



AUT

School of Economics

Working Paper Series

**Liquidity, Monetary Policy and Unemployment: A
New Monetarist Approach**

Mei Dong & Sylvia Xiaolin Xiao

2017/07

Liquidity, Monetary Policy and Unemployment: A New Monetarist Approach*

Mei Dong[†] Sylvia Xiaolin Xiao[‡]

September 18, 2017

Abstract

We discover a consumption channel of monetary policy in a model with money and government bonds. When the central bank withdraws government bonds (short-term or long-term) through open market operations, it lowers returns on bonds. The lower return has a direct negative impact on consumption by households that hold bonds, and an indirect negative impact on consumption by households that hold money. As a result, firms earn less profits from production, which leads to higher unemployment. The existence of such a consumption channel can help us understand the effects of unconventional monetary policy.

JEL classification: E24, E40, E50

*We would like to thank Chris Edmond, Pedro Gomis-Porqueras, Susumu Imai, Benoit Julien, Chao Gu, Tim Kam, Gordon Menzies, Guillaume Rocheteau, Jeff Sheen, Chi Chung Siu, John Wooders, Stephen Williamson, Randall Wright, Yajun Xiao and seminar participants at Deakin U., Monash U., RMIT, Tsinghua U., U. of Adelaide, U. of Melbourne, U. of New South Wales, U. of Queensland, U. of Technology, Sydney, U. of Wisconsin-Madison, FRB of St. Louis, the 26th Australian PhD Conference in Economics and Business, Sydney Macro Reading Group and 2014 Chicago Fed Summer Workshop on Money, Banking, Payments and Finance for helpful suggestions and comments. Dong acknowledges financial support under Australian Research Council's DECRA scheme (project number: DE120102589). Xiao acknowledges the hospitality and support during her visits at U. of Wisconsin-Madison, FRB of St. Louis and U. of Melbourne during Jan., 2014 to June, 2015, and the funding support from Dan Searle Fellowships from August, 2015 to August, 2017.

[†]Department of Economics, University of Melbourne; Email: mei.dong@unimelb.edu.au.

[‡]School of Economics, Auckland University of Technology; E-mail: sylvia.xiao@aut.ac.nz.

Key words: interest rate, monetary policy, consumption, unemployment

1 Introduction

We develop a model with money and government bonds to study how a change in the supply of government bonds through open market operations (OMOs) affects consumption and unemployment through a consumption channel. Conventional monetary policy generally targets some short-term interest rates by conducting OMOs. During the recent Great Recession, targeted short-term interest rates in several advanced economies have been cut close to zero.¹ This limits a central bank's ability to further lower the short-term interest rate to stimulate the economy. Instead of targeting short-term interest rates, the central banks of the US, Japan and some European countries all conducted *unconventional monetary policy* by either purchasing long-term government bonds or other government-guaranteed private securities in financial markets. The goal is to directly lower long-term interest rates in financial markets.

An important feature of conventional and unconventional monetary policy is that they are essentially about adjusting the supply of government bonds through OMOs. Hence, it is necessary for a model to have money and bonds in order to understand the effects of such policy. A few recent papers in monetary theory including Williamson (2012) and Rocheteau et al. (2017) consider multiple assets, but most of these models do not have unemployment. Given that one of the US Federal Reserve's mandated objectives is the achievement of "full employment," we integrate a labor market model with genuine unemployment into a microfounded monetary model with multiple assets.

¹In 2008, the US Federal Reserve cut the Federal Fund Rate to zero. This zero-lower-bound problem is also observed in Japan and various European countries. In Japan, as early as in 1995, the Bank of Japan cut the short-term interest rate almost to zero, which lasted until 2016. After the Great Recession, the European Central Bank cut the short-term target rate to zero.

There are two key elements of our model. The first is that money and bonds are valued by households because they can facilitate transactions in the goods market. The coexistence of money and bonds makes the model suitable to consider OMOs as central bank's swaps of government bonds and money. Monetary policy can affect households' portfolio decisions if the policy changes the relative return of these assets. The second key element is that the labor and goods markets are connected. Firms bring production from the labor market for sale in the goods market. Households can purchase goods for consumption in the goods market using money or government bonds. This link between the labor and goods markets provides a channel through which monetary policy affects unemployment.

Our model builds on Berentsen et al. (2011). We first add short-term government bonds in addition to money. In such an environment, the central bank can adjust the supply of short-term bonds as a monetary policy instrument, i.e., OMOs. We find that different cases of monetary equilibrium exist depending on the relative supply of short-term bonds. OMOs can affect the economy only when the supply of bonds is scarce, but not too scarce. In this case, the return on bonds is higher than that on money. Households that have access to bonds use only bonds to trade in the goods market, and households that do not have access to bonds use money. When the central bank reduces the supply of government bonds by purchasing bonds, the price of bonds increases and the short-term interest rate decreases. For households that use bonds, the lower interest rate directly induces them to hold fewer bonds and consume less. As a result, firms' profits from selling to these households decrease. In the labor market, lower profits discourage firms from entering and raise unemployment. In the goods market, households face fewer trading opportunities and this will lower the marginal benefit of holding money. This general equilibrium effect indirectly makes households that use money hold less money and consume less, which further lowers firms' profits and raises unemployment.

We focus on the effects of OMOs through a consumption channel. That is, the change in the supply of bonds affects the return on bonds and consumption by bond holders in the goods market. This in turn can affect labor market outcomes, which further affects the return from holding money. Therefore, real balances also respond to OMOs. Our model provides a clear transmission channel of OMOs to the real side of the macroeconomy. A permanent decrease in the supply of government bonds has a negative impact on employment. The conventional view is that a central bank's purchase of government bonds would lower interest rates and thus stimulate investment. Our model does not have this investment channel. As this investment channel has been identified in recent studies such as Rocheteau and Rodriguez-lopez (2014), we argue that the consumption channel is complementary to the investment channel.

During the Great Recession, several central banks choose to purchase long-term government bonds or other government-guaranteed private securities. To address the effects of this unconventional policy, we extend the basic model by adding long-term government bonds. We consider the long run effects of unconventional policy where the central bank changes the supply of long-term government bonds. When the short-term interest rate is close to zero, the central bank can buy or sell long-term government bonds to directly adjust the long-term interest rate. Through the consumption channel, unconventional policy lowers the long-term interest rate and households' consumption. As a result, equilibrium unemployment increases. We find that a positive lower bound on the long-term interest rate exists because long-term bonds are less liquid than short-term bonds.

We contribute to the monetary theory literature by providing a framework to analyze how monetary policy especially OMOs affects unemployment. We find that OMOs may not affect consumption and unemployment when the supply of government bonds is either too low or too high. When OMOs have a real effect, the consumption channel indicates that the central bank's withdrawal of either short-term

or long-term government bonds lowers consumption and raises unemployment. Moreover, there exists a positive lower bound on the long-term interest rate if a central bank changes the supply of long-term bonds.

Our paper is related to two broad lines of literature. The first line uses search and matching theory to integrate the goods and labor markets. In Berentsen et al. (2011) and Bethune et al. (2015), a medium of exchange is essential to facilitate transactions in the goods market. Monetary policy is modeled as adjusting the growth rate of money supply. These models provide implications on the effects of monetary policy on unemployment. We introduce government bonds so that we can consider OMOs as an alternative monetary policy tool.

There is also a recent growing literature that studies the interaction between the product and labor markets. For example, Kaplan and Menzio (2016) show how multiple equilibria arise when employed workers and unemployed workers have different shopping patterns. They find that high unemployment can be a self-fulfilling outcome. See also Bai et al. (2017) and Hall (2017). Relative to this literature, we introduce money and bonds into the product market in order to address how monetary policy affects the interaction between the goods and labor markets.

The second line of research involves microfounded models of assets and liquidity. Williamson (2012) and Rocheteau et al. (2017) are most closely related to our paper. Williamson (2012) develops a model with money, government bonds and private equity to study the effects of both conventional and unconventional monetary policy. He models unconventional monetary policy as purchasing private equity. Rocheteau et al. (2017) focus on conventional OMOs and show how market structures and liquidity properties of money and bonds matter for understanding the effects of OMOs. See also Mahmoudi (2013) and Williamson (2013) for more references. However, these papers do not consider unemployment.

Rocheteau and Rodriguez-lopez (2014) build a model with an over-the-counter

financial market and a labor market. The model includes various types of assets with different acceptability, i.e., money, government bonds and private assets, and distinguishes public and private liquidity. The main result is that an increase in public liquidity through a higher supply of real government bonds raises the real interest rate, crowding out private liquidity and increasing unemployment. The main theme is closely related to our paper, but they focus on the investment channel of monetary policy by showing how monetary policy lowers the interest rate and stimulates investment demand. Wen (2013) and Herrenbrueck (2013) develop models to understand unconventional monetary policy. They calibrate to the US data and find that unconventional policy can effectively stimulate investment under certain conditions. In their models, there is no explicit unemployment because production occurs in competitive markets.

