Application Problems vs Pure Mathematics Questions: Students' Perspectives

Sergiy Klymchuk, Kerri Spooner

Auckland University of Technology, New Zealand

The paper presents results of two case studies with undergraduate students majoring in applied mathematics and engineering. The first case study deals with students' preferences and difficulties with application problems and pure mathematics questions in their courses. The students were taking a second year course on ordinary differential equations (ODE). The students were asked about their preferences in dealing with application problems and pure mathematics questions and their difficulties with solving application problems not only in the ODE course but also in other courses. The second case study deals with students' preferences for application problems and pure mathematics questions in their first and second year engineering mathematics courses. The students were majoring in engineering. Students' responses are presented and discussed in the paper.

Introduction and Background Literature

1. Students' difficulties

University students' difficulties with solving application problems and more generally with mathematical modelling were analysed in a number of papers (e.g Chaachoua and Saglam 2006; Dominquez, Garza and Zavala, 2015; Kadijevich, 2009; Klymchuk & Zverkova, 2001; Klymchuk et al, 2010; Rowland and Jovanoski, 2004; Rowland, 2006; Soon, Tirtasanjaya Lioe, & McInnes, 2011).

Examination of the papers showed there are at least four main areas of difficulty. These areas are lack of strategies to approach problems, challenges with context of problems, a general inability to translate a problem situation from the real world to mathematics and difficulties with choosing and using appropriate formula.

Challenges with context of problems include language difficulties, limited understanding of the situation's context and limited translation skills between the context and mathematics. Studies have shown there is difficulty with interpreting the contextual language used to describe a situation (Kadijevich, 2009). Klymchuk et al (2010) found that just under half of the tertiary students in their study did not understand the wording of the problem. Kadijevich (2009) found that when year one tertiary students were not familiar with the context they made inappropriate assumptions concerning some of the important variables in the situation. Klymchuk et al (2010) and Soon et al (2011) discuss how some tertiary students do not know how to convert between words and mathematics. These challenges when present inhibit students from starting an application or modelling activity (Kadijevich, 2009).

A difficulty, which partially can be contributed to challenges with the context of a problem, is the struggle to choose and use appropriate formula. Soon et al (2011) demonstrate that even though students maybe familiar with formulas they have difficulty recalling them and do not know when to use them in an applied situation. To be able to choose and use the appropriate formula, links between the context and the mathematics need to be recognised. These relations involve knowledge concerning how physical processes, physics and mathematics of the particular context are interconnected. Studies involving tertiary students have demonstrated that students do not tend to link between physical processes, physics and mathematics (Chaachoua and Saglam, 2006; Dominquez, Garza and Zavala, 2015; Rowland and Jovanoski, 2004; Rowland, 2006). Chaachoua and Saglam (2006) carried out a study investigating linking physics and mathematics**.** The study showed that first year university students have weak links between mathematics and physical situations. Two questions were asked. The first question required mathematical procedural skills to be applied to a routine mathematics problem. The second question, to interpret the solutions from the first question in a real situation. Forty-six students attempted the first question and out of these only eight attempted the second question. Out of the eight who attempted the second question nobody went through the process of moving from a real situation to a mathematical model. The study showed that students carry out mathematical operations in isolation from applications. The absence of relations between mathematics and other situations within other disciplines, namely physics in this instance, shows that students are not developing transverse competencies. It appears that understanding the context of the problem and its connection with mathematics maybe a crucial area in reducing students' difficulties with application and modelling problems.

2. Why use applications?

What is the purpose of including applications and modelling problems in a mathematics course, in this case an ODE course? It would stand to reason that a course in applied mathematics by mere implication of the word 'applied' would include applications or at the very least mathematics that is useful for applying to real situations. This aside, one of the main reasons is to demonstrate how mathematics is linked to real situations. Context based applications allow students to encounter the way abstract concepts are connected to practical examples and allows mathematics to becomes real, providing a hands on experience for concepts to become tangible mathematical ideas (Dindyal & Kaur, 2010; Julie, 2002). This is not to say that mathematical ideas removed from context are intangible. Modelling and the use of applications is about making connections between the real world and mathematics, bringing mathematics into the student's world (Dindyal & Kaur, 2010; Julie, 2002). This all contributes to understanding the concepts of mathematics relevant to the problem and provide experience of using mathematics to explain things (Treilibs, Burkhardt, & Low, 1980). Armstrong and Bajpai (1988) believe that without sufficient practical experience pupils are unable to relate abstract mathematical concepts to any form of reality. The use of applications provides one way of doing this. "Applications and modelling activities should be seen as embedded within mathematics (and thus the curriculum) and then together the two become an effective lens for describing and analysing the real world" (Stillman, 2007, p. 466). Using real applications helps students to understand the abstract concepts, giving concrete examples to explain things through.

