# THE ECKERT NUMBER PHENOMENON

## Experimental investigations on the heat transfer from a moving wall in the case of a rotating cylinder

M A GSCHWENDTNER

*University of Canterbury / New Zealand*

mag@mech.canterbury.ac.nz

This work was carried out at the University of the Federal Armed Forces in Munich / Germany.

The Eckert number phenomenon was investigated theoretically by Geropp in 1969 and describes a reversal in heat transfer from a moving wall at an Eckert number Ec ≈ 1. In this report the Eckert number phenomenon is confirmed experimentally for the first time. For that purpose the heat transfer from a heated, vertically rotating cylinder in a crossflow was investigated. In order to perform the experiments in a range where the predicted phenomenon occurs, extreme rotational speeds were necessary. A heating concept had to be developed which allowed an input of heating power independent of the speed and which therefore had to be contact-free. The results show, among other things, that the temperature difference between the wall and the surrounding fluid has a significant effect on the predicted reversal of heat transfer at the wall. Moreover, maximum heat transfer occurs at an Eckert number Ec ≈ 0.3, which is of great importance for the cooling of hot surfaces in a gas-flow.

*Eckert number* *Nusselt number* *heat transfer* *rotating cylinder* *adiabatic wall temperature* *boundary layer* *dissipation* *frictional heat*

## List of symbols

| | |
|---|---|
| $A$ | Area |
| $c_f$ | Friction coefficient |
| $c_p$ | Specific isobaric heat capacity |
| $c_{1...5}$ | Constants |
| $D$ | Cylinder diameter |
| $Ec$ | Eckert number |
| $h$ | Heat transfer coefficient |
| $k$ | Thermal conductivity |

| | |
|---|---|
| $l$ | Cylinder length |
| $Nu$ | Nusselt number |
| $P$ | Power |
| $Pr$ | Prandtl number |
| $\dot{Q}$ | Heat flow |
| $Re$ | Reynolds number |
| $T$ | Temperature |
| $\Delta T$ | Temperature difference |
| $Tu$ | degree of turbulence |
| $v$ | Velocity |
| $W$ | Width of wind-tunnel |
| $\delta$ | Boundary layer thickness |
| $\varphi$ | Angle |

### Subscripts

| | |
|---|---|
| $ad$ | adiabatic |
| $corr$ | Corrected |
| $cs$ | Cross-sectional |
| $el$ | Electric |
| $t$ | Turbulent |
| $W$ | Wall |
| $h$ | Convective |
| $\Omega$ | Rotational |
| $\infty$ | Main flow |

## Introduction

The title, the Eckert number phenomenon, sounds mysterious. A phenomenon is an appearance of something of which the cause is in question or unknown. In natural sciences experiments are carried out in order to identify correlations of physical laws. Quite often, however, the results of these experiments raise even more new questions than giving answers. In this case, the Eckert number phenomenon had its origin in experiments which took an unexpected course.

In the 1960s experiments were carried out to investigate the cooling of commutators. These devices are cylindrical parts on electric motors by which electric power is transmitted to the rotor. Due to friction, the commutator gets hotter at the slip-ring-contacts the faster the rotor spins. In order to avoid surface temperatures becoming too high, the commutator has to be cooled. However, since rotation in quiescent air does not reject enough heat, the device needs to be cooled by forced convection.

First experimental work on axial and radial cooling of commutators was carried out by Yildiz [1], who found out that at a fast rotating cylinder the heat transfer is only determined by the rotational Reynolds number, which relates in a dimensionless form to the peripheral velocity of the cylinder. However, he discovered that the heat transfer could not be increased any further above a rotational Reynolds number $Re_{\Omega} = 2.5 \times 10^6$.

This observation induced Geropp [2] to carry out a theoretical study about the correlations of the Nusselt, Reynolds and Eckert numbers. His main focus was on the high rotational speed of an infinitesimally long cylinder in quiescent air where the heat is created by dissipative effects and therefore the Eckert number (defined as the ratio of kinetic energy at the wall to the specific enthalpy difference between wall and fluid) becomes an important factor for the heat transfer. Based on the boundary-layer equations, Geropp formed a theory which supports Yildiz' observations in that the heat transfer stagnates at a particular rotational Reynolds number. Moreover, he predicted that the heat transfer even changes its direction at an Eckert number $Ec \approx 1$. According to Geropp, this reversal will occur at a rotational Reynolds number $Re_{\Omega} \approx 6.9 \times 10^6$. This, then, is the actual phenomenon: A rotating body is not cooled any longer; it takes up heat despite the fact that its surface temperature is still higher than ambient temperature.

The main goal of this work was to confirm the existence of the Eckert number phenomenon experimentally for the first time and to investigate the contributing heat transfer processes in the case of a heated, rotating cylinder in a crossflow. For this purpose extensive investigations of the influence of the fluid-dynamic variables such as gas velocities, temperatures and temperature gradients (which are most significant for the heat transfer) were carried out and provided some interesting insights into the complex interactions taking place in the surroundings of a rotating cylinder. These results, however, will be reported in a separate paper [3]. Here, only the background and the fundamental conclusions of the experimental results will be given as an introduction to the Eckert number phenomenon.

## Theoretical background

### *Geropp´s theory*

Based on the fundamental equations for velocity and temperature fields in conjunction with empirical formulae for wall shear stress, Geropp [2] deduced the relations for the turbulent heat transfer of a heated, horizontal cylinder which is rotating freely in space. He refers to experiments of various authors, mainly of Yildiz. Geropp's work was focused on high rotational Reynolds numbers, where the frictional heat created by dissipation and thereby the Eckert number, gains influence on the heat transfer. The dimensionless Eckert number

$$Ec = \frac{\frac{1}{2}v^2}{c_p\left(T_W - T_\infty\right)} \qquad (1)$$

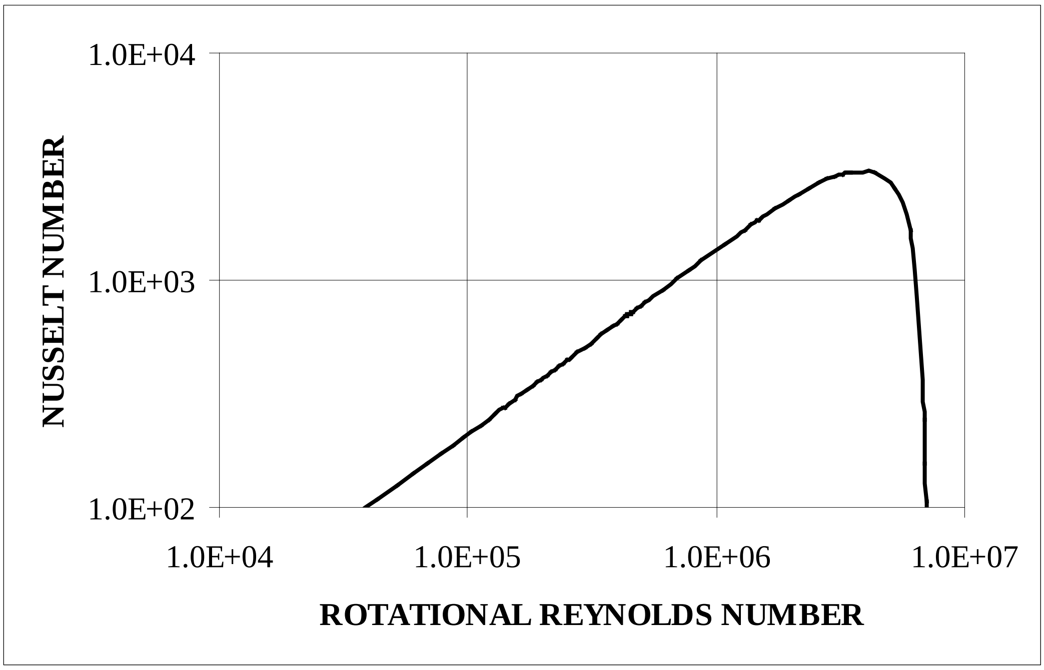
therefore plays an important role, representing the ratio of kinetic energy at the wall to the specific enthalpy difference between wall and fluid.