Our paper is also related to the literature on market segmentation. Papers studying OMOs from the perspective of limited participation of agents in the asset market or transaction cost of transferring money between the assets market and goods market include Alvarez et al. (2001, 2002, 2009), and Kahn (2006). Most of them use CIA models, and they focus on the short-run effects of OMOs. We abstract from any short-run effects of OMOs and focus on the long-run effects.

The rest of the paper is organized as follows. Section 2 describes the model's environment. Section 3 introduces the basic model. Section 4 characterizes monetary equilibria and analyzes the effects of OMOs. Section 5 focuses on unconventional monetary policy by extending the basic model to incorporate long-term government bonds. We conclude in Section 6. Proofs and detailed derivations can be found in the Appendix.

2 Environment

Time is discrete and continues forever. As in Berentsen et al. (2011), there are three subperiods in each period: there is a search and matching labor market in the first subperiod; a decentralized goods market in the second subperiod; and a frictionless centralized market in the last subperiod. We refer to these markets as labor, goods and centralized markets hereafter. There are two types of agents: firms and households, indexed by f and h .² The measure of households is 1, while the measure of firms is arbitrarily large, but not all firms are active. In addition, there exists a government which is a consolidated fiscal and monetary authority. All government asset transactions take place in the centralized market.

In the first subperiod, unemployed households and vacant firms search and match bilaterally to create a job. Let $e = 1$ if a household and a firm are matched, and $e = 0$ if unmatched. Once matched, the match produces output y . The wage w is determined by generalized Nash bargaining. The match may break up at an exogenous rate δ . If unmatched, the household is unemployed, and will receive unemployment benefits, κ . Households receive w or κ in the subsequent centralized market.

In the second subperiod, all households enter the goods market as buyers. The utility from consuming q units of the goods is $v(q)$, where $v(0) = 0$, $v'(0) = \infty$ and $v'' < 0 < v'$. Only firms with output from the labor market are active as sellers while those unmatched firms skip the goods market. For active firms, the cost of selling q units of goods is $c(q)$ in terms of y , where $c(0) = 0$, $c' > 0$ and $c'' \geq 0$. Buyers and sellers are matched randomly and bilaterally. The terms of trade are determined by bargaining in all meetings.

In the goods market, the roles of households and firms create the double coincidence problem. Since households cannot store any good, barter is impossible. Lack

²In our model, households are buyers in the goods market. We can also interpret households as financial investors who need to choose a portfolio of assets. See Rocheteau et al. (2017) for a detailed discussion about different interpretations of households in a similar framework.

of commitment and lack of record-keeping imply that pure credit is not viable in the goods market. These frictions make assets essential as a medium of exchange to facilitate trade. We assume that there are two permanent types of households, depending on whether households can use bonds in the goods market. A fraction ω of households can use only money, whom we label as type-1 households, i.e., $h = 1$. The remaining fraction, $1 - \omega$, of households can use both money and bonds.³ We label these households as type-2 households, i.e., $h = 2$. We can view type-2 households as those who have access to financial assets.

All agents can enter the centralized market in the last subperiod, where a numeraire good x is produced and traded in this competitive market. We assume that this numeraire good is nonstorable. A household's utility from consuming x units of the numeraire goods is x . If x is negative, it means that households produce x . This linear utility makes households' asset portfolios tractable. Firms with $e = 1$ sell inventory (if $c(q) < y$), rebalance asset portfolios and pay wages and dividends to households.⁴ Firms with $e = 0$ can choose to create a new vacancy at a cost k . All agents discount between the centralized market and the next labor market at rate β .

The government is active only in the centralized market. In the benchmark model, the government issues money and short-term government bonds. It can also adjust the supply of short-term bonds through OMOs. Let M_t be the money supply measured in the beginning of the period t . The net growth rate of the money supply is π . Short-term bonds are one-period nominal bonds. Bonds that are issued at some discount price in period t would pay 1 unit of money in period $t + 1$. Let B_t be the supply of bonds in period t . We focus on the steady states from now on, so we drop the time

³We model the liquidity difference between money and bonds through their roles as a medium of exchange. Money is more liquid than bonds as money can be used by all households, while only type-2 households can use bonds. Alternatively, one can model the liquidity difference between money and bonds through their roles as collateral (See Rocheteau et al., 2017). In that way, short-term bonds are less liquid as a collateral asset than money and long-term bonds are less liquid as a collateral asset than short-term bonds. We use the first interpretation in this paper for simplicity.

⁴As in Berentsen et al. (2011), we assume that firms are equally owned by households.

subscript so that there is no confusion. It is useful to define the nominal interest rate of short-term bonds. From the Fisher equation, i is defined as $1 + i = (1 + \pi)/\beta$. Let i_s be the nominal interest rates of short-term bonds, $1 + i_s = \phi_m/\phi_s$, where ϕ_s is the price of bonds in terms of x and ϕ_m is the price of money in terms of x . As in Silveira and Wright (2010), we define the spread as the normalized nominal return difference between money and bonds. The spread of short-term bonds s_s is

$$(1) \quad s_s = \frac{i - i_s}{1 + i_s}.$$

3 Model

The value functions for the labor, goods and centralized markets are U_e^j , V_e^j , W_e^j , respectively, where $j \in \{1, 2, f\}$ and $e \in \{0, 1\}$. We begin with the value functions for households and firms in the centralized market, and then move to the following labor and goods markets.

3.1 Households

A household entering the centralized market with type $j \in \{1, 2\}$, employment status e and a portfolio (m, b_s) , chooses x and the portfolio of money and bonds (\hat{m}, \hat{b}_s) for the next period,

$$W_e^j(m, b_s) = \max_{x, \hat{m}, \hat{b}_s} \left\{ x + (1 - e)\chi + \beta U_e^j(\hat{m}, \hat{b}_s) \right\}$$

$$\text{st. } x + \phi_m \hat{m} + \phi_s \hat{b}_s + T = ew + (1 - e)\kappa + \Delta + \phi_m m + \phi_s b_s,$$

where χ is the value of leisure. The LHS of the budget constraint is total expenditure, which includes the consumption of x , the value of money and bonds carried to next period, and taxes T . The RHS is total income, which includes wage w or unemploy-

ment benefit κ , firms' dividends Δ , and the value of money and bonds. Notice that the value of b_s in terms of x is $\phi_m b_s$ as 1 unit bond pays 1 unit money at maturity. The value of \hat{b}_s in terms of x is $\phi_s \hat{b}_s$ as new bonds are issued at the price ϕ_s .

Substituting x from the budget constraint into the value function, we obtain

$$(2) \quad W_e^j(m, b_s) = I_e + \phi_m m + \phi_m b_s + \max_{\hat{m}, \hat{b}_s} \left\{ -\phi_m \hat{m} - \phi_s \hat{b}_s + \beta U_e^j(\hat{m}, \hat{b}_s) \right\},$$

where $I_e = ew + (1-e)(\kappa + \chi) + \Delta - T$. The envelop conditions give $\partial W_e^j(m, b_s)/\partial m = \partial W_e^j(m, b_s)/\partial b_s = \phi_m$. As in Lagos and Wright (2005), quasi-linear preferences in the centralized market imply that W_e^j is linear in (m, b_s) , and the choice of (\hat{m}, \hat{b}_s) is independent of (m, b_s) .

For a household in the following labor market,

$$\begin{aligned} U_1^j(\hat{m}, \hat{b}_s) &= \delta V_0^j(\hat{m}, \hat{b}_s) + (1 - \delta) V_1^j(\hat{m}, \hat{b}_s), \\ U_0^j(\hat{m}, \hat{b}_s) &= \lambda_h V_1^j(\hat{m}, \hat{b}_s) + (1 - \lambda_h) V_0^j(\hat{m}, \hat{b}_s), \end{aligned}$$

where λ_h is the endogenous job creation rate. Let (u, v) denote the measures of unemployed households and vacancies. The matching function $\mathcal{N}(u, v)$ exhibits constant returns scale. We have $\lambda_h = \mathcal{N}(u, v)/u = \mathcal{N}(1, \tau)$, where $\tau = v/u$ is the labor market tightness.

Moving to the goods market, households become buyers while firms with $e = 1$ become sellers. Each household is matched randomly with a firm. Given that the measure of households is 1 and the measure of firms with $e = 1$ is $1 - u$, the matching function is $\mathcal{M}(1, 1 - u)$, which also has constant returns to scale. Recall that there are two types of households. Type-1 households can use only money to trade. Their value function is

$$V_e^1(\hat{m}, \hat{b}_s) = \alpha_h \left[v(q^1) + W_e^1(\hat{m} - d^1, \hat{b}_s) \right] + (1 - \alpha_h) W_e^1(\hat{m}, \hat{b}_s),$$

where $\alpha_h = \mathcal{M}(1, 1 - u)$ is the household's probability of meeting a firm and (q^1, d^1) are the terms of trade. That is, the household uses d^1 units of money to exchange for q^1 units of goods. For type-2 households that can use both money and bonds, they have

$$V_e^2(\hat{m}, \hat{b}_s) = \alpha_h \left[v(q^2) + W_e^2(\hat{m} - d^2, \hat{b}_s - \mu_s) \right] + (1 - \alpha_h) W_e^2(\hat{m}, \hat{b}_s),$$

where (q^2, d^2, μ_s) are the terms of trade. The household uses d^2 units of money and μ_s units of short-term bonds to exchange for q^2 units of goods.

