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Inverting Non-minimum Phase FIR Transfer Functions

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Abstract The approximate solution to the inverse of a non-minimum phase finite-impulse response (FIR) transfer function is given. Such a problem arises in many areas which include room-acoustics and channel equalisation. Unlike previous work the method does not require multiple sensors or the use of all-pass transfer function networks.

Keywords Non-minimum Phase, Inversion, Impulse Response, Signal Processing

1. Introduction

FIR transfer functions are more than often the impulse response of a communication channel or acoustic environment found experimentally. In the case of minimum-phase FIR transfer functions their inverse is easily found, but for the non-minimum phase case the inverse becomes unstable. Previous work in the area[1] involves splitting the transfer function into the combination of an all-pass transfer functions and a minimum-phase transfer function. The minimum-phase part is then inverted leaving the all-pass transfer function which has magnitude unity (and hence does not effect the magnitude response of the inverse filter). Provided the group-delay is constant this method is satisfactory. However, as discussed in[1] if the all-pass transfer function does not have constant group-delay then the conventional method will have a noticeable non-linear phase miss-match. The importance of a linear phase solution is that frequencies are scaled by a proportional amount when passing through the channel and filter, thus preserving the shape of the original signal. Linear phase is equivalent to having a pure time-delay (constant group-delay) which will for example introduce no perceptual distortion in a speech signal.

The method used here recognises that the inverse of a non-minimum phase impulse-response can either be an unstable system or a stable yet uncausal system. The uncausal part is then used by employing a time-delay and shifting this part back into positive time. Unfortunately the uncausal impulse response has an infinite impulse response so it must truncated making the method approximate depending on the length of the delay. This method is not new in principle, having been used for decades in adaptive filtering using least-mean squares (LMS) algorithms[2, 3]. In audio acoustics, the equalization of loudspeaker characteristics faces similar problems[4] and in sound rendering[5]. In the control literature similar problems are often encountered[6]. Techniques already exist for two or more sensors[7], but this explicit solution is new and only requires one signal-transmission channel.

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2. Mathematical Preliminaries

If a polynomial defined as $X(z^{-1}) = x_0 + x_1 z^{-1} + x_2 z^{-2} + \ldots + x_n z^{-n}$ of degree n with real coefficients has all its roots within the unit circle in the z plane, then it is termed strict sense minimum phase. No zeros are assumed to be on the unit circle. For simplicity $X(z^{-1})$ is often written as X, omitting the complex argument z^{-1}. The conjugate polynomial $X^*(z) = x_0 + x_1 z + x_2 z^2 + \ldots + x_n z^n$ is strict sense non-minumum phase having all of its roots outside the unit circle on the z plane. The reciprocal polynomial is defined as $\tilde{X}(z^{-1}) = x_0 z^{-n} + x_1 z^{-(n-1)} + x_2 z^{-(n-2)} + \ldots + x_n = z^{-n} X^*(z)$ which has all its roots outside the unit circle provided $X(z^{-1})$ is strict sense minimum phase. The zeros of $\tilde{X}$ are the zeros of X reflected in the unit circle. Similarly $\tilde{X}^*(z^{-1}) = z^n X(z^{-1})$ has all its roots within the unit circle. For polynomials which are strict sense non-minimum phase, we can factorise $X(z^{-1}) = X_1(z^{-1}) X_2(z^{-1})$ where X_1 is strict sense minimum phase and X_2 is strict sense non-minimum phase.

3. Theory

Begin with the FIR transfer functionof a transmission channel as described by the product of two polynomials

$$H(z^{-1}) = B_1(z^{-1})B_2(z^{-1}) \quad (1)$$

where [8] $B_1(z^{-1})$ is strict sense minimum phase and $B_2(z^{-1})$ is strict sense non-minimum phase. Their polynomial dimensions are nb_1 and nb_2 respectively.

