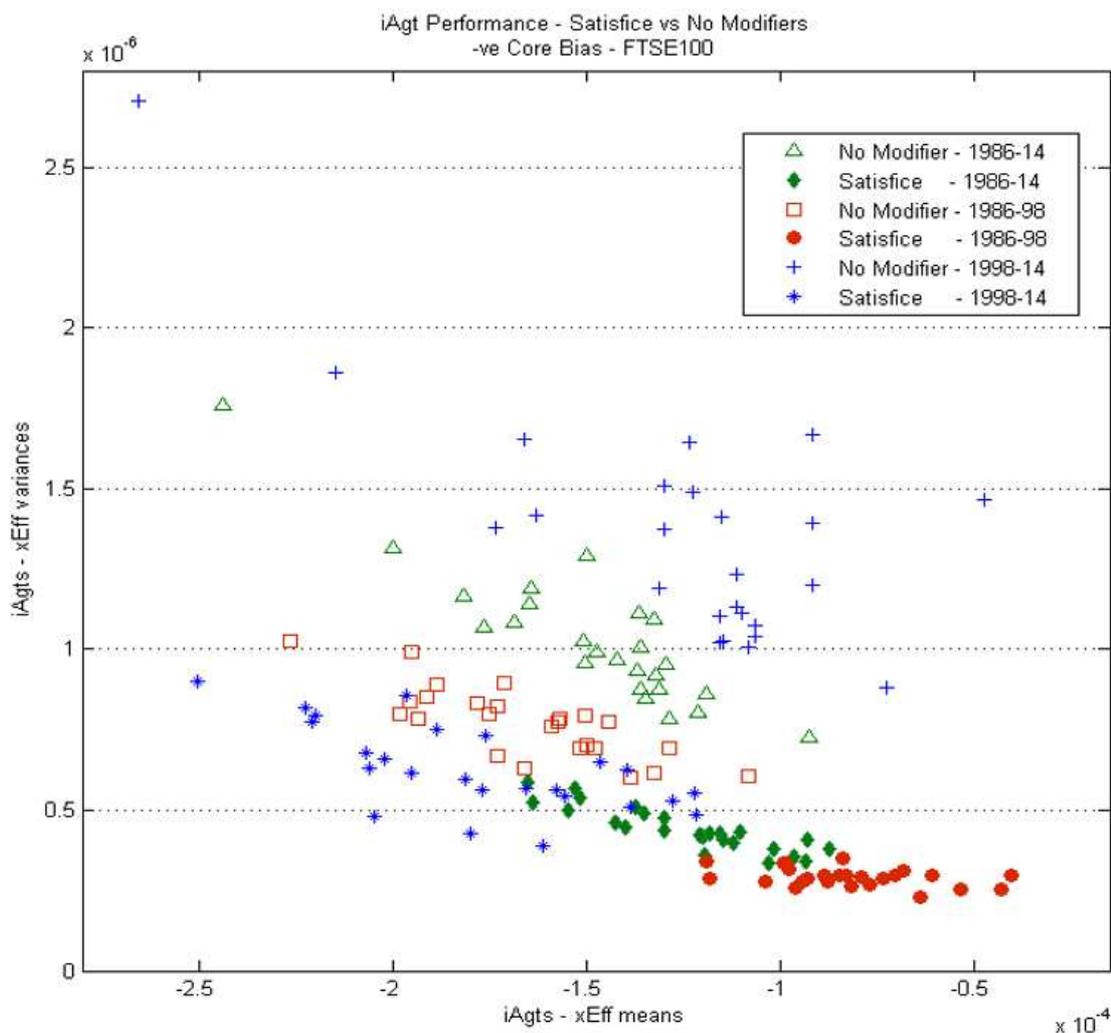
Table 6.7.: Modifier Performance - Statistical Tests



**Figure 6.15.:** Stop Loss Modifier -  $X_{eff}$  Means vs.  $X_{eff}$  Variances,  $+ve$  Bias iAgt, GBPUSD

Figure 6.16 shows equivalent results for the satisficing modifier in iAgt with  $-ve$  bias in FTSE100. Here the results do not immediately appear to be as clear, however this is largely because there is less spatial movement in the population values between different phases.

For the  $+ve$  bias, stop loss sample there is clear separation of the iAgt population by type. In all cases the  $X_{eff}$  variances of iAgt with modifiers is lower, however in the different periods recorded the relationship of effective mean returns to return variances changes markedly. Between 1986 and 1998, satisficing,  $-ve$  bias iAgt outperformed non-satisficing iAgt both in terms of returns and variance of returns (although they all had negative returns), however from 1998 to 2014 although satisficers had lower variance of returns, they also had much lower returns.



**Figure 6.16.:** Satisficing Modifier -  $X_{eff}$  Means vs.  $X_{eff}$  Variances,  $-ve$  Bias  $iAgt$ s, FTSE100

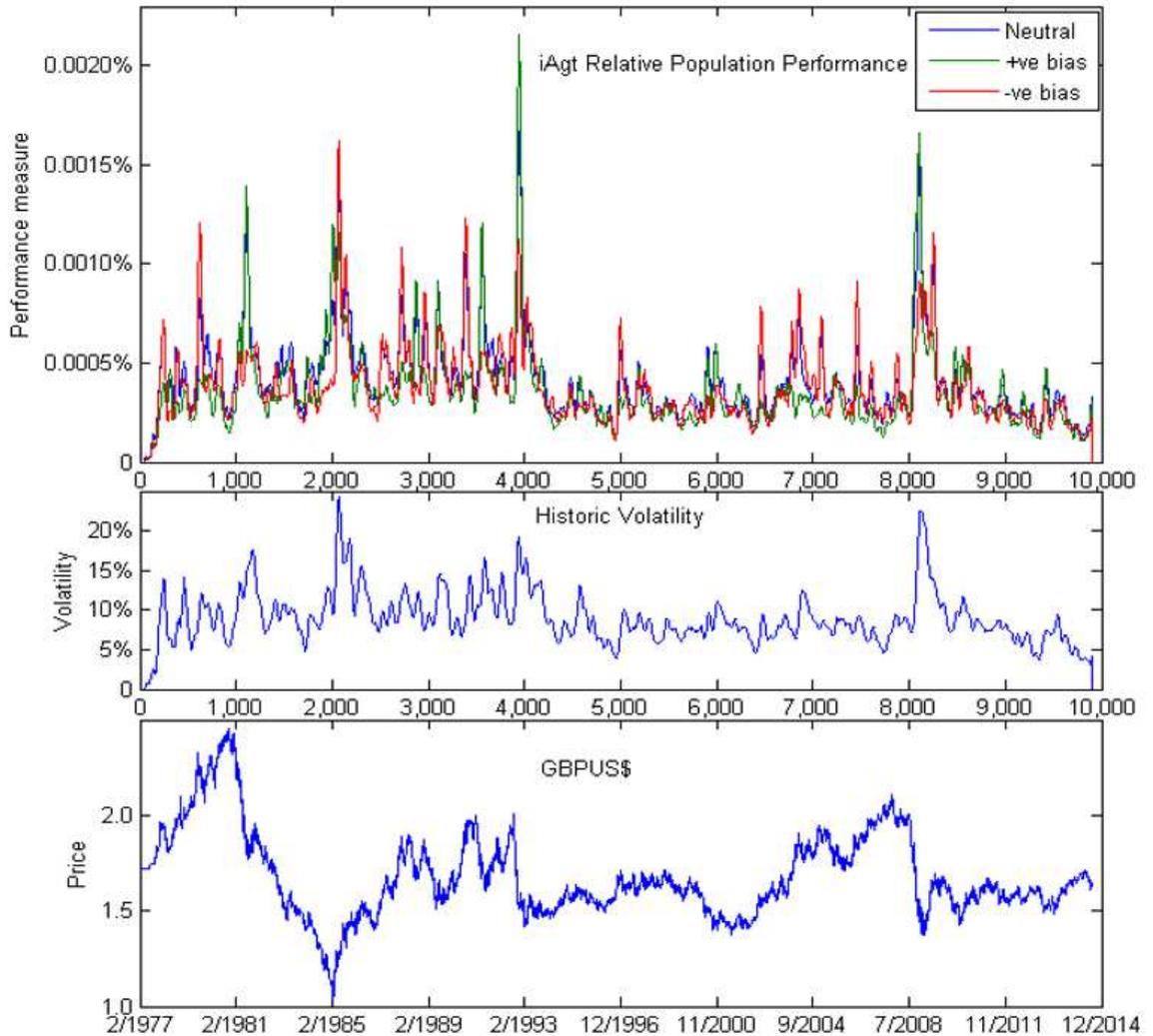
Both scatter-plots also emphasise an important general feature of modifier behaviour in terms of underlying markets, specifically that performance changes over time, reflecting the complex adaptive nature of economic systems as participants co-evolve.

As an analysis of summary return statistics, the work here helps quantify general characteristics for preference modifying behaviours in economic preference expression. These modifiers are seen to reduce the variance of agent risk-adjusted returns. In the case of satisficing heuristics however the underlying cause of this reduction is not clear. To do this it is necessary to look at agent level performance within sample runs particularly in terms of market volatility and periods of significant differential performance. This is the work set out in the next section.

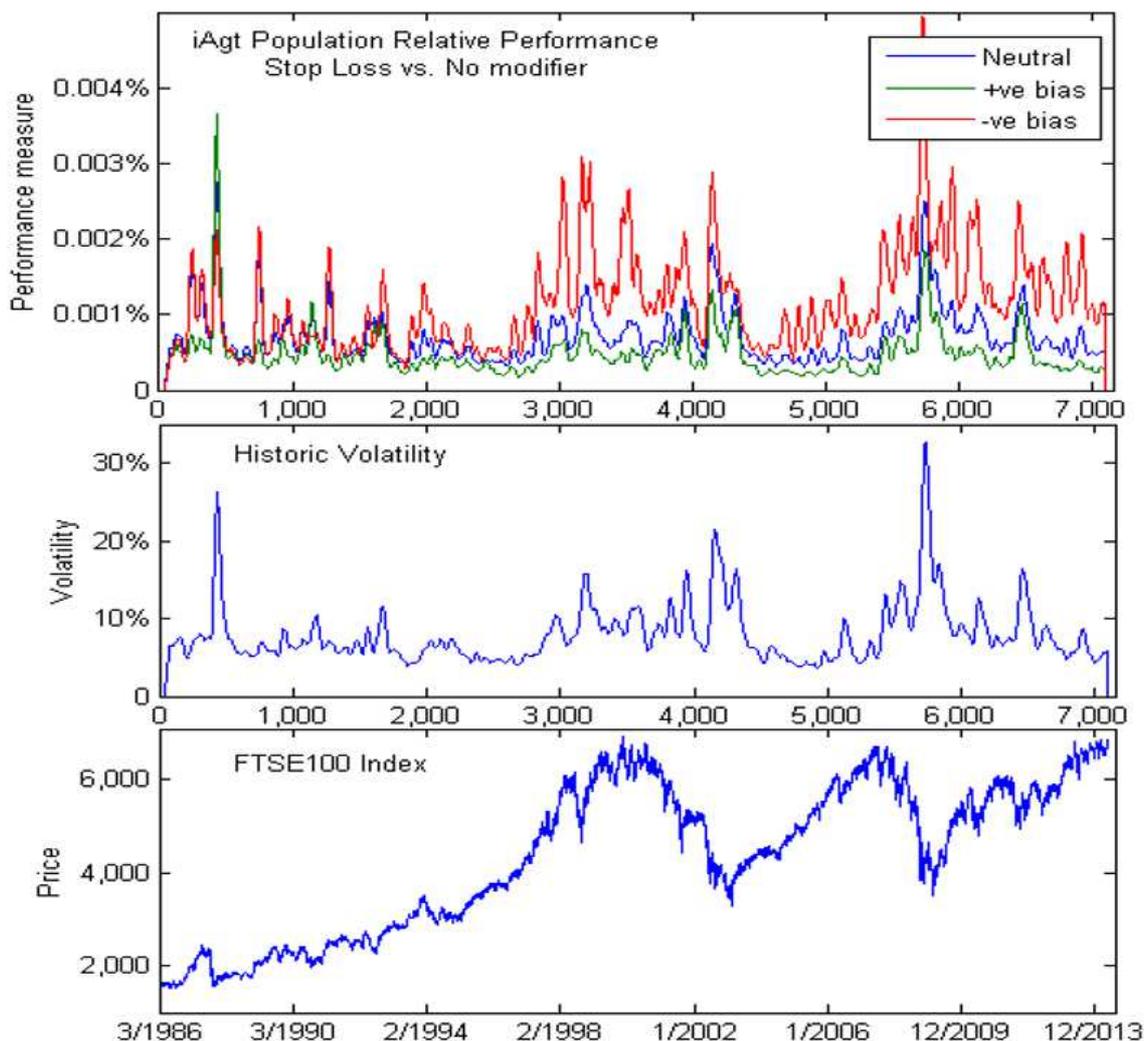
### Population Risk-Adjusted Relative Performance & Agent-Level Analysis

The summary risk-adjusted return measures and statistical tests presented in the previous sections provide some level of assurance that observed preference modifier effects (on returns; volatility of these returns; and cumulative profitability) are

not anomalous. However temporal information is lost in these analyses: assuming a role for preference modifiers in both risk management and uncertainty mitigation it is important to know when and where to look in order to understand how modifiers function. For this population and agent-level analysis retaining temporal information is necessary. Cumulative P&L histories are of limited use as they do not readily show changes in performance. Population relative performance measures offer a better tool for examining the data and are used in following analysis.



**Figure 6.17.:** Population Relative Performance - GBPUSD



**Figure 6.18.:** Population Relative Performance - FTSE100

Figures 6.17 & 6.18 show example plots of the population relative performance measure,  $I_X$ , against historic market volatility for GBPUSD and FTSE100 iAgtS with and without stop loss preference modifiers (similar results were found for satisficing and trailing stop). Three populations from separate trials made up of MC agents are shown: a neutral bias MC population, a *+ve* bias population and a *-ve* bias population. These figures show a high degree of correlation between market volatility and increases in relative risk-adjusted performance across the agent populations. This is confirmed in Table 6.8 which shows correlation coefficients for  $I_X$  vs. historic volatility.<sup>22</sup>

**Relative Risk-Adjusted Return Surfaces** Given the length of the timeseries picking out specific features in plots themselves of  $I_X$  is challenging and potentially misleading, a key value of charts like Fig 6.17 and Fig 6.18 is to help identify

<sup>22</sup>Note that this table also reports  $I_X$  using Benink's excess return measure for comparison. Here the results are much less clear and reflect the instability & insensitivity of the measure found in trials.

		MC neutral	MC <i>+ve</i> bias	MC <i>-ve</i> bias	Mean
GBPUSD	$I_X$ - gross returns	0.81	0.89	0.87	0.85
	$I_X$ - excess returns	0.91	0.41	0.33	0.55
FTSE100	$I_X$ - gross returns	0.70	0.83	0.74	0.76
	$I_X$ - excess returns	0.237	0.71	0.64	0.53

**Table 6.8.:** Historic Volatility vs. PRP Measure Correlation - Stop Loss Modifiers

areas of interest for closer examination and analysis. Plots of relative risk-adjusted return surfaces, as introduced Chapter 5, help to visualise agent performance within populations. In the experiments here (with deliberately rudimentary, minimal core preferences; limited modifier structure; and no learning mechanisms) one would not expect significant differentiation between agents of the same type in the iAgt population. However as with the SFASM reconstruction, if there were any anomalous behaviour worth further investigation this would allow it to be targeted.

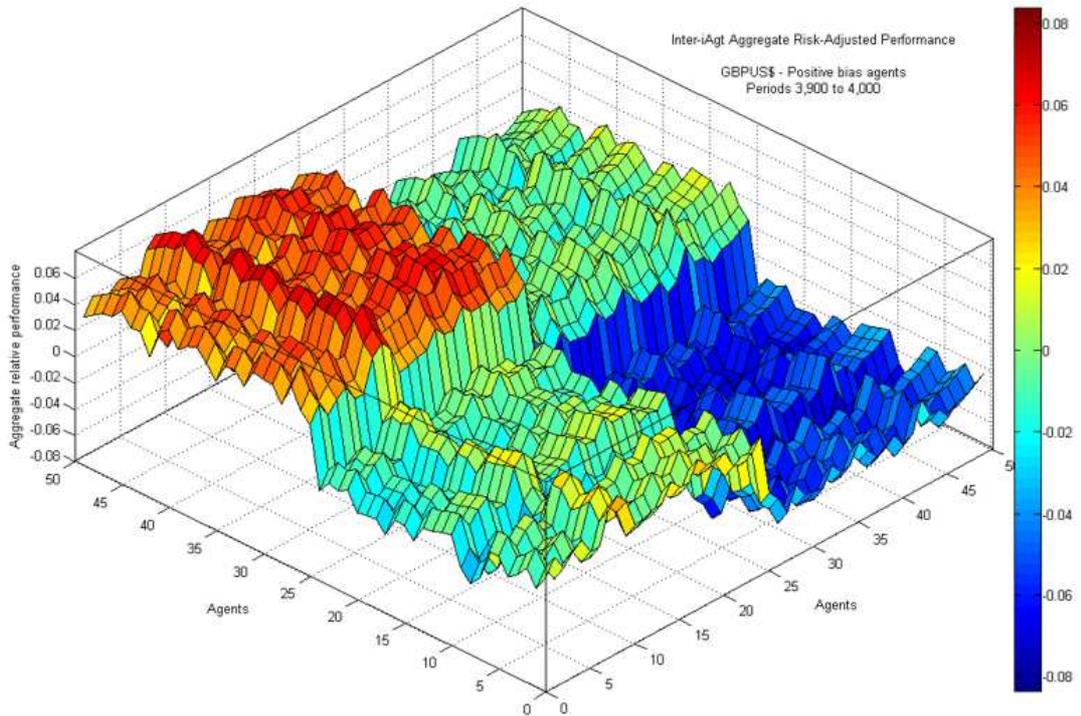
Figure 6.19 shows a sample for *+ve* bias MC iAgts with and without modifiers from the PRP history in Fig 6.17. This window, covering the period 3,900-4,000 in 1993, shows a clear separation of iAgt performance by type, but little differentiation between types.<sup>23</sup> Here the stop loss iAgts sharply outperform as stop loss behaviours come into play. The equivalent surface plot for *-ve* bias iAgts shows little differentiation between stop loss and no modifier iAgts. Both these effects are to be expected given the earlier analysis. A plot of mean iAgt  $X_{eff}$  scores covering the period, Figure 6.20, confirms this.

An equivalent performance surface chart for FTSE100, for periods 5,900-6,000 in 2009, for *-ve* bias MC iAgts captures a period in which stop loss agents slightly underperform agents without modifiers. Here the differentiation is less clear as the stock market had a period of sharp decline so that preference modifying behaviour acts as a drag on performance while reducing volatility of returns.

### Group 1 & 2 Experiments - Final Discussion

The experiments and analyses in this section have focused on exploring and confirming observations of the interaction between preference modifying behaviours and minimal core preferences with different cognitive bias structures. The analyses moved from high level data of cumulative and historic profitability for agents to quantifying and describing behaviours across agent populations using the agent-level measures and tools developed earlier in the work.

<sup>23</sup>In this chart iAgts 26 - 50 are stop loss *+ve* bias agents, while 1-25 have no modifier.



**Figure 6.19.:** Inter-iAgt Relative Performance Surface,  $I_X$ , GBPUSD *+ve* Bias MC iAgt, Window Periods 3,900-4,000

Overall the results confirm a potentially important role for at least some preference modifying behaviours, particularly stop loss, in risk management and uncertainty mitigation in the form of reduced return variances, while highlighting weaknesses in other modifiers such as satisficers. It remains the case that the core preferences for iAgt in these experiments were deliberately & unrealistically weak. Although the SHaaP architecture allows more sophisticated core preferences to be used, and subsumed, by economic agents, the aim here was to systematically examine modifier behaviours in relative isolation before introducing more sophisticated core preferences - these will be introduced in future work.