Let $S^1 = v(q^1) - \phi_{m+} d^1$ and $S^2 = v(q^2) - \phi_{m+}(d^2 + \mu_s)$ be the trading surplus for type-1 and type-2 households, respectively. Here ϕ_{m+} refers to the value of money in the following period. Using the linearity of W_e^j , we can rewrite U_e^j for $j \in \{1, 2\}$ as

$$(3) \quad U_e^j(\hat{m}, \hat{b}_s) = \alpha_h S^j + \phi_{m+}(\hat{m} + \hat{b}_s) + \mathbb{E}W_e^j(0, 0),$$

where $\mathbb{E}W_e^j(0, 0)$ is the expectation with respect to next period's employment status. It is clear that $\partial U_e^j / \partial \hat{m}$ and $\partial U_e^j / \partial \hat{b}_s$ do not depend on the employment status. We then substitute (3) into the maximization problem of (2),

$$(4) \quad W_e^j(m, b_s) = I_e + \phi_m(m + b_s) + \beta \mathbb{E}W_e^j(0, 0) \\ + \max_{\hat{m}, \hat{b}_s} \left\{ -\phi_m \hat{m} - \phi_s \hat{b}_s + \beta \left[\alpha_h S^j + \phi_{m+}(\hat{m} + \hat{b}_s) \right] \right\}.$$

From (4), the choice of (\hat{m}, \hat{b}_s) is independent of e and (m, b_s) . Hence, households of the same type take the same portfolio of money and bonds out of each centralized market.

3.2 Firms

Firms do not carry any money or bonds out of the centralized market since they would not use it in the subsequent markets. For a matched firm with inventory ξ , money balances m and short-term bonds b_s , its value function in the centralized market is

$$W_1^f(\xi, m, b_s) = \xi + \phi_m m + \phi_m b_s - w + \beta U_1^f.$$

As firms do not carry any assets, we omit the state variables in U_e^f and V_e^f without loss of generality. Depending on a firm's employment status, the firm's value function in the following labor market is

$$\begin{aligned} U_1^f &= \delta V_0^f + (1 - \delta) V_1^f, \\ U_0^f &= \lambda_f V_1^f + (1 - \lambda_f) V_0^f, \end{aligned}$$

where $\lambda_f = \mathcal{N}(u, v)/v = \mathcal{N}(1, \tau)/\tau$, is the endogenous job filling rate. Only firms with $e = 1$ produce y and participate in the subsequent goods market.

In the goods market, a firm may meet a type-1 household or a type-2 household. The firm's value function is

$$(5) \quad V_1^f = \omega V_1^{f1} + (1 - \omega) V_1^{f2},$$

where

$$\begin{aligned} V_1^{f1} &= \alpha_f W_1^{f1}[y - c(q^1), \phi_{m+d^1}, 0] + (1 - \alpha_f) W_1^{f1}(y, 0, 0), \\ V_1^{f2} &= \alpha_f W_1^{f2}[y - c(q^2), \phi_{m+d^2}, \phi_{m+\mu_s}] + (1 - \alpha_f) W_1^{f2}(y, 0, 0). \end{aligned}$$

Here $\alpha_f = \mathcal{M}(1, 1 - u)/(1 - u)$ is the firm's probability of trade. It costs a firm $c(q^j)$ units of goods produced in the labor market to sell q^j units of goods for $j \in \{1, 2\}$.

The firm can carry the rest $y - c(q^j)$ as inventory to the subsequent centralized market. Using the linearity of W_1^f in (x, m, b_s) , we rewrite (5) as $V_1^f = y - w + \alpha_f S_f + \beta U_1^f$, where $S_f = \omega [\phi_{m+} d^1 - c(q^1)] + (1 - \omega) [\phi_{m+} d^2 + \phi_{m+} \mu_s - c(q^2)]$ is the firm's expected surplus from trading in the goods market.

The free entry condition in the centralized market implies that firms with $e = 0$ can choose to enter the centralized market by paying the entry cost k . Thus we have

$$W_0^f = \max \left\{ 0, -k + \beta \lambda_f V_1^f + \beta (1 - \lambda_f) V_0^f \right\},$$

where $V_0^f = W_0^f = 0$ in equilibrium. It follows that $k = \beta \lambda_f V_1^f$. As in Mortensen and Pissarides (1994), we can derive

$$(6) \quad k = \frac{\beta \lambda_f (y - w + \alpha_f S_f)}{1 - \beta(1 - \delta)}.$$

Recall that firms pay out profits as dividends in the centralized market. The aggregate profit by all firms is $(1 - u)(y - w + \alpha_f S_f) - vk$. For a household that owns shares of all firms, the dividend income is $\Delta = (1 - u)(y - w + \alpha_f S_f) - vk$.

3.3 Government

The government is a consolidated fiscal and monetary authority. Without loss of generality, suppose that the government has a balanced budget in every period. The government budget constraint is

$$(7) \quad \phi_m (M - M_-) + \phi_s B_s + T = G + \phi_m B_{s-} + u\kappa.$$

Here a subscript "–" denotes variables associated with the previous period. The LHS of (7) shows total revenue, which includes the value of newly issued money and bonds plus tax revenue. The RHS represents total expenditure, which includes government

purchases G , the value of previously issued government bonds and unemployment benefits.

The central bank can either adjust the growth rate of the money supply or the relative supply of money and bonds. Let σ_s denote the ratio of short-term government bonds to money. The central bank commits to monetary policy where the money supply grows at $1 + \pi$, and the ratio of short-term bonds to money is σ_s . Mathematically,

$$(8) \quad \frac{M}{M_-} = 1 + \pi \text{ and } \frac{B_s}{M} = \sigma_s.$$

By the Fisher equation, changing π is equivalent of changing i . We model OMOs as changes in σ_s . See also Williamson (2012) and Rocheteau et al. (2017) that define OMOs in a similar way. The interpretation of σ_s is the steady-state ratio of bonds supply held by the public (households in our model) to money supply. When we consider a change in σ_s , we essentially compare two steady states with different values of σ_s . For example, a decrease in σ_s implies that the new steady state has a lower supply of bonds because the money supply is determined by π . Fewer bonds are available for households to hold. We follow the arguments in Rocheteau et al. (2017) to justify such a choice to model OMOs. Traditional OMOs where the central bank decreases the supply of bonds are achieved through injection of money. We can allow for such OMOs, but we implicitly assume that the change in money supply associated with OMOs is sterilized by government's taxes T . That is, if OMOs inject money, we can let taxes retire the newly injected money to ensure the growth rate of money supply is $1 + \pi$. In this way, OMOs involve swaps of money and bonds, but we can still consider changes in π and σ_s as separate monetary policy parameters.

4 Equilibrium

The terms of trade in three markets are determined as follows: agents are price takers in the centralized market, and bargain over the terms of trade in the labor and goods markets. In this section, we solve for equilibrium conditions in all markets and define a stationary monetary equilibrium. Then we use the model to analyze the effects of monetary policy.

4.1 Goods Market Equilibrium

A generic way to define the bargaining solution is that for $j \in \{1, 2\}$, a household pays $g(q^j)$ in real terms to purchase q^j units of goods, where $g(\cdot)$ depends on the specific bargaining protocol.⁵ For example, the bargaining protocol could be Kalai bargaining or generalized Nash bargaining. Let θ denote the household's bargaining power. With Kalai bargaining, for the case $d^j = \hat{m}$ and $\mu_s = \hat{b}_s$, we have,

$$(9) \quad g(q^j) = \phi_{m+}(\hat{m} + \hat{b}_s \cdot \mathbf{I}^j) = (1 - \theta)v(q^j) + \theta c(q^j),$$

where \mathbf{I}^j is an indicator, with $\mathbf{I}^1 = 0$ and $\mathbf{I}^2 = 1$. In case that either $d^j < \hat{m}$ or $\mu_s < \hat{b}_s$, we have (9) and $q^j = q^*$ where q^* solves $v'(q) = c'(q)$.

For now, we use the general bargaining solution where the payment for exchanging q^j units of goods is $g(q^j)$. Note that another implicit constraint associated with the bargaining problem is $c(q^j) \leq y$. It means that a firm's supply of q^j is restricted by y produced in the labor market. We assume that y is big enough so that this constraint never binds.