By using real applications to make connections between mathematical ideas and the real world we are developing mathematically versatile citizens. This is in line with mathematical practices in real life and the work place, which is context specific and applied to real situations (Julie, 2002; Robitaille & Dirks, 1982). The importance of building competent mathematical citizens is imperative in the survival of the commercial world (Lingefjard, 2006).

From a student perspective, the use of applications provides real world contexts which can provide motivation to explain and learn mathematics. Treilibs et al (1980) believe that the use of realistic problems provides extra motivation for studying and acquiring mathematical skills. By studying real life situations that students are familiar with, we can provide for some students more motivation for engagement compared to presenting abstract skills out of context (Kaiser, Lederich & Rau, 2010; Treilibs et al, 1980). That is not to say that abstract mathematics cannot be taught in motivating ways. In Germany there are some educational groups which have implemented genuine modelling. Results from one such group found that over “the course ……. it was found that students who were initially sceptical about the problem and mathematics in general abandoned their existing notions and prejudices and became significantly involved in the solution of the problem” (Kaiser et al, 2010, pp. 236-237). As we develop models there is increasing student engagement with mathematics as its need becomes apparent. The search to find the mathematics to model and solve the problem provides the motivation to learn further mathematics (Lingefjard, 2006; Rowland & Jovanoski, 2004) with the situation itself providing motivation to learn the mathematics necessary to understand and solve it. Particular contexts can provide motivation for the study of some mathematical disciplines, for example modelling a parachute jump can provide the context for learning kinematics (Lingefjard, 2006).
While real life context can provide motivation for some students there is evidence that this is not the case for all students. While there is research showing familiar contexts that are part of students’ life experiences can have a positive effect on engagement (Kaiser & Schwarz, 2006) other research suggests over-familiar contexts having a negative effect (Busse, 2011). A study conducted by Busse (2011) showed contexts were interpreted very individually. This is supported also by Treilibs et al (1980). A study by Kaiser and Schwarz (2006) also showed that for some students the absence of a unique solution, common in real world problems, and the need to include common sense is seen as an additional barrier having a negative effect on engagement (Busse, 2011).

This paper presents results of two case studies with undergraduate students majoring in applied mathematics and engineering. The students were asked about their preferences in dealing with application problems and pure mathematics questions and their difficulties with solving application problems

Case Study – 1

The first case study involved seven undergraduate students majoring in applied mathematics from a New Zealand university. At that university there were three other majors in the mathematical sciences – analytics, computer science, and astronomy. There was not pure mathematics major. The students were taking a second-year course on ordinary differential equations (ODE). The course covered the three main approaches in teaching ODE – graphical, analytical and numerical. It contained about two thirds pure mathematics questions and one third application problems. The research question was to check whether students majoring in applied mathematics preferred application problems to pure mathematics questions and identify their difficulties in solving application problems.

On the day of the data collection (a questionnaire) seven out of eight students were present in the class. Three students were female and four male. Three students were domestic and four international. Students’ average grades in all previous courses ranged from B (good) to A+

(excellent). All seven students answered the questionnaire and gave their comments. The participation in the study was voluntary and the response rate was 100%.

The following (anonymous) questionnaire was given to the students.

1. The questionnaire

Question 1. What questions are more interesting for you to solve - pure mathematics questions or application problems? Why?

Question 2. What questions are more difficult for you to solve - pure mathematics questions or application problems?

Question 3. What are your main difficulties (if any) in solving application problems?

Question 4. What do you do to overcome those difficulties?

Question 5. What is approximately your average grade in university mathematics courses so far (e.g. A+, A, A-, B+, B, B-, etc.)?

Question 6. What is your gender?

Question 7. Are you a domestic student or an international?

2. Results and Discussion

Question 1. What questions are more interesting for you to solve - pure mathematics questions or application problems? Why?