The experiments here did not consider adaptation or learning for iAgt populations, nor did it look in detail at sAgt choice behaviours in subsuming iAgt preferences. These areas are considered in the experiments in the following sections.

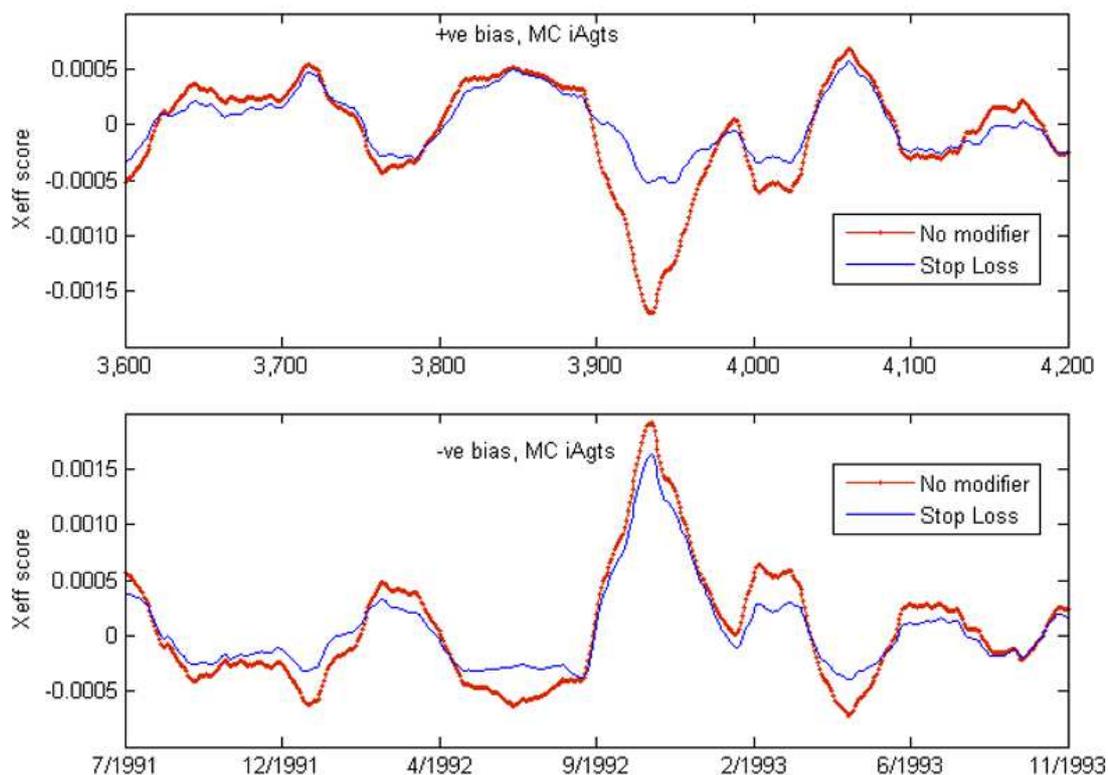


Figure 6.20.: GBPUSD Stop loss  $X_{eff}$  Micro-structure, +ve Bias iAgtS

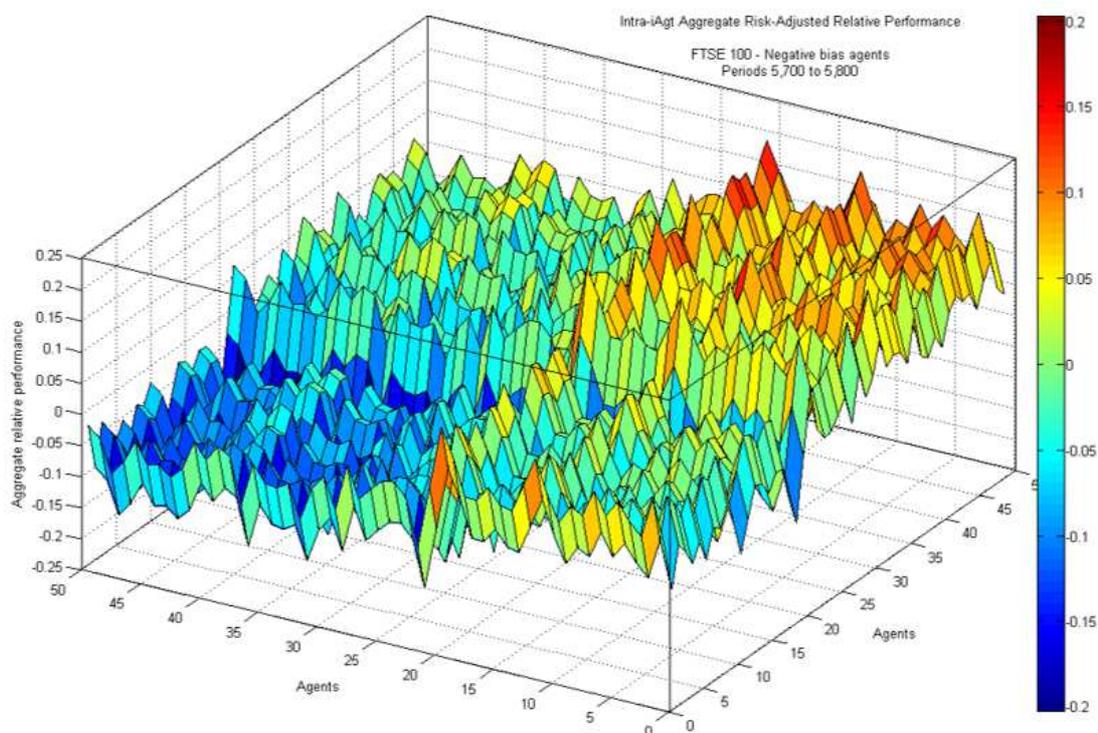


Figure 6.21.: Inter-iAgt Relative Performance Surface,  $I_X$ , FTSE100 -ve Bias MC iAgtS, Window Periods 5,700-5,800

#### 6.4.4. Learning, Adaptation & Preference Modifiers

**MC Neutral iAgt & Stop Loss With PSO Learning** In the face of minimal core preferences and the absence of learning or adaptation mechanisms economic agents in the previous experiments were surprisingly successful. In particular the observed reduction in volatility of risk-adjusted returns was quite robust. The parameterisation protocols & calibration process from the trial experiments in Section 6.4.2 seem to have been effective, however this does not allow for adaptation in the face of changing market conditions and participant behaviours. It is an open question for uncertainty mitigation & heuristic preference modifiers whether there are optimal parameter values which can actually be learned, or whether they remain consistent over time.

The experiments in this section explore adaptation for modifier parameter settings using a basic PSO and variants of this algorithm in a controlled setting. Here the focus is on stop loss as a modifier, reflecting not only the findings from the earlier experiments which showed clear performance benefits in terms of risk adjusted returns, but also a need to limit the scope of exploration at this stage. In introducing a simple, PSO learning algorithm, the main experimental question is what effect this has in terms of adaptation, resilience and overall economic performance. A basic PSO algorithm is tested followed by variants using two modifications designed to combat early lock-in effects. These modifications were described in Section 6.3.3 and are set out below.

#### Experimental Protocols & Structures

The systematic, exploratory approach from the experiments in Sections 6.4.2 & 6.4.3 was applied here. A series of experimental runs were carried out in which PSO learning was applied to rule packet population of each iAgt in a simulation independently and the results compared to iAgt without learning. In this series the form of PSO (basic; with velocity noise; with gBest decay; and with both velocity noise and decay) was changed and the results collated.

The experiments here were limited to neutral bias, MC iAgt with stop loss modifiers as a demonstration of the effect of such learning mechanisms. For each experiment,

- In each iAgt population, 25 iAgt of each PSO variant type were instantiated using the default parameters set out in Table 6.2.  $X_{eff}$  windows for iAgt and rulepackets were set to 15 periods.
- Preference modifiers were initialised with the parameters established in the calibration runs in Section 6.4.2 and set out in Table 6.3 using the approach detailed in Section 6.3.2, so that the mode for stop loss modifiers in GBPUSD was set to 0.5% and the scalar to 1.
- A PSO algorithm was implemented as set out below for rulepacket learning. This was set up in the 2 forms described in Section 6.3.3.1,
  - A basic, 'classical' form, following Kennedy & Eberhart's formulation[70, 71]

- A modified form in which gBest and pBest parameters were allowed to decay as a form of forgetting, and noise could be added to particle velocity when this had approached zero. These modifications were set up so that they could be activated separately or together.
- The PSO parameters used in these experiments are shown in Table 6.9.

PSO parameters	Default setting
PSO frequency	1/20
PSO start period	trading start +100
Cognitive factor, $c_{cog}$	1.50
Social factor, $c_{soc}$	1.50
Momentum weight, $\omega$	0.90
Decay rate, $\alpha_d$	20
Velocity noise factor, $r_{noise}$	0.40
Velocity noise hurdle	0.0001

**Table 6.9.:** PSO Parameters

PSO variants	Notes
PSO basic	Basic PSO implementation - see Section 6.3.3.1
PSO - decay	pBest & gBest scores exponentially weighted decay - see Section 6.3.3.2
PSO - decay & noise	decay, plus noise added to stationary particle velocity
PSO - gBest decay & noise	gBest decay only, plus noise added to stationary particle velocity

**Table 6.10.:** PSO Variants

## PSO Experimental Results

**Mean Net Profit** Figure 6.22 shows net profit time series for neutral bias, MC iAgt populations with stop loss for each PSO variant set out in Table 6.10 and for iAgt with no learning mechanism in each market.

It is immediately apparent that, although there is some differentiation between iAgt populations with and without active PSO, overall the general features of the results from the main experiments in Sections 6.4.2 & 6.4.3 are repeated. After a period of positive general return performance this effect disappears, and in the case of FTSE100 runs net gains are eroded over time.

More importantly in all but one case, in terms of cumulative net P&L, an iAgt population with no active PSO appears to consistently perform as well as, and in general outperform iAgt with PSO. Only for PSO with decay and noise active did an iAgt population outperform.

Analysis of effective returns and variance of expected returns is shown in Tables 6.11 & 6.12, which include statistical tests for significance comparing PSO variant populations to non-PSO iAgt.<sup>24</sup>

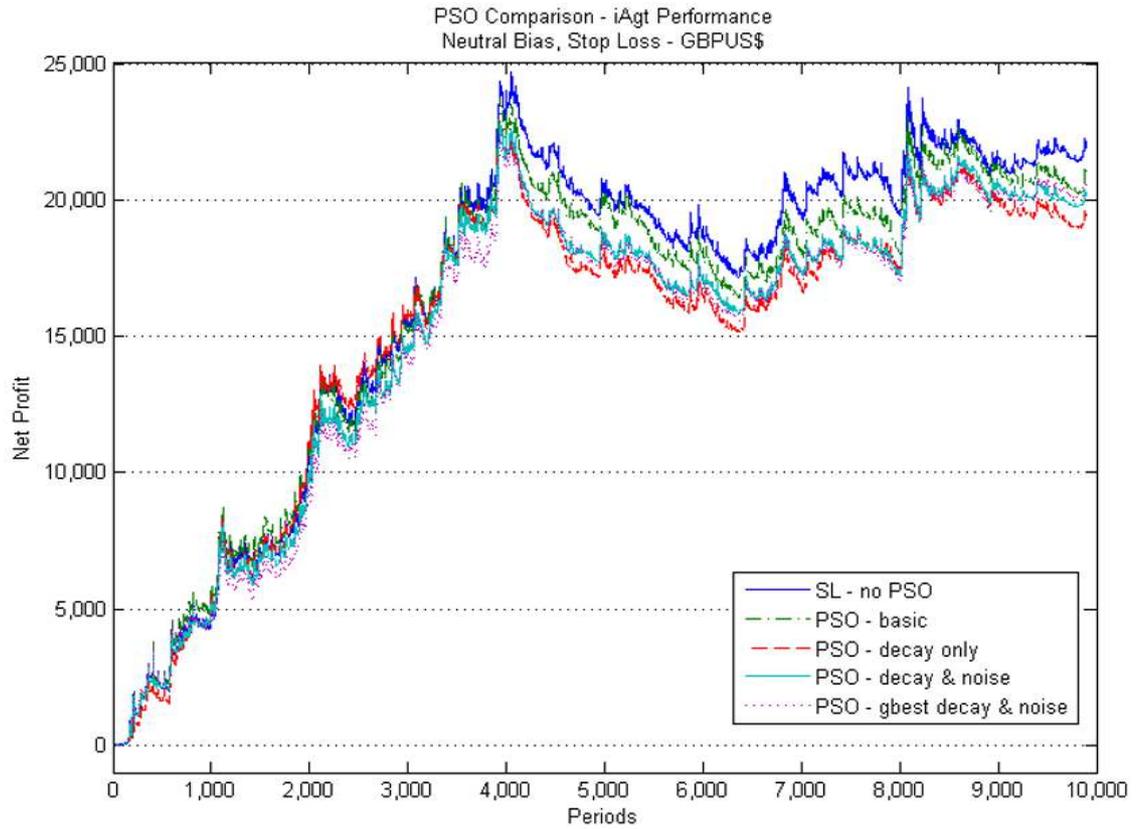
This provides a better picture of PSO performance. At a 1% significance threshold only 4 instances reject  $H_0$  all of which are for GBPUSD, that iAgt mean  $X_{eff}$  variance is lower for PSO than without, while there are no significant instances where a null hypothesis for iAgt mean  $X_{eff}$  is rejected. If significance level is dropped to 5% there is no change for mean  $X_{eff}$ , however for mean  $X_{eff}$  variance a total of 11 instances out of the 24 samples<sup>25</sup> meet, or reasonably closely approach, this threshold. All but 2 of these instances are in the 1st phase of the sample runs - again leading to the speculation that in the late 1990s there was some change in the structure of the markets or in the performance of preference modifying behaviours.

Superficially this would tend to suggest some value for PSO introduced adaptation for stop loss as a preference modifier - recall that equivalent investment returns a rational investor should typically prefer the return with lesser volatility. Here, for MC preferences & stop loss, we appear to have no significant difference between returns, but a number of cases where the volatility of returns is lower.

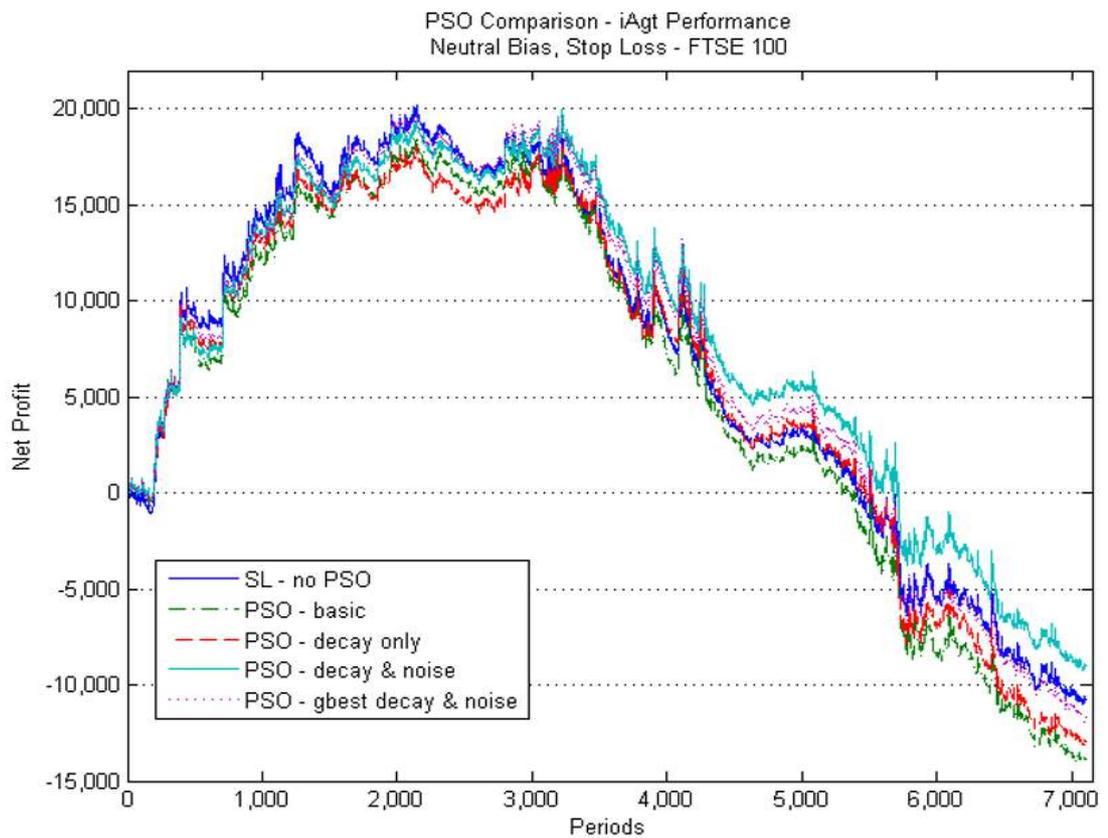
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<sup>24</sup>In these tests a 1-tail test is used for  $X_{eff}$  variance since for stop loss we are most interested in reduction in volatility of returns and risk-adjusted performance, while for mean agent  $X_{eff}$  a 2-tail test is applied. These are the same tests as for the experiments in Section 6.4.3.

<sup>25</sup>Obviously there is a caveat here in that the 'Whole Series' sample may not properly be considered to be a separate sample.



(a) PSO Comparison - GBPUSD



(b) PSO P&L Comparison - FTSE100

**Figure 6.22.:** PSO Performance Comparison - GBPUSD & FTSE100

## 6.4 Main Experiments

However overall effect size needs to be considered: in contrast to the results reported in Section 6.4.3, here effect size seems relatively small particularly for GBPUSD, even where significant at either threshold. In the end the results are somewhat equivocal, and for this adaptation mechanism operating on this preference modifier and core preference combination at least it may be difficult to justify invoking the learning mechanism. This does not of course mean that it will not perform better in other agents with other preferences, or in different markets.