As a result of his calculated temperature distribution, Geropp deduced that the temperature gradient at the wall changes its sign and therefore will cause a change in the direction of the heat flow. Geropp´s deduction of the Nusselt number supports Yildiz´ observation that from a certain rotational Reynolds number, the heat transfer does not increase any further. Furthermore, the theory predicts a decrease of the heat transfer down to a condition at which the heat flow actually reverses. Geropp gives the following function for the Nusselt number

$$Nu = \frac{\frac{1}{2} c_f Re_\Omega Pr}{c_1 + c_2 \sqrt{\frac{c_f}{2}}} \left\{ \frac{1}{Pr_t} - Ec \left[ c_3 + c_4 \sqrt{\frac{c_f}{2}} + c_5 \frac{c_f}{2} \right] \right\}, \quad (2)$$

whereby $c_f$ is a friction-coefficient according to Dorfmann´s [4] wall shear stress law, $Pr_t$ is the turbulent Prandtl number and $c_1$ to $c_5$ are constants. In this equation the heat transfer changes its sign once the Eckert number reaches a value around unity and a cooling of the cylinder wall leads to a heating despite the fact that the wall temperature is still above the fluid temperature. Since there was no experimental data available for this range of Eckert numbers, Geropp extrapolated the function $Ec = f(Re_\Omega)$ from Yildiz´ data and specified the Reynolds number $Re_\Omega = 6.9 \times 10^6$ where the Nusselt number becomes zero (Figure 1).

**Figure 1**

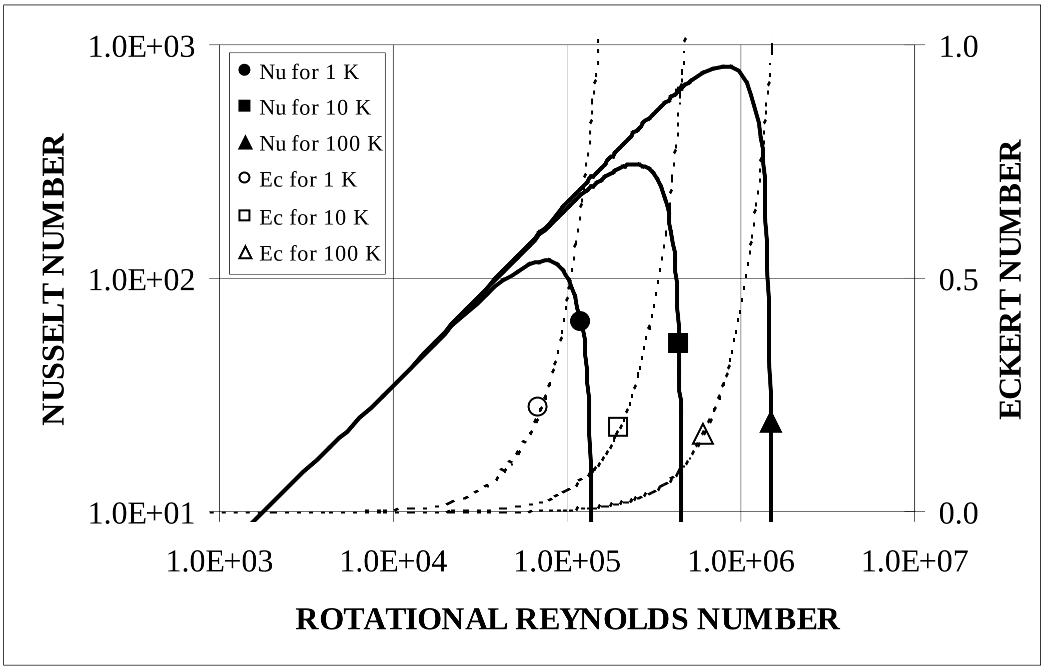



Geropp explains the so-called Eckert number phenomenon as being the result of dissipation created by shear stresses in the fluid at the wall. The reduction in heat transfer is determined by the hypothetical "adiabatic wall temperature” $T_{ad}$ [5] which the cylinder would take on if it were insulated. With a wall temperature $T_W$ being smaller than the “adiabatic wall temperature”, heat is being transferred to the cylinder even if $T_W$ is above the fluid temperature.

### *Further conclusions from Geropp´s theory*

Additional conclusions can be drawn from Geropp´s equations. Firstly, together with the temperature difference $\Delta T = (T_W - T_\infty)$, an additional arbitrary parameter for the calculation of the rotational Reynolds number at which the reversal in heat transfer occurs, is hidden in the Eckert number. From this it obviously follows that a boundary Reynolds number, at which the Eckert number phenomenon should occur - as mentioned in Geropp´s publication - does not exist. However, there is <u>one</u> function of the Nusselt number dependent on the rotational Reynolds number for any given temperature difference $(T_W - T_\infty)$. In figure 2, three of these functions are shown on a logarithmic scale for three chosen temperature differences, according to Geropp´s theory (the function of the corresponding Eckert numbers is plotted with dotted lines).

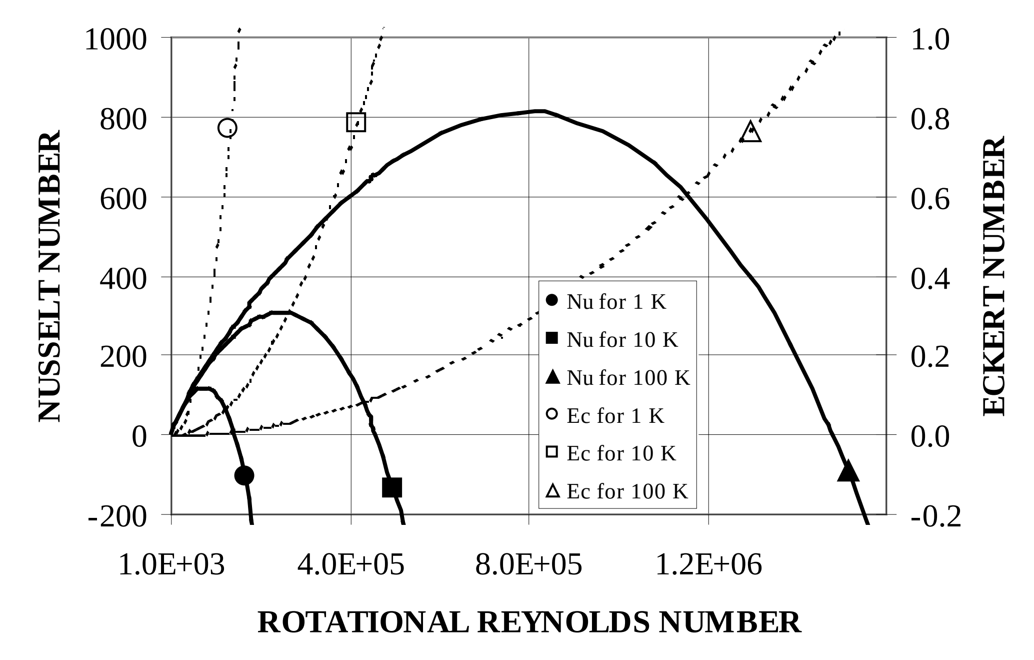
**Figure 2**




It can be seen that the rotational Reynolds numbers at which the Nusselt numbers suddenly fall away correspond to the Eckert numbers being approximately one. All the experimental results from authors quoted by Geropp occur on the straight line before the drop in heat transfer. The reason why none of these previous experiments reached the range where the drop in heat transfer occurs is that the necessary rotational Reynold numbers were inaccessibly high for the underlying temperature differences. In other words, in the ignorance of the Eckert number phenomenon, all authors chose temperature differences too high for their experimental capabilities (Geropp´s function in figure 1 refers to a temperature difference of $\Delta T$ = 2165 K!).

