

# A Simple Sizing Optimization Method for Wind-Photovoltaic-Battery Hybrid Renewable Energy Systems

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**Abstract**-This paper presents a simple methodology to optimize the size of a hybrid wind generator (WG), photovoltaic (PV) module and battery storage system for a given demand. The method utilizes typical meteorological year (TMY) data to calculate hourly power output of a PV module and a WG throughout the year. By changing the combination of PV and WG, the generated energy is matched with the hourly average load of a year. This is done in such a way that the maximum of the total energy deficit in a cluster of hours in between hours of excess energy generations becomes minimum. The required number of batteries is calculated from that maximum of the total energy deficit among these clusters. The combination of WG, PV and battery that satisfies the desired loss of power supply probability (LPSP) and has the lowest total cost is considered as the optimum. A case study has been carried out to size a hybrid renewable energy system (HRES) optimally. The size obtained by this method is verified using an iterative algorithm and a genetic algorithm (GA). It is found that all of these methods give the same result for the same demand.

**Keyword:** Wind/PV hybrid renewable energy system (HRES); Iterative algorithm; PV module; Optimization; Genetic Algorithm (GA).

## I. INTRODUCTION

Global environmental concerns, rapid depletion of fossil fuels and ever increasing energy demand necessitate looking for alternative energy sources with a competitive cost. With extensive research, photovoltaic (PV) and wind turbine generators (WG) have been developed to such an extent that they promise an economical alternative to conventional energy generating systems. However, their nature makes them somewhat unpredictable and costly as an alternative to conventional sources when each of them is used alone. The solution of this problem is to integrate them in the presence of storage components. Such systems are known as hybrid renewable energy system (HRES). The complementary energy generation capability of PV and WG enables users to eliminate the weakness associated with each other. Further battery banks can be used as a backup supply to satisfy loads when energy from the renewable sources is not available.

An optimum mix and selection of each component of a HRES is crucial in order to make the system reliable and cost effective. Sizing optimization methods can be applied to keep the lowest investment cost with adequate and full use of resources. Recently several optimization methods have been

recommended for sizing a standalone HRES. An algorithm for sizing a PV array with a given WG in a Wind-PV hybrid system is presented by Borowy and Salameh in [1]. It does not consider the size of battery storage and overall cost of the system. Reliability of the system is not also taken into account. The same authors present a methodology to optimize the number of PV modules and batteries for a given WG and desired loss of power supply probability (LPSP) [2]. The cost function is minimized in process of optimization.

Markvart [3] provides a graphic construction technique to optimize the size ( $m^2$ ) of PV and wind turbine generator. System cost is minimized while generation of PV and WG are kept higher than the demand at any time of the day. Chedid and Rahman utilize linear programming techniques to minimize the cost by optimizing the size of PV and of WG in [4]. Total cost in this case consists of both initial cost and yearly operation and maintenance cost. An iterative technique for optimization is presented by Yang, Lu and Zhou [5]. The system cost is minimized by selecting the number of PV module, WG and battery that ensure a desired LPSP requirement. Genetic algorithm (GA) is introduced by Xu, Kang, Chang, and Cao [6] for selecting type and number of WG, PV and battery to satisfy a constant load demand. Tilt angle of PV is kept constant and installation height of WG is not considered. Kourtroulis, Kolokotsa, Potirakis and Kalaitzakis [7] utilize GA and propose a methodology to suggest among a list of commercially available system devices the optimal number and type of unit ensuring 20 years total system cost is minimized while total demand is completely fulfilled. The method also considers WG height and PV tilt angles. Yang, Zhou, Lu and Fang [8] show that annualized cost of the system can be reduced further with a compromise reliability index, LPSP.

A simple and faster optimization technique for hybrid renewable energy system is the main focus of research in this field. In previous studies several attempts have been made to develop a simplified method. Kellogg et al. [9] presents a simple method to size a wind-PV-battery hybrid system based on balance of energy. This method is simple and easy to implement but it does not consider the seasonal changes throughout the year. The system might be oversized in attempting to achieve balance of energy throughout the year. This paper attempts to develop a simple method to optimize the size of WG, PV and battery system using an iterative method.

Figure 1 shows a simple Wind-PV-Battery hybrid system. WG and PV are connected to a 24-V DC bus through control unit using appropriate ac/dc and dc/dc converters. Their loss and efficiency are considered when power output from the WG and PV are calculated.