	Mean $X_{\text{eff}}$ (e-06)			$X_{\text{eff}}$ Variances Mean (e-06)		
	1975-14	1975-93	1993-14	1975-14	1975-93	1993-14
No PSO	19.76	52.33	-2.32	0.021	0.031	0.014
PSO - basic	18.80	50.95	-3.00	0.021	0.029	0.014
PSO - decay	17.60	48.46	-2.02	0.019	0.026	0.012
PSO - decay & noise	18.28	49.42	-2.83	0.019	0.026	0.013
PSO - gBest decay & noise	18.53	49.63	-1.67	0.024	0.030	0.013

PSO comparison - GBPUSD

	Whole Series				0-4,000				4,000-end			
	$X_{\text{eff}}$ means		$X_{\text{eff}}$ variances		$X_{\text{eff}}$ means		$X_{\text{eff}}$ variances		$X_{\text{eff}}$ means		$X_{\text{eff}}$ variances	
	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value
PSO - basic	0	0.538	0	0.266	0	0.669	0	0.109	0	0.658	0	0.636
PSO - decay	0	0.635	1	1.9E-06	0	0.812	0	0.002	0	0.313	1	1.7E-06
PSO - decay & noise	0	0.105	1	1.8E-05	0	0.125	1	1.2E-05	0	0.620	0	0.123
PSO - gBest decay & noise	0	0.855	0	0.189	0	0.710	0	0.596	0	0.793	0	0.058

Table 6.11.: PSO Variant Performance & Statistical Tests - GBPUSD

	Mean $X_{\text{eff}}$ (e-06)			$X_{\text{eff}}$ Variances Mean(e-06)		
	1986-14	1986-98	1998-14	1986-14	1986-98	1998-14
No PSO	-16.77	53.21	-67.93	0.067	0.061	0.066
PSO - basic	-23.07	50.24	-76.67	0.069	0.054	0.072
PSO - decay	-21.09	48.77	-72.14	0.069	0.054	0.073
PSO - decay & noise	-14.69	53.38	-64.46	0.061	0.054	0.061
PSO - gBest decay & noise	-18.72	54.74	-72.43	0.065	0.050	0.066

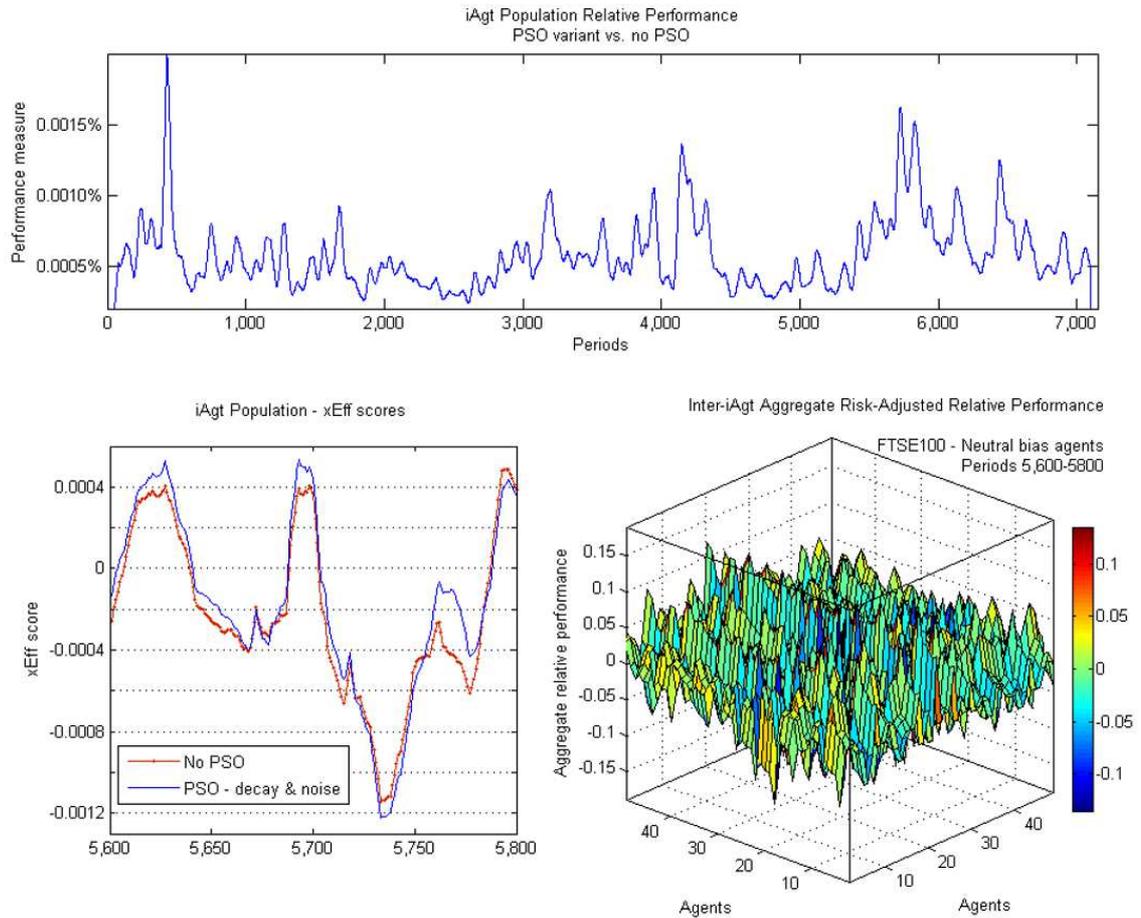
PSO comparison - FTSE100

	Whole Series				0-3,000				3,000-end			
	$X_{\text{eff}}$ means		$X_{\text{eff}}$ variances		$X_{\text{eff}}$ means		$X_{\text{eff}}$ variances		$X_{\text{eff}}$ means		$X_{\text{eff}}$ variances	
	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value	$H_1$ 2-tail	P value	$H_1$ 1-tail	P value
PSO - basic	0	0.186	0	0.725	0	0.450	0	0.047	0	0.232	0	0.926
PSO - decay	0	0.093	0	0.801	0	0.132	0	0.062	0	0.214	0	0.933
PSO - decay & noise	0	0.796	0	0.033	0	0.183	0	0.056	0	0.321	0	0.247
PSO - gBest decay & noise	0	0.627	0	0.065	0	0.963	0	0.330	0	0.545	0	0.086

Table 6.12.: PSO Variant Performance & Statistical Tests - FTSE100

**PSO Ineffectiveness & Modifier Behaviours** The relative lack of impact from any form of the PSO applied in this study is somewhat surprising. Aggregate PRP plots of iAgt population relative performance show little differentiation, with incremental periods of out- and under-performance between agent types over time. Similarly, aggregate PRP time series and surface plots identify no anomalous or exceptional behaviours between agent types.

Figure 6.23 shows an example of this lack of evidence from a FTSE100 run where the PSO variant with both decay and noise is compared to iAgt with no PSO.



**Figure 6.23.:** PSO Aggregate Population Relative Performance Plot Example - FTSE100

**PSO Temporal Dynamics** Given the apparent lack of effectiveness of the basic PSO, this raised the question of its swarm behaviour during simulation runs, which in turn after examining its temporal dynamics prompted the development of the PSO variants reported. Analysis for all the PSO types is shown in this section showing characteristic differences between these variants' temporal dynamics. Figure 6.24 shows the evolution of the stop loss threshold parameter over the course of sample runs for GBPUSD & FTSE100, plotting mean and standard deviation values across iAgt populations.

- **PSO - BASIC.** Although the swarm standard deviation is stable throughout the sample run there is little exploration of the parameter space and mean threshold values are also stable throughout each run.
- **PSO - DECAY ONLY.** For this variant lock-in occurs early in each simulation run and the standard deviation of threshold values collapses to zero.
- **PSO - DECAY & NOISE.** There appears to be reasonable exploration of the parameter space throughout simulation runs, although the swarm standard deviation falls away somewhat.

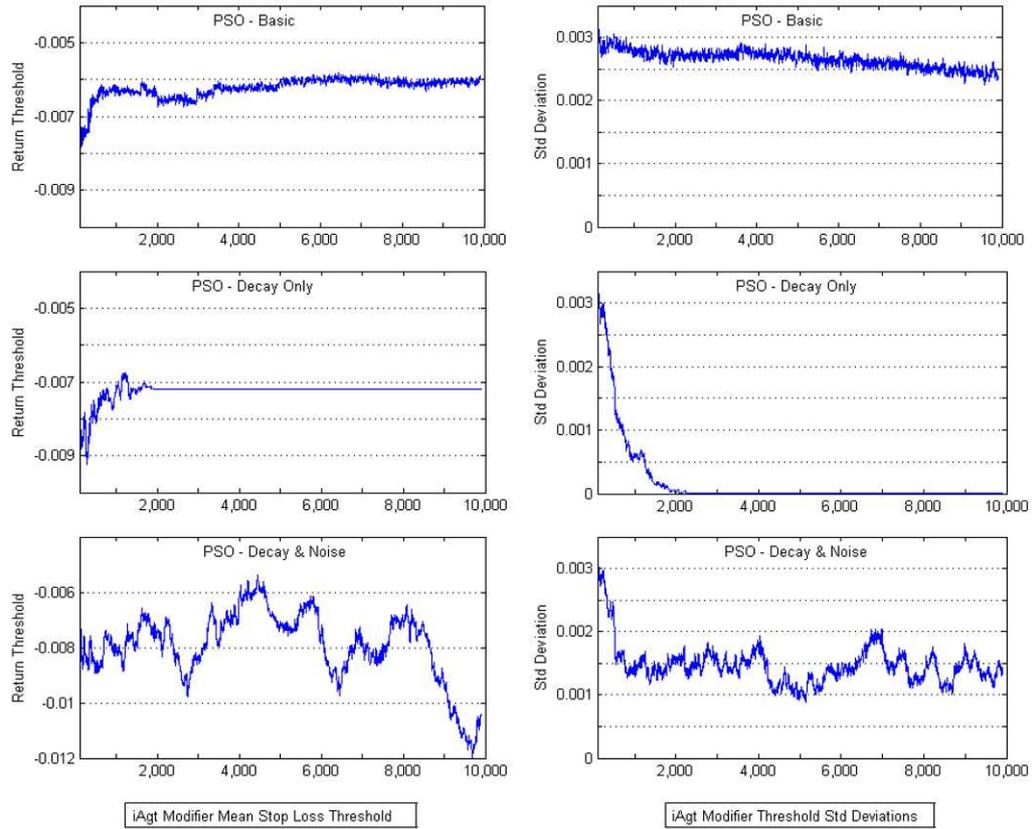
Despite differences in characteristics, including the intuitively attractive result for the 'decay & noise' PSO variant where it seems to explore the parameter space, as shown Tables 6.11 & 6.12 this did not translate into particularly meaningful iAgt performance benefits, which is interesting in itself.

**Discussion** The relatively poor performance of PSOs when compared to iAgt with no adaptation mechanism in their rulepacket populations raises some interesting questions and points to potential future research directions.

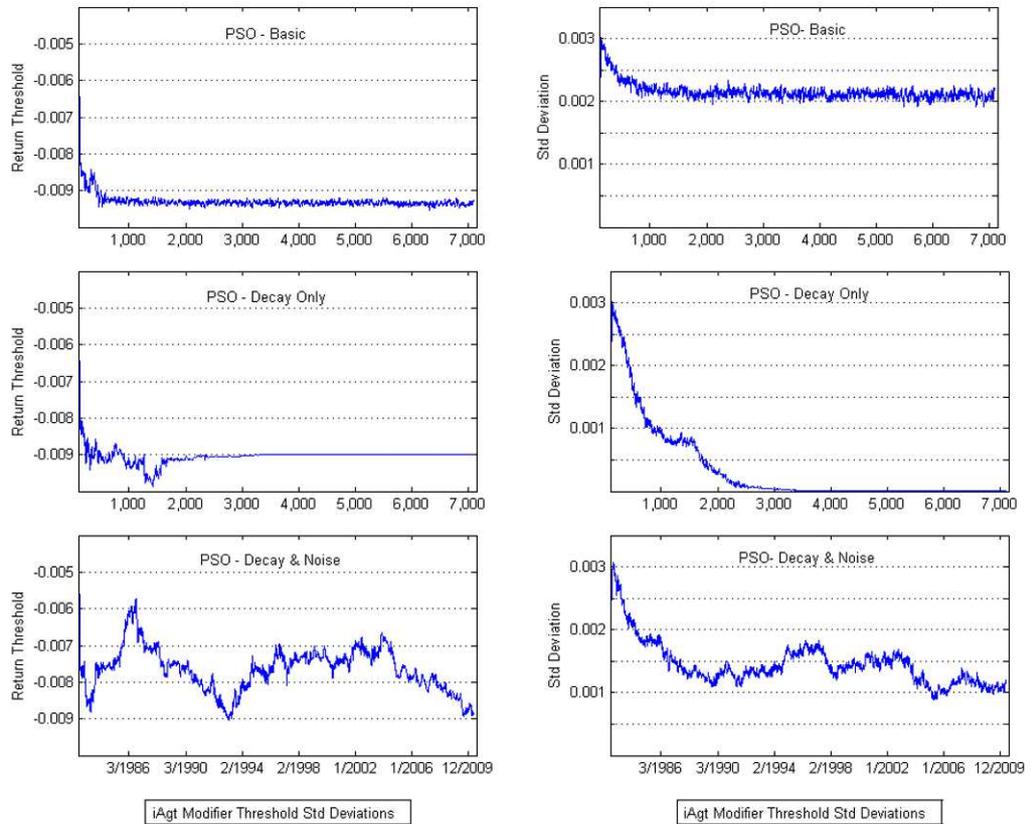
- What is the learning task?
- Is it reasonable or appropriate?
- Is a better distinction between risk based performance and uncertainty required?

It is worth considering the subject of the learning task again. A basic problem in modelling economic preferences is that there is a confounding effect from common underlying situated performance metrics: ultimately for core preferences and preference modifiers in situated environments these relate to economic performance over time.

In the experiments with MC preferences here the position is radically simplified - no attempt is made to optimise core choice behaviours as these are simple biases, so in practice here only the modifier is adjusted. Where core preferences are more sophisticated, or better fitted to the state of the market, and make few errors, then their returns will be high and modifiers like stop loss rarely triggered. Where poorly fitted, a dual learning process is required adjusting both core preferences and modifiers, however unless separate metrics can be identified then it is difficult to see how this can usefully be done. Investigations of potential solutions to this issue, such as structural choice heuristics and asymmetric risk-adjusted performance measures will form part of future work in this area and are discussed in the final part of this chapter.



(a) PSO Characteristics - GBPUSD



(b) PSO Characteristics - FTSE100

Figure 6.24.: PSO Temporal Characteristics

Given an averred role for preference modifiers in mitigating uncertainty, where underlying asset return distributions are unknown and subject to change, the learning task may ultimately be unreasonable, or possibly superfluous - at least for stop loss. Very rough settings for stop loss heuristics may be sufficient to deal with uncertainty, if a sufficiently spread out group of candidate solutions is set around the limits of expected acceptable risk distributions for an agent's core preferences as expressed via its investment strategy. Certainly here an initial, reasonably disperse rulepacket population seems to perform at least comparably to populations with PSO.

Based on the results here, stop loss certainly seems to satisfy Gigerenzer's definition of a 'fast and frugal' heuristic. In terms of ecological rationality, as a subsumed preference modifying behaviour in economic preference expression, adaptation may not therefore be a priority if sufficient domain knowledge is available to augment the initial parameterisation process. It may also point to an uncertainty mitigation function for population-based subsumptive structures as structural meta-heuristics themselves: given the prevalence of this type of organisational structure in investment banking and trading, this seems not unreasonable and warrants further investigation.

#### **6.4.5. Agent-Subsuming Agents - Choice & Meta-Heuristics**

This section considers additional layers of subsumption in preference expression and choice within the SHaaP architecture. Preliminary results are presented for agent-subsuming, senior agents making choices based on the risk-adjusted performance of populations of iAgtS with heterogeneous biases of the kind presented in Section 6.4.3. Subsumptive hierarchies of this type are commonly observed in financial institutions and corporate structures<sup>26</sup> where it can involve oversight and management heuristics to control behaviour expression, or, as in the case here, a form of copy-cat heuristic where successful behaviours are replicated and repeated.

This experiment revisits the early trials and experiments from Section 6.4.3 where limited results for this type of agent-subsuming agent were reported. However those were of limited value since they were for only single instances of sAgtS: here this weakness is addressed with more meaningful populations of sAgtS and the results analysed.

In the earlier FTSE100 experiments sAgt performance appeared to be dominated by overall market trend, and sAgtS were unable to outperform iAgtS with a consistent strong *+ve* bias. However for GBPUSD sample runs sAgtS appeared able to choose successfully between preference behaviours amongst iAgtS with heterogeneous cognitive biases, outperforming their iAgt population over extended periods. This choice behaviour is the focus of the work in this section, exploring a population of sAgtS subsuming populations of heterogeneously cognitive biased iAgtS investing in GBPUSD markets.

A secondary observation from earlier results was the effect of changing the  $X_{eff}$  window used by sAgtS for monitoring their subsumed inner agent populations. For the single sAgtS used in the Group 1 study of GBPUSD markets with stop loss iAgt populations decreasing the  $X_{eff}$  window from 40 periods to 15 periods appeared

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<sup>26</sup>See Chapter 4 for discussion of subsumptive structures in economic systems and entities.

to improve overall performance. The original intuition in choosing sAgt population parameters was that longer monitor periods would give sAgts more stable data and better inform preference choice. However the trial indications contradicted that suggesting this as an interesting area to investigate.

### Experimental Protocol

In this experiment sAgt performance in the GBPUSD forex market was examined for two values of the iAgt  $X_{eff}$  window - 15 periods and 40 periods. The original experiment gave results for only a single sAgt in each sample run as iAgt subsuming behaviours were the primary focus, here the experiment is repeated with multiple sAgts in each population. Due to memory constraints<sup>27</sup> the sAgt population was limited to 10 agents for each sample run, and each sAgt's iAgt population was also limited to 10, each of which had rulepacket populations reduced to 30 from 60. To give results for 20 sAgts of each type as a reasonable sample size, each run was repeated with a new set of seed values for initialising the random number generators in the architecture<sup>28</sup> and the aggregate results used in the analysis.

Otherwise the basic experimental protocol is the same as for the Group 1 experiment in Section 6.4.3 - the main parameter settings are shown again in Table 6.13. Following the findings of the PSO study in the previous section, there is no explicit learning or adaptation mechanism, since there was little observed gain from these mechanisms. This also serves to limit the number of new factors to consider in the experiments.

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<sup>27</sup>Recall that each sAgt subsumes an iAgt population, each of which has its own population of rulepackets and each rulepacket has a set of rule copies. In the default configuration used in the experiments thus far this means each sAgt potentially has 15,000 rules at any time engaged in paper trades or actual transactions.

<sup>28</sup>See Table A.14 for these values.

Parameters	Value	Notes
sAgt	20	Total number of sAgt
sAgt & iAgt portfolio trades	20	Maximum number of open positions at any time
Inner Agent population size	10	iAgt in each sub-population within sAgt
Rulepackets per iAgt	60	Rulepackets available to each iAgt
Rules per rulepacket	10	Number of rules in each packet, identical by preferences
Agent $X_{eff}$ period	15 or 40	Performance window for iAgt, monitored by sAgt
Agent $X_{eff}$ eMav	10	Exponential moving average period for $X_{eff}$ score
Rules $X_{eff}$ period	15	Performance window for rules, monitored by iAgt
Rules $X_{eff}$ eMav	10	Exponential moving average period for $X_{eff}$ score
Poison mean	10	Mean of Poisson distribution for maximum trade lengths
Learning period	10	Mean period between learning algorithm runs

Table 6.13.: sAgt Window Comparison - Experimental Parameters

## Results & Discussion

Figure 6.25 shows the monitor window effect for the 2 sAgt types in a sample run with 4 heterogeneous iAgt biases (neutral,  $+ve$ ,  $-ve$  & weak  $+ve$ ) - here the mean cumulative net P&L for each agent type is plotted. Both sAgt populations can be seen to have generally similar performance patterns, although a clear differential emerges over the course of the sample run.

Both sAgt groups outperform their iAgt populations producing stable returns in a variety of conditions and can also be seen to respond to sharp market moves, apparently switching preference behaviours, such as in late 2009 where  $-ve$  bias iAgt successfully capture a steep sell-off in the exchange rate.

Once again however cumulative net P&L performance changes in the mid-1990s. Until this point it appears that sAgt can successfully switch between iAgt opportunistically, producing stable low volatility returns. After period 4,000 in the GBPUSD timeseries however this effect fades. While it could be attributed to the same change in iAgt performance necessarily being passed on to the subsuming sAgt, examining Fig 6.25 again between periods 5,000 and 7,000 shows poor sAgt performance in the face of reasonable iAgt performance for some agent types.

Table 6.14 shows the underlying performance statistics for sAgt in this experiment, confirming the general risk-adjusted return profile shown in the chart. The shorter  $X_{eff}$  monitor window appears to be beneficial both in terms of mean performance and in the volatility of returns, although the effect size for  $X_{eff}$  variance is not enormous - it is statistically significant in all cases. The effect is present throughout the entire experimental run - sAgt using a 15 period monitor window for their iAgt populations consistently perform better both in terms of mean risk-adjusted return and volatility of these returns.