Also, the logarithmic scale in figure 1, as used by Geropp, conceals another conclusion which becomes obvious in a linear scale as shown in figure 3: The line of the Nusselt-function is a symmetric curve to the vertical line through the maximum value.

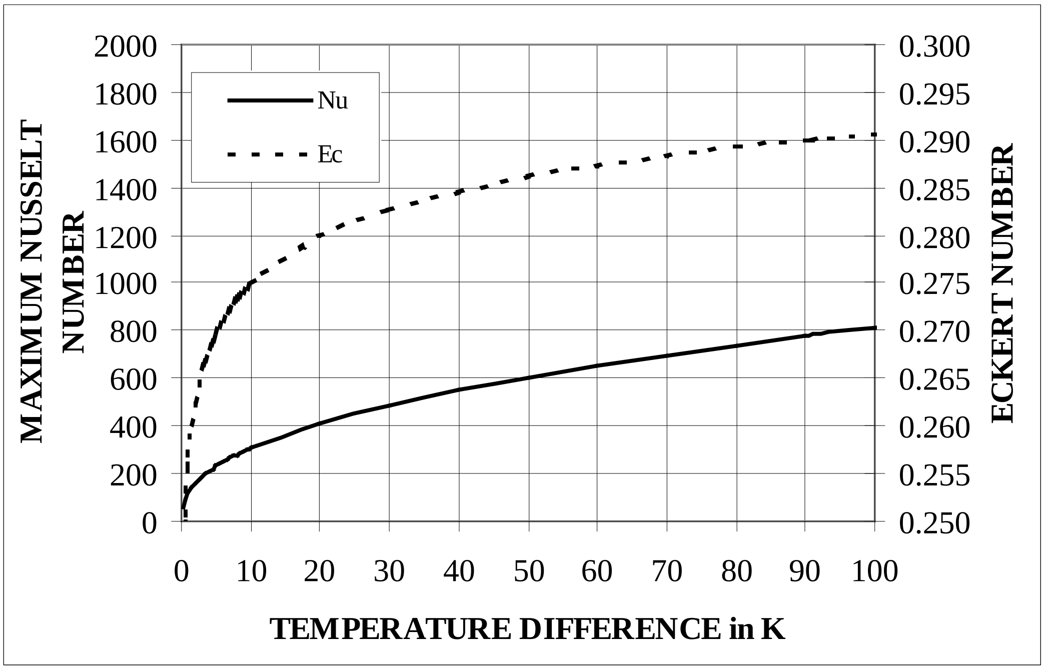
**Figure 3**




This means that the heat transfer decreases beyond the maximum value at the same rate as it previously increased. A sudden drop in heat transfer therefore does not exist. Negative values for Nusselt numbers, as shown in figure 3, can be understood as a reversal of the heat-flow.

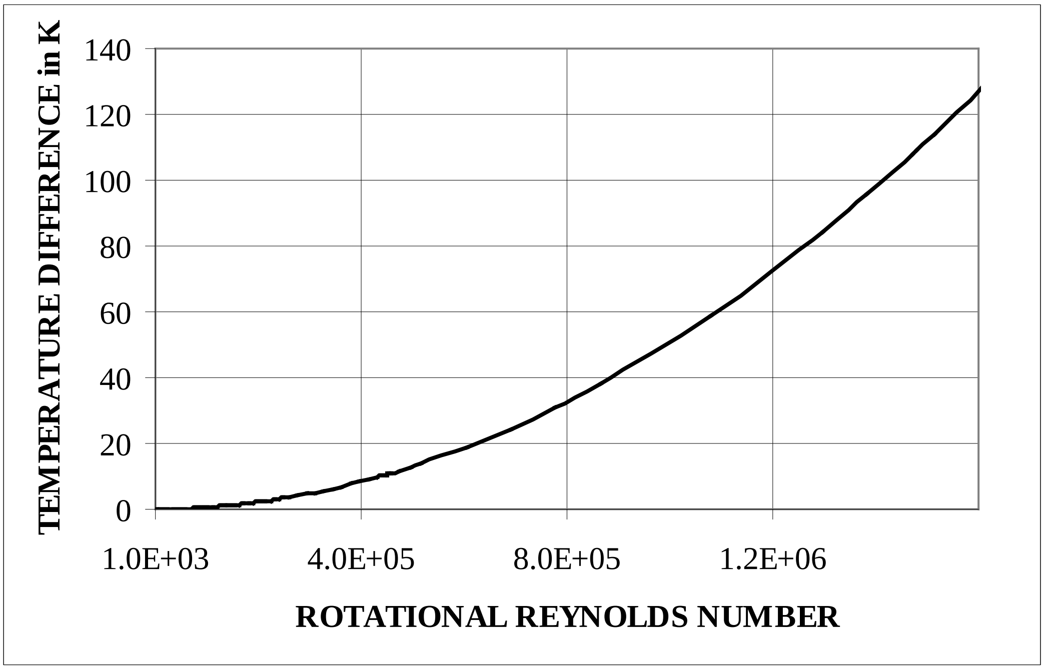
The linear display in figure 3 allows an additional discovery. It is striking that the maximum Nusselt number at a given temperature difference roughly occurs at a constant Eckert number. From Geropp´s theory another functional relation can therefore be deduced: There is <u>one</u> function for the maximum Nusselt number and the respective Eckert number dependent on the temperature difference $(T_W - T_\infty)$. Figure 4 illustrates the form of these functions as a function of the temperature difference. This value for the Eckert number approaches 0.3 for large temperature differences (> 1000 K).

**Figure 4**




Finally it can be concluded from Geropp´s equations that the "self-heating-temperature" $T_s$ is only a function of the rotational Reynolds number. This means that the heating of the cylinder by dissipation is independent of the temperature difference $(T_W - T_\infty)$ and therefore is a process of its own without a direct influence on the heat transfer. Only the interaction of the temperature differences $(T_W - T_\infty)$ and $\Delta T_s = (T_S - T_W)$ determine an increase or decrease in heat transfer. The increase in "self-heating" is shown in figure 5 for the range of rotational Reynolds numbers in the previous graphs.

**Figure 5**

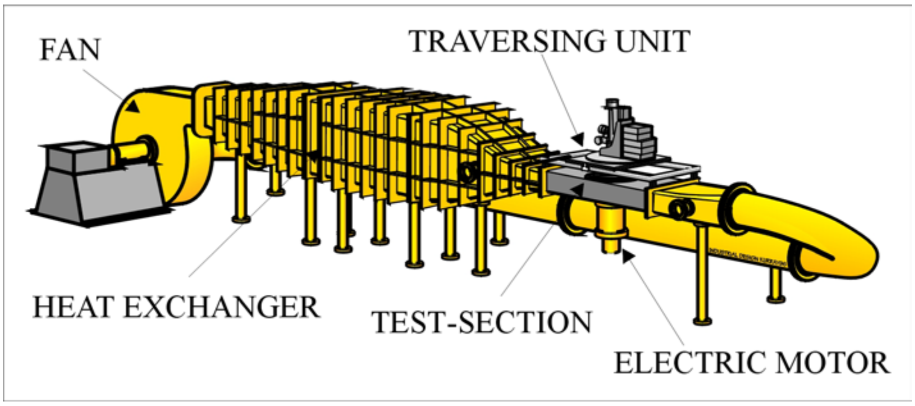



## Experimental apparatus

### *Overall setup*

The goal was to obtain experimental confirmation of the Eckert number phenomenon. If successful, this would show that the rotation of a heated cylinder has a supportive effect on the heat transfer initially, followed by a decrease and finally a reversal of the heat flow between the cylinder wall and the surrounding fluid as the rotational speed increases. In order to understand the complex fluid-dynamic and thermodynamic processes occurring at a heated rotating cylinder, extensive experiments were necessary.