We require an inverse transfer-function

$$F(z^{-1}) = \frac{1}{H(z^{-1})} = \frac{1}{B_1(z^{-1})B_2(z^{-1})} \quad (2)$$

To proceed further we note that $B_2B_2^* = \tilde{B}_2\tilde{B}_2^*$ and substitute $B_2 = \frac{\tilde{B}_2\tilde{B}_2^*}{B_2^*}$ into (2) giving

$$F(z^{-1}) = \frac{B_2^*}{B\tilde{B}_2\tilde{B}_2^*} \quad (3)$$

Now the reciprocal polynomial $\tilde{B}_2$ is stable having all of its roots within the unit circle, but $\tilde{B}_2^*$ is not. Therefore expand in positive powers of z

$$\frac{B_2^*}{\tilde{B}_2^*} = d_0 + d_1 z + d_2 z^2 + \ldots \quad (4)$$

For a non-minimum phase polynomial B_2 of degree nb_2 the expansion (4) is easily found by a simple recursion. For example if

$$B_2 = \alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \ldots + \alpha_{nb_2} z^{-nb_2} \quad (5)$$

then $d_0 = \frac{\alpha_0}{\alpha_{nb_2}}$

$$d_i = \frac{(\alpha_i - \sum_{j=0}^{nb_2-1} \alpha_{nb_2-i+j} d_j)}{\alpha_{nb2}} \quad i = 1, 2 \ldots nb_2 \quad (6)$$

and

$$d_i = -\frac{1}{\alpha_{nb_2}} \sum_{j=0}^{nb_2-1} \alpha_j d_{i+j-nb_2} \text{ for } i > nb_2 \quad (7)$$

Delaying by ℓ terms

$$\frac{B_2^*}{\tilde{B}_2^*} z^{-\ell} = d_0 z^{-\ell} + d_1 z^{-\ell+1} + \ldots + d_\ell + \ldots d_{\ell+1} z + d_{\ell+2} z^2 + \ldots \quad (8)$$

Which is a Laurent series with negative (causal) and positive (uncasual) powers of z.

$$\frac{B_2^*}{\tilde{B}_2^*} z^{-\ell} = \tilde{D}_\ell + E^* \quad (9)$$

The reciprocal polynomial $\tilde{D}_\ell = d_0 z^{-\ell} + d_1 z^{-\ell+1} + \ldots + d_\ell$, whereas $E^* = d_{\ell+1} z + d_{\ell+2} z^2 + \ldots$ is an infinite polynomial. However, if the delay ℓ is large enough we can approximate $E^* \to 0$ since this is part of a convergent power-series whose higher terms will converge to zero. So (3) becomes

$$F(z^{-1}) \approx \frac{\tilde{D}_\ell}{B_1\tilde{B}_2} \text{ for large } \ell \quad (10)$$

which is the inverse filter. It would be possible of course to express this as an FIR transfer function by using polynomial division and further truncation.

4. Example

Consider an FIR transfer function with $H(z^{-1}) = 1 - 2.6z^{-1} + 1.2z^{-2}$ or $B_1(z^{-1}) = 1 - 0.6z^{-1}$ and $B_2(z^{-1}) = 1 - 2z^{-1}$. Find its inverse and use a delay of $\ell = 6$.

Solution: Use the fact that $B_2^* = 1 - 2z$, $\tilde{B}_2 = -2 + z^{-1}$, $\tilde{B}_2^* = -2 + z$. Now expand

$$\frac{B_2^*}{\tilde{B}_2^*} z^{-5} = \frac{1-2z}{-2+z} z^{-6} = -0.5z^{-6} + 0.75z^{-5} + 0.375z^{-4} + 0.1875z^{-3} + 0.0937z^{-2} + 0.04687z^{-1} + 0.023437 + \ldots \mathrm{E}^*$$

So that $\tilde{D}_\ell = -0.5z^{-6} + 0.75z^{-5} + 0.375z^{-4} + 0.1875z^{-3} + 0.0937z^{-2} + 0.04687z^{-1} + 0.0234375$

Substituting into (10) we find the inverse transfer function as

$$F(z^{-1}) \approx -0.5 \frac{(-0.5z^{-6} + 0.75z^{-5} + 0.375z^{-4} + 0.1875z^{-3} + 0.0937z^{-2} + 0.04687z^{-1} + 0.0234375)}{(1-0.5z^{-1})(1-0.6z^{-1})}$$

For comparison purposes we use the alternative approach [1] which employs an all-pass transfer function.

$$F'(z^{-1}) = \frac{0.5}{(1-0.5z^{-1})(1-0.6z^{-1})}\frac{(1-2z^{-1})}{2(1-0.5z^{-1})}$$

Now when the original channel transfer-function $H(z^{-1}) = 1 - 2.6z^{-1} + 1.2z^{-2}$ is applied by convolution with the inverse filter, unity gain should be achieved up to half-sampling frequency. A comparison of the magnitude and phase inverse transfer functions are shown in Figs 1 and 2 respectively. Fig 1 shows a comparison for different values of delay showing that the amount of ripple reduces considerably as the delay increases.