As in Lagos and Wright (2005), the bargaining solution must be $d^1 = \hat{m}$ and $q^1 = g^{-1}(\phi_{m+}\hat{m})$ for type-1 households. Given this, we move back to the centralized

⁵See Gu and Wright (2016) for a detailed discussion on various mechanisms determining the terms of trade.

market and solve for (\hat{m}, \hat{b}_s) in (4) for type-1 households. The FOC with respect to \hat{m} yields

$$(10) \quad i = \alpha_h(u)\lambda(q^1).$$

We use $\lambda(q^j) = v'(q^j)/g'(q^j) - 1$ to denote the liquidity premium in a meeting with a type- j household. For type-2 households, they can use both money and bonds. Notice that the return on bonds must be no lower than the return on money. When $i_s > 0$, type-2 households would choose $\hat{m} = 0$ and an interior solution for \hat{b}_s solves

$$(11) \quad s_s = \alpha_h(u)\lambda(q^2).$$

When $i_s = 0$, we have $s_s = i = \alpha_h(u)\lambda(q^2)$, which we discuss in more detail later. In (10), i is the marginal cost of spending 1 more unit of money for type-1 households, while the RHS is the marginal benefit of spending 1 more unit of money. Similarly, in (11), s_s is the marginal cost of spending 1 more unit of short-term bonds for type-2 households, while the RHS is the marginal benefit of spending 1 more unit of short-term bonds. For any u , (10) and (11) determine (q^1, q^2) . The labor and goods markets are linked: more unemployment reduces the number of firms entering into the goods market and hence reduces the trading probability for households, which will further affect equilibrium (q^1, q^2) .

4.2 Labor Market Equilibrium

In the labor market, wage is determined by generalized Nash bargaining. Let η be the bargaining power of a firm. Following Mortensen and Pissarides (1994), we can solve for

$$(12) \quad w = \frac{\eta[1 - \beta(1 - \delta)](b + \chi) + (1 - \eta)[1 - \beta(1 - \delta - \lambda_h)](y + \alpha_f S_f)}{1 - \beta(1 - \delta) + (1 - \eta)\beta\lambda_h}.$$

Substituting (12) into (6), the free entry condition becomes

$$(13) \quad k = \frac{\eta\lambda_f(u)[y - \kappa - \chi + \alpha_f(u)S_f]}{r + \delta + (1 - \eta)\lambda_h(u)}.$$

The flow condition in the labor market implies that $(1 - u)\delta = \mathcal{N}(u, v)$. This implicitly defines $v = v(u)$. The free entry condition determines u , given (q^1, q^2) . This establishes another link between the labor market and goods market. Compared with the free entry condition in Berentsen et al. (2011), the firm's expected trading surplus S_f in (13) is the expected surplus from trading with two types of households.

4.3 Equilibrium Allocation

In any monetary equilibrium, i_s is endogenously determined given (i, σ_s) . The no-arbitrage condition implies that i_s must not be lower than the nominal return of money (i.e., 0). In addition, i_s cannot exceed i . Therefore, in equilibrium $0 \leq i_s \leq i$. We now define general equilibrium.

Definition 1 *Given (i, σ_s) , a stationary monetary equilibrium is a list (q^1, q^2, i_s, u) such that (i) given $u, (q^1, q^2)$ solves (4), and i_s satisfies (1); (ii) given (q^1, q^2, i_s) , u satisfies (13); and (iii) asset markets clear.*

Proposition 1 *Stationary monetary equilibrium exists if $k < \eta(y - \kappa - \chi)/(r + \delta)$ and $\pi \geq \beta - 1$. The Friedman rule ($i = 0$) implements the efficient allocation where $q^1 = q^2 = q^*$ and u is solved from (13).*

The proof of existence follows Berentsen et al. (2011). If k is too high, entry would be too costly for firms. Both the labor and goods markets will shut down. Therefore, a monetary equilibrium does not exist. Notice that there always exists a non-monetary equilibrium for any k . When a monetary equilibrium exists, it need not be unique. In general, there might be multiple equilibria due to the strategic

complementarity between firm entry and household portfolio decisions. If a monetary equilibrium is not unique, we will focus on the equilibrium with the lowest u in the following analysis. Under the Friedman rule, consumption in the goods market is efficient by using money. Bonds are not valued and $i_s = 0$. For $i > 0$, we have three cases of monetary equilibrium depending on the values of (i, σ_s) .

When $i > 0$ and σ_s is small, the equilibrium return on bonds is $i_s = 0$. In this case, a scarce supply of bonds makes the price of bonds high. Money and bonds earn the same nominal return, 0, and become perfect substitutes for type-2 households. The economy is in a liquidity trap. From (10) and (11), $q^1 = q^2 = q^\ell$ is solved from $i = \alpha_h(u)\lambda(q^\ell)$. The model reduces to Berentsen et al. (2011). This case with a liquidity trap exists if and only if $i > 0$ and $\sigma_s \leq (1 - \omega) / \omega$.

When $i > 0$ and σ_s is big, the equilibrium value of i_s can reach its upper limit $i_s = i$. It is costless for households to hold bonds. Therefore, type-1 households are willing to hold any amount of bonds and type-2 households hold bonds at least to buy q^* in the goods market. In terms of allocation, $q^1 = q^{1p}$ is solved from (10) and $q^2 = q^*$. This type of monetary equilibrium exists if and only if $i > 0$ and $\sigma_s \geq \sigma^*$, where

$$(14) \quad \sigma^* = \frac{(1 - \omega) g(q^*)}{\omega g(q^{1p})}.$$

That is, when σ_s is big enough, the supply of bonds is abundant and the return on bonds is high. We label this case as the plentiful bonds case.

When $i > 0$ and the value of σ_s is neither too small nor too big, the equilibrium return i_s is between 0 and i . The return on bonds is higher than that of money so that type-2 households would prefer to hold only bonds. Meanwhile, the return on bonds is not so high and type-2 households hold a finite amount of bonds. The aggregate demand for bonds in real terms is $(1 - \omega) \phi_{m+} \hat{b}_s = (1 - \omega) g(q^2)$, which equals the

supply of bonds in real terms $\phi_{m+}B_+$. Type-1 households hold only money. The aggregate demand for money in real terms is $\omega\phi_{m+}\hat{m} = \omega g(q^1)$, which equals the aggregate supply $\phi_{m+}M_+$. Recall that $B_+/M_+ = \sigma_s$. It implies that

$$(15) \quad \frac{(1-\omega)g(q^2)}{\omega g(q^1)} = \sigma_s.$$

The equilibrium allocation (q^1, q^2, i_s, u) is solved from (10), (11), (13), and (15). This case exists if and only if $i > 0$ and $(1-\omega)/\omega < \sigma_s < \sigma^*$. Compared to the case with plentiful bonds, the supply of bonds is relatively scarce and the return on bonds is low. Therefore, we label this case as the scarce bonds case. We summarize these three cases of monetary equilibrium in the following proposition.

Proposition 2 *For $i > 0$, three cases of monetary equilibrium exist: (1) the liquidity trap case exists if and only if $\sigma_s \leq (1-\omega)/\omega$; (2) the scarce bonds case exists if and only if $(1-\omega)/\omega < \sigma_s < \sigma^*$; and (3) the plentiful bonds case exists if and only if $\sigma_s \geq \sigma^*$.*

Figure 2 shows how the existence of these three cases of monetary equilibrium depends on the values of (i, σ_s) . Notice that q^{1p} decreases as i increases. It implies that σ_s^* is an increasing function of i . For $i > 0$, the economy is in a liquidity trap when σ_s is very small. As σ_s increases and the supply of bonds increases, the economy moves to the scarce bonds case, where the return on bonds is positive but still lower than i . As σ_s further increases and bonds become abundant, the return on bonds reaches its upper limit i .

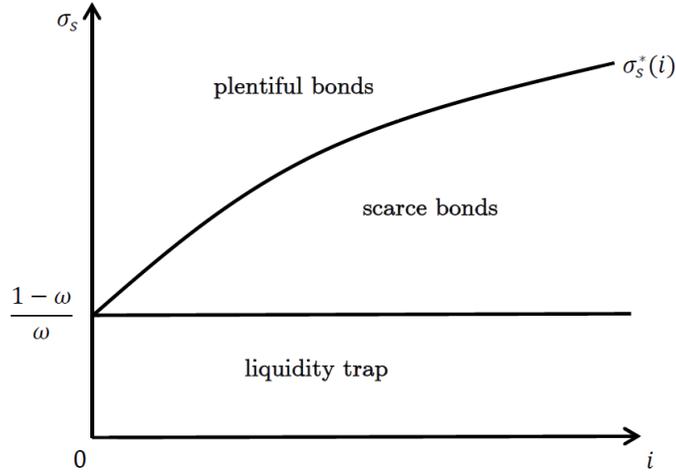


Figure 1: Three cases of monetary equilibrium

Apart from (i, σ_s) , ω is also an important parameter that affects the existence of different equilibrium cases. A higher ω represents a bigger fraction of households that use only money. It follows that $(1 - \omega) / \omega$ is smaller and the region for the liquidity trap case will shrink. A higher ω also shifts down $\sigma_s^*(i)$. With fewer households using bonds, it is more likely bonds become plentiful and the equilibrium reaches the plentiful bonds case.