## 6.4 Main Experiments

	Window	Mean $X_{eff}$ (e-06)			$X_{eff}$ Variances Mean (e-06)			Mean iAgt P&L		
		1975-14	1975-93	1993-14	1975-14	1975-93	1993-14	1975-14	1975-93	1993-14
sAgt	- 15 periods	32.23	96.87	-11.60	0.058	0.080	0.039	38,081.8	47,720.7	-9,638.9
	- 40 periods	21.40	87.77	-23.48	0.067	0.095	0.044	24,084.1	43,038.0	-18,954.0

Window - 15 vs. 40											
Whole Series				0-4,000				4,000-end			
$X_{eff}$ means		$X_{eff}$ variances		$X_{eff}$ means		$X_{eff}$ variances		$X_{eff}$ means		$X_{eff}$ variances	
H <sub>1</sub> 2-tail	P value	H <sub>1</sub> 1-tail	P value	H <sub>1</sub> 2-tail	P value	H <sub>1</sub> 1-tail	P value	H <sub>1</sub> 2-tail	P value	H <sub>1</sub> 1-tail	P value
1	6.4E-10	1	1.2E-11	1	0.0001	1	2.5E-12	1	3.5E-08	1	3.8E-08

**Table 6.14.:** sAgt  $X_{eff}$  Window - Summary Statistics & Test Results

This is illustrated quite clearly in Fig 6.26 which plots mean  $X_{eff}$  against  $X_{eff}$  variance for each sAgt in the sample run, emphasising from a rational investor's standpoint that given lower volatility of returns and higher overall expected return the shorter monitor window should be preferred.

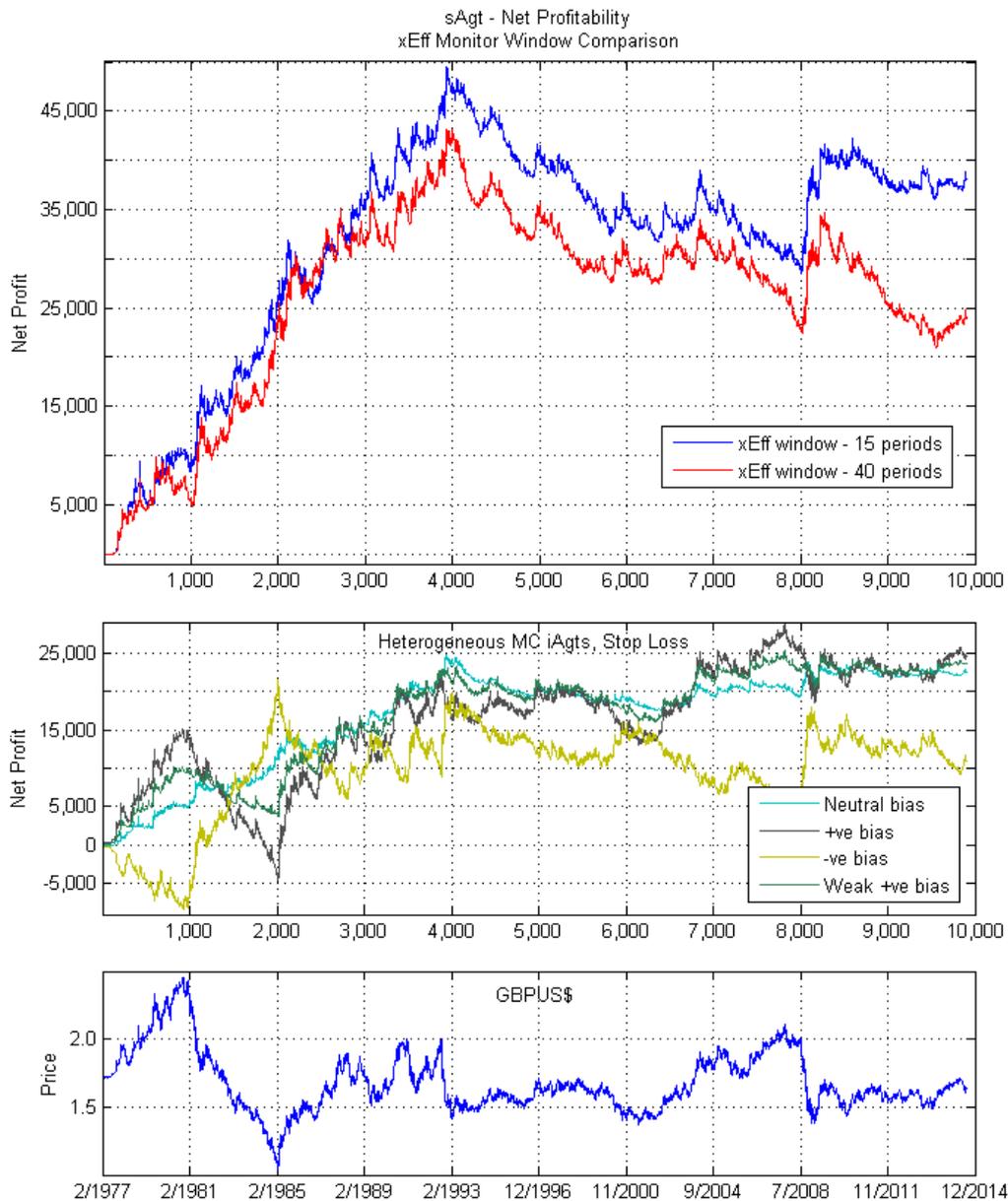
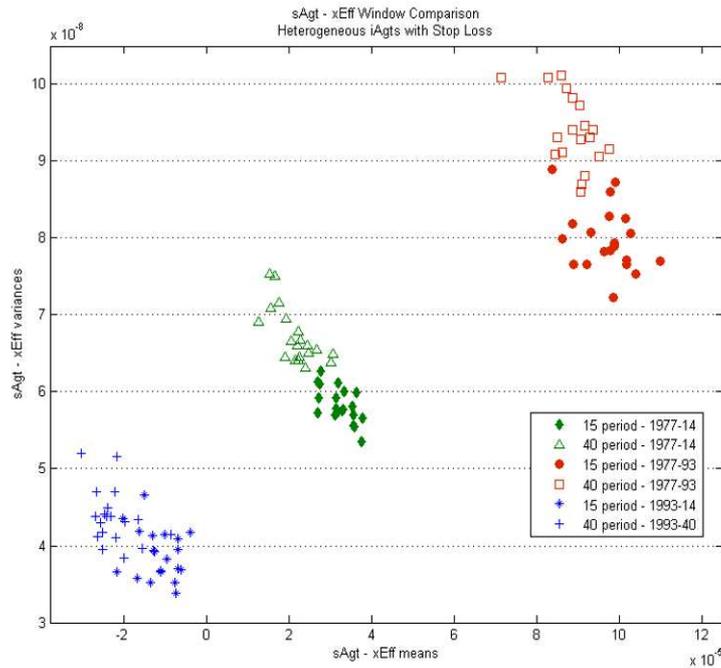


Figure 6.25.: sAgt  $X_{eff}$  - Cumulative P&L Effects



**Figure 6.26.:** sAgt  $X_{eff}$  Window - Risk-Adjusted Return Descriptive Statistics

**Choice Protocols & Performance** Agent-level analysis of choice behaviours is important to understanding the dynamics of sAgt performance. Within the SHaaP model here, sAgt monitor their iAgt population performance - ranking these agents according to their rolling  $X_{eff}$  performance using either 40 period or 15 period windows. In each trading period where they can trade they copy preferences from the best performing iAgt at that time. In each simulation run the top iAgt type in each period was recorded for each sAgt in the population, reflecting the choice available at that time.

Figure 6.27 explores the dynamics of top iAgt type distribution changes across both sAgt populations over a subsection of the simulation run encompassing both a highly volatile period and subsequent, more stable, price action. sAgt using a narrower monitoring window see changes in iAgt performance sooner affecting their choices, and that this shorter window is more sensitive to short term market movements and volatility. When the underlying calculation method for  $X_{eff}$  as a risk-adjusted performance measure is considered<sup>29</sup> this is not a surprise since it functions as a form of averaging and there is more inertia in wider windows. However the overall long term impact on mean risk-adjusted returns, variance of returns and cumulative P&L is more unexpected.

The example in Fig 6.27 shows that although increased responsiveness correlates with some periods of underperformance for narrower windows, in general sAgt using a 15 period monitoring measure outperform exhibiting higher risk-adjusted performance for most of the sample shown here<sup>30</sup>.

<sup>29</sup>See Section 3.3.5.1 in Chapter 3 and Appendix A.2 to review the detailed methods.

<sup>30</sup>Note, again, to avoid confusion, that the  $X_{eff}$  score used to compare iAgt & sAgt in all these charts and tables is calculated off a common rolling 40 day period. The varied window under test in this section is that used by the agents themselves to monitor internal performance.

An interesting observation highlighted by this chart is the dominance of simple  $-ve$  and  $+ve$  bias preferences. Neutral and weak  $+ve$  bias preferences hardly figure, and when they do it is only apparent for short periods at low frequencies in either sAgt population. In particular, Neutral bias iAgt performance is relatively stable both for rolling risk-adjusted returns, and more strikingly in terms of cumulative net P&L as shown in Fig 6.25. Referring back to the Group 2 experiment results comparing modifier performance to iAgt without modifiers, it can be seen in Table 6.6 that  $X_{eff}$  score variance is lower in general across all preference package combinations, although the mean score is more mixed, particularly in FTSE100 where  $+ve$  bias agents dominate. These lower variance scores are likely in part a function of the same effect for 'No Modifier, Neutral Bias' agents noted in Section 6.4.3.

When comparing sAgt risk-adjusted returns to their subsumed iAgt populations it is worth noting that, given dominant use of  $+ve$  and  $-ve$  bias preferences sAgt returns are consistently less volatile than either of these types of iAgt.

However, in terms of sAgt choice, it is somewhat concerning that for similar risk-adjusted return scores preferences with lower volatility of returns do not appear to be selected. The risk-adjusted performance measure itself may be a contributing factor to this blindness: a potential failing is that it treats positive and negative returns symmetrically, while its parameterisation, following Dacorogna's recommended settings[34], is somewhat arbitrary. Both these aspects require further investigation.

Similarly the sAgt decision rules in this experiment are deliberately rudimentary and do not attempt to differentiate beyond a simple ranking between relative iAgt performance. These issues are discussed in more detail in the final section of this chapter in terms of future work and development of the SHaaP architecture.

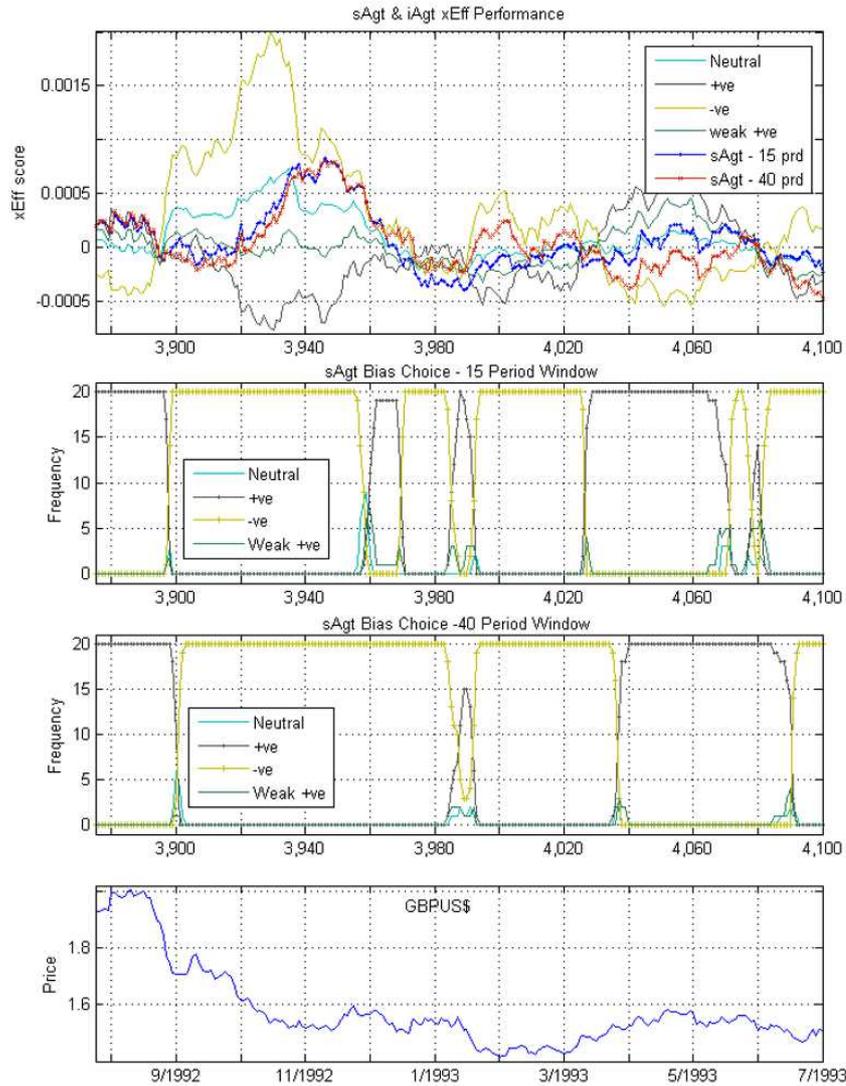


Figure 6.27.: sAgt  $X_{eff}$  Window Comparison - Granular iAgt Switching Patterns

**SHaAP Population Parameter Effects** Interestingly the reduction in iAgt rulepacket population size appears to have had little impact on overall performance. This raises interesting questions in terms parameterisation and performance robustness. The original configuration was chosen to generate sufficient candidate preference packages for each iAgt to make a reasonable selection. Even with only 30 rulepackets per iAgt this gives a substantial amount of information on packet performance: each packet having 10 rule copies, given an expected life of 10 periods for a transaction with no preference modifier and a trade latency of 2 periods, this means each agent is overlooking the conflated results of approximately 150,000 paper transactions in a single GBPUSD sample run, a number which is higher when preference modifiers are used since they act here only to shorten trade life. Of course, MC preferences are deliberately atypical (at least in terms of expected behaviours by educated investors) - they generate a very high number of trading signals relative to what one would expect from more sophisticated core preferences using market or economic information to inform decisions. However it does point to an important area for future work with more realistic core preferences - i.e. the parameterisation and robustness of the subsumptive population structure in the architecture.

## 6.5. Discussion

The case study experiments in this chapter focused on the role of simple heuristic preference modifiers operating on minimal core preferences, exploring the behaviour of populations of agents subsuming these preference behaviours, and themselves subject to subsumption. This serves two purposes. Agent-based models exploring preference modifying behaviours are notably rare in the literature, despite perceived benefits to such granular, bottom-up approaches - the work here begins at least to address this absence. The second objective was to provide a comprehensive case study demonstrating the SHaaP architecture in action and allowing it to be critically evaluated. Both objectives appear to have been well served by the experiments reported here.

### Preference Modifiers - Fast, Frugal & Ecologically Rational?

Agents in the studies were imbued with deliberately minimal core preferences and their economic preference expression observed as these MC preferences were subsumed within preference modifying structures and latterly by higher levels of subsumption. No claim is made that the core preferences, or those core preferences subsumed by modifying behaviours, or by other agents are in themselves realistic, or should be used in anger for investment purposes. The aim here was to allow preference modifying behaviours to be studied in relative isolation, as a necessary step towards more sophisticated core preference behaviours within subsumption hierarchies.

Situated performance measures and simulation environments were used to validate and explore the results of a series of experiments in a SHaaP architecture implementation. A basic question beginning this chapter was the role of such modifiers in risk management and uncertainty mitigation. While the experiments have not directly, and certainly not definitively, answered this question, they represent a first stage in a new approach to modelling such behaviours. Simple heuristic preference modifiers such as stop loss appear to improve economic performance in uncertainty prone environments, and are quick to apply while not imposing significant computational burdens, i.e. they are fast and frugal. Based on the results here, they at least complement more complex risk management tools, and in terms of ecological rationality there is a strong case for further exploration.

**Risk-adjusted Returns & Uncertainty Mitigation** The experiments showed not only that preference modifying behaviours can materially reduce volatility of investment returns, but in some cases can shape preference expression to yield consistent positive return profiles over extended periods. This is a striking result and appears to be supported at least to some degree by Kaminsky & Lo's recent empirical study[65]. In their work, retroactively analysing model portfolios and return processes they found some states in which stop loss policies could generate positive returns and reduce volatility. However an important difference in their modelling approach is that effectively it simply documents the return profiles of fixed investment strategies over time rather than allowing agents to dynamically select from a population of core preference and preference modifying behaviours, as is the case

in the SHaaP architecture. Nevertheless Kaminsky & Lo's work is interesting with value in considering modifier parameterisation, particularly insofar as they attempt to fit their findings to a theoretical framework. In this thesis no attempt is made to fit the findings within a theoretical framework - this is weakness, at least according to some academic criteria, in the approach reflecting its exploratory nature. Tractability is an issue with any rich ABM which is not highly abstracted, and models in the SHaaP architecture are no exception. The emphasis on validation in the methodology via situation and realistic operational performance measures addresses this to some extent, but as a limitation it cannot be ignored.

In the discussion following the initial experiments it was suggested that with the advent of real time data feeds and algorithmic trading models that their impact on markets may have changed market micro structure, hence the change in performance post-1993 for foreign exchange and post-1998 for the stock index experiments. As an inference this is difficult to validate, however the shift in market behaviours it illustrates is interesting, and certainly points to some underlying structural change over time.

**Learning & Adaptation** Although agents in the SHaaP architecture can choose dynamically from their population of candidate behaviours allowing a form of adaptation to market conditions, beyond recent performance within these populations, agents in the initial experiments have no memory and do not actively learn or adapt. The basic PSO implemented was tested alongside two PSO variants operating on stop loss preference modifier parameters so that the effect of an active adaptation mechanism could be explored. The results for these studies were surprisingly poor in terms of PSO performance. Although exhibiting interesting dynamics there was little evidence of performance improvement or adaptation over the SHaaP implementation with no learning algorithm.

Rather than describing the PSO performance as 'poor', an alternate interpretation of the results might be that the original population structure itself provides a robust and effective structural adaptation meta-heuristic in environments prone to uncertainty and change. Unlike evolutionary algorithms where candidate solutions may be replaced as part of a learning or optimisation process where previously successful historic behaviours may be lost, in the SHaaP architecture candidate solutions are not replaced and so remain available for conditions where they are viable. A downside of this structure is the absence of an explicit innovation mechanism in terms of behaviours, however a suitably deep subsumptive hierarchy may go some way to meeting this deficiency: multiple subsumption layers of behavioural elements may yield intelligent adaptive and behaviourally innovative responses in situated environments as Brooks proposes[25, 21].

A second possible interpretation, presented in the discussion of the PSO experiment results, may be that preference modifiers such as stop loss, which are activated in times of increased volatility and uncertainty, are not particularly suitable candidates for optimisation or learning. This would explain why the PSO had little final effect on modifier performance and the population-based SHaaP structure is resilient. It would also provide, for this type of modifier at least, an escape from the confounding effect of a common performance metric for both core preferences and preference

modifiers.<sup>31</sup> If a modifier is cast in an uncertainty mitigation role, and if as a result it is not a suitable candidate for learning or adaptation algorithms, then core preference performance may be addressed in relative isolation, which will be important when more realistic, sophisticated core preferences are used.

All three interpretations may of course have some degree of validity and future work will revisit and explore these issues.

**Subsumption Hierarchies** The final part of the case study in this chapter demonstrated a subsumptive hierarchy in operation as senior, agent-subsuming agents expressed behaviours subsumed from their inner agent populations. These senior agents showed superior performance to their subsumed populations both in terms of ongoing risk-adjusted returns and volatility of those returns. A clear sensitivity to recent performance was also seen. Although the results were encouraging and yielded some interesting behaviours, it was obvious that at times the choice heuristic failed to discriminate between long term high quality returns from some agent groups. The heuristic itself is a simple copycat behaviour, ranking iAgt risk-adjusted performance over time, so this weakness is not altogether surprising.

The subsumption levels had limited depth and more extended hierarchies are an obvious area for future work. An attractive domain driven approach will be to add additional structural subsumptive layers for capital allocation and secondary risk and uncertainty management. Alongside more realistic core preferences this gives a pathway to developing practical, empirically validated models for decision support. However, given the ease of implementing compound choice behaviours inside a subsumption hierarchy and rapid increases in combinatorial complexity, it is important to continue the principled approach developed here to allow meaningful inferences to be made.

### **Experimental Limitations, Criticisms & The SHaaP Architecture**

The experiments presented were deliberately limited in scope, the aim being to explore preference modifiers in a situated environment, while at the same time to allow a detailed examination of the architecture itself in operation.