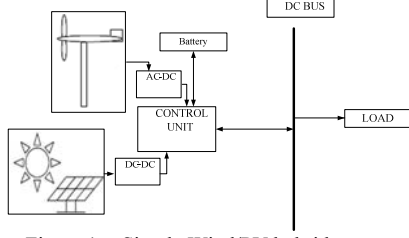


Figure 1: Simple Wind/PV hybrid system

## II. SYSTEM MODEL

### A. Wind generating model

The power output from a wind turbine generator is site dependent. Hourly wind speed at a specific height can be sourced from the US Department of Energy [10]. Wind speed at any height can be calculated using equation (1) [11].

$$v_h = v_{ref} \left( \frac{h}{h_{ref}} \right)^\alpha \quad (1)$$

where  $v_{ref}$  and  $v_h$  are wind speed at reference height and at hub height respectively and  $\alpha$  is a power law exponent ( $\sim 1/7$  for open space). The power output  $P_w$  ( $W/m^2$ ) from a wind turbine depends on wind speed and can be calculated using equation (2) [10].

$$\begin{aligned} P_w &= 0 & v(t) < v_{ci} \\ P_w &= a v^3(t) - b P_r & v_{ci} \leq v(t) < v_r \\ P_w &= P_r & v_r \leq v(t) < v_{co} \\ P_w &= 0 & v(t) \geq v_{co} \end{aligned} \quad (2)$$

where  $a = \frac{P_r}{(v_r^3 - v_{ci}^3)}$ ,  $b = \frac{v_{ci}^3}{(v_r^3 - v_{ci}^3)}$ ,  $P_r$  is the rated power,  $v_{ci}$ ,  $v_{co}$  and  $v_r$  are cut-in, cut-out and rated speed of the wind turbine respectively. Actual power output from the wind turbine is given by equation (3) [12]:

$$P_{WG} = P_w A_{WG} \eta \quad (3)$$

where  $A_{WG}$  is the total swept area of WG and  $\eta$  is the efficiency of the wind turbine generator and corresponding converters.

### B. Photovoltaic module model

The maximum output power of a PV array at any time of a day can be expressed in equation (4) [7]:

$$\begin{aligned} P_{PV}(t, \beta) &= N_s \cdot N_p \cdot V_{OC}(t) \cdot I_{SC}(t, \beta) \cdot FF(t) \\ I_{SC}(t, \beta) &= \{I_{SC-STC} + K_I [T_c(t) - 25^\circ\text{C}]\} \frac{G(t, \beta)}{1000} \\ V_{OC}(t, \beta) &= \{V_{OC-STC} - K_V T_c(t)\} \\ T_c(t) &= T_A + (NCOT - 20^\circ\text{C}) \frac{G(t, \beta)}{800} \end{aligned} \quad (4)$$

where  $I_{sc}(t, \beta)$  is short circuit current (A),  $I_{SC-STC}$  is short-circuit current under STC (A),  $G(t, \beta)$  is the global irradiance ( $W/m^2$ ) incident on the PV module placed at tilt angle  $\beta^0$ ,  $K_I$  is the short-circuit current temperature coefficient ( $A/^\circ\text{C}$ ),  $V_{OC}(t, \beta)$  is the open circuit voltage (V),  $V_{OC-STC}$  is the open circuit voltage temperature coefficient ( $V/^\circ\text{C}$ ),  $T_A$  is the ambient temperature ( $^\circ\text{C}$ ),  $NCOT$  is the nominal cell operating temperature,  $FF$  is the fill factor specified by the manufacturer,  $N_s$  and  $N_p$  are the number of series and parallel connected solar module [7].

### C. Battery model

Current state of charge (SOC) is used to determine the charging and discharging of a battery. At any time  $t$ , SOC is defined by the equation (5) [13]:

$$SOC(t) = SOC(t-1) \cdot \left(1 - \frac{\sigma \Delta t}{24}\right) + \frac{I_{batt.}(t) \Delta t \eta_{batt.}}{C_{batt.}} \quad (5)$$

where  $\sigma$  is the self-discharge rate (0.2% per day) [13],  $C_{batt.}$  is the nominal capacity of a battery. As in [13] the battery charge efficiency ( $\eta_{batt.}$ ) is set to round-trip efficiency and the discharge efficiency is set to 1. The battery current of a hybrid WG-PV at any time is given by:

$$I_{batt.}(t) = \frac{P_{PV}(t) + P_{WG}(t) - P_{Load}(t)}{V_{batt.}(t)} \quad (6)$$

where  $P_{PV}(t)$  and  $P_{WG}(t)$  is the instantaneous solar power and wind power respectively,  $P_{LOAD}(t)$  is the demand and  $V_{batt.}(t)$  is the battery voltage. Maximum allowable depth of discharge (DOD) of a battery is assumed to be 0.8. Thus  $SOC_{min}$  is 0.2 and  $SOC_{max}$  is 1 when battery is fully charged.