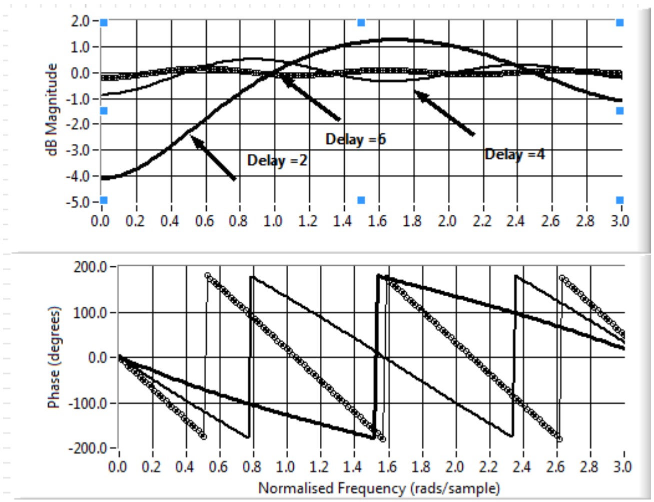

Figure 1. Magnitude and phase of equalised impulse response using new method

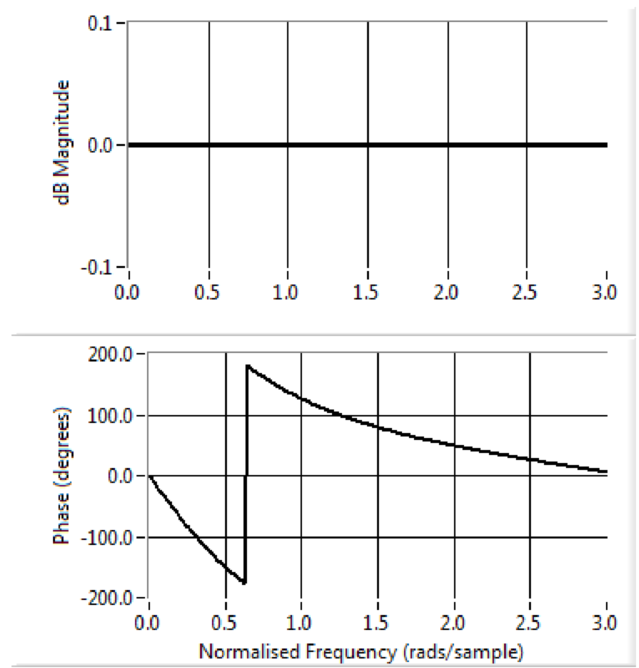

Figure 2. Magnitude and phase of equalised response using all-pass method

It can be seen that there is some 0.5dB peak-to peak ripple in the magnitude for the new case when the delay is 4 whilst for the all-pass method the magnitude is perfectly flat. However, the phase for the all-pass method is non-linear whereas for the new method the phase is linear. Furthermore the ripple for the new method is further reduced by making the delay term bigger. For example, for a delay of 6 samples the dB ripple is down to 0.1dB peak-to peak whilst still maintaining linear phase.

5. Conclusions

A single sensor approach to the inversion of non-minimum phase FIR transfer functions has been shown. The method is computationally simple and does not rely on all-pass transfer functions. Instead, it uses the uncausal part of the transfer function inverse itself which is shifted into positive time by using a time-delay. This gives rise to an equalised transfer function which has a small amount of ripple yet is linear phase. For small impulse responses it becomes quite an easy task to find the inverse and this is of some theoretical importance, However, for much larger impulse responses which could be met in an acoustic environment,the method requires the factorization of large polynomials into minimum and non-minimum phase polynomials, which is in itself quite a computational task. Therefore an efficient computational algorithm needs to be found for performing this task without having to find all of the individual roots of the polynomials themselves.

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