The central bank can adjust either i or σ_s as its monetary policy parameter. We find that the effects of inflation are similar to the previous findings in the literature. A rise in i decreases households' incentives to hold money and therefore reduces consumption in the goods market. Firms' profits fall, which reduces their incentives to enter into the labor market. Unemployment rises and we obtain a positive relationship between inflation and unemployment as in Berentsen et al. (2011).

It is more interesting to examine the effects of adjusting the supply of bonds σ_s . Clearly, in both the liquidity trap case and the plentiful bonds case, changing σ_s is irrelevant because it does not affect the allocation. Only in the scarce bonds case is σ_s relevant. Consider a decrease in σ_s as an example. The central bank essentially

decreases the supply of bonds to the public. This open market purchase of bonds will drive up the price of bonds and lower the return on bonds. The nominal interest rate i_s decreases. In response, type-2 households hold fewer bonds and consume less q^2 . This reduction of liquidity in the goods market leads to a decrease in entry of firms in the labor market. Unemployment rises, which implies a reduction of sellers in the goods market. It follows that the return from holding money falls and type-1 households reduce their demand for real balances. Therefore, q^1 also decreases and this leads to a further rise in unemployment.

OMOs affect the real side of the economy through the linkage between the goods and labor markets. We label this channel as the consumption channel because the central bank's swap of money for bonds (or vice versa) affects the return on bonds and households' consumption. In our model, type-2 households hold bonds and their consumption is directly affected by the OMO. Changes in consumption by type-2 households in turn affect labor market outcomes, which changes the return from holding money in the goods market. Consumption by type-1 households is thus indirectly affected by the OMO. This will again influence labor market outcomes. It is important to note that the interaction between the goods and labor markets is the key to understand the overall effects of OMOs. Proposition 3 formally describes the effects of monetary policy.⁶

Proposition 3 *Consider (i, σ_s) as monetary policy parameters: (1) the liquidity trap case: $\partial q^1/\partial i < 0$, $\partial q^2/\partial i < 0$, and $\partial u/\partial i > 0$; $\partial q^1/\partial \sigma_s = \partial q^2/\partial \sigma_s = \partial u/\partial \sigma_s = 0$; (2) the scarce bonds case: $\partial q^1/\partial i < 0$, $\partial q^2/\partial i < 0$, and $\partial u/\partial i > 0$; $\partial q^1/\partial \sigma_s > 0$, $\partial q^2/\partial \sigma_s > 0$, and $\partial u/\partial \sigma_s < 0$; and (3) the plentiful bonds case: $\partial q^1/\partial i < 0$, $\partial q^2/\partial i = 0$, and $\partial u/\partial i > 0$; $\partial q^1/\partial \sigma_s = \partial q^2/\partial \sigma_s = \partial u/\partial \sigma_s = 0$.*

⁶Notice that changing i or σ_s can move the economy from one case to another case, as can be seen from Figure 2. For example, when i rises, the equilibrium moves from the liquidity trap case to the scarce bonds case and then to the plentiful bonds case. Our comparative statics results focus on the effects of changing i or σ_s within in each case.

In addition to the connection between the goods and labor markets, the other key assumption for our result is that both money and bonds are valued by households, but are not perfect substitutes. If money and bonds are perfect substitutes, then we would have the results as in Wallace (1981) that OMOs are neutral. We assume that two permanent types of households have access to different assets. In this way, money and bonds differ in terms of their liquidity properties and returns. We need type-1 households to ensure money is always valued. We also need type-2 households so that bonds are valued by some households. One can consider several other assumptions about households. For example, we can assume that households are homogeneous and can use bonds with some probability. There will be two types of meetings in the goods market depending on whether a household uses bonds. We can alternatively assume that households are homogeneous but can use bonds at a cost. This way essentially endogenizes ω .

We choose to have two permanent types of households for several reasons. The first is that this assumption of permanent types delivers sharp analytical results that highlight the consumption channel of OMOs. While the consumption channel still exists using the other two assumptions, neither way can provide clean analytical results for the effects of OMOs on unemployment. If households can use bonds with some probability $1 - \omega$, the same decrease in σ_s would lead to a fall in q^2 . However, since households choose a portfolio of money and bonds before going to the goods market, the lower return on bonds raises the relative benefit of using money, which makes households hold more money and q^1 increase. The overall effect of σ_s on u depends on the value of ω . Our numerical example shows that a decrease in σ_s raises unemployment when ω is low, but reduces unemployment when ω is high. If households can use bonds at a cost, there is an endogenous ω that makes households indifferent between using bonds and money. In equilibrium, a fraction ω of households use money and the rest use bonds. Since ω also depends on the value of σ_s , it is less

clear how σ_s affects consumption and unemployment, although our numerical example shows that a lower σ_s leads to a higher u . We feel that having permanent types of households is the simplest model to deliver the consumption channel.

Another reason is that households in reality tend to be heterogeneous. As mentioned earlier, we view type-2 households as those who have access to financial assets. Schuh and Stavins (2015) find that 91.3% of households in the US had a bank account in 2013. Badarinza et al. (2016) document that 94% households in the US participate in asset markets using the 2010 wave of the US Survey of Consumer Finance. These results are more likely to reflect permanent differences in households rather than differences caused by random shocks.

The last reason is more technical. If households are homogeneous and can use bonds with some probability, the equilibrium of the liquidity trap case occurs only when the supply of bonds is zero. When bonds earn the same return as money, all households hold only money because bonds are not accepted in type-2 meetings. With permanent types, even when bonds have the same return as money, type-2 households are still indifferent between using money and bonds. Monetary equilibrium with a liquidity trap exists in a larger set of the parameter space. See also Rocheteau et al. (2017) for a discussion of the liquidity trap case using permanent types of agents.

5 Unconventional Monetary Policy

In the benchmark model, OMOs involve the central bank adjusting the supply of short-term bonds. The recent Global Financial Crisis made a few central banks including the US Federal Reserve purchased a large amount of long-term government bonds. This type of monetary policy is labeled as unconventional monetary policy. In the section, we incorporate long-term government bonds to address the effects of

unconventional monetary policy.⁷ These long-term bonds are perpetual bonds (like Consols) that pay 1 unit of money in every future period.

The nominal interest rate on long-term government bonds i_ℓ and the spread s_ℓ are defined as

$$(16) \quad 1 + i_\ell = \frac{1 + \phi_{\ell+}/\phi_{m+}}{\phi_\ell/\phi_m} \text{ and } s_\ell = \frac{i - i_\ell}{1 + i_\ell},$$

where ϕ_ℓ/ϕ_m represents the nominal value of long-term bonds. Type-1 households can still use only money whereas type-2 households can use money and both types of bonds in the goods market. When traded in the goods market, we assume that long-term bonds are not as liquid as short-term bonds so that type-2 households can use only a fraction γ of their long-term bonds to buy goods.⁸ The central bank can potentially change the relative supply of long-term bonds as its monetary policy parameter. Suppose that the central bank commits to

$$(17) \quad \frac{B_\ell}{M} = \sigma_\ell,$$

in addition to (8). Now monetary policy parameters include $(i, \sigma_s, \sigma_\ell)$.

We discuss the main characterization of the extended model and leave details about derivation in Appendix D. In the goods market, type-2 households can use any assets. When solving the bargaining problem, an additional asset constraint $\mu_\ell \leq \gamma \hat{b}_\ell$ exists, which reflects that the household can use only a fraction γ of long-term bonds in transactions. Type-2 households consume q^* whenever any of the asset constraints

⁷Our model focuses on the steady state long run equilibrium. One may question that unconventional monetary policy should be considered as short run stabilization policy. Given that the Fed has implemented this type of unconventional monetary policy for more than 7 years and the Bank of Japan has used similar policies for about two decades, it is useful to understand the long run implications of such policies.

⁸See Nosal and Rocheteau (2013) and Rocheteau et al. (2015) for similar approaches to model the liquidity difference of different assets. See Li et. Rocheteau (2012) for more discussion on how to endogenize the liquidity differences.

is not binding. If all asset constraints are binding, there is a FOC with respect to \hat{b}_ℓ ,

$$(18) \quad s_\ell = \gamma \alpha_h(u) \lambda(q^2).$$

It is immediate from (11) and (18) that

$$(19) \quad \frac{s_\ell}{s_s} = \gamma \text{ or } i_\ell = \frac{(1 - \gamma) i + (\gamma + i) i_s}{1 + \gamma i + (1 - \gamma) i_s}.$$

For both short-term bonds and long-term bonds to be held by type-2 households, the spread of long-term bonds must be lower than the spread of short-term bonds. That is, long-term bonds must have a higher return than short-term bonds. This type of positive term premium is also found in Williamson (2013) and Geromichalos et al. (2013).