## III. OBJECTIVE FUNCTION FORMULATION

A typical constrained single variable optimization problem can be stated as follows (7):

$$\text{maximize/minimize } f(x) \quad (7)$$

subject to the constraints:

$$x_{min} \leq x \leq x_{max} \quad (8)$$

where  $f(x)$  is an objective function. The objective function of a HRES is taken to be 20-years total system cost including capital cost, operation and maintenance cost and it is expressed in equation (9),

$$\begin{aligned} f(N_{PV}, N_{WG}, N_{batt.}) &= -[N_{PV}(C_{PV} + 20 \cdot M_{PV}) \\ &+ N_{WG}(C_{WG} + 20 \cdot M_{WG}) + N_{batt.}(C_{batt.} \\ &+ y_{batt.} \cdot C_{batt.} + (20 - y_{batt.} - 1)M_{batt.})] \end{aligned} \quad (9)$$

subject to the constraint:

$$N_{PV} \geq 0, \quad (10)$$

$$N_{WG} \geq 0, \quad (11)$$

$$N_{batt.} \geq 0, \quad (12)$$

where,  $N_{PV}$  ( $N_s \times N_p$ ) is no. of photovoltaic modules,  $N_{WG}$  is no. of wind turbine generators,  $N_{batt.}$  is no. of batteries  $C_{PV}$  is capital cost of a photovoltaic module,  $C_{WG}$  is capital cost of a WG,  $C_{batt.}$  is capital cost of a battery,  $M_{PV}$  is annual maintenance and operation cost of photovoltaic module,

$M_{WG}$  is annual maintenance and operation cost of WG,  $M_{batt.}$  is annual maintenance and operation cost of battery and  $y_{batt.}(=10)$  is expected number of battery replacement in 20 years period.

The loss of power supply probability (LPSP) is incorporated with the cost function so that the designed system can meet desired reliability. LPSP is defined as the ratio of total loss of power supply (LPS) to total demand for a considered period of time  $T(=8760 \text{ hrs})$  as shown in equation (13)[2].

$$LPSP = \frac{\sum_{t=1}^T LPS_t}{\sum_{t=1}^T E_t} \quad (13)$$

In this paper the probabilistic approach is used to calculate the LPSP in order avoid computational burden. Thus LPSP is defined as follows in equation (14) [13].

$$LPSP = \frac{\sum_{t=1}^T \text{Power failure time}}{T} \quad (14)$$

If the total available power from the WG, PV and storage is less than the demand at any time then the system will not able to meet that demand at a particular hour. This period is termed as power failure time. The power available at any time from the hybrid system is expressed in equation (15).

$$P_{available}(t) = P_{PV}(t) + P_{WG}(t) + k \cdot V_{batt.} \quad (15)$$

$$\begin{aligned} & \text{Min}[I_{batt.max} \\ & = \frac{0.2 \cdot C}{\Delta t}, \frac{C \cdot (SOC(t) - SOC_{min})}{\Delta t}] \end{aligned}$$

#### IV. SYSTEM SIZE OPTIMIZATION

The required number of PV modules, WGs and batteries for a given demand load are calculated as follows:

(i) hourly energy output from individual wind generator and PV module for a typical year is calculated using wind speed and solar insolation of the site,

(ii) different combinations of WGs and PV modules are used in order to match the generation with the given hourly load of a year. For each of the combinations, there will be energy deficit during several consecutive hours in between the hours of excess energy generation. This cluster of energy deficit cannot be supplied by renewable sources.

(iii) total energy deficit in each of these clusters throughout the year is calculated and maximum amount of deficit among these is determined,

(iv) the combination of WG and PV is selected which minimizes the maximum deficit. The amount of maximum deficit is used to determine the storage size i.e., the size of the battery,

(v) loss of power supply probability (LPSP) and 20-years total cost of the hybrid renewable power system are calculated for each of these combinations and

(vi) optimal combination is selected based on desired LPSP and minimum total cost of the system.

#### V. CASE STUDY

Auckland, New Zealand is used as the site for a case study. Latitude and longitude of the given site is  $36.86^\circ \text{ S}$  and  $174.78^\circ \text{ E}$  respectively. Weather data for the location, for a typical weather year was taken from US Department of Energy website [10]. Figure 2, Figure 3 and Figure 4 show hourly global horizontal radiation ( $\text{W/m}^2$ ), temperature ( $^\circ\text{C}$ ) and wind speed ( $\text{m/s}$ ) respectively of the proposed site.