Functionally the architecture itself proved resilient, and the findings, despite experimental restrictions, demonstrated interesting subsumptive and population-based behaviours. Given the prevalence of both types of structure in real world economic entities and their apparent robustness in these experiments, further study is warranted. Future work will need to relax the constraints imposed here, adding more sophisticated core preferences and additional layers of subsumption.

In the experiments, although in some experiments iAgt populations were described as heterogeneous this was arguably generous. There were only 4 iAgt types in each population and, biases aside, their core preferences were not differentiated or designed to show intelligent behaviours. Additional and more diverse preferences would allow sAgt subsumption of iAgt population behaviours a wider range of expression.

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<sup>31</sup>This problem & possible solutions forms part of the overall SHaaP architecture discussion below.

Only two effective layers of subsumption were invoked in the simulations: iAgtS subsumed core preferences within preference modifying behaviours, and were themselves subsumed by sAgtS. More layers, and larger candidate populations of heterogeneous behaviours provide an enormous range of scope for exploring economic preference expression as compound preference structures are developed.

There was no interaction between iAgtS, nor was there between sAgtS. Their economic expression was limited to responses to their economic environment in the form of the financial time series as it was presented to them, while their decisions, given their size relative to the environment was assumed not to have a material impact on that environment. In the context of these experiments this is a reasonable abstraction, particularly since it is to a large extent how these markets themselves are observed to function. However, for smaller markets, or markets where direct interactions are critical to functional preference expression across agent populations, this will be a consideration. Similarly for models used in policy design applications, where the interactions of heterogeneous populations of agents face structural and regulatory modification, inter-agent interaction will be an important consideration. Cascade effects and radical changes in previously stable relationships and market correlations have been symptoms of a number of economic crises and SHaaP models exploring these are likely to be a significant research area.

Structural modifier effects were not explored at all here. Alongside policy-based regulatory structures, their interaction with economic preference expression in systems of agents are an important area for study. In the early chapters, this was identified as a significant omission and source of frustration in reviewing academic models. Although the impact of this absence in such models can be argued as minor given their academic context, if real applications and decision support are envisaged they cannot be ignored. In the current implementation all the programming structures necessary for such modifiers have been put in place ready for this next stage of work.

Beyond these deliberate experimental limitations there are two main areas of concern in the approach developed in the SHaaP architecture.

- The lack of an overall theoretical framework to underpin the design rationale. This was referred to above in terms of preference modifiers, but applies to the SHaaP architecture as a whole. Given the inherent complexity of an architecture made up of populations of component elements subject to subsumption, in which each element may encapsulate behaviours ranging from simple heuristic to unboundedly rational computational decision rules, tractability is inevitably sacrificed. While individual components may fit inside frameworks of their own, such as neoclassical utility maximisation models, in larger subsuming structures this breaks down.

In the earlier chapters of this thesis highly abstracted approaches attempting to fit to theoretical frameworks at the expense of empirical validation metrics were criticised. The SHaaP architecture sacrifices the former in favour of the latter. This does not mean theoretical frameworks should be ignored, rather the approach favoured here is to develop & validate situated models of economic preference expression and use these to develop an operationally meaningful framework. This thesis is an early stage of that work.

- Performance measure confounding effects. As a hierarchical structure of sub-

sumptive layers with common risk-adjusted performance measures, there is no obvious way to separate or attribute economic contributions to specific preference expression components. However this is a problem shared by any model, agent-based, or otherwise which seeks to use realistic performance measures to directly inspect preference expression. Abstractions which ignore agent-level behaviours in favour of theoretical constructs have limited operational relevance, while models focusing on optimising forecasting behaviours are not actually dealing with preference expression per se.

The main risk-adjusted measure,  $X_{eff}$ , used in the study is itself open to criticism in that it treats profits and losses symmetrically and its parameterisation is somewhat arbitrary. Asymmetric measures have been suggested as practical improvements[34], penalising negative returns more heavily than positive returns. A trial measure,  $R_{eff}$ ,<sup>32</sup> has now been implemented, but the initial results showed little benefit in terms of the confounding issue here, and in fact its parameterisation is also rather arbitrary and its theoretical basis a potential red herring.

A better subsumptive approach, where components are subject to this confounding effect, may be to maintain and monitor performance of independent component populations, equivalent to the rulepacket structure already in place in the architecture, selecting from these in the subsumption process.

Ultimately however this issue is a problem for any situated, realistic, agent-based model for economic preference expression, just as it is for risk and trading managers in actual application. At this stage a solution is not obvious, making it an interesting research topic in itself.

## 6.6. Summary & Conclusions

This chapter presented a series of original experiments exploring the role of preference modifier behaviours in situated environments, demonstrating the use of subsumptive and population-based structural meta-heuristics in modelling economic preferences within the SHaaP architecture.

As heuristics modifying risk and uncertainty exposures, preference modifiers are worthy of study in their own right, although the relative lack of research in this area belies their potential significance in day to day risk management. Such modifiers may often be simple to describe, however they are not necessarily simple to investigate, something the work in the case study and in developing the experimental architecture has confirmed. Core preferences and preference modifiers, by definition, interact and generally share common performance metrics so that exploring their functional characteristics is challenging - which of course goes some way to explaining the relative absence of work in this area.

The experiments and exploratory analyses presented here provide an extensive, systematic, comparative study of preference modifying behaviours in combination with deliberately minimal core preferences. This study used the new population relative performance measures developed in the thesis in combination with visualisation

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<sup>32</sup>See Appendix A.2 for a full description and comparison to  $X_{eff}$ .

techniques and statistical analysis of risk-adjusted returns to explore agent-level and population behaviours in detail, building on the experience and techniques developed in Chapter 5.

Given the simplicity of these MC preferences the experimental findings were quite striking. Not only was it possible to successfully demonstrate uncertainty mitigation and effects in agent behaviours, but in some circumstances consistent profitable investment behaviours were also observed for extended periods. As a demonstration of subsumptive structures in economic preference expression this is valuable, however the persistence of this profitable behaviour and its subsequent disappearance is an interesting finding in itself: as the models were situated using real historic financial time series, this lends itself to immediate operationally meaningful interpretation.

Later experiments in the case study introduced agent adaptation via a PSO algorithm. The relative failure of adaptation to improve agent performance may point to the robustness of the population-based preference structure in the SHaaP implementation used here. However this may also be an effect limited to preference modifier behaviours and emphasised by the use of MC preferences. Future work should investigate this further, invoking more sophisticated core preferences and alternate adaptation mechanisms.

Deeper subsumption hierarchies with agent-subsuming agents were also explored. Analysis of preference expression and choice behaviours for these types of agents was instructive demonstrating again subsumption as an effective structural meta-heuristic, however the scope of these experiments was limited and there is significant room for future work here.

As interesting as the particular experimental findings for preference modifier behaviours were, particularly from an ex-practitioner's point of view, equally important in the case study was the demonstration of the SHaaP architecture - employing subsumption and population-based structural meta-heuristics in modelling economic preference expression. The architecture itself is a principal research contribution in the thesis, and the experimental data from the experiments here provide the basis for investigating and documenting its characteristics. The robustness of agent performance in the absence of explicit adaptation or learning methods, and when preference population sizes are degraded suggest significant areas for future work in exploring the architecture and its behaviour.

Overall the case study work in this chapter highlights the potential value of subsumptive and population-based meta-heuristic structures in modelling preference expression. The experiments support the methodological approach in the thesis: combining a clear experimental architecture with realistic performance measures, exploratory tools, and situation to allow systematic, principled investigation of economic preference expression. There are significant opportunities for future work developing the architecture and exploring situated behaviours. The continued emphasis on situation as a basic starting point to validation is important, allowing behaviours to be extracted for detailed analysis while at the same time remaining immediately relevant in developing decision support models.

## 7. Discussion & Conclusions

*'The Universe is an unsettlingly big place, a fact which for the sake of a quiet life most people tend to ignore.*

*Many would happily move to somewhere rather smaller of their own devising, and this is what most beings in fact do.'*

Douglas Adams, The Hitchhiker's Guide to the Galaxy

### Overview

This thesis has been concerned with addressing practical issues in modelling economic preferences using agent-based models by applying domain knowledge from experience of real economic systems and behaviours. A cornerstone of the approach presented here is that, although they may have value in their own specific circumstances, highly abstracted academic models developed in isolation from situated environments frequently have little direct economic or operational relevance in practical contexts. At the same time abstraction and the use of rational representative agents as stand-in's for actual behaviours in top-down models, or in homuncular versions in ABMs, denude these models of explanatory value.

Ultimately the conclusion is that, while abstraction might allow a quiet life by simplifying experimental problems and allowing tractability, on its own, moving 'somewhere rather smaller of our own devising' is not a viable solution to understanding behaviours in economic systems, or developing useful decision support tools. The real world is where models need to be situated as part of their validation, and the SHaaP architecture is designed for that purpose - abstraction comes later.

This chapter concludes the work in the thesis, drawing together the themes presented thus far, identifying and revisiting the contributions of the work here, potential applications, and future work.

### Subsumption & Realism in Agent-Based Economic Preference Expression

This thesis began by describing fundamental problems in modelling economic preference expression in terms of risk while ignoring uncertainty exposures as a disruptive characteristic of real systems. The distinction between risk and uncertainty was presented as a critical issue in modelling reflexive systems of economic preference expression.

Comparing academic descriptions of economic behaviour and rationality to direct experience of sophisticated computational investment models applied in practical situations, it became clear that the academic agent-based models typically lacked

any meaningful degree of situation or realistic performance measures. Similarly they failed to recognise functional preference modifying structures necessary to operate in the real environments. Beyond simple operational logistics, some such structures may also have roles in uncertainty mitigation, akin to those identified by Knight in 1921[74]. As apparently rediscovered knowledge this has become particularly relevant since the global financial crisis.

This led to discussion of the potential strengths and apparent methodological weaknesses of bottom-up agent-based approaches to modelling economic systems, in particular echoing criticisms such as Berg & Gigerenzer's[11] description of 'as-if' neo-classical economics in behavioural finance models. The case study in Chapter 5 showed not so much that the thinking behind the SFASM was flawed, rather that designing and testing & validating ABMs is hard. Combinatorial complexity inherent to ABMs and large volumes of experimental data mean that an ongoing forensic approach to exploratory data analysis is advocated in this thesis. Lacking suitable, realistic agent-level performance measures the solution frequently seems to have been to make the problem easier to handle, while forgoing critical exploratory & forensic analysis. The approach presented here has been to apply experience of practical solutions and performance measures to agent-based models. New situated agent-level exploratory data analysis tools were developed and tested in Chapter 5 for use in evaluating and investigating agent-based models of economic preference expression.

This domain knowledge driven approach was then extended to the design of the SHaaP architecture in Chapter 4 inspired by subsumptive structures in nature, and following Brooks'[25] insights from evolutionary robotics. The architecture yields distinct benefits: functional decomposition of economic preference expression into explicit individual component behaviours allows the possibility of systematic, principled investigations of economic preferences which are immediately operationally meaningful. It finally opens up the possibility of moving experimental economics away from considering just choices towards, as Simon[129] suggested, closer alignment with other social sciences by looking at underlying processes.

Gigerenzer and others have identified a number of simple heuristics which appear to function effectively in economic contexts, in some cases outperforming more traditional, unboundedly rational models of preferences. These fast and frugal heuristics are appealing since they also allow for boundedly rational economic actors to both be present and ecologically successful. However a weakness in the case for such heuristic preferences is the somewhat evangelical suggestion by some researchers that they are a holistic solution to preference expression. Although not unusual in promoting scientific debate, in terms of ecological rationality it fails to acknowledge situations and periods where hyper-rational risk-based models are both appropriate and sufficiently resilient, when subsumed by uncertainty mitigation behaviours, to be favoured.

The exploratory experiments in Chapter 6 showed agents with subsumptive, simple heuristic preference structures exhibiting apparently intelligent economic behaviour without the need for sophisticated, unboundedly rational homunculi at the agents' cores. At the same time, they provided evidence for the role of preference modifiers in uncertainty prone environments. These studies are interesting in themselves since this area has seen little empirical work with agent-based models. Situating the

models and using realistic, agent-level exploratory data analysis tools supports their immediate relevance to direct application, although this will still require substantial development to particular use cases.

The more important outcome of the experiments, however, was to demonstrate models of economic preference expression in action in an effective subsumptive architecture. The results here were very encouraging: SHaaP models show potential for operationally meaningful models of economic preferences and economic systems with a high degree of explanatory value. By allowing both simple heuristic and computational models within subsumptive structures, component elements can be explored separately before their interactions are systematically evaluated in larger, more complex models. From an AI standpoint this is a compelling proposition. Brooks has claimed emergent intelligent behaviours built up from lower level, well understood components, which are demonstrated to be resilient in situated environments, are a strong basis for principled AI research, and that is the case argued throughout this thesis.

### **Model Development - Situation, Subsumption & Validation**

The case has been made in the thesis that typical agent-based academic research methodologies, focused on developing unsituated, abstracted models and attempting to relate them to situated behaviours, are inherently weak in terms of validation and verification, if not basically flawed. The approach advocated here, beginning with situated models, allows movement in the opposite direction - from situated to unsituated - which should allow closer alignment of experimental studies and practical applications.

Systematic validation and testing of core preferences, preference modifiers and subsuming preference structures has enormous potential, allowing new, subsumptive structures to be explored in a principled manner both in academia and in practice. This is quite different from either abstracted, top-down approaches to model construction or highly parameterised systems dynamics models: for the former the degree of abstraction makes application to real world scenarios difficult or impossible to justify; for the latter, the degree of parameterisation makes it possible to game the simulation and fit modellers' assumptions and desired outcomes to derive a required result.

Situation supports validation, while subsumption enables verification, and, combined with a population-based approach to model design, allows progressive development of sophistication and complexity within models. Taken together this is quite different from existing academic attempts at interfacing with models in applied environments.

Taking functional behaviours which have some level of demonstrable validation and incorporating them in unsituated models still allows the possibility of unsituated simulations generating artificial financial timeseries for examination, but with a basic level of validation not previously available. Agent behaviours in such simulations already have some level of established operational meaningfulness which carries through to exploratory 'what-if' scenario analysis. For policy makers, regulators and risk managers alike this is an important step, narrowing the distance between academia and practice.

## Ongoing & Future Work

While the work here has certainly moved beyond 'proof of concept' as far as the architecture and methodological approach described are concerned, it is still at a relatively early stage. The potential scope for experiments is of course large, so future work described here is limited to my own particular research interests - focusing specifically on further systematic development and exploration of the architecture; subsumption in economic preference expression & uncertainty mitigation structures; and practical decision support applications. Other work in collaboration with other researchers and practitioners is likely to be a fruitful additional source of material.

With this in mind, several distinct directions can be identified. Some of this work is already in progress or planning, while some, such as decision support applications, is further out and will follow logically as the other research outputs are generated and evaluated.

### ARCHITECTURE DEVELOPMENT & EXPLORATION.

*Performance Measures & Structural Decision Processes.* The subsumption hierarchy experiments, with agent-subsuming sAgs in the case study in Chapter 6 and subsequent preliminary experiments with compound preference modifier structures, highlight the problem of differentiating between contributions of different subsumed components to overall performance when they share common metrics. In discussing these findings it was suggested that an asymmetric risk-adjusted performance measure might allow better differentiation between effective preference modifier activity and core preference behaviours. One such measure has now been implemented in the architecture and preliminary experiments using it are under way.

An alternate approach, monitoring segregated core preference and preference modifier performance across heterogeneous populations offers potential. Structurally this is appealing since it reflects observed behaviours in real economic entities. Both approaches will be explored in the planned future work.

*Robustness & Population Design.* The sAgt experiments also showed robust agent performance, or at least little sensitivity, to changes in rule and subsumed agent population sizes. This may be specific to the preferences in the case study, however this aspect of the architecture is worth investigation since it impacts on overall model design and on practical applications. Systematic investigation with minimal core preferences, more sophisticated core, and compound preference structures is planned. As a basic structural component to any SHaaP model, it will naturally form a part of the analysis of each case study as it is developed.

### COMPOUND PREFERENCE STRUCTURES & SUBSUMPTION HIERARCHIES.

*Core Preferences & Compound Structures.* The experiments in the main case study were strictly limited to observing agents using preference modifying heuristics with minimal core preferences. This was a necessary step in a principled approach to investigating these types of heuristic. Future work will consider more sophisticated, more realistic core preferences as part of overall preference expression. This will also allow more heterogeneous subsumed agent populations within subsumption hierarchies.

*Subsumption Hierarchies & Agent Subsumption.* The agent-subsuming agents in the case study experiments employed what was essentially a copycat heuristic. This

is a valid structure comparable to many proprietary trading and social learning behaviours in situated environments. However an equally commonly observed behaviour is an operational portfolio approach, in which subsumed agents are treated as part of a portfolio, allocated capital and governed according to their success. Here overall economic preference expression for the subsuming agents is the aggregate output of their subsumed populations. It seems possible that as a form of structural choice heuristic this may to some extent address and obviate performance measure differentiation issues. Its population-based portfolio structure may itself constitute a form of uncertainty mitigation behaviour - retaining knowledge while (with appropriate preference modifiers) self-limiting economic exposure. The underlying programming elements for this type of behaviour have been built into the current SHaaP implementation and comparative studies of sAgt subsumption are planned with heterogeneous preferences.

#### DECISION SUPPORT APPLICATIONS.

These are discussed in the following section, however they definitively form part of the future work, albeit, as part of the systematic development approach described in the thesis, they follow logically from the work already presented and planned here.

**Scope & Range of Application** SHaaP architecture-based models may be extended beyond purely financial systems and markets (though these are sufficiently broad and deep as to satisfy most requirements): corporate governance, venture capital investment heuristics, and policy design for regulatory & legal systems all have potential areas of interest for this approach to modelling economic preference expression, especially where actors are heterogeneous and where uncertainty exposures are present. To date the research in the thesis and the formulation of the SHaaP architecture as an experimental framework have concentrated on financial markets as economic systems with specific case studies exploring preference modifying heuristics. In developing ABMs of economic preferences the tendency to gravitate towards trading behaviours and artificial financial markets is natural given the widespread presence of actual actively traded markets, availability of rich, comparative data, and their significance in global economies. However this is a potential distraction, and in any case a subset of more general work in identifying, exploring, and validating preference expression behaviours.

Although the architecture could in principle be useful in developing resilient trading systems that is not, and has never been, its main purpose.<sup>1</sup> The strength of SHaaP models studying these behaviours is their explanatory value, which in turn informs the design and development of preference structures with deeper layers of subsumption. These are important steps towards building operationally meaningful models of larger systems, which in turn have value when overlaid with preference modifying regulatory and system constraints. As a principled, bottom-up approach this offers regulators and policy makers potentially important decision support tools.

Haldane[52] as a Bank of England deputy governor has already identified a need to move beyond regulatory regimes using mainly risk-based analytical models and

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<sup>1</sup>There are quite obviously many fully situated forms of such systems in the form of 'black box', algorithmic hedge fund models which operate at some level in this space, though their resilience is another question.

rules to a more heuristic framework recognising uncertainty exposures, a proposal supported by Neth et al[99]. In reality such frameworks must be hybrids, since that is both ecologically rational and because some component exposures actually are suitable for risk-based models. The structural subsumption meta-heuristic in the SHaaP architecture handles both cases, being agnostic to the form of encapsulated and subsumed preferences, and preference modifying structures.

Crucially this also allows, at least potentially, economic preference expression for non-financial entities to be modelled. Corporate business rules, which otherwise would have to be abstracted for inclusion in traditional risk models, can be subsumed with minimal abstraction within larger preference structures. This allows operational exposures, regulatory and governance requirements to be mapped in appropriately granular detail more suitable to capturing uncertainty and exploring the trade-off between balance sheet efficiency and resilience. Similarly legal structures and constraints, important modifying layers in preference expression, are likely better represented explicitly as part of a subsumptive structure rather than by encapsulation, or attempted encapsulation, in all encompassing quantitative descriptions.