**Figure 6**




The experiments were carried out in a closed-circuit with a rotating cylinder vertically mounted in the test-section (figure 6). The wind-tunnel was gas-tight up to a pressure of 2 bar, and air velocities of 70 m/s could be reached at ambient pressure. The circulating gas (air in this case) could be kept at a constant temperature by means of a heat exchanger [6]. The cylinder was driven by an electric asynchronous motor, which was designed originally for high-speed aluminium milling, allowing speeds up to 30,000 rpm. The motor was mounted at the bottom of the test-section, while a highly precise optical glass-window was let into the top, allowing access for optical measurements. Above the test-section, a four-axes traversing unit with stepping motors was mounted in order to position the measuring devices, mainly optical equipment.

In order to examine the influence of rotation on the flow conditions in the surroundings of the cylinder, a knowledge of the characteristic flow parameters is of great importance. Gas velocities were therefore measured with a two-dimensional Laser-Doppler-velocimeter (LDV) which is a highly complex optical measuring technique, but offers the advantage of accurate measurements without disturbing the sensitive flow. The driving force for heat transfer is temperature gradient. To determine the gradients, an optical measuring technique was developed, based on the deflection of a light beam in a temperature field according to Schmidt´s analysis [7]. Furthermore, the real-time observation of temperature fields in selected areas should allow insights into the actual processes around the cylinder. With a Michelson interferometer, a third optical method was employed, with which the fluctuations of isothermals could be recorded on video. Finally, in order to determine the heat transfer in terms of Nusselt numbers as a function of the Eckert number (including rotational speed and temperature difference), the electrical heating supply and the surface temperature of the rotating cylinder were measured.

### *Rotating cylinder*

The centrepiece of the apparatus was the rotating cylinder which spanned vertically the total height of the test section. Because of the high centrifugal acceleration of more than 25,000 g at the circumference, the cylinder was made of high strength aluminium ($AlCuMg_2$) licensed for aeronautical applications. The surface was machined to a concentricity of ± 1 µm and was polished. The cylinder could therefore be regarded as hydraulically smooth.

High rotational speeds demanded high mechanical strength of the rotating system, especially of the heating supply. The development of a new concept was necessary to provide both high mechanical stability and meet the demand of good and reliable control accuracy. Therefore, the complete heating supply was moved from the rotating to the non-rotating side, and the rotating system was designed to be as simple as possible. The solution was to heat up the cylinder via radiation from inside by means of a high-performance heating cartridge, while the relevant wall-surface temperatures were measured from outside by infrared thermometers.

**Figure 7**

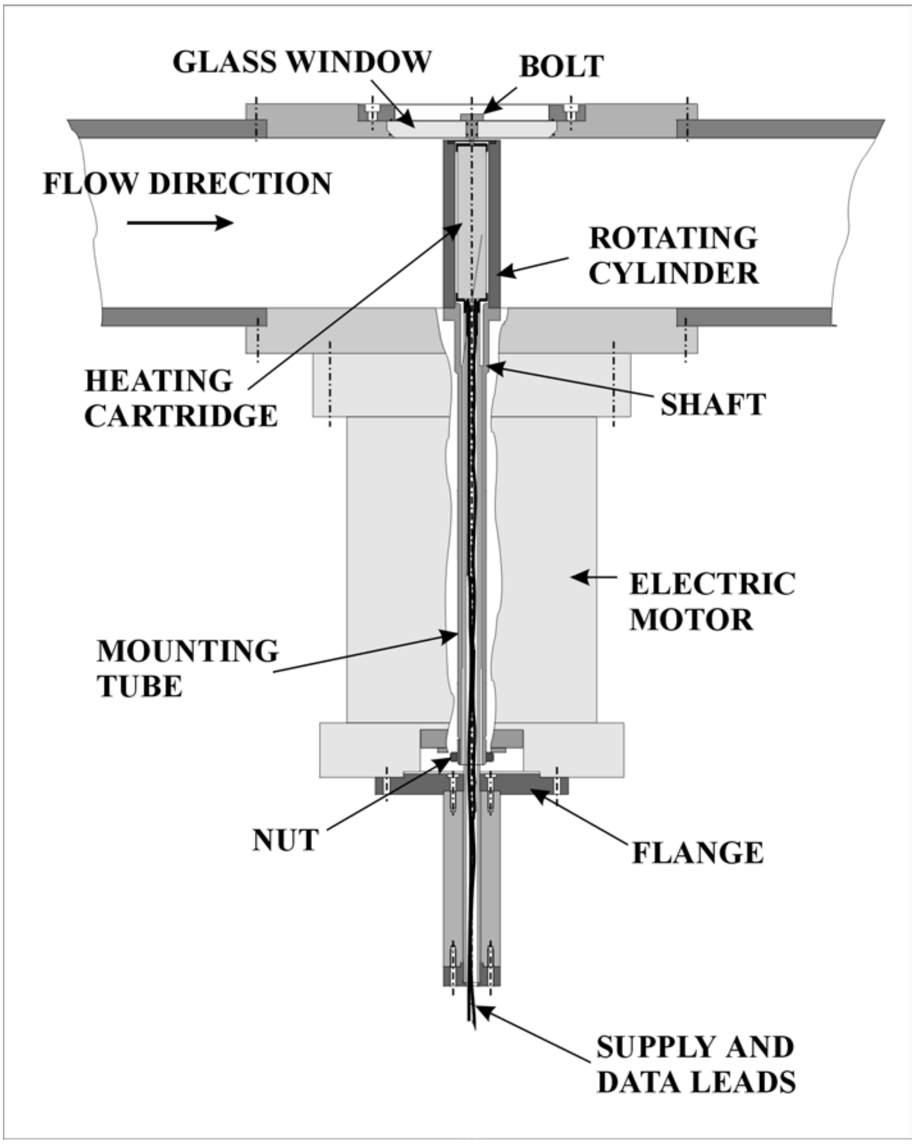



Figure 7 shows the set-up of the cylinder heating system. Mounted to the bottom of the electric motor is a massive flange which serves both for holding and leading a stainless steel tube through the hollow shaft of the rotor with a clearance of 2 mm. The heating device is soldered to the top end of the tube, whilst the respective supply and sensor cables are led down through the tube. Since the freely standing section of the tube is relatively long, the top of the heating device had to be positioned by a bolt which was inserted into the center of the glass-plate, to avoid swinging of the tube caused by vibrations. The heating cartridge was designed for 500 W and could be kept at constant temperature by a self-optimising controller. A thermocouple inside the heating cartridge measured the inner temperature and forwarded it to the controller. Since the thermal resistance between the thermocouple and the outside of the cylinder is relatively high and the heat control therefore reacts slowly, it was more convenient to run the device at constant voltage and to wait for temperature equilibrium. Temperature fluctuations could be tolerated, since only the temperature difference between the cylinder and the fluid was relevant, and this difference was optically measured instantaneously (in real time).

Three infra-red thermometers were set into the wall of the test-section to measure the mean wall temperature of the bottom, middle and top sections of the cylinder. With a focal length of 15 mm and a distance of half the test section width from the cylinder, each of the three thermometers covered exactly a third of the height of the cylinder. The three measured areas at the top, the middle and the bottom served to detect any temperature variation across the cylinder wall. By entering the absorptivities, the devices could be set to the surface to be measured. The calibration was performed with a certified Pt100 platinum resistance thermometer. The reliability of the sensors was confirmed in that all three devices had to be set to the identical absorptivity in order to return the reference temperature and the deviation of the sensors never exceeded 0.1 °C.

## *Experimental strategy*

Gerop´s theory was the keystone for the design of the experiments. Since it was the goal to confirm the Eckert number-phenomenon in practice, i.e. the decrease or even reversal in heat transfer, the experiments had to be carried out at Eckert numbers around unity. In order to reach this value, it was of critical importance to know that apart from the circumferential velocity of the cylinder, which was high but limited, an additional parameter was available in the form of the temperature difference between the cylinder wall and the fluid. For experimental reasons it was advantageous to choose this temperature difference to be as large as possible, which consequently meant operating at the maximum rotational speed of the experimental apparatus. With a maximum rotational speed of 30,000 rpm, the temperature difference between cylinder wall and surrounding fluid should not exceed 5 K according to Gerop's theory, in order to show the dependence of the Nusselt numbers on the rotational Reynolds number. The disadvantage of this procedure was that the absolute error of the temperature measurements was larger;

however, the advantage was that the temperature dependency of fluid properties could be neglected at these small temperature differences.