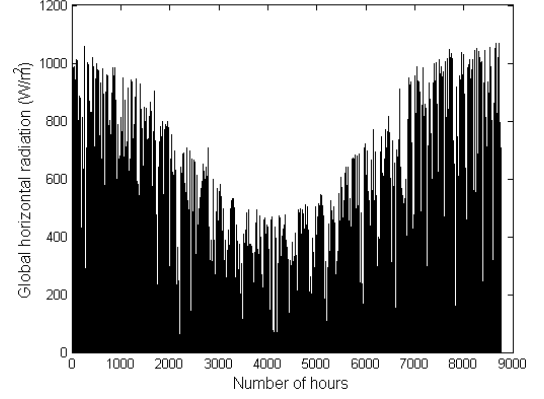


Figure 2: Hourly global horizontal radiation ( $\text{W/m}^2$ )

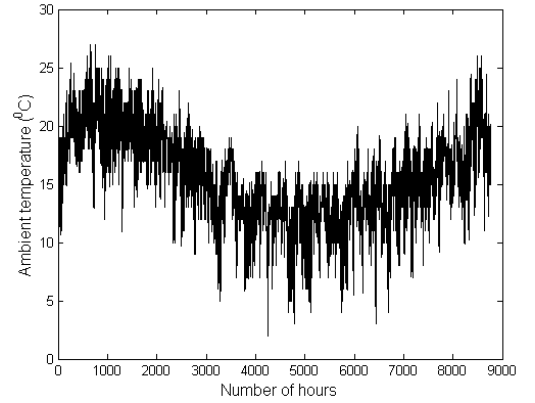


Figure 3: Hourly Temperature of the year ( $^\circ\text{C}$ ).

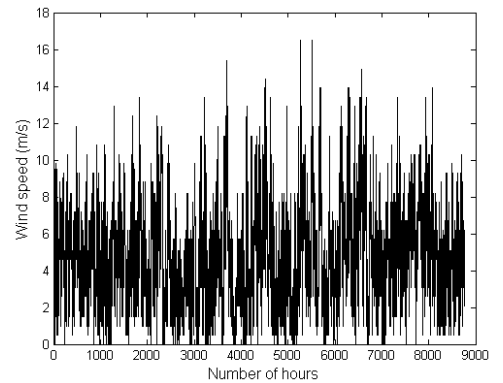


Figure 4: Hourly wind speed ( $\text{m/s}$ ).

The developed methodology could be applied to size the HRES for any load using the given TMY data. A typical hourly domestic load [7] as shown in Figure 5 is used for

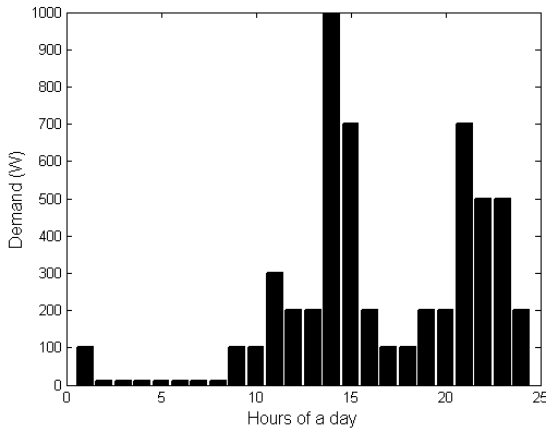


Figure 5: Hourly load of a day

sizing a HRES in this paper. This load is to be matched with the generation from different combinations of wind generator and photovoltaic module.

Table I and Table II show the component details used in this optimization.

TABLE I. SPECIFICATION OF THE PV

Sl. No.	V <sub>OC</sub> (V)	I <sub>SC</sub> (A)	V <sub>max</sub> (V)	I <sub>max</sub> (A)	P <sub>max</sub> (W)	Cap. Cost(\$)
01.	64.8	6.24	54.7	5.86	320	1008

TABLE II. SPECIFICATION OF THE WG

Sl No.	Power (W)	h <sub>low</sub> (m)	h <sub>high</sub> (m)	Cap. Cost (\$)
01.	1000	11	35	3400