Proposition 1 still holds with the addition of long-term bonds. For $i > 0$, we classify different cases of monetary equilibrium depending on the values of (σ_s, σ_ℓ) as follows. First, consider an extreme case where $\sigma_\ell = 0$. If the supply of long-term bonds is 0, the model is essentially our benchmark model, where we have three cases of monetary equilibrium depending on the value of σ_s . In Figure 2, we draw different cases of monetary equilibrium in the space of (σ_s, σ_ℓ) for any given $i > 0$. The three segments on the vertical axis represent the liquidity trap, scarce bonds and plentiful bonds cases as σ_s increases from 0 to infinity. Next, consider the other extreme case where $\sigma_\ell = \infty$. If the supply of short-term bonds is 0, the economy resembles the benchmark economy except that long-term bonds are not as liquid as short-term bonds. Therefore, depending on the relative supply of long-term bonds, σ_ℓ , we have three cases which we show on the horizontal axis in Figure 2. That is, when σ_ℓ is small, $i_\ell = 0$ and we have the liquidity trap case. When σ_ℓ is big, the supply of long-term bonds is abundant so that $i_\ell = i$. This is the plentiful bonds case. When σ_ℓ is of intermediate value, we have the scarce bonds case and $0 < i_\ell < i$.

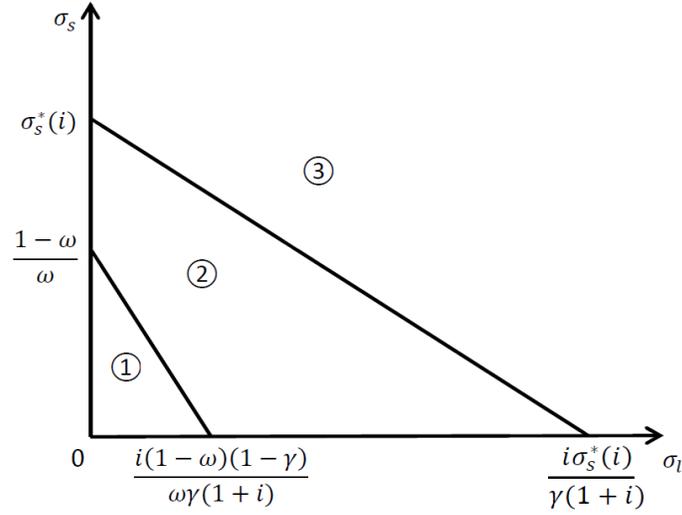


Figure 2: Equilibrium with long-term bonds

Finally, consider the case where $\sigma_s > 0$ and $\sigma_\ell > 0$. There are again three subcases. In Figure 2, area 1 shows the region where (σ_s, σ_ℓ) is small. The returns on the bonds are such that $0 = i_s < i_\ell = (1 - \gamma) i / (1 + \gamma i) < i$. Long-term bonds offer a higher return, but type-2 households are indifferent between long-term bonds and the other two types of assets because long-term bonds are less liquid. Area 2 in Figure 2 illustrates the second subcase where the combination of (σ_s, σ_ℓ) gives rise to $0 < i_s < i_\ell < i$. Type-2 households strictly prefer to hold bonds, but are indifferent between the two types of bonds. When the supply of bonds is abundant as show in area 3, both bonds yield high returns and $0 < i_s = i_\ell = i$. Here long-term bonds have the same return as short-term bonds. Type-2 households are indifferent between the two types of bonds because it is not costly to hold bonds.

A new case that is worth discussing is the equilibrium represented by area 2 in Figure 2. It requires

$$(20) \quad \frac{\sigma_s + \gamma(1+i)\sigma_\ell}{i(1-\gamma)} \geq \frac{1-\omega}{\omega} \text{ and } \sigma_s + \frac{\gamma(1+i)\sigma_\ell}{i} \leq \sigma_s^*(i).$$

Type-2 households hold a portfolio of short-term and long-term government bonds to use in the goods market. Therefore, $g(q^2) = \phi_{m+}\hat{b}_s + (\phi_{m+} + \phi_{\ell+})\gamma\hat{b}_\ell$. The equilibrium conditions that characterize $(q^1, q^2, s_s, s_\ell, u)$ are (10), (13), (18), (19) and

$$(21) \quad g(q^2) = \left[\frac{\omega\sigma_s}{1-\omega} + \frac{\omega\sigma_\ell\gamma(1+i)}{(1-\omega)(i-s_\ell)} \right] g(q^1),$$

where (21) is derived from the asset market clearing conditions. Short-term bonds have a return premium because they are more liquid than long-term bonds.

Given that both i_s and i_ℓ are positive, the central bank can adjust either σ_s or σ_ℓ when conducting OMOs. In practice, it might be more common for central banks to change σ_s when $i_s > 0$. However, historical evidence documented by D'Amico et al. (2012) reveals that the Fed did long-term bonds transactions between 1942 and 1951, which could directly affect the long-term interest rate. In addition, when i_s is close to 0, it might be appealing to resort to unconventional policy to adjust i_ℓ . Proposition 4 shows how conventional and unconventional policy affects consumption and unemployment.

Proposition 4 *When (20) is satisfied, $\partial q^1/\partial\sigma_s > 0$, $\partial q^2/\partial\sigma_s > 0$, and $\partial u/\partial\sigma_s < 0$; and $\partial q^1/\partial\sigma_\ell > 0$, $\partial q^2/\partial\sigma_\ell > 0$, and $\partial u/\partial\sigma_\ell < 0$.*

The qualitative effects of σ_s and σ_ℓ on (q^1, q^2, u) remain the same as before. When the central bank decreases the supply of bonds, the nominal interest rate decreases. The lower interest rate induces type-2 households to hold fewer bonds and cut back consumption of q^2 . The decrease in q^2 has a negative impact on employment in the labor market, which indirectly reduces the trading opportunities of type-1 households in the goods market. Through this general equilibrium effect, type-1 households also hold less money and consume less q^1 . As a result, employment further decreases. The consumption channel may sound counter-intuitive, but the essence is that if monetary policy changes returns on assets and assets are not perfect substitutes, it could affect

the portfolio choices by households and therefore affect the macroeconomy. Such a consumption channel exists only in models where households face non-trivial portfolio choices.⁹

One interesting implication from the extended model is that there exists a positive lower bound on the long-term interest rate. If the central bank keeps reducing the supply of long-term bonds by lowering σ_ℓ , the equilibrium eventually moves from area 2 to area 1. Then the economy is in a liquidity trap where $i_s = 0$ and

$$\underline{i}_\ell = \frac{(1 - \gamma) i}{1 + \gamma i}.$$

Type-2 households are indifferent between money and both types of bonds. Notice that long-term bonds still earn a positive interest rate because they are less liquid than money and short-term bonds. This positive lower bound \underline{i}_ℓ depends on the inflation rate and the liquidity of long-term bonds. A higher inflation rate (high i) or a lower liquidity (lower γ) of long-term bonds leads to a higher bound.

6 Conclusion

We build models where money and bonds coexist to examine the effects of monetary policy on macroeconomic performance such as consumption and unemployment. In the benchmark model with money and short-term government bonds, we find that a lower supply of government bonds can lower the short-term interest rate. The lower interest rate induces households that use bonds to reduce their consumption. Households that do not use bonds also lower their consumption through an indirect

⁹There is some empirical literature about the impacts of low interest rates during the Great Recession on household consumption and unemployment, such as Mian et al. (2013), Mian and Sufi (2014), and Keys et al. (2015). Particularly, Mian and Sufi (2014) show the housing net worth channel played a significant role in the sharp decline in the U.S. employment during 2007-2009. Although our model does not have the exact "housing net worth channel", the empirical results from Mian and Sufi (2014) still provide support for the link between household consumption and employment. See also Maggio et al. (2015).

general equilibrium effect. The lower consumption by households reduces firms' profits and leads to higher unemployment in the economy. We highlight that the effects of such OMOs are through a consumption channel. When the economy's short-term interest rate is close to zero, the central bank can resort to unconventional monetary policy by adjusting the long-term interest rate. By purchasing long-term government bonds, the long-term interest rate is lowered, which again leads to lower consumption and higher unemployment.

When assessing the effectiveness of unconventional monetary policy, it is more common to focus on the investment channel where a lower interest rate could stimulate investment demand and output. Our model uncovers a new channel that works through consumption demand. In contrast to the effects on investment, the lower interest rate has negative effects of consumption and employment. We view this consumption channel as being complementary to the investment channel. It would be useful to construct models where both the consumption channel and the investment channel are present to evaluate the effectiveness of unconventional monetary policy. We leave this for future research.