Usually, the heat transfer at a standing surface is described by the Nusselt number

$$Nu := \frac{hD}{k} \ , \qquad (3)$$

defined by the heat transfer coefficient $h$ according to Newton's law of cooling. However, Newton´s approach does not provide a reversal of the heat flow $\dot{Q}$ being

$$\dot{Q} = hA(T_W - T_\infty) \qquad (4)$$

at a constant temperature difference $(T_W - T_\infty)$, as both the area $A$ has a positive sign and the heat transfer coefficient $h$ is positively defined as the thermal conductivity of the boundary layer thickness $\delta$ by

$$h := \frac{k}{\delta} . \qquad (5)$$

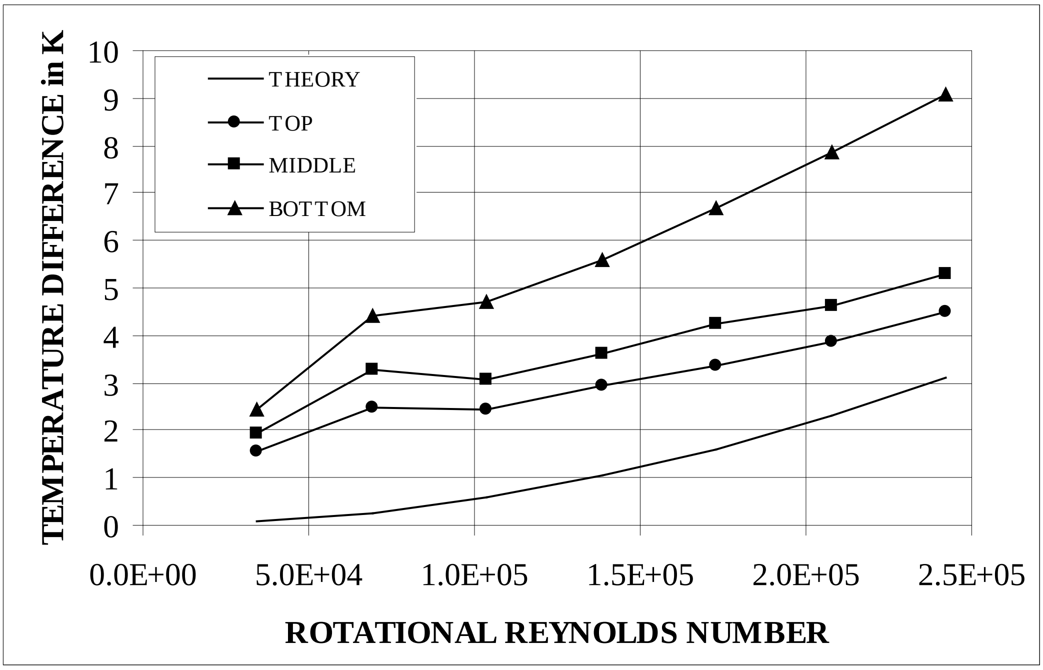
However, in the case of a moving wall the definition of the Nusselt number as

$$Nu := \frac{\dot{Q}}{d\pi l(T_W - T_\infty)} \qquad (6)$$

by means of the heat flow $\dot{Q}$, as can be derived from a dimensional analysis, also allows negative Nusselt numbers, since the heat flow $\dot{Q}$ can now change its direction and therefore its sign. With the definition by the heat flow $\dot{Q}$ in which dissipative effects of rotation are accounted for, now not only a local but also a global change of direction can be described. For the determination of the Nusselt number, therefore, three variables have to be measured with the heat flow $\dot{Q}$ and the two temperatures $T_W$ and $T_\infty$. Although the temperatures were able to be measured with infrared thermometers, the heat flow $\dot{Q}$ could only be determined indirectly.

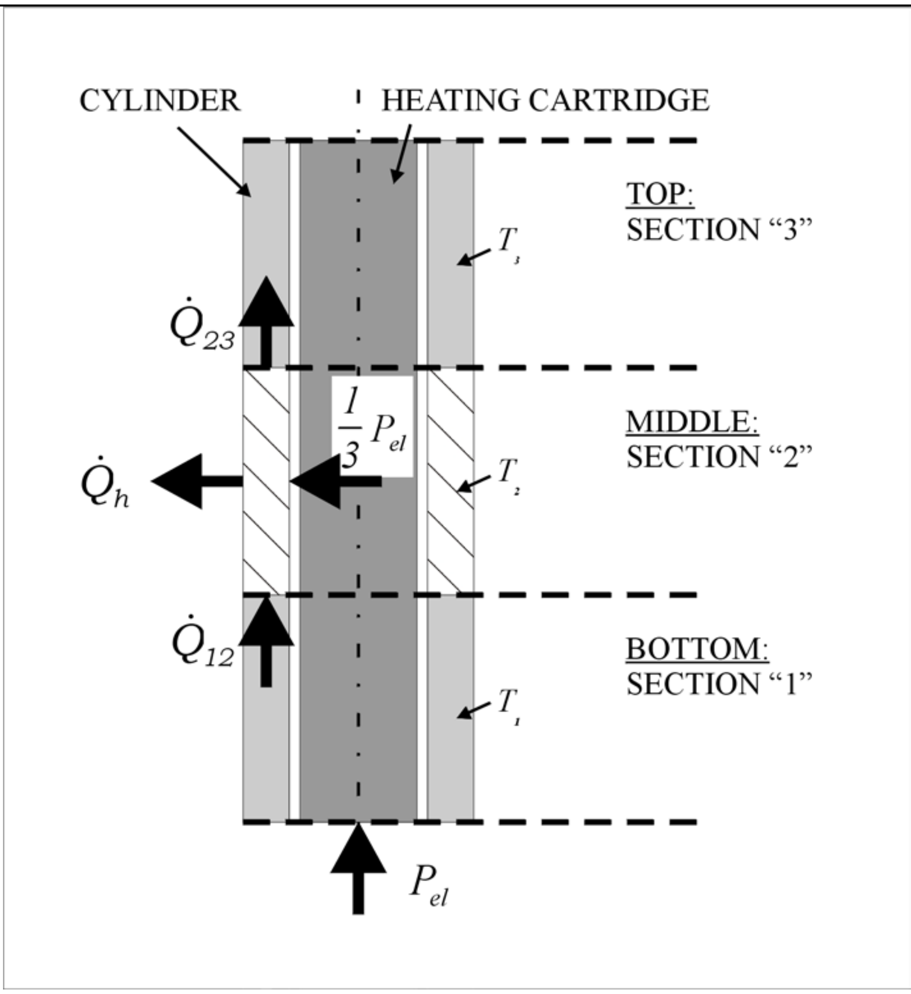
This was because, in the experiments, the waste heat of the electric motor played a non-negligible role. The axial conductive heat flow along the shaft on which the cylinder was mounted increased with rising speed and contributed additionally to the heating of the cylinder. Since only a few Watts were necessary for the desired small temperature differences, the contribution of this waste heat had to be taken into account. However, the problem was that this amount of heat could not be measured or isolated with comparative reference measurements since an additional speed-dependent effect was involved, namely the "self-heating" caused by dissipation in the boundary layer.