A combination of different numbers of WG and PV was tried to match the energy generation with the load. For each of these combination, there was energy deficit in a group of consecutive hours in between hours of excess generation. The total amount of energy deficit in each of these clusters throughout the year was calculated and maximum amount of deficit in a cluster among these was determined. The combination of WGs and PV modules was selected which minimizes this maximum deficit. A simple spreadsheet was created for this calculation. A sample of this calculation is shown in Table III. The first two columns show energy generation of a PV module and a WG respectively and the third column shows demand. In the next column total renewable energy generation from a combination of 32 PV modules and a WG is calculated. The difference between the generation and demand in each hour is found in the fifth column and negative sign before the energy difference indicates a deficit. The consecutive hours of energy deficit are clustered in between the cluster of excess energy generation hours. The total energy deficit in each of these groups is calculated throughout the year as shown in the last column. As renewable sources are unable to meet these deficit directly, energy must be supplied from the energy storage system i.e.,

batteries. The generated excess energy could be stored by charging the battery or generating hydrogen. The maximum numerical value of the total energy deficit among all of these group was therefore used to determine the size of the battery system. It is important to ensure that the excess energy is enough to recharge the battery whenever energy is discharged from it. To ensure minimal loss of power and longer life of the battery, the size may be kept twice the size of the maximum energy deficit.

TABLE III. CALCULATION OF TOTAL ENERGY DEFICIT IN A GROUP

PV Gen. (W)	WG Gen. (W)	Demand (W)	Total Gen.	Diff of Gen. and Demand	Deficit	Total deficit in series
-	-	-	-	-	-	-
1.89	146.20	200	206.76	6.76	0	
0	72.45	700	72.45	-627.54	-627.54	
0	38.43	500	38.43	-461.56	-461.56	
0	0	500	0	-500	-500	
0	0	200	0	-200	-200	
0	0	100	0	-100	-100	
0	0	10	0	-10	-10	
0	0	10	0	-10	-10	
0	0	10	0	-10	-10	
0	0	10	0	-10	-10	-
1.85	0	10	59.35	49.35	0	1929.11
-	-	-	-	-	-	-
-	-	-	-	-	-	-

The battery specification of Table IV was used to calculate the number of required batteries in the system. As each of the battery has a terminal voltage of 12V, two batteries are connected in series to match with the dc bus voltage of 24 volt.

TABLE IV: BATTERY SPECIFICATION.

Battery Model	Price (\$)	Voltage (V)	Capacity (Ah)
Surrette 12-CS-aaPS	1239	12	357

Number of batteries required is calculated as shown in equation (16).

$$Number\ of\ batteries = \frac{2 \times E_{sd}}{V_t \times C \times DOD} \quad (16)$$

where,

$N_s$ =number of series connected battery,

$E_{sd}$ = maximum total energy deficit among all the groups throughout the year,

$V_t$ =terminal battery voltage,

$C$ =Battery Capacity and

$DOD$ =Depth of discharge.

Several combinations were found that can meet the load with less number of WG, PV and batteries however all the combinations may not fulfill the loss of power supply probability (LPSP). A simulation program was subsequently developed in MATLAB to calculate LPSP for all of these combinations. The combination with minimum total system cost and with desired LPSP was selected as the optimum size that can fulfill the demand.

## VI. RESULT AND DISCUSSION

Table V below shows the combination WG, PV and battery which fulfill the demand with less number of storage element.

TABLE V: COMBINATION WITH LESS NUMBER OF BATTERIES

WG No.	PV No.	Battery No.	Total cost (NZ\$)	LPSP
1	20	4	83234.04	0.1094
1	25	4	89282.04	0.0369
1	30	4	95330.04	0.00244
1	31	4	96539.64	0.0013
1	32	4	97749.24	0
1	33	4	98958.84	0
1	34	4	100168.4	0
1	35	4	101378	0
2	10	6	102699.1	0.2267
2	20	4	87314.04	0.1116
2	25	4	93362.04	0.0054
2	30	4	99410.04	0

For each of the combinations total cost of the system and LPSP were calculated. Total cost including the capital cost and lifetime maintenance, replacement cost and operation cost were taken into account. The annual operation and maintenance cost of each components is taken 10% of its capital cost. Total life time of the system was considered to be 20 years. The system was sized for LPSP of zero ie., the load will be always satisfied. The number of WG, PV and batteries were found 1, 32 and 4 that satisfy LPSP of zero value with minimum total cost.

TABLE VI: COMPARATIVE RESULT

Sl. No.	Method	No. of WG	No. of PV	No. of Bat.
1.	This Method	1	32	4
2.	GA	1	32	4
3.	Iterative Method	1	32	4

## VII. CONCLUSION

A simple and easy to implement size optimization technique is presented in this paper. The proposed method utilizes TMY data to calculate PV module and WG output. Loss of power supply probability (LPSP) is used for the measure of system reliability and total cost of the system is used for economical constraint of the system design. The result obtained by this method is verified using iterative algorithm and genetic algorithm. All methods give the same result for the given load.

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