

The role of Kemeny's constant in properties of Markov chains

Jeffrey J Hunter

School of Computing and Mathematical Sciences, Auckland University of Technology, New Zealand
(email: jeffrey.hunter@aut.ac.nz)

Abstract:

In a finite m -state irreducible Markov chain with stationary probabilities $\{\pi_i\}$ and mean first passage times m_{ij} (mean recurrence time when $i = j$) it was first shown, by Kemeny and Snell (1960) that $\sum_{j=1}^m \pi_j m_{ij}$ is a constant, K , not depending on i . This constant has since become known as Kemeny's constant, (Grinstead and Snell (1997)). We consider a variety of techniques for finding expressions for K , derive some bounds for K , and explore various applications and interpretations of this result. Interpretations include the expected number of links that a surfer on the World Wide Web, located on a random page needs to follow before reaching a desired location (Levene and Loizou (2002)), as well as the expected time to mixing in a Markov chain (Hunter (2006)). Various applications have been considered including some perturbation results (Hunter (2006), Catral, Kirkland, Neumann and Sze (2010)), mixing on directed graphs (Kirkland (2010)), and its relation to the Kirchoff index of regular graphs (Palacois (2010), Palacois and Renom (2010)). In a further extension, it is shown that typically the variances of the mixing times depend on i , (Hunter (2008)).

Keywords:

Markov chains, Kemeny's constant, mixing times, perturbations, regular graphs

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