**Figure 8**




The solution to this problem resulted from the observed axial temperature difference at the cylinder wall. With increasing rotational speed, a gradient in the wall temperature can be observed from the bottom to the top. In figure 8 the temperature differences between the top, middle and bottom sections of the unheated cylinder towards the surrounding fluid are plotted against the rotational speed. Additionally, the theoretical "adiabatic wall temperature" is shown which the cylinder would take on if it were insulated. It is striking that all three cylinder temperatures resemble the theoretical curve. The increasing drift between the temperatures can be explained by the increased waste heat at higher speeds. The wanted heat flow $\dot{Q}_h$, transferred from the cylinder to the fluid, can be derived finally from an energy balance according to figure 9:

**Figure 9**




$$\dot{Q}_h = \frac{1}{3}P_{el} + \dot{Q}_{12} - \dot{Q}_{23}, \qquad (7)$$

with the added and deduced heat flows

$$\dot{Q}_{12} = kA_{cs}\frac{(T_1 - T_2)}{\frac{1}{3}l} \qquad (8)$$

and

$$\dot{Q}_{23} = kA_{cs} \frac{\left(T_2 - T_3\right)}{\frac{1}{3} l}, \qquad (9)$$

with $A_{cs}$ being the cross-sectional area of the cylinder-ring section, $l$ being the length of the cylinder and the electric power $P_{el}$ being supplied to the heating cartridge. For the heat flow $\dot{Q}_h$ it follows finally that:

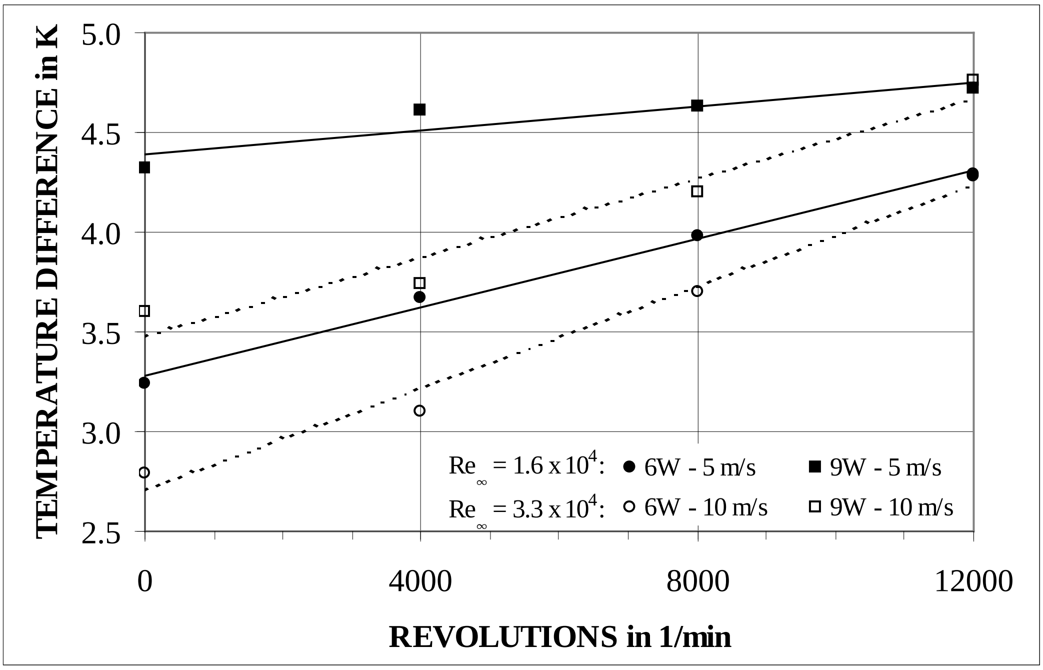
$$\dot{Q}_h = \frac{1}{3} P_{el} + \frac{3kA_{cs}}{l}\left(T_1 - 2T_2 + T_3\right). \qquad (10)$$

The strategy for the heat transfer experiments was that a constant heating rate was set until temperature equilibrium was reached. This solved both the controlling problem due to thermal inertia of the heating system, and also the case of a constant wall heat flux seemed to be closer to reality than a constant wall temperature. During the heating phase, the three cylinder temperatures and the fluid temperature were recorded during the approach to thermal equilibrium.

## Experimental results

The heating rate was increased in 3 W steps from 0 to 12 W, and the rotational speed of the cylinder was increased in steps of 4000 rpm. The flow velocity was varied in the lower range of  cylinder speeds, in order to show that beyond a certain velocity ratio $\Omega$ ($\Omega$ being the ratio of circumferential velocity to the main flow velocity) rotation dominates the heat transfer in contrast to forced convection. In figure 10, the temperature difference is shown as a function of the cylinder speed for two different heating rates. The solid line represents a flow velocity of 5 m/s whereas the dotted line represents 10 m/s. It can be seen that in the range between 8,000 and 12,000 rpm, which is equivalent to velocity-ratios $\Omega$ of 2 to 3 at a flow velocity of 10 m/s, the respective measured temperature differences converge to the same value. This confirms the observation of various authors that the flow velocity loses influence on the heat transfer above a velocity ratio $\Omega$ = 2. All further experiments were therefore carried out at a flow velocity of 10 m/s.

## Figure 10




To be able to make a comparison with Geropp´s prediction of the dependence of the Nusselt number on the temperature difference $(T_W - T_\infty)$, similar temperature differences had to be selected from the array of measured values. The aim was to collect data to show this Nusselt number dependence at temperature differences $\Delta T$ = 1, 2 and 5 K. To do so it was necessary to interpolate the obtained data points in a linear function.

## Figure 11

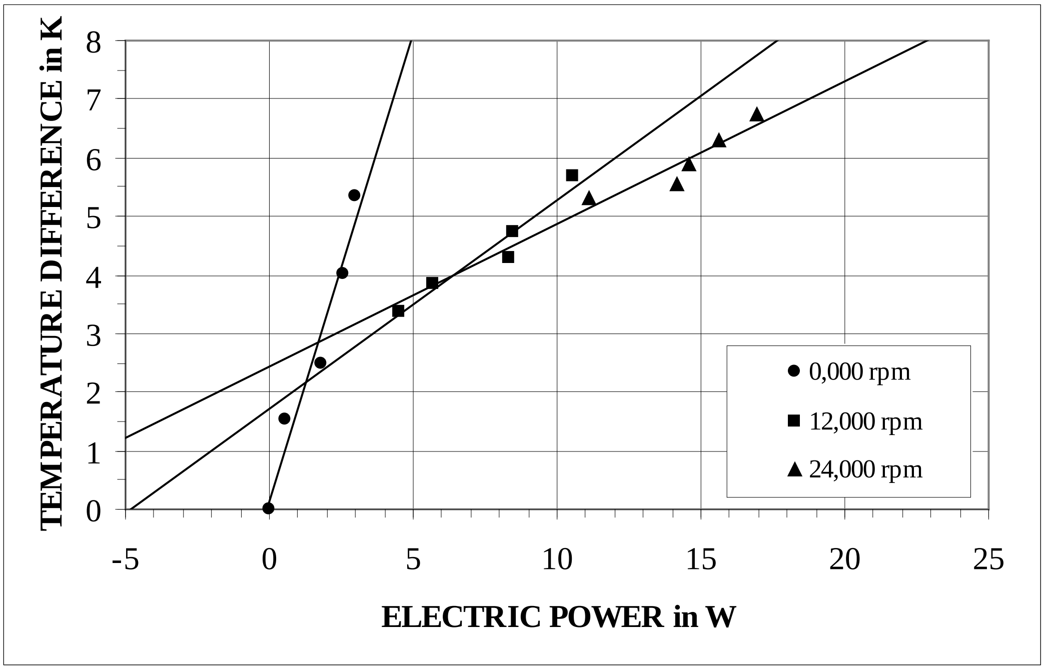
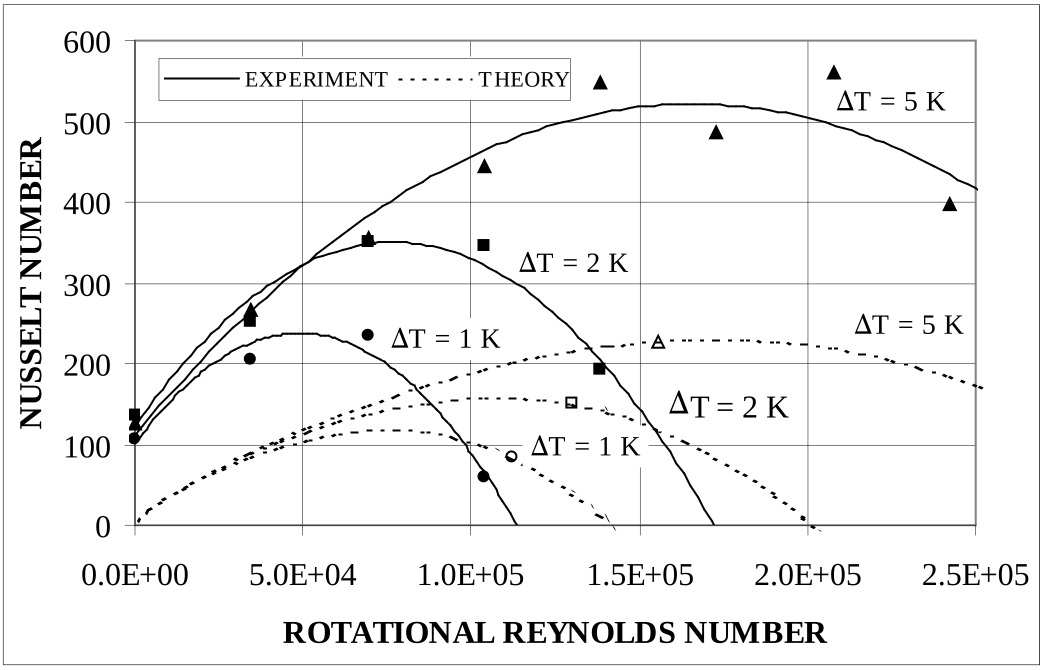