Our expectations were that the students would be more interested in solving application problems rather than pure mathematics questions taking into account their major. However, four out of seven students reported that pure mathematics questions were more interesting for them. The comments were: "I like seeing and learning mathematical concepts at their most 'abstracted' form"; "I am more interested in just calculating – not how we can apply"; "because they are easier to understand and tell you exactly what to do, and once you know what you are doing, solving problems become more interesting"; "I guess that I would like to concentrate on the mathematical structures without the distraction of the application details (for now)". The three students who preferred application problems made the following comments: "Because application problems can be related to the real life, and help me to understand the mathematical theory"; "although I love pure mathematics I find an application easier to understand. I like to know where it used in the 'real' life"; "I was never taught pure mathematics properly, so I don't have a good base, so it takes me a long time to grasp even simple stuff".

Question 2. What questions are more difficult for you to solve - pure mathematics questions or application problems?

Three out of seven students reported that application problems were more difficult for them to solve than pure mathematics questions, for two students pure mathematics questions were more difficult and the remaining two students did not see any difference ("I am fairly OK with both"; "I don't think either of them are more difficult").

Question 3. What are your main difficulties (if any) in solving application problems?

There were two categories of difficulties. The first category was reported by three students and it was related to the language or wording of the problem. The comments were: "For the application problems I need to understand the meaning of the words fully and get fully meaning of the questions. Sometimes I cannot get the full idea of the meaning of the application problems, and may miss some words"; "I feel difficult to get implication of the statement, it is difficult for me to find relationship between numbers and real situation"; "interpreting the problem. Trying to figure out what exactly the question wants us to find out". The second category was reported by four students and it was dealing with special knowledge, e.g. applying necessary formulas/laws. The comments were: "When the application is in a field I am not terribly familiar with, e.g. physics, especially electrical circuits where I have no idea what units/symbols/terminology are"; "remembering the right formula to apply to the application problem"; "remembering small rules"; "actually, the only application problems that I found too difficult were those related to electrical engineering, where I have no knowledge of the subject".

Question 4. What do you do to overcome those difficulties?

Students' comments to this question fell into three categories. The first was about 'more practice' and five students made the following comments about it: "Practice more and more a lot, and read the question carefully each time, or ask my classmates to get some ideas"; "I try to do more application problems to gain practice and get familiar with such problems"; "simply keep studying until I can see the problem clearly"; "I often look through as many worked examples as possible"; "practice, practice, practice. I believe that to overcome difficulties in maths you need to practice". The second category was dealing with formulas and there were two comments: "usually I can just follow the given formula in the question so it's not too much of an issue"; "I carry around a formula sheet with all sort of formulas". The third category related to getting help from others: "Ask other people to get idea of how to answer and how to think"; "ask my classmates to get some ideas".

Contrary to our expectations, more than half of the students reported that they were more interested in solving pure mathematics questions rather than application problems. This is in spite of the fact that the students were majoring in applied mathematics. Partially it can be explained by the absence of pure mathematics major at that particular university and some students did not want to enrol to another university where pure mathematics major was offered. Those students liked calculations and the abstract nature of pure mathematics according to their comments. The remaining minority of the students were more interested in solving application problems due to connections to 'real life'. In terms of difficulty to solve, the comments were almost evenly distributed: three for application problems, two for pure mathematics questions and two with no difference. Regarding their difficulties in solving application problems the comments were also split almost evenly: three were related to the wording and four to special knowledge of a particular area of application. To overcome difficulties associated with solving application problems the vast majority of the students (five out of seven) indicated more practice as a remedy.

Case Study – 2

The second case study involved 80 engineering students: 51 participants were taking first year engineering mathematics course (mainly single variable calculus) and 29 participants were taking second year engineering mathematics course (mainly multivariable calculus). Each course contained about three quarters of pure mathematics questions and one quarter of application problems. The research question was to check whether students majoring in engineering preferred application problems to pure mathematics questions. The participation in the study was voluntary and the response rate was 96%. A combination of a convenience and judgement sampling methods was used to select the participants.

The participants were asked the following questions.

1. First year engineering mathematics course

Which questions do you prefer to solve: application problems or pure maths questions?

For example:

<u>An application problem:</u>

The rate of decay of the mass of a radioactive substance is proportional to the mass of the substance present. Initially the mass of the substance was 165 mg. At the end of 1.5 years, 112 mg remain. How much will there be after 3.5 years?

<u>An equivalent pure maths question:</u>

Find the solution of the initial value problem $\frac{dy}{dx} = ky, y(0) = 165$ at the point $x = 3.5$.