## Final Remarks

The main contributions of the work in this thesis were set out in Chapter 1 and presented in detail in the rest of the thesis, however it is worth revisiting the key components with some final remarks.

The work here began by noting frustration with existing methodologies. Although many models demonstrated academic rigour and strong theoretical groundings, they appeared to lack operational meaningfulness and explanatory value. Equally importantly, in many cases ABM researchers failed to perform or to report exploratory data analysis necessary to validate, or indeed to verify their findings and justify the inferences they presented.

The SHaaP architecture and the exploratory tools developed in the course of the research address these issues. This work began with the observation that simple heuristic rules have a place in economic preference expression, before moving on to the realisation that economic preferences could usefully be represented in a subsumptive structure. This is a major difference to traditional experimental approaches - simple rules and behaviours testable in situated environments subsumed into potentially sophisticated compound preferences.<sup>2</sup> The major downside to this approach is the combinatorial complexity of these models limiting tractability - however this is a problem common to many agent-based models and ignoring or resorting to abstraction to avoid it is not a satisfactory solution. By explicitly recognising subsumptive components which may themselves be tractable the problem is obviated to some extent - indeed the interactions and expression of subsumed layers in preferences as a whole is a major area of interest.

Taken together, although the specific work on preference modifiers and uncertainty mitigation is clearly important, the key contributions of the SHaaP architecture are

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<sup>2</sup>Once again it is again important to recognise Rodney Brooks contribution in this regard.

modelling economic preferences as subsumptive structures in a clear experimental framework, and the introduction of realistic, situated performance measures and exploratory tools which allow substantially greater methodological rigour in validating and verifying experimental results.

By complementing, augmenting, or (of course) potentially subsuming extant models and frameworks, the hope is that we can end up with more operationally meaningful models for both academics and practitioners, while the universe may seem less unsettlingly large.

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# Nomenclature

ABM	Agent-based Model
ACE	Agent-based Computational Economics
AFM	Artificial Financial Market
AI	Artificial Intelligence
ARCH	auto-regressive conditional heteroskedacity
CAS	Complex Adaptive Systems
CFTC	U.S. Commodity Futures Trading Commission
ema	exponential moving average
EMH	Efficient Markets Hypothesis
forex	foreign exchange
FTSE100	FTSE100 equity index
GBPUSD	British Pound vs. US Dollar foreign exchange rate
gearing	A leverage measure, reflecting the strength of an investment signal.
GFC	Global Financial Crisis
HFT	High Frequency Trading
iAgt	Inner Agent - Main class of economic agent in the SHaaP architecture
IDE	Integrated Development Environment
LTCM	Long-Term Capital Management
MC	Minimal Core
MPT	Modern Portfolio Theory
PRP	Population relative performance.
PSO	Particle Swarm Optimisation
REH	Rational Expectations Hypothesis
Repast	Recursive Porous Agent Simulation Toolkit

RepastJ Repast Java

RepastS Repast Symphony

sAgt Senior agent - agent-subsuming economic agent in the SHaaP architecture

SEC U.S. Securities and Exchange Commission

VaR Value at risk

vNM von Neumann-Morgenstern

ZI Zero Intelligence

ZIP Zero Intelligence Plus

# A. Appendix

## A.1. An Example of a Trading Model

From Dacorogna *et al* [34]. This sets out the component elements of the gearing calculator for a moving average based trading model, RTT, a single horizon, trend following model which initiates trades when a monitored indicator (in this case the difference between the single period closing price of the asset,  $x_t$ , and an exponential moving average, MA ) crosses a triggering threshold. Dacorogna states that as the model below is set out it is momentum based - it will initiate trades if according to its indicators the price movement in the time series is trending - but that in extreme movements it will generate anti-trend signals, i.e. it will recommend a sell following a very sharp rally or a buy following a sharp drop in prices.

At any time  $t$ , the gearing function of the model is

$$g_t = \text{sign}(I_x(t))f(|I_x(t)|c(I(t))) \quad (\text{A.1})$$

where

$$I_x(t) = x_t - MA(\tau = 20\text{days}, 4; x) \quad (\text{A.2})$$

MA is the moving average returned from an underlying asset return or price time series where  $MA(\tau, n) = \frac{1}{n} \sum_{k=1}^n EMA[\tau', k]$  with  $\tau' = \frac{2\tau}{n+1}$  and EMA an exponential moving average.

$$f(|I_x(t)|) = \begin{cases} 1 & \text{if } |I_x(t)| > b \\ 0.5 & \text{if } a < |I_x(t)| < b \\ 0 & \text{if } |I_x(t)| < a \end{cases} \quad (\text{A.3})$$

and

$$c_t(I) = \begin{cases} +1 & \text{if } |I_x(t)| < d \\ -1 & \text{if } |I_x(t)| > d \text{ and } g_{t-1} \cdot \text{sign}(I_x(t)) > 0 \text{ and } r_l > P \end{cases} \quad (\text{A.4})$$

where  $a < b < d$ ,  $r_l$  is the return of the last deal and  $P$  is the profit objective.  $f(|I_x(t)|)$  gives a signal strength at time  $t$ , while  $c_t(I)$  acts as the contrarian strategy.

Under this set of rules a contrarian strategy can only be initiated if the profit objective has been reached on the last trade. The other parameters  $a$  and  $b$  depend

on the risk position of the model, such that,

$$a(t) = \begin{cases} a & \text{if } g_{t-1} \neq 0 \\ 2a & \text{if } g_{t-1} = 0 \end{cases} \quad (\text{A.5})$$

and  $b = 2a$ . An additional set of rules are invoked if the gearing calculator is recommending a trade, i.e.  $g_t \neq 0$ , and volatility in the market has been 'low',

$$a(t) = \begin{cases} a & \text{if } |x_e - x_t| > v \\ 10a & \text{if } |x_e - x_t| < v \end{cases} \quad (\text{A.6})$$

where  $x_e$  is the log of the entry price on the last initiated trade (presumably the one before the current one) and  $v$  is a threshold parameter (Dacorogna suggests a typical value would be low,  $v$  less than 0.5%). The result is that for periods where there is not much price variation. This is very clearly a rudimentary rule of thumb at best, but may be a little more elegant than it appears at first sight as it links recent trading activity to a very crude volatility measure.

The performance measures  $X_{eff}$  and  $R_{eff}$  (see the following section) are used to optimise performance of parameters  $\tau$ ,  $a$ ,  $d$  and  $v$ .

The model has two auxiliary parameters: the stop loss,  $S$ , the return at which an open position is automatically closed whatever the market conditions and the profit objective,  $P$ , - Dacorogna suggests that a typical objective on a trade would be a gross return of 3%. As in all Dacorogna's models no positions are retained overnight one would assume that this profit objective is a simple gross number and also that there is some rule to close out all positions profitable or not before the end of each trading day. Dacorogna states that the last two parameters,  $S$  and  $P$ , are only set after the others have been found.

## A.2. Strategy-based, Risk-adjusted Performance Measures - $X_{eff}$ and $R_{eff}$ .

Both of these return measures are proposed in Dacorogna et al, Chapter11, Pgs 305-309. These are designed to model single model performance based on profitability, not forecast accuracy. They do not incorporate trading signals which have been rejected by the trade filters incorporated in Dacorogna's overall design infrastructure.

For a given model, total return  $R_T$  is given by

$$R_T = \sum_{j=1}^n r_j \quad (\text{A.7})$$

where  $n$  is the number of transactions during period  $T$ ,  $j$  is the  $j^{th}$  transaction and  $r_j$  is the return for the  $j^{th}$  transaction. The cumulative return,  $\tilde{R}_T$ , includes any unrealised profit or loss for open trades,  $r_c$ , and thus  $\tilde{R}_T = R_T + r_c$ . These total return measures are independent of trading frequency. The change in total return,  $X_{\Delta t} = \tilde{R}_t - \tilde{R}_{\Delta t}$ , for a given model between reference points is used in the performance measures below.

### A.2.1. $X_{eff}$ - A Symmetric Effective Returns Measure

The 'effective return',  $X_{eff}$  is the total return for a given period adjusted,  $X_{\Delta t}$ , adjusted to allow for constant risk aversion.  $X_{\Delta t}$  for a model is given by,

$$X_{eff} = \bar{X}_{\Delta t} - \frac{\alpha \sigma_{\Delta t}^2}{2}. \quad (\text{A.8})$$

The  $\frac{\alpha \sigma_{\Delta t}^2}{2}$  element acts as a risk premium component modifying the original return. This measure is arrived at by assuming that  $X_{\Delta t}$  follows a Gaussian random walk with mean,  $\bar{X}_{\Delta t}$ . The expected utility,  $u(X_{\Delta t})$ , for an observation is taken to be  $-exp(-\alpha X_{\Delta t})$ , a CARA function (A.3.1 below). Here  $\alpha$  is a risk aversion coefficient, such that the expected utility,  $\bar{u}$ , is

$$\bar{u} = u(X_{\Delta t}) exp\left(\frac{\alpha^2 \sigma^2}{2}\right) \quad (\text{A.9})$$

if  $\sigma^2$ , the variance of  $X_{\Delta t}$  is

$$\sigma^2 = \frac{n}{n-1} (\overline{X_{\Delta t}^2} - \bar{X}_{\Delta t}^2) \quad (\text{A.10})$$

Under this measure, different values of  $\Delta t$  cannot be directly compared and so must be converted to some standard (usually annual) period. A simple annualisation factor,  $A_{\Delta t}$  (the number of observation periods per year), allows this. Using Eqtn A.8 gives,

$$X_{eff,ann,\Delta t} = A_{\Delta t} X_{eff} = \bar{X} - A_{\Delta t} \frac{\alpha \sigma_{\Delta t}^2}{2}. \quad (\text{A.11})$$

Here  $\bar{X}$ , the annual return is now independent of the observation period, however the risk term still is related to  $\Delta t$ . To deal with this Dacorogna suggests taking a weighted sample of returns over different values of  $\Delta t$ , with the weightings 'chosen' to the 'relative importance of the time horizons' within the model.

Some points to note. In favour of this  $X_{eff}$  measure, is that unlike the Sharpe Ratio it is stable at small variances. However, decisions such as values for weightings of observation periods and for the coefficient of risk aversion are still at the hands of the modeller. Insofar as these are applied to real trading models (and as Dacorogna suggests) these parameters are useful targets for optimisation algorithms, while at the same time bearing in mind risks of data-snooping and over-fitting. Another point is that risk aversion is assumed constant throughout with losses treated equivalently to profits. The  $R_{eff}$  measure attempts to address this.

### A.2.2. $R_{eff}$ - An Asymmetric Effective Returns Measure

Similar to  $X_{eff}$ , however it attempts to treat losses differently from profits. In  $R_{eff}$  as presented the coefficient of risk aversion is taken to be higher for negative returns than positive returns, analogous to Tversky and Kahneman's loss aversion and prospect theory. The resultant risk measure is applied to returns over time,  $\Delta R_t$ , penalizing losses in unprofitable periods, (drawdowns) more than it rewards profits .

Following Keeney & Raiffa[66], for utility,  $u(\Delta\tilde{R})$ , the coefficient of risk aversion,  $\alpha$ , is given by,

$$\alpha = -\frac{\frac{d^2u}{d(\Delta\tilde{R})^2}}{\frac{du}{d\tilde{R}}}$$

where

$$\alpha = \begin{cases} \alpha_+ & \text{for } \Delta\tilde{R} \geq 0 \\ \alpha_- & \text{for } \Delta\tilde{R} < 0 \end{cases}$$

and  $\alpha_+ < \alpha_-$ . From this utility functions for  $\Delta\tilde{R}$  may be derived,

$$u = u(\Delta\tilde{R}) = \begin{cases} -\frac{e^{-\alpha_+\Delta\tilde{R}}}{\alpha_+} & \text{for } \Delta\tilde{R} \geq 0 \\ \frac{1}{\alpha_-} - \frac{1}{\alpha_+} - \frac{e^{-\alpha_-\Delta\tilde{R}}}{\alpha_-} & \text{for } \Delta\tilde{R} < 0 \end{cases} \quad (\text{A.12})$$

Inverting this formula gives return values *from* utilities.

$$\Delta\tilde{R} = \Delta\tilde{R}(u) = \begin{cases} -\frac{\log(-\alpha_+u)}{\alpha_+} & \text{for } u \geq -\frac{1}{\alpha_+} \\ -\frac{\log(1-\frac{\alpha_-}{\alpha_+}-\alpha_-u)}{\alpha_-} & \text{for } u < -\frac{1}{\alpha_+} \end{cases} \quad (\text{A.13})$$

The problem with this is that these functions are dominated by losses in the tail of the distribution and given observed leptokurtosis in financial return data, an assumption of a Gaussian distribution may not be acceptable in a trading model. Dacorogna suggests taking explicit utilities from sample observations across specific

trading intervals as a starting point rather than rely on an assumed distribution as in  $X_{eff}$ .

The suggested approach is to take multiple observations of length,  $\Delta t$ , for a particular (relatively) large sample period,  $T$ , and aggregating within those observations weighted by how much of the sample period they overlap or are in the sample period. Taking many observations which overlap each other increases the sensitivity of the sample to large drawdowns.<sup>1</sup> Thus, utility for the  $j^{th}$  observation in a sample with observation interval  $\Delta t$  is

$$u_{\Delta t,j} = u(\tilde{R}_{t,j} - \tilde{R}_{t,j-\Delta t}) \quad (\text{A.14})$$

and

$$\text{mean utility, } u_{\Delta t} = \frac{\sum_{j=1}^{N_j} v_j u_{\Delta t,j}}{\sum_{j=1}^{N_j} v_j}, \quad (\text{A.15})$$

where  $N_j$  is the number of observations of size  $\Delta t$  and  $v_j$  is a weighting factor (equal to one for observation entirely within the sample period  $T$  and a the fraction of an observation time in terms of  $\Delta t$  that lies within the sample period for observations which only partially lie in the period, i.e. at the ends).

Applying this mean utility in Eqtn A.13 gives a value for the typical, risk adjusted, effective return measure,  $\Delta \tilde{R}_{eff,\Delta t} = \Delta \tilde{R}(u_{\Delta t})$ , for a horizon  $\Delta t$ . As with  $X_{eff}$  this must be annualized and the same approach is applied as in Section A.2.1, which together with weighting for different horizons (values of  $\Delta t$ ) gives

$$R_{eff} = \frac{\sum_{i=1}^n w_i \tilde{R}_{eff,ann,\Delta t_i}}{\sum_{i=1}^n w_i}. \quad (\text{A.16})$$

As before, weights are chosen to fit the model, as are values for  $\alpha_+$  and  $\alpha_-$ .

The approach has merit in that its embedded structure acts as a filter for models which are subject to high volatility of return and drawdowns. A drawback is the need for explicit utilities in constructing the performance measure, however from a practitioner's point of view the benefit of explicitly trapping for large drawdowns is an intuitive benefit.

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This idea of a drawdown is particularly important for fund management and trading in that many banks and funds have specific organisational rules which allow investors (or the bank) to withdraw funds from (or fire) traders when profits fall by a specific trigger amount from current or all time highs. This is quite different from looking at performance numbers which summarize average returns over extended periods. The market and therefore traders and fund managers are very concerned with ways of capturing large single losses and avoiding them.

The Calmar Ratio also attempts to capture drawdowns in a summary performance measure, using the maximum drawdown in a given period vs. returns in that period.

## A.3. Risk Aversion & Utility Functions

### A.3.1. CARA (constant absolute risk aversion) Functions.

Very commonly used. An important subset of utility functions because their ranking is complete. They have the form,

$$u(x) = -ae^{-\alpha x} + b \quad (\text{A.17})$$

where the coefficient of absolute risk aversion,  $\alpha_A$  (also known as the Arrow-Pratt coefficient of absolute risk aversion), has the form,

$$\alpha_A = \alpha = -\frac{u''(x)}{u'(x)} \quad (\text{A.18})$$

Here it is only the absolute risk that is important, independent of any *changes* in wealth.

### A.3.2. CRRA (constant relative risk aversion) Functions.

Utility functions where risk aversion is dependent the risk in proportion to wealth. The coefficient of relative risk aversion,  $\alpha_R$ , (also known as the Arrow-Pratt coefficient of relative risk aversion), is of the form,

$$\alpha_R = -x \frac{u''(x)}{u'(x)}. \quad (\text{A.19})$$

Common CRRA functions are

$$u(x) = \frac{x^{1-\alpha} - 1}{1-\alpha}, \quad 0 < \alpha < 1$$

$$u(x) = \ln(x), \quad \alpha = 1$$

where  $\alpha_R = \alpha$ .

## A.4. Repast Modelling Notes for the SHaaP Architecture

This appendix contains some basic modelling notes for the SHaaP architecture within Repast Symphony. It also includes some practical notes specific to the SHaaP implementation used in the case study experiments in Chapter 6. Full archive copies of the SHaaP source code used in the thesis, sHaaP\_v1, and data files from the experimental runs have of course been created for storage.

### A.4.1. Versions & Download

Repast Symphony for Java is available in downloadable packages for Windows, Linux and Mac OS X from the Repast project website, <http://repast.sourceforge.net/> - other versions for Python and High Performance Computing can also be downloaded there. The version used in this thesis was Repast Symphony 2.2 64-bit released in June 2014, an incremental upgrade from versions 2.1 and 2.0 on which the original models were developed. Testing is currently under way to ensure there are no issue with the upgrade, but there is no evidence of any problems at this stage.

The Windows package comes bundled with Eclipse IDE and Java 7.

### A.4.2. Repast Model Setup

The Repast project site contains extensive documentation covering the various software packages and setup guides. It also provides worked examples which walk users through the entire process for implementing basic Repast models - a good place to begin being 'Repast Java Getting Started'<sup>2</sup>. Additional model examples provided for the Repast Symphony curated by Nick Collier, Rick Riolo, and Michael North, core Repast developers, can be found at <https://code.google.com/p/repast-demos/>: these proved invaluable as supplementary material to the main documentation, helping to illustrate practical implementations of Repast concepts.

As part of the general model setup, which is also common to SHaaP model implementation, some basic housekeeping steps have to be followed when a new model is initiated - these are well covered in the site documentation, however some specific items are noted below.

#### A.4.2.1. Repast Symphony Projects in Eclipse

Repast projects are set up in a fairly typical manner to other Java projects using the Eclipse IDE bundled with the Repast platform. From the File menu select New, then Other, and then Repast Symphony Project, which then runs through a setup wizard for the project and creates all the necessary working project folders.

If running the project without using the extra graphical interfaces available in RepastS, then two files need to be deleted after a new project has been set up: in the

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<sup>2</sup><http://repast.sourceforge.net/docs/RepastJavaGettingStarted.pdf>

main source folder these are 'ModelInitializer.groovy' and 'ModelInitializer.agent'. In the SHaaP implementations here these files are deleted.

### A.4.2.2. sHaaP.rs Folder

This folder contains XML files automatically generated by Symphony with runtime information for models. Some manual updates need to be carried out to the context.XML and scenario.xml files when setting up a new model or importing a project, if for instance one wanted to set up and modify or extend a specific SHaaP instance.

*Context.xml* The context ID needs to be set to match the top level package name, in this case *sHaaP\_v1* and class to *sHaaP\_v1.ShBuilder*.

*Parameters.xml* This file contains parameters for user input within simulation runs. Default settings for these parameters can be directly overwritten here.

*User\_path.xml* Contains the model name, which again needs to be set to the top level name of the model package, in this case *sHaaP\_v1*.

### A.4.2.3. File Output & Capturing Data

Data from simulations is captured and stored in a two stage process, both parts of which are carried out from the simulation GUI.

Firstly, a *Data Set* is created. This can either record aggregate or non-aggregate data for particular object types. Aggregate data operates over all instances of the particular object providing summary statistical information for that class. Non-aggregate data records data fields as specified for each instance of the class according to the schedule specified setting up the data set and uses the public methods of that class.<sup>3</sup>

Secondly a *File Sink* is created and linked to a specific Data Set. A file is created during each simulation and output to disk as comma separated values for later analysis. In the experiments in Chapter 6 for example a data set was created for each agent type, and for each agent instance of that type in the population values for required data fields were saved.