Figure 11 shows the measured temperature differences for three rotational speeds and their linear trendlines. The respective heating rates were determined using an energy-balance on the middle section of the cylinder as mentioned above. It should also be mentioned that the negative values in figure 11 can only be realised with cooling the cylinder. However, it can be concluded from the form of these functions that an extrapolation of the data taken in the range of added heat down to the range of rejected heat (negative heating rate $\dot{Q}$) is justified. This is also supported by the reference case of the non-rotating cylinder, in which the function goes through the origin of the coordinate system as no additional heat is created by dissipation in this case.

**Figure 12**




The calculated Nusselt numbers derived from the three temperature differences are compared with the theoretical ones in figure 12. By comparing the experimental results with the theory the following statements can be made:

- The overall trend of the measured values is similar to the theoretically predicted ones, which confirms the existence of the Eckert number-phenomenon with its reversal in heat transfer.
- The measured values are twice as high as the theoretical ones.

In order to explain this discrepancy it is useful to look at the heat transfer measurements of the non-rotating cylinder as a reference, to exclude any systematic errors in the experiments. To do so, the measured Nusselt number of *Nu* = 125 at a Reynolds number $Re_\infty = 1.6 \times 10^4$ has to be corrected to the theoretical case of an infinitely long, freely rotating cylinder in an ideal crossflow in order to be comparable at all.

Using Morgan´s [8] correlation

$$\frac{\Delta Nu}{Nu_{Tu=0}} = 2{,}42(Tu)^{2/3} \qquad (11)$$

for the influence of the degree of turbulence of the crossflow, an increase of the Nusselt number by 28 % is the result at a flow velocity of 5 m/s with a degree of turbulence of 4 % measured by LDV.

A cylinder diameter of $D$ = 50 mm and a length of $l$ = 148 mm yields an aspect ratio of approx. 3. Referring to Quarmby and Al-Fakhri´s [9] suggested correlation, the given geometry increases the heat transfer by 5 % in contrast to the infinitely long cylinder.

Of critical influence on the Nusselt number is the blockage ratio which is defined by the ratio of cylinder diameter $D$ to the width $W$ of the test-section. Morgan [8] therefore proposes the correlation

$$\frac{\Delta Nu}{Nu_{D/W=0}} = \left(\frac{v_{corr}}{v_\infty}\right)^{0,633} - 1, \quad (12)$$

in which $v_{corr}$ and $v_\infty$ are the corrected and measured velocities respectively. Employing this ratio for Hiwada´s [10] approach

$$\frac{v_{corr}}{v_\infty} = \left(1 + \left(\frac{D}{W}\right)^{1/2}\right)^{5/4} \qquad (13)$$

the heat transfer is improved by 27 % at a given blockage ratio of 12.5 %. However, these three influences are not independent of each other. As a first order approximation, however, it can be assumed that their influences can be added together. Thus, the Nusselt number is increased by 60 %. This means that the measured Nusselt number of 125 should be corrected down to a value of $Nu$ = 78. According to Churchill and Bernstein [11], a theoretical Nusselt number of $Nu$ = 71 can be expected for the given Reynolds number $Re_\infty = 1.6 \times 10^4$. A comparison of the corrected Nusselt number and the theoretical one shows good agreement, especially since the influence of free convection and surface-condition of the cylinder were neglected.

This reduction of a measured value to an ideal case, which allows the comparison with other experiments, demonstrates quite vividly what influence each flow-parameter has on the heat transfer. However, the application of the above demonstrated correction is impossible for the rotating cylinder as no investigations of the influence of external parameters exist in the case of rotation. Even so, it seems plausible that for the same reason external parameters (degree of turbulence, geometry etc.) contribute to an increase of Nusselt numbers, in contrast to theory, in the case of the rotating cylinder as they do for the non-rotating cylinder.