Please circle one: Application problem Pure maths question

Why? Please circle all that apply:

More relevant More interesting More useful Easier Other (please specify)

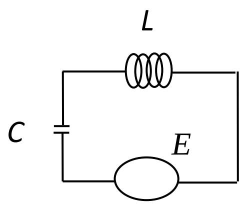
2. Second year engineering mathematics course

Which questions do you prefer to solve: application problems or pure maths questions?

For example:

<u>An application problem:</u>

An electric circuit consists of an inductor (with inductance L = 1 henry), a capacitor (with capacitance C = 0.0625 farads), and a power supply $E(t)$ = 8 volts at time t seconds, $t \geq 0$. No current is flowing in the circuit at time t = 0. Use Laplace transforms to find $i(t)$ - the current in this circuit at time t. Use the Kirchoff's second law: $L\frac{di}{dt} + \frac{1}{C}\int_0^t i(\tau)d\tau = E(t)$.



An equivalent pure maths question:

Use Laplace transforms to solve the integro-differential equation $\frac{dy}{dx} + 16\int_0^x y(s)ds - 8 = 0$ with the initial condition $y(0) = 0$.

Please circle one: Application problem Pure maths question

Why? Please circle all that apply:

More relevant More interesting More useful Easier Other (please specify)

3. Results and Discussion

As the question was the same for both groups - first year and second year students (just different examples of mathematics problems due to the level of the courses) we combined students' responses. Surprisingly more than half of engineering students preferred pure maths problems to application problems (43 versus 37). Their reasons are presented in table 1.

Reasons	Application Problems	Pure Maths Questions
More relevant	17	12
More interesting	19	2
More useful	12	11
Easier	5	25
Other		2

Table 1. Students' preferences of solving problems in their engineering mathematics courses.

Students' preference to pure maths problems most likely was dominated by the common belief that pure maths version of an application problem is easier to solve. It was reported by 25 students compared to only 5 students who indicated that applications problems are easier for them than pure maths questions. This is in spite of the fact that students found application problems more relevant to them (17 vs 12), interesting (19 vs 2) and useful (12 vs 11).

Common comments of the students who preferred pure maths questions to application problems are as follow: "We appear to be mainly taught and given examples in the 'pure' format. The 'application' format is intimidating given the lack of experience"; "Application problems are prejudiced against those who have limited knowledge of application subject"; "Several disciplines do engineering mathematics-2 paper including mechanical, electrical, fluid mechanics, etc. Having a pure maths paper will mean everyone is assessed fairly"; "Pure maths questions are more to the point and do not confuse students"; "We have to take in so much in such a short time. Having to then apply that to real world problems does not increase our understanding of solving equations. It only increases our subject matter"; "If taught the application way that would be fine for an assignment but definitely not for an exam".

Common comments of the students who preferred application problems to pure maths questions are as follow: “You are able to apply the maths you are learning to real world circumstances, and relate the maths you are learning”; “”Because we don’t do so many application problems I don’t feel I understand them. If we did more I/we would get better as doing them. I also think they are more relevant”; “Application problems are explained in English rather than maths language which is easier to late and understand”; “But it would help if the first questions of the particular section isn’t very complicated. Wording must be perfect”.

Conclusion

Taking into account that students were majoring in applied mathematics and engineering it was unexpected to see that more than half of the participants in both case studies preferred pure maths questions to application problems. One of the main reasons was that students find pure maths questions are easier to solve than application problems even when an application problems is mathematised to a large extend. This is partially due to the language difficulties in the application problems. This is consistent with another study that involved about 200 first-year engineering students from New Zealand and Germany (Klymchuk et al, 2010). About 95% of the students in that study were not able to solve a routine application problem on a test because of the unusual wording of the problem. In some cases special knowledge can be attributed to a lack of understanding of the context of the application problem.

Application is more than just applying a skill set. It is also about understanding the context so that appropriate skills can be applied accordingly. A deeper understanding of the situations is needed rather than more practice of ‘types’ problems. With more practice we run a risk of the application problem becoming a routine problem. Hence the opportunity to choose and apply the appropriate knowledge to a new application is lost. A deeper appreciation of context will enable links between the situation and its relevant formula to be established. Original applications will become accessible to students. The understanding of context allows mathematics to become connected to the real world and no longer an isolated, abstract activity. Although this approach is well established by the Realistic Mathematics Education (RME) framework (e.g. Gravemeijer, 1994; Gravemeijer and Doorman, 1999) it still requires more research.

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