In creating these Data Sets and File Sinks it is worth noting that it is important to save the project workspace before running the next simulation, otherwise these structures will not be retained.

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<sup>3</sup>As a note, for a particular child class such as the T1\_iAgt class this will mean all public methods returning values can be accessed including public methods of the abstract super class Agent. ArrayLists cannot be returned, which makes things more difficult when one is interested in intra-agent objects such as rulepacket learning for compound preference modifiers. Although this no doubt could be engineered, the question arises of when this level of detail is useful - it would certainly slow simulations dramatically.

## A.5. SHaaPa - Classes, Methods & Interfaces

This appendix sets out the overall package and class structure used in the SHaaP architecture implementation used in the experiments in the thesis, together with various model control and utility structures.

The architecture is implemented as a Repast Symphony project using the Eclipse IDE bundled with Repast. The overall Java project sHaaP\_v1 is split into six Java packages segregating functional classes as shown in Table A.1. These form the basis of the SHaaP architecture implementation detailed in Chapter 6 following the overall design set out in Chapter 4.

New experimental designs will necessarily require modifications in the form of concrete implementations of the abstract classes and interfaces described here specific to each experiment, however these core principles of the SHaaP architecture should be accessible with reasonable ease and robustness.

Package	Description
sHaaP_v1	Main project package - contains only ShBuilder, the top level class for simulation.
sHaaP_v1.agents	Agent classes & collections.
sHaaP_v1.assets	Asset, asset state, specification & time series classes.
sHaaP_v1.common	Common parameter classes, global variables & utility classes.
sHaaP_v1.prefs	Preference, preference modifier classes & collections.
sHaaP_v1.strat	Rule, rulepacket & trade classes.

**Table A.1.:** sHaaP\_v1 Java Package Structure

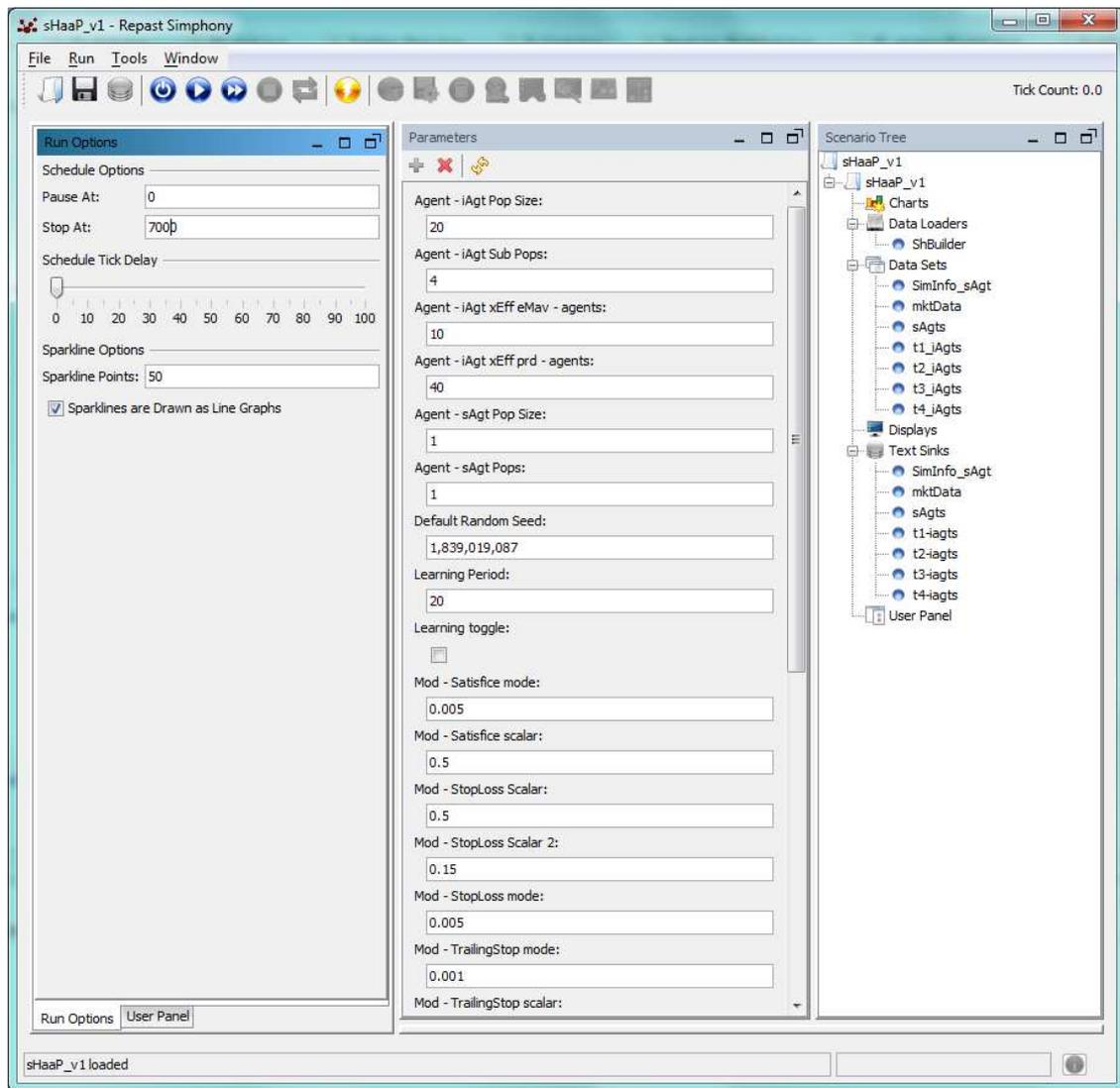
### A.5.1. Classes

#### A.5.1.1. ShBuilder

This is the main class called by RepastS to initiate and run Symphony models. It contains the methods and objects necessary to set up the GUI, agent contexts and top level scheduling. In SHaaP the top level of agents are the sAgtS and these are created in this class and added to the main context object. At the start of a simulation run the GUI is created and parameter field values created in the GUI to match the method calls in ShBuilder can be adjusted to modify the simulation parameters. Figure A.1 shows a screen capture of the GUI.

After all parameters are established

A schedule object 'schedule' is created within ShBuilder and forms operational backbone of each simulation. Scheduling within Repast Symphony in the SHaaP architecture is performed using the annotated method 'mainScheduleSteps()'. This method



**Figure A.1.:** Symphony GUI screen capture

updates the asset information and calls all the agents within a simulation which perform the behaviours necessary to express preferences and any other operational actions.

#### **A.5.1.2. Inner Agents & Senior Agents**

Table A.2 shows the agent and agent related classes in the SHaaP architecture. Tables A.3 & A.3a show the main methods for the Agent class and for the T1\_iAgt class as a representative example of a concrete extension of Agent. Get & set methods are not shown in detail since there are too many to be useful, similarly methods which are essentially just function calls on repetitive code are not listed but are available in the source code archive. The inner classes used to construct portfolio preference modifiers are shown in Table A.3b.

Class		Description
Senior_Agent	Abstract class	Subsuming agent class
S1_sAgt	Extends Senior_Agent	Child class of Senior Agent class
Agent	Abstract class	Main agent class for inner agents
T1_iAgt	Extends Agent	Child class of Agent class
T2_iAgt	Extends Agent	Child class of Agent class
T3_iAgt	Extends Agent	Child class of Agent class
T4_iAgt	Extends Agent	Child class of Agent class
AgentPopulation	Abstract class	Collection class for Agent type
InnerAgt_Pop	Extends AgentPopulation	Child class of AgentPopulation

**Table A.2.:** Agent Classes

Agent: methods		Description
initialisePortfolio()	abstract	Set up agent portfolio.
generatePrefsAndPkts()	abstract	Generate preferences & rulepackets
bopActions()	concrete	Scheduling: beginning of period actions. Updating rules, calculating performance & learning.
eopActions()	concrete	Scheduling: end of period actions. Update portfolio.
updateState()	concrete	Call bopActions() & eopActions() methods.
updatePortfolio()	concrete	Update agent portfolio. Update existing open trades: check preferences and modifying behaviours. Check to start new trades.
rulePktLearning()	concrete	Rulepacket population-based learning - PSO.
calc_pktPopMod_stats()	concrete	Calculate summary statistics for pref. modifiers in the rule packet population.
init_Mods_PsoParams()	concrete	Initialise modifier parameters for PSO.
updateBestPktPrefs()	concrete	Update best rulepacket preference objects.
calc_iAgt_NetPosn()	concrete	Calculate iAgt net portfolio position.
calcNetPflioVal(0	concrete	Calculate iAgt net portfolio value.
calcFriction()	concrete	Calculate trade friction value.
setAgt_mAvs()	concrete	Set moving average periods.
Getters & Setters	concrete	General get & set methods.

**Table A.3.:** Agent Class Methods

Concrete class - T1\_iAgt. Extends the Agent class creating inner agents (iAgt). As a base level concrete classes of Agent represent the basic, minimum for economic operational functionality. The abstract Agent class provides the core functionality and subsumes core preferences, behaviours, expression and learning. The child class, of which T1\_iAgt is an example instantiates Agent with the addition of inner classes of structural preference modifying behaviours such as budgeting and basic position limit constraints. In this way a standard template is formed for adding and calling these features. The child class initialises all the core preferences and heuristic preference modifying behaviours which are held in ArrayLists

Senior Agents (sAgt) are a subsuming agent class. As shown in the schematic diagrams of Chapter 4, Fig 4.4, sAgt subsume populations of iAgt integrating their behaviours with supervisory & regulatory routines as well as their own preferences

(a) T1\_iAgt Methods

T1_iAgt : methods		Description
init_AgentParams()	concrete	Initialise agent parameters
generatePrefsandPkts()	concrete	Creates preferences and rule packets
initializePortfolio()	concrete	Create template rules for portfolio array
init_LearningParams()	concrete	Initialise learning parameters
Getters & setters()	concrete	General get and set methods.

(b) T1\_iAgt Inner Classes

T1_iAgt: inner classes		Description
Budgeter	Implements Budget_PrefMod	Portfolio preference modifier - budgeting
EntryHurdle_Mod	Implements Pflio_PrefMod	Portfolio preference modifier - trade entry
MinMax	Implements Pflio_PrefMod	Portfolio preference modifier - position limits

**Table A.4.:** T1\_iAgt Methods & Inner Classes

and preference modifiers. Table A.5 shows the main methods of the abstract class Senior Agent, while Table A.6 shows the main methods and inner classes of S1\_sAgt, a concrete child class of Senior Agent. It should be noted that at this time no learning methods are implemented for Senior Agents although these may be added as the architecture is developed and suitable protocols designed.

## A.5.2. Interfaces

Java interfaces are used to add behaviours and types to preference classes, both core preferences and preference modifiers, allowing polymorphism and multiple modifiers to be passed to agent classes. The interfaces used in the current SHaaP implementation are listed in Table A.7.

Interfaces are necessarily abstract, so when added to a preference class all methods must be implemented forcing a relatively standard structure on modifier design which helps to standardise economic agent and SHaaP model design. The main functional methods for these interfaces are shown in Tables A.9 to A.12.

Senior Agent: methods		Description
create_AgtPops()	abstract	Create iAgt populations & sub-populations
EntryHurdle_Mod	abstract	Abstract modifier inner class
Maximum	abstract	Abstract modifier inner class
BOPactions()	concrete	Beginning of period actions
EOPactions()	concrete	End of period actions
calc_sAgtNetPosn()	concrete	Calculate net portfolio position
calcFriction()	concrete	Calculate friction for trading
chooseActive_iAgt()	concrete	Pick best performing iAgt to use preferences in portfolio trading
initializeSuperAgtPflio()	concrete	Initialise sAgt portfolio
initialise_Top_iAgtArray()	concrete	Initialise ArrayList for best performing iAgts
meanAbsPosn()	concrete	Calculate mean absolute exposure
set_sPflioRulePrefs(0	concrete	Get preferences & modifiers from top iAgt rules & set for trading
sortAgtsBy_ID()	concrete	Sorts ArrayLists of iAgts by agent ID
sortAgtsBy_xrEff()	concrete	Sorts ArrayLists of iAgts by performance score
update_sAgtPflio()	concrete	Update sAgt portfolio
update_iAgtPopulations()	concrete	Update iAgt populations
Getters & Setters	concrete	General get & set methods

**Table A.5.:** Senior Agent(sAgt) Main Methods & Inner Classes

Interface	Description
Budget_PrefMod	Mixed modifier type: partly structural, partly risk modifying.
CorePreferences	Only core preference interface, requiring all interface methods to be implemented
PrefModifier	Main preference modifier interface, used in StopLoss_RiskMod
Pflio_PrefMod	Simple portfolio preference modifier interface.
Structural_PrefMod	Structural modifier interface - used for market imposed constraints such as trade latency.

**Table A.7.:** Interface List

It can be seen that these interfaces share common basic functional methods. The variation in get & set methods comes from experience in reuse of particular interfaces over time, ensuring these methods are included at the time of creation and, if

(a) S1\_sAgt Main Methods

S1_sAgt: methods		Description
create_AgtPops	concrete	Create iAgt populations & sub-populations
Getters & setters	concrete	General get & set methods

(b) S1\_sAgt Inner Classes

S1_sAgt: inner classes		Description
sAgtBudgeter	Implements Budget_PrefMod	Portfolio preference modifier - budgeting
EntryHurdle_Mod	Implements Pflio_PrefMod	Portfolio preference modifier - trade entry
MinMax	Implements Pflio_PrefMod	Portfolio preference modifier - position limits

**Table A.6.:** S1\_sAgt Main Methods

automatically generated, speeding up development with fewer errors.

Budget_PrefMod: methods	Description
check_PrefMod_Behaviour()	Check for modifier action
setPrefMod()	Set modifier parameters
updatePrefModCriteria()	Update modifier criteria

**Table A.8.:** Budget\_PrefMod Interface Methods

CorePreferences: methods	Description
initialisePrefs()	Initialise preferences
updateMktState()	Update asset state
tradeEntry()	Trade entry preferences
getTrdChoice()	Get trade choice
tradeExit()	Trade exit preferences'
getExitAmt()	Get exit amount
tradeExitFlag()	Trade exit indicator
getFullExitFlag()	Full exit indicator
partialExit()	Partial exit behaviour
getExitFrac()	Get exit fraction
Getters & Setters()	General get & set methods

**Table A.9.:** CorePreferences Interface Methods

Pflio_PrefMod: methods	Description
initPrefMod()	Initialise preference modifier.
check_PrefMod_Behaviour()	Boolean: check if preference modifying behaviour is activated.
updatePrefModCriteria()	Update conditions for modifier activation.

**Table A.10.:** Pflio\_PrefMod Interface Methods

PrefModifier: methods	Description
initRiskMod()	Initialise modifier
updateRiskModCriteria()	Update modifier criteria
check_RiskMod_Behaviour	Check modifier criteria for activation
Getters & Setters	General get & set methods for modifier type, learning parameters etc.

**Table A.11.:** PrefModifier Interface Methods

Structural_PrefMod: methods	Description
checkTrdLatency()	Check trade latency.
get_latencyOn_Togl()	Check latency activation.
getTrdLatency()	Get latency period.
setTrdLatency()	Set latency period.
set_LatencyTogl()	Set latency activation status.

**Table A.12.:** Structural\_PrefMod Interface Methods

### A.5.3. Preferences - Interfaces & Classes

Class name		Description
Core	Abstract class	Abstract class. No methods, passes Core type.
CorePak	Concrete Class Class	Core preference collection class. Holds an array of core preference objects.
CoreParams	Abstract Class	Core parameter class.
ModParams	Abstract Class	Preference modifier parameter collection class.
SL_Params	Extends ModParams	Stop-loss parameter class.
Str1_StructuralMod	Extends StructuralMod	Structural latency modifier
Trailing_Stop	Implements PrefModifier	Trailing stop-loss risk modifier
Trailing_Stop2	Implements PrefModifier	Trailing stop-loss risk modifier - no entry threshold trigger
ZI_Core	Extends Core implements CorePreferences	ZI core preferences - fixed trade maximum trade length
ZI_Core2	Extends Core implements CorePreferences	ZI core preferences - Poisson distribution determined maximum trade length
ZI_CoreParams	Extends CoreParams	ZI preferences parameter collection class

Table A.13.: Main Preference Classes

### A.5.4. Class Schematics

#### A.5.4.1. RulePacket

RulePacket is the abstract class for rulepackets. Concrete instances, such as P1\_strategyPacket, contain a number of instances of the same rule, sharing core and risk modifying preferences. Rulepackets allow rule performance to be evaluated, learning to occur, and preferences to be exported to iAgts and sAgts for use in trading.

#### A.5.4.2. RuleStrat

RuleStrat is the abstract class for rules. Concrete instances, such as R1\_Strategy, populate RulePacket instances. Each instance has basic behaviours as well as core and risk modifiers common to its rulepacket.

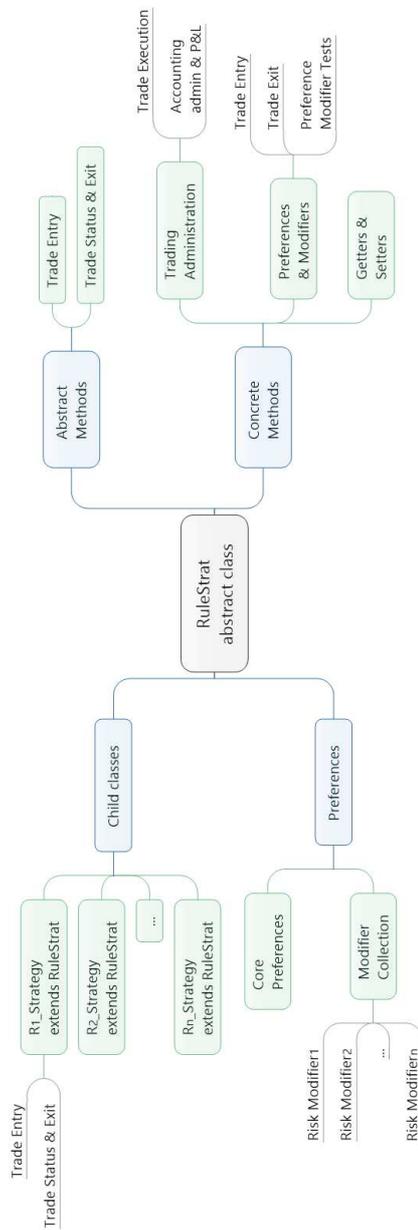


Figure A.2.: RulePacket Class

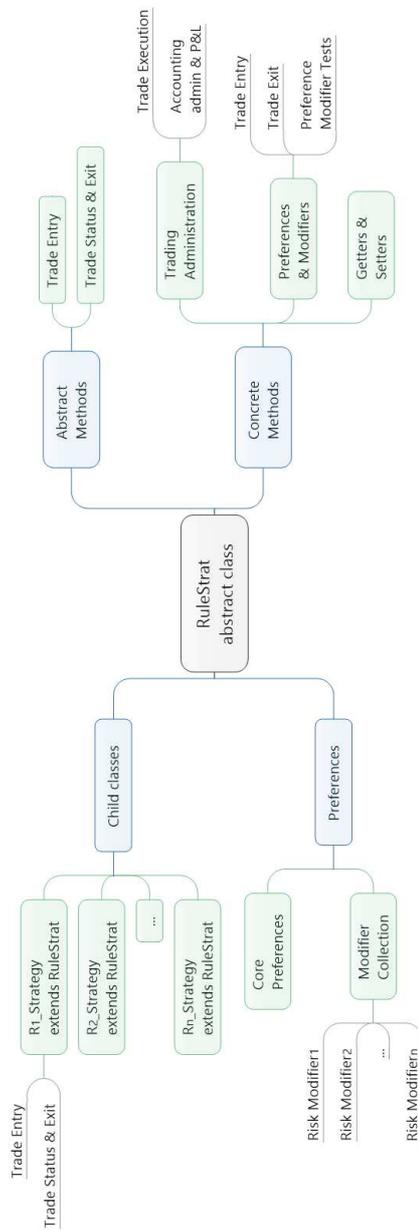


Figure A.3.: RuleStrat Class

### A.5.5. Common & Utility Classes

These classes are found in the sHaaP\_v1.common package.