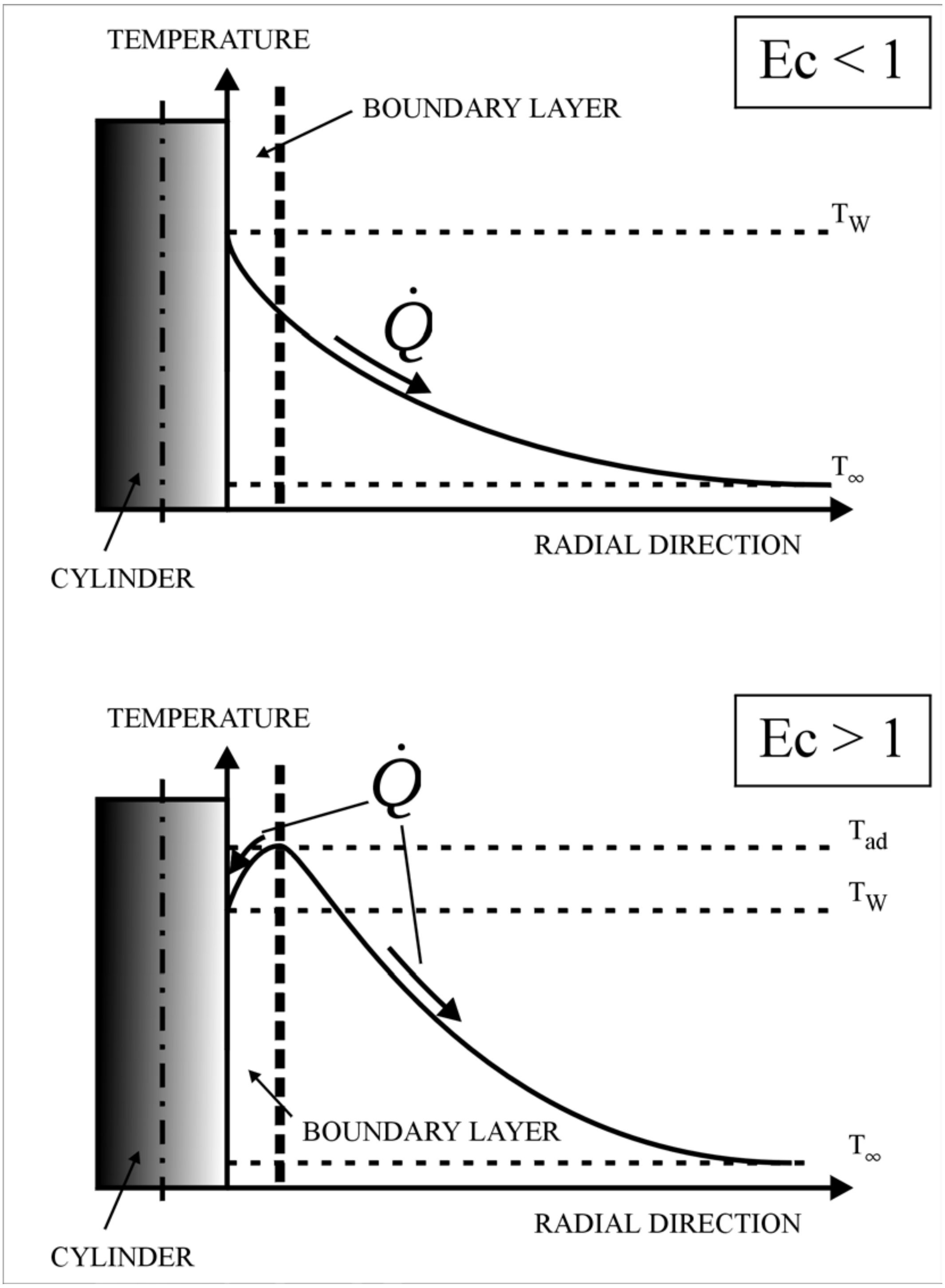
The scattering of the measured values also needs closer investigation. For the determination of the Nusselt numbers shown in figure 12, it was necessary to extrapolate the supplied heating rates down to lower temperature differences (figure 11). The relatively small scattering of the Nusselt numbers obtained at $\Delta T = 5$ K indicates that part of the error is caused by extrapolation, since the experiments were mainly carried out temperature differences around 5 K. The measured values can be approximated by a polynomial function of second order. However, because of the scattering of the values, such functional fitting also has to be viewed with scepticism. In other words, a comparison of occurring maxima and zero-crossings of the experimentally obtained functions and the theoretical ones seem dubious. Despite these reservations, however, it is important to note that after reaching the maximum, a decrease in heat transfer occurs in accordance with the predictions of Geropp´s theory.

## A scenario for the Eckert number phenomenon

Concluding from Geropp's considerations and the supporting experiments, the following scenario can be established for the heat transfer of a cylinder with constant temperature difference $(T_W - T_\infty)$ and increasing rotational speed: Initially rotation has a positive effect on the heat transfer. However, with increasing circumferential speed, the sheer stress due to viscosity of the fluid creates more and more dissipation. The maximum heat transfer occurs when the latter effect is still small while the former continues to increase. With a further increase of the rotational speed, however, dissipation plays the dominating role. In this regime the location where most of the dissipation occurs can be regarded as a local heat source. Since dissipation takes place mainly where the greatest velocity gradients are, this location is not situated at the wall where the fluid adheres, but in the boundary layer. In other words, the boundary layer encloses a virtual, concentric area around the cylinder which takes on the " adiabatic wall temperature " $T_{ad}$ . Figure 13 illustrates radial temperature profiles at a heated cylinder. The diagram at the top represents the ordinary case at Eckert numbers < 1, with the temperature decreasing from $T_W$ and asymptotically approaching ambient temperature $T_\infty$. With rotation and Eckert numbers > 1, however, dissipation within the boundary layer acts like a heat source at the temperature $T_{ad}$ and creates an additional temperature gradient towards the cylinder wall (diagram at the bottom of figure 13).

For the heat transfer, the consequence is that the cylinder does not "see" the temperature of the surroundings, that is $T_\infty$, but the thin annular area around the cylinder with the " adiabatic wall temperature " $T_{ad}$. With this temperature increasing, the effective temperature difference becomes smaller and so does the heat transfer. Once the " adiabatic wall temperature " reaches the value of the cylinder wall (at an Eckert number $Ec \approx 1$), the temperature gradient reverses and the cylinder is heated by the locally-created entropy production rate caused by friction within the boundary layer. At the same time, a temperature difference $(T_{ad} - T_\infty)$ exists for the surrounding fluid. Therefore a portion of the dissipation being produced within the boundary layer is also transferred to the fluid.

**Figure 13**




As a result of these considerations, it becomes clear why Eckert numbers greater than unity have never been reached in experiments. If one wants to keep the chosen temperature difference $(T_W - T_\infty)$ at a constant level, one has to start rejecting the entropy-rate transferred to the cylinder from $Ec \approx 1$ on, which is impossible without an active cooling system.

## Conclusions

1. Gerópp´s theory describes the turbulent heat transfer at a rotating, infinitely long cylinder in quiescent air. It can be concluded from his equations that a reversal of the heat transfer can be expected at an Eckert number $Ec \approx 1$, which is defined as the ratio of the specific kinetic energy and the specific enthalpy. Geropp suggests a boundary Reynolds number of $Re_\Omega \approx 6.9 \times 10^6$ for the Nusselt number to change its sign. From his analysis further conclusions can be drawn:
2. With the temperature difference $\Delta T = (T_W - T_\infty)$, an additional parameter is hidden in the Eckert number. This means that a general boundary Reynolds number does not exist, but there is an individual function of the Nusselt number for each temperature difference.
3. There is <u>one</u> function for the maximum Nusselt number and the respective Eckert number dependent on the temperature difference $(T_W - T_\infty)$. For practically relevant temperature differences, the maximum heat transfer occurs at Eckert numbers $Ec \approx 0.3$.
4. The “self-heating” is a process of its own and is only a function of the rotational Reynolds number. In other words, the creation of dissipation is independent of the temperature difference between cylinder wall and fluid, but the interaction of $\Delta T = (T_W - T_\infty)$ and the “adiabatic wall temperature difference” $\Delta T_{ad} = (T_{ad} - T_W)$ determines the heat transfer.
5. The definition of the Nusselt number via the heat flow $\dot{Q}$ as opposed to the heat transfer coefficient $h$, takes account for a change of the direction of the heat flow. Thus, not only a local but also a global change of the heat flow direction can be described.
6. Experiments qualitatively confirm Geropp´s theory and the existence of the Eckert number-phenomenon, although the measured values are nearly twice as high as the theoretical ones. However, looking at the non-rotating case as a reference, this discrepancy can be explained by well known effects like the degree of turbulence, blockage and aspect ratio which each increases the heat transfer.

## Captions to figures

Figure 1: The Nusselt number as a function of the rotational Reynolds number according to Geropp´s theory

Figure 2: Nusselt and Eckert numbers for $\Delta T$ = 1, 10 and 100 K according to Geropp´s theory

Figure 3: The functions Nu($Re_\Omega$) and Ec($Re_\Omega$) of figure 2 in a linear scale

Figure 4: The maximum Nusselt and Eckert number as a function of the temperature difference $(T_W - T_\infty)$

Figure 5: The "adiabatic wall temperature" $\Delta T_{ad}$ of the cylinder against the rotational Reynolds number

Figure 6: The wind-tunnel

Figure 7: Principle set-up of the heating system

Figure 8: The measured wall temperature of the cylinder dependent on the rotational speed

Figure 9: Energy balance at the middle section of the cylinder

Figure 10: The influence of the velocity ratio on the heat transfer

Figure 11: The measured temperature differences against the determined heating power for three rotational speeds at a crossflow Reynolds number $Re_\infty = 1.6 \times 10^4$

Figure 12: The measured Nusselt numbers with their trendlines (solid) at a crossflow Reynolds number $Re_\infty = 1.6 \times 10^4$ and the theoretical Nusselt numbers according to Geropp (dotted)

Figure 13: The temperature profile at a wall at $Ec < 1$ and $Ec > 1$

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