A Proof of Proposition 3

The equilibrium values of (q^1, q^2, s_s, u) are determined by (10), (11), (15), and

$$(22) \quad H(u) = \omega [g(q^1) - c(q^1)] + (1 - \omega) [g(q^2) - c(q^2)],$$

where (22) is derived from (13) and

$$H(u) = \frac{k[r + \delta + (1 - \eta)\lambda_h(u)] - \eta\lambda_f(u)(y - \kappa - \chi)}{\eta\lambda_f(u)\alpha_f(u)}.$$

Since $\lambda'_h(u) < 0$, $\lambda'_f(u) > 0$ and $\alpha'_f(u) > 0$, we know that $H'(u) < 0$. We reduce the equations system to two equations (10) and (22) to solve for (q^1, u) , where q^2 is a function of q^1 through (15). Then q^2 is derived from (15) and s_s can be derived from (11). Taking full derivation of (10) and (22), we have

$$\begin{aligned} \frac{\partial q^1}{\partial \sigma_s} &= \frac{\omega\alpha'_h\lambda_1g_1(g'_2 - c'_2)}{D} \simeq -D, \\ \frac{\partial u}{\partial \sigma_s} &= -\frac{\omega\alpha_h\lambda'_1g_1(g'_2 - c'_2)}{D} \simeq D, \end{aligned}$$

where

$$D = -\alpha_h\lambda'_1g'_2H' - \omega\alpha'_h\lambda_1[g'_2(g'_1 - c'_1) + \sigma_s g'_1(g'_2 - c'_2)].$$

If we graph (10) and (22) on the (u, q^1) space, we know from (10)

$$\frac{dq^1}{du} = -\frac{\alpha'_h\lambda_1}{\alpha_h\lambda'_1} < 0.$$

It implies that (10) is downward sloping. Moreover, when $u \rightarrow 0$, q^1 is derived from $i = \alpha_h(1)\lambda(q^1)$, which should be a finite number. When $q \rightarrow 0$, u should approach

1. From (22), we have

$$\frac{dq^1}{du} = \frac{g'_2 H'}{\omega g'_2 (g'_1 - c'_1) + \omega \sigma_s g'_1 (g'_2 - c'_2)} < 0,$$

which means that (22) is also downward sloping. Moreover, when $u \rightarrow 0$, $H(u)$ approaches infinity. It follows that q^1 should approach infinity as well. The intersection of the two curves gives equilibrium (u, q^1) . If monetary equilibrium exists, there is at least one solution at which (22) is steeper than (10). If monetary equilibrium is unique or if we focus on the equilibrium with the smallest q^1 , then it must be true that (22) is steeper than (10) at the equilibrium allocation. Mathematically,

$$-\frac{\alpha'_h \lambda_1}{\alpha_h \lambda'_1} > \frac{g'_2 H'}{\omega g'_2 (g'_1 - c'_1) + \omega \sigma_s g'_1 (g'_2 - c'_2)}.$$

After rearranging, this exactly implies that $D < 0$.

We use (15) to derive

$$\frac{\partial q^2}{\partial \sigma_s} = -\frac{\omega g_1 [\alpha_h \lambda'_1 H' + \omega \alpha'_h \lambda_1 (g'_1 - c'_1)]}{(1 - \omega) D}.$$

Notice that $D < 0$ implies that the $\alpha_h \lambda'_1 H' + \omega \alpha'_h \lambda_1 (g'_1 - c'_1) > 0$ and therefore we obtain

$$\frac{\partial q^1}{\partial \sigma_s} > 0, \quad \frac{\partial q^2}{\partial \sigma_s} > 0, \quad \text{and} \quad \frac{\partial u}{\partial \sigma_s} < 0.$$

From (11), we have

$$\frac{\partial s_s}{\partial \sigma_s} = -\frac{\omega \alpha_h g_1 \{ \alpha_h \lambda'_1 \lambda'_2 H' + \alpha'_h [\omega \lambda_1 \lambda'_2 (g'_1 - c'_1) + (1 - \omega) \lambda'_1 \lambda_2 (g'_2 - c'_2)] \}}{(1 - \omega) D}.$$

The sign of $\partial s_s / \partial \sigma_s$ is not clear.

Recall that $s_s = (i - i_s) / (1 + i_s)$. It follows that $\partial s_s / \partial \sigma_s \simeq -\partial i_s / \partial \sigma_s$. Another way to see the sign of $\partial i_s / \partial \sigma_s$ is the following. From the equilibrium exis-

tence condition, we know that for any $i > 0$, liquidity trap equilibrium exists when $\sigma_s \in [0, (1 - \omega) / \omega]$ and plentiful bonds equilibrium exists when $\sigma_s \in [\sigma_s^*(i), \infty)$. Also note that $i_s = 0$ when $\sigma_s = (1 - \omega) / \omega$, and $i_s = i$ when $\sigma_s = \sigma_s^*$. When $\sigma_s \in [(1 - \omega) / \omega, \sigma_s^*]$, the equilibrium is a scarce bonds equilibrium. From the equilibrium conditions, we have $i_s \rightarrow 0$ when $\sigma_s \rightarrow (1 - \omega) / \omega$ and $i_s \rightarrow i$ when $\sigma_s \rightarrow \sigma_s^*$. When monetary equilibrium is unique, there is one i_s for any given σ_s . If it is also true that there is one σ_s for any given i_s , then we know that $i_s(\sigma_s)$ must be either strictly increasing or strictly decreasing.

To prove that there is one σ_s for any given i_s is equivalent to prove that the solution (q^1, q^2, u) to (10), (24) and (22) is unique for any given i_s or s_s . In general, it is not guaranteed that there exists a unique solution. However, when the solution is unique, we can argue that $i_s(\sigma_s)$ must be strictly monotonic. Given the values of the end points, it is only possible that $i_s(\sigma_s)$ is a strictly increasing function. That is

$$\frac{\partial i_s}{\partial \sigma_s} > 0 \text{ and } \frac{\partial s_s}{\partial \sigma_s} < 0.$$

When $0 = i_s < i_\ell < i$, the return on short-term bonds is 0. The return on long-term bonds is positive. However, since long-term bonds are less liquid than short-term bonds, it is not clear how type-2 households choose among money, short-term bonds and long-term bonds. Recall that for both short-term bonds and long-term bonds to be held by type-2 households, we have (19). When $i_s = 0$, i_ℓ must be $(1 - \gamma) i / (1 + \gamma i)$ so that type-2 households hold both types of bonds. It follows that [1] when $0 = i_s < i_\ell < (1 - \gamma) i / (1 + \gamma i)$, type-2 households would not hold long-term bonds and monetary equilibrium is the same as the liquidity trap equilibrium in the model without long-term bonds; and [2] when $0 = i_s < (1 - \gamma) i / (1 + \gamma i) < i_\ell < i$, type-2 households hold only long-term bonds. In this case, short-term bonds and money are dominated by long-term bonds for type-2 households. If the economy is

in this equilibrium, the government cannot further lower i_s when conducting OMOs. Instead, the government can rely on changing i_ℓ to affect the economy. We argue that this type of monetary policy resembles unconventional monetary policy conducted by central banks in U.S. and other advanced economies.

To understand the effects of changing σ_ℓ , we gather the equilibrium conditions (10), (22), (18), and

$$(23) \quad s_\ell = i - \frac{\omega\gamma\sigma_\ell(1+i)g(q^1)}{(1-\omega)g(q^2)}.$$

Here (23) is derived from (31). Substitute s_ℓ from (23) into (18),

$$(24) \quad i - \frac{\omega\gamma\sigma_\ell(1+i)g(q^1)}{(1-\omega)g(q^2)} = \gamma\alpha_h(u)\lambda(q^2).$$

Now we use (10), (24) and (22) to solve for (q^1, q^2, u) . Taking full derivation against these three equations, we have

$$\begin{aligned} \frac{\partial q^1}{\partial \sigma_\ell} &= -\frac{\omega(1-\omega)\gamma(1+i)\alpha'_h\lambda_1g_1(g'_2 - c'_2)}{D} \simeq D, \\ \frac{\partial q^2}{\partial \sigma_\ell} &= \frac{\omega(1+i)\gamma g_1[\alpha_h\lambda'_1 H' + \omega\alpha'_h\lambda_1(g'_1 - c'_1)]}{D}, \\ \frac{\partial u}{\partial \sigma_\ell} &= \frac{\omega(1-\omega)\gamma(1+i)\alpha_h\lambda'_1g_1(g'_2 - c'_2)}{D} \simeq -D, \end{aligned}$$

where

$$\begin{aligned} D &= (1-\omega)\gamma\alpha'_h(g'_2 - c'_2)[\omega\sigma_\ell(1+i)\lambda_1g'_1 - (1-\omega)\alpha_h\lambda'_1\lambda_2g_2] \\ &\quad - (1-\omega)[\alpha_h\lambda'_1H' + \omega\alpha'_h\lambda_1(g'_1 - c'_1)][\gamma\alpha_h(\lambda'_2g_2 + \lambda_2g'_2) - ig'_2]. \end{aligned}$$

To find the sign of D , we adopt the approach used in the proof of the basic model. Instead of three equations, we reduce the system to two equations (10) and (22) to solve for (q^1, u) . From (10) and (18), we have $s_\ell = \gamma i\lambda_2/\lambda_1$, which can be substituted

into (23). In (22), we view q^2 as a function of q^1 solved from

$$(25) \quad g(q^2) = \frac{\omega\gamma\sigma_\ell(1+i)\lambda_1g_1}{(1-\omega)i(\lambda_1-\gamma\lambda_2)}.$$