- *ShAgtParams* & *ShPrefParams*. The main parameter classes for the architecture - split into two interconnected functional classes to make them manageable. Where not overwritten in the GUI or by batch files at runtime these provide the default starting parameters for agents in the simulations.

*ShAgtParams*. Holds the main non-preference parameters for agents. *ShPrefParams* holds preference related parameters for agents.

The main purpose of these two classes is to keep these parameters in a central location where any changes to default parameters are easily located and identified. From experience a major source of errors, both here and in other researchers' models comes from hard-wired changes made locally within program structures.

- *ShWorld*. Sets up the economic environment, creating asset and timeseries classes for access during simulations.
- *ShGlobals*. Sets up Symphony environment variables.
- *ShUtil*. A collection of utility functions used throughout the SHaaP models. These include basic statistical & accounting functions. All the risk-adjusted performance measures & tools developed in the research are also found here allowing easy access and (again) reducing potential coding errors if these were to be included directly as method calls within models.
- *Distrib*s. Holds pseudo-random number generators and distributions. See below - for full details.

Name	Type	Default seed	Secondary seeds	Use
riskMod	normal	933,145	266,641	Risk modifiers
riskMod2	normal	784,221	66,737	Risk Modifiers
prefEntry	uniform	26,546	973,501	Core preference - trade entry
prefExit	Poisson	401,328	489	Core preference - trade exit
randSeq	uniform	670,513	665,779	General use
randSeq2	uniform	803,271	865,288	General use
randSeq3	uniform	117,420	611,891	General use

**Table A.14.:** Distributions

**Distribution Class** The stand-alone distribution class, *Distrib*s, holds the various pseudo-random number generators and distributions used by SHaaPa. This class uses the cern.jet library bundled with RepastS. Table A.14 shows the pre-set distributions used in the current work though of course more can be added. Seed values are presented either as batch file parameters and/or via the GUI (see Section for a screen shot of the input layout).

It will be noted that there are several independent instances of some types of distribution, particularly in this case uniform. This is to segregate these sequences

with a view to replicability where different algorithms may use the same type of distribution and may be desirable to use a fixed process for a particular element allowing the rest to vary.

### A.5.6. Switches & Toggles

A set of logical switches and toggles are used to activate and deactivate various model components, such as turning on or off rulepacket PSO learning, the type of performance metric used or specific modifier elements. These switches are listed with their default settings in Table A.15.

Switch/Toggle	Default	Description
xEffrEFff_togl	True	Switches between $X_{eff}$ & $R_{eff}$ performance measures. True = $X_{eff}$ .
xEff_emav_togl	True	Exponential moving average vs rolling arithmetic average. True = exponential.
pctMarginFlag	False	Margin calculated as a percentage of exposure vs. fixed charges. True = percentage. Currently unused.
agt_TrdLatency_Togl	True	Agent Trade latency activation. True = activated.
iAgt_RiskModTogl	True	Activates risk modifiers. True = activated.
sAgtRiskMod_Togl	True	Activates sAgt risk modifiers. True = activated.
iAgt_EntryFlag	False	iAgt trade Entry Hurdle modifier. True = activated.
sAgt_EntryFlag	False	sAgt trade Entry Hurdle modifier. True = activated.
iAgt_MinMaxFlag	False	iAgt MinMax portfolio modifier activation. True = activated.
sAgt_MinMaxFlag	False	sAgt MinMax portfolio modifier activation. True = activate.
iAgt_LearningTogl	False	Activates iAgt level learning. True = activated. Currently unused.
sAgt_LearningTogl	False	Activates iAgt level learning. True = activated. Currently unused.
slLogN_togl	True	Choose Stop Loss threshold return from LogN distribution or uniform. True = LogN.
psoTogl	True	RulePacket PSO activation. True = activated. Overwritten at runtime.
pso_velNoise_Togl	True	Velocity noise switch. Introduces noise into PSO. True = activated.
pso_decay	True	PSO decay switch. Decays pBest & gBest values. True = activated.
miniPopMode	False	PSO minipopulation switch. Currently unused.

**Table A.15.:** Toggles & Switches

### A.5.7. Contexts & Projections

*Contexts* are a key feature of Repast simulations. They provide the basic data structure in which all agents and model components and data objects are held both in terms of modelling and the Java environment. The central class to the SHaaPa architecture, *ShBuilder*, begins by initialising the SHaaP context, *mainContext*.

Contexts can be nested hierarchically with objects occurring in more than one context. Although this structure is not currently used in SHaaPa it may be useful in future versions and extensions as a logical organisational structure to improve robustness and agent manipulation.

RepastS also allows *projections* - functionally these are named sets of data structures allowing relationships between context members to be defined. This allows for instance graph and network relationships to be specified for a context and agents within that context. Contexts can also include data layers accessible by agents within the context according to their location. These features offer powerful configuration options and flexibility in delineating agent relationships.

For the current SHaaP architecture implementation the structure is kept deliberately simple. Only the main single level context is used. Later versions will consider the networking functionality.

## A.6. SHaaPa - Default Parameter Settings & Practical Notes

This appendix contains the default SHaaP architecture settings used in the case study experiments in Chapter 6, as well as some practical notes specific to this study.

A large number of practical decisions are required for any simulation, situated or unsituated. As discussed in the main body of the thesis, while these decisions may not seem material it is important to document these decisions since they may have unintended effects within experimental designs. These decisions are generally documented within the body of the SHaaP implementation code, building on the experience of attempting to understand the original Santa Fe Artificial Stock Market models which suffered from a lack of both suitable structure and documentation in archived copies of their code.

The overall code structure and object oriented design mean that most decisions in the SHaaP implementation are relatively easy to follow, and also made model extension fairly simple and robust. Throughout the implementation attempts to standardise method calls and class design to promote extensibility and transparency.

The main programming issue throughout the development of the Java implementation was dealing with 'copy by reference' errors: inner agents and senior agents behaviours involve choosing preferences from sub-populations - this means the preference behaviours need to be copied as new instances not simply references to sub-population instances otherwise performance monitoring is compromised and any learning or adaptation mechanisms would also be vulnerable. The somewhat complicated ArrayList method designs within these agents address this issue without impacting materially on overall simulation speed.

### A.6.1. Agent Population Structure

Table A.16 shows the default population settings for SHaaP models: sAgt; iAgt; and rules.

Population Structure	Value	Variable Name
sAgt population numbers	1	sAgt_Pops
sAgt per population	1	sAgt_perPop
iAgt sub-populations per sAgt	4	iAgt_subPops
iAgt sub-population size	10	iAgt_subPopSize
RulePackets per iAgt	40	pktsPerAgt
Rules per rulepacket	20	rulesPerPkt

**Table A.16.:** Agent & Rule Population Structure

### A.6.2. Default Economic Preference Variable Settings

Table A.17 shows the default portfolio settings for iAgt & sAgt. These provide the economic constraints on investment decisions by agents. Some elements such

as trade length are overwritten at run time, depending on toggle & switch settings in ShAgtParams and ShAgtPrefParams. Table A.18 shows risk aversion, preference and performance measure settings.

	Value	iAgt	sAgt
Start cash	\$100,000	iAgt_StartCash	sAgt_StartCash
Basic trade size	\$3,000	iAgt_tradeSize	sAgt_tradeSize
Portfolio trade limit	20	iAgt_pflioTrades	sAgt_pflioTrades
Borrowing limit	\$0	iAgt_BorrowingLimit	sAgt_BorrowingLimit
Maximum long position	10	iAgt_maxPosn	sAgt_maxPosn
Maximum short position	-10	iAgt_minPosn	sAgt_minPosn
Trade latency	2	iAgt_trdLatency	sAgt_trdLatency
Bid-Offer Spread	0	bidOfferPct	bidOfferPct
Brokerage	0	agt_brokerage	brokerageAmt
Max trade length	50	max_TradeLength	max_TradeLength
Trading start period	500/600	iAgt_Trading Start	sAgt_TradingStart

**Table A.17.:** Investment Portfolio Default Settings

		iAgt	sAgt
Symmetric risk aversion	0.50	iAgt_riskAv	sAgt_riskAv
Profit risk aversion	0.05	iAgt_riskAv_pos	sAgt_riskAv_pos
Loss risk aversion	0.40	iAgt_riskav_neg	sAgt_riskAv_neg
Entry hurdle	0.00	iAgt_entryHurdle	sAgt_entryHurdle
Exit hurdle	0.20	agt_exitHurdle	agt_exitHurdle

Business days per annum	252	ann_trdDays
Annualisation factor	15.87	annFac_days
Return period	1	agt_returnPeriod
Return eMav period	10	agtRtn_emavPeriod
$X_{eff}$ & $R_{eff}$ window	40	agt_xrEffWindow
$X_{eff}$ & $R_{eff}$ eMav period	10	agt_xrEff_eMavPeriod

**Table A.18.:** Performance Metric & Risk Preference Settings

### A.6.3. Preference Modifier Settings

These tables show the settings for Stop Loss, Trailing Stop & Satisficing preference modifiers.

Variable	Values	Variable Name
P(Exit)	0.07	pExit
P(Buy vs Sell)	0.50	buySell_bias
P(do nothing)	0.60	doNthg_frac

**Table A.19.:** ZI Core Default Settings

Modifier Type	Variable	Value	Variable name
Stop Loss	mode	0.005	stopLossMode
	scalar	0.500	stopLossScalar
Trailing-stop	mode	0.002	trailStopMode
	scalar	0.500	trailStopScalar
Satisficer	mode	0.0025	satisficeMode
	scalar	0.500	satisficeScalar

**Table A.20.:** Preference Modifier Default Settings

Variable	Value	Variable Name
PSO frequency	1/20	pso_freq
PSO start period	600	psoStart
Cognitive factor, $c_{cog}$	1.50	pso_cogFacP
Social factor, $c_{soc}$	1.50	pso_socFacG
Momentum weight, $\omega$ , maximum	0.95	pso_omMax
Momentum weight, $\omega$ , minimum	0.05	pso_omMin
Decay rate, $\alpha_d$	20	pso_decayRate
Velocity noise factor, $r_{noise}$	0.40	pso_velNoise
Velocity noise hurdle	0.0001	pso_velHurdle

**Table A.21.:** PSO Default Parameters

## **A.6.4. Other Practical Notes**

### **Scheduling**

Sample Start Periods. The overall scheduling protocol is set out in Fig. 4.2, however in addition in the case study experiments a 500 period initial stub is allowed before iAgt trading commences in each sample run. This is to allow sufficient time for technical market measures to be built up such as long term moving averages and oscillators, it is not for training of the agents. In fact, in the experiments presented in the thesis no technical indicators are used in the minimal core preferences, however this stub was left in to cater for later sample runs where more sophisticated preferences may be used.

Senior Agent Trading. This is arbitrarily set to 100 periods after iAgt begin trading to allow some trading performance history to be built up and reasonable selection amongst iAgt populations to be made.

### **Trading & Transactions**

Many academic models introduce terms for market features such as friction and structural latency in some attempt at realism. These are allowed for in the case study models, looking forward to future simulations, but are set to the values in zero in the case study experiments (see Table A.17).

### **Rulepacket & Preference Selection**

Currently this is set to be strictly elitist - in each period where an agent, or agent-subsuming agent, can select preferences it simply picks the best performing candidate available. In the SHaaP implementation this has been set up to allow a probabilistic choice amongst groups of candidate solutions, however this is not enabled in the case study experiments, so that their initial scope is deliberately restricted.

## A.7. SFASM Reconstruction Notes

### A.7.1. Reconstruction pathway

This required several attempts and many revisions to the model since my route was to write the model from scratch.

**Stage(1)** Main source materials for the reconstruction were the originators' journal papers [85, 3], subsequent review papers by LeBaron [81] and Tesfatsion [142] including notes from her website of agent-based resources [141]. The resulting runs bore little resemblance to those described by the authors.

**Stage(2)** Using source code available provided by Paul Johnson at SourceForge, who maintains a support site of materials relating to the SWARM implementation of the SFASM, I attempted to identify problems and potential misinterpretations in my understanding of the model. It quickly became obvious that based on this code, there a considerable number of important structural features were present which were not documented in the journal papers. Additionally, some sections of code appeared to directly contradict explicitly defined aspects of the model outlined in the papers. Two versions were available - 1.0 and 2.4. It was assumed, as it turns out mistakenly, that version 1.0 was closer chronologically to the original code and therefore closer to the model behind the published results. Attempts to reconcile these differences and reproduce time series with the properties described by Arthur et al failed.

**Stage(3)** Having variously revisited my coding and interpretation of the published model description and having eliminated a number of bugs, the conclusion was that more information was needed. Given that the original work was published in the last century and failed attempts to get information from authors, the best strategy appeared to be to contact authors of recent work on the model, since their understanding would be fresh and they must have had working models.

**Stage(4)** Separate doctoral work by Norman Ehrentreich [39] and by Thomas Badgruber [6] formed the basis of the final stage of reconstructing a working SFASM. From these it has finally been possible to produce a version which can be claimed to be close to the original. No attempt is made here to claim that it is an exact copy for several reasons: most prosaically it is written on a totally different platform; secondly as set out in the following subsection there are a number of points where it is still unclear what parameters were used and for low  $k$  values the Matlab model remains less stable in terms of time series statistics than the original model.

### A.7.2. SFASM Parameters & Variable Tables

Main Variable Table. All these variables which are core to the model are fully specified in the literature, although in some versions of the released code values for  $\rho$ , the autoregressive parameter, were apparently calculated and the models highly unstable till this was hard-wired. However as principal parameters for the model

these are well reported and documented. The values for  $f$  and  $g$  are generated as part of the derivation of the REE conditions.

**Table A.22.:** SFASM Main Variables

SFASMv2.4	Matlab	Documented	Param	Consistent	Main parameters
numBFagents	Nind	yes	25	yes	Number of agents
numfcasts	nRls	yes	100	yes	Rules per agent
(equal to $N$ )	N	yes	25	yes	Risky shares in system, $N$
baseline	mn_div	yes	10	yes	Mean dividend, $d$
initholding	shares	yes	1	yes	initial stock per agent
initialcash	cash	yes	20,000	yes	initial units of cash per agent
intrate	rfr	yes	0.10	yes	Risk free interest rate, $r_f$
rho	rho	yes	0.95	yes	Autoregressive parameter, $\rho$
gauss	eVar	yes	0.0743	yes	Dividend variance, $\sigma_\varepsilon^2$
-	reeVar	yes	4.00	yes	REE variance, $\sigma_{p+d}^2$
rea	f	yes	6.3333	yes	REE proof constant, $f$
reb	g	yes	16.688	yes	REE proof constant, $g$
[a_min, a_max]	a_rng	yes	[0.7,1.2]	yes	Forecast bit 'a' range
[c_min, c_max]	b_rng	yes	[-10.0 , 19.002]	yes	Forecast bit 'b' range
lamda	rAv	yes	0.5	yes	risk aversion, $\lambda$
tauv	wgt	yes	75	yes	weighting constant for exponential MA of forecast accuracy

The main market generated values are the stock price and the stock dividend in each period, agent demand is calculated using each agent's expectation for the successive period. Again the mechanisms and formulae generating these are well documented and consistent through all implementations.

**Table A.23.:** Computationally Generated Variables

SFASM v2.4	Matlab	Documented	Setting	Consistent	Description
price	px	yes	$p_t$	yes	stock price in period $t$
dividend	d	yes	$d_t$	yes	stock dividend in period $t$
demand	AgtDmd	yes	$x_{i,t}$	yes	agt $i$ demand in period $t$
wealth	AgtWlth	yes	$W_i$	yes	total agent wealth after taxes - the tax calculation is arbitrary and documented only in code
global_mean	glob	no	calculated	unspecified	Exponentially smoothed moving average of clearing price
variance	defVar	no	calculated	unspecified	default rule variance - multi-stage calculation process

The documentation becomes less clear in the parameterisation and the computational routines behind the genetic algorithm and new rule generation. As discussed in Chapter 5, parameters controlling the GA, rule eligibility for trading, activation and new rules were either undocumented or conflicted within code, with journal descriptions or subsequent research.

**Table A.24.:** Trading Constraints

SFASM v2.4	Matlab	Documented	Pilot Study Setting	Consistent	Description
maxbid	maxTrd	yes	10	yes	maximum trade per period
minholding	maxShrt	yes	-5	yes	maximum short position
mincash	minCash	yes	-2,000	yes	minimum cash
sptype	specialist	yes	slope, eta	unspecified	auction process described, but detailed description present only in source code
variance	defVar	no	calculated	unspecified	default rule variance - multi stage calculation process

**Table A.25.:** Rule Parameters & Controls

SFASM v2.4	Matlab	Documented	Setting	Consistent	Description
mincount	minCount	no	2	unspecified	Minimum times a rule must be activated to be eligible to forecast.
initvar	initVar	yes	4.0		initial rule variance
conbits	clBits	yes	12	yes	Total condition bits - actually 16 bits in v2.4 but contained fillers
bitprob	[pTrue,pFalse]	yes	0.1	yes	probability of a bit set to 1 or 0 - .05 for pTrue & pFalse
<i>(1-bitprob)</i>	pHash	yes	0.9	yes	probability a bit is set to 'don't care', hash value (NaN in Matlab)
-	nTek	yes	4	yes	technical bits
-	nFnd	yes	6	yes	fundamental bits
-	fBits	yes	2	yes	forecast bits
bitcost	cost	yes	.005	yes	specificity cost for set bits in rules

Table A.26.: GA parameters

SFASM v2.4	Matlab	Documented	Setting	Consistent	Parameters
gafrequency	k	yes	250 or 1000	yes	GA freq, $k$
firstgatime	GStart	no	1,001	unspecified	GA start time, 501 periods after initialisation stub.
longtime	badRlCount	yes	4,000	yes	Generalisation trigger periods.a
genfrac	genFrac	yes	0.25	yes	Fraction of bits to set to 'don't care' in generalisation
poolfrac	GGap	yes	0.2	yes	Fraction of rules for replacement in GA
newfrac	GGap	yes	0.2	yes	Fraction of new rules produced in GA
pcrossover	pXvr	yes	0.1	yes	Probability of crossover
$(1-pcrossover)$	$(1-pXvr)$	yes	0.9	yes	Probability of mutation
plinear	migOp	yes	1/3	yes	probability for forecast bit crossover
prandom	migOp	yes	1/3	yes	probability for forecast bit crossover
pmutation	pMut[1]	yes	0.03	yes	Individual condition bit mutation probability
$(transitionmatrix)$	migOp	yes	1/3	yes	condition bit transition probability factor (see Section 3.4.2.1
plong, pshort	pMut[2]	yes	0.2	yes	Forecast bit mutation probability
nhood	mutRngOp	yes	0.001	no	scaling factor for forecast mutation - .001 reported in journal (equivalent to 0.05%) vs .1 in Badegruber's v2.4 code.

### A.7.3. Other Parameterisation Omissions

- Technical bit market information. The journals described the use of moving average technical information bits. In the coded versions the option to use simple (arithmetic) or exponentially smoothed moving averages was available as a switch.
- Maximum and minimum stock prices. Arbitrarily set to 500 and 0.001 respectively. Clearly non-negative pricing seems like a reasonable constraint to impose, although it does raise questions about the demand function used by agents, highlighting its artificial nature and the fact that the agents are not rational about investment or trading in any operationally meaningful sense. The upper price limit is less easily explained and may have originated from a simple coding requirement, however setting it to relatively low levels could affect the forecast error measure used in agent learning.
- GA start time. Although the GA could be allowed to operate from the start of trading, with no forecast accuracy updates, i.e. no history to work from, the choice of suitable parent rules could be envisaged leading to extra brittleness.
- The GA short jump mutation operator 'nhood'. This may be a typographical error. Specified as 0.05% in the 1999 journal paper in all versions of the code implemented it appears set at 5%.