From (10), we have

$$\frac{dq^1}{du} = -\frac{\alpha'_h\lambda_1}{\alpha_h\lambda'_1} < 0.$$

It means that in the (u, q^1) space, (10) is downward sloping. Moreover, when $u \rightarrow 0$, q^1 is derived from $i = \alpha_h(1)\lambda(q^1)$, which should be a finite number. When $q \rightarrow 0$, u should approach 1. From (22), we have

$$(26) \quad H'(u) = \omega(g'_1 - c'_1)\frac{dq^1}{du} + (1-\omega)(g'_2 - c'_2)\frac{dq^2}{du},$$

where dq^2/du can be derived from (25)

$$(27) \quad \frac{dq^2}{du} = \frac{\overbrace{\omega\gamma\sigma_\ell(1+i)(\lambda'_1g_1 + \lambda_1g'_1) - (1-\omega)i\lambda'_1g_2}^{\Phi_1}}{\underbrace{(1-\omega)i(\lambda_1g'_2 - \gamma\lambda'_2g_2 - \gamma\lambda_2g'_2)}_{\Phi_2}} \frac{dq^1}{du}.$$

We can show that $\Phi_1 > 0$ and $\Phi_2 > 0$. Substituting (27) into (26), we reach

$$\frac{dq^1}{du} = \frac{H'\Phi_2}{\omega(g'_1 - c'_1)\Phi_2 + (1-\omega)(g'_2 - c'_2)\Phi_1} < 0.$$

It implies that in the (u, q^1) space, (22) is also downward sloping. Moreover, when $u \rightarrow 0$, $H(u)$ approaches infinity. It follows that q^1 should approach infinity as well.

We know that both (10) and (22) are downward sloping in the (u, q) space. In addition, (22) must be above (10) at $u \rightarrow 0$. The intersection of the two curves gives equilibrium (u, q^1) . If monetary equilibrium exists and is unique (or we focus on the equilibrium with the smallest q^1), it must be the case that (22) is steeper than (10)

at the equilibrium allocation. Mathematically, it must be true that

$$(28) \quad \frac{H'\Phi_2}{\omega(g'_1 - c'_1)\Phi_2 + (1 - \omega)(g'_2 - c'_2)\Phi_1} < -\frac{\alpha'_h\lambda_1}{\alpha_h\lambda'_1}.$$

After some algebra, one can find that (28) exactly implies that $D > 0$. When $D > 0$, one can also show that the numerator in $\partial q^2/\partial\sigma_\ell$ must be positive. We can conclude that

$$\frac{\partial q^1}{\partial\sigma_\ell} > 0, \quad \frac{\partial q^2}{\partial\sigma_\ell} > 0, \quad \text{and} \quad \frac{\partial u}{\partial\sigma_\ell} < 0.$$

B Solving the Model with Long-term Government Bonds

With long-term bonds, we update (4) as

$$(29) \quad W_e^j(m, b_s, b_\ell) = I_e + \phi_m(m + b_s) + (\phi_m + \phi_\ell)b_\ell + \beta\mathbb{E}W_e^j(0, 0) \\ + \max_{\hat{m}, \hat{b}_s, \hat{b}_\ell} \left\{ -\phi_m\hat{m} - \phi_s\hat{b}_s - \phi_\ell\hat{b}_\ell + \beta \left[\alpha_h S^j + \phi_{m+}(\hat{m} + \hat{b}_s) + (\phi_{m+} + \phi_{\ell+})\hat{b}_\ell \right] \right\},$$

where $S^2 = v(q^2) - \phi_{m+}(d^2 + \mu_s) - (\phi_{m+} + \phi_{\ell+})\mu_\ell$, with μ_ℓ denoting the amount of long-term bonds used by type-2 households in goods market. For a firm, the expected trading surplus in the goods market is $S_f = \omega[\phi_{m+}d^1 - c(q^1)] + (1 - \omega)[\phi_{m+}(d^2 + \mu_s) + (\phi_{m+} + \phi_{\ell+})\mu_\ell - c(q^2)]$. The government budget constraint becomes

$$(30) \quad \phi_m(M - M_-) + \phi_s B_s + \phi_\ell(B_\ell - B_{\ell-}) + T = \phi_m B_{s-} + \phi_m B_{\ell-} + u\kappa.$$

In (30), newly issued long-term government bonds $\phi_\ell(B_\ell - B_{\ell-})$ contributes to government's revenue and payment incurred by the outstanding long-term bonds $\phi_m B_{\ell-}$ contributes to government's expenditure.

In the goods market, type-2 households can use any assets. Let $(q^2, d^2, \mu_s, \mu_\ell)$

denote the terms of trade in the goods market for type-2 households. The general bargaining solution is $g(q^2) = \phi_{m+} (d^2 + \mu_s) + (\phi_{m+} + \phi_{\ell+}) \mu_\ell$, where $d^2 \leq \hat{m}$, $\mu_s \leq \hat{b}_s$ and $\mu_\ell \leq \gamma \hat{b}_\ell$. When all asset constraints are binding, Kalai bargaining solution gives

$$g(q^2) = \phi_{m+}(\hat{m} + \hat{b}_s + \gamma \hat{b}_\ell) = (1 - \theta)v(q^2) + \theta c(q^2).$$

We can then derive the FOC with respect to \hat{b}_ℓ as given by (18).

In the extended model, the Friedman rule ($i = 0$) still achieves the efficient allocation. For $i > 0$, different cases of monetary equilibrium exist depending on the values of $(i, \sigma_s, \sigma_\ell)$. We have the following nine cases for $i > 0$.

(1) When $\sigma_\ell = 0$ and $0 \leq \sigma_s \leq (1 - \omega) / \omega$, long-term bonds do not exist. Monetary equilibrium is the liquidity trap case in the benchmark model. In this case, $i_s = i_\ell = 0$.

(2) When $\sigma_\ell = 0$ and $(1 - \omega) / \omega < \sigma_s < (1 - \omega) g(q^2) / [\omega g(q^1)]$ where (q^1, q^2) are solved from $\alpha_h(u) \lambda(q^1) = i$, $\alpha_h(u) \lambda(q^2) = 0$ and (22), long-term bonds do not exist. Monetary equilibrium is the scarce bonds case in the benchmark model. In this case, $0 = i_\ell < i_s < i$.

(3) When $\sigma_\ell = 0$ and $\sigma_s \geq (1 - \omega) g(q^2) / [\omega g(q^1)]$ where (q^1, q^2) are solved from $\alpha_h(u) \lambda(q^1) = i$, $\alpha_h(u) \lambda(q^2) = 0$ and (22), long-term bonds do not exist. Monetary equilibrium is the plentiful bonds case in the benchmark model. In this case, $0 = i_\ell < i_s = i$.

(4) When $\sigma_s = 0$ and $\sigma_\ell \leq (1 - \omega) (1 - \gamma) i / [\omega \gamma (1 + i)]$, short-term bonds do not exist. The supply of long-term bonds is low and $i_\ell = (1 - \gamma) i / (1 + \gamma i) < i$. Type-2 households are indifferent between money and long-term bonds. Monetary equilibrium resembles the liquidity trap case in the benchmark model.

(5) When $\sigma_s = 0$ and $(1 - \omega) (1 - \gamma) i / [\omega \gamma (1 + i)] < \sigma_\ell < (1 - \omega) i g(q^2) / [\omega \gamma (1 + i) g(q^1)]$, where (q^1, q^2) are solved from $\alpha_h(u) \lambda(q^1) = i$, $\gamma \alpha_h(u) \lambda(q^2) = 0$ and (22), short-term

bonds do not exist. The equilibrium return of long-term bonds is $0 < (1 - \gamma) i / (1 + \gamma i) < i_\ell < i$. Monetary equilibrium resembles the scarce bonds case in the benchmark model, where now only long-term bonds are held by households. In equilibrium, type-1 households hold money and type-2 households hold long-term government bonds to trade in the goods market. It follows that $g(q^2) = (\phi_{m+} + \phi_{\ell+}) \gamma \hat{b}_\ell$. We gather the equilibrium conditions (10), (13), (18), and the market clearing condition (17) that determine equilibrium (q^1, q^2, u, s_ℓ) . Notice that (17) implies that

$$(31) \quad g(q^2) = \frac{\omega \gamma \sigma_\ell (1 + i_\ell)}{(1 - \omega) i_\ell} g(q^1),$$

where i_ℓ is a function of s_ℓ from (16).

(6) When $\sigma_s = 0$ and $\sigma_\ell \geq (1 - \omega) i g(q^2) / [\omega \gamma (1 + i) g(q^1)]$ where (q^1, q^2) are solved from $\alpha_h(u) \lambda(q^1) = i, \alpha_h(u) \lambda(q^2) = 0$ and (22), short-term bonds do not exist. The supply of long-term bonds is high enough so that $i_\ell = i$. Monetary equilibrium resembles the plentiful bonds case in the benchmark model, where now only long-term bonds are held by households.

(7) When $\sigma_s > 0, \sigma_\ell > 0$ and $\sigma_s + \gamma(1 + i) \sigma_\ell / [(1 - \gamma) i] \leq (1 - \omega) / \omega$, both types of bonds exist. The relative low supply of bonds leads to a low return and $0 = i_s < i_\ell = (1 - \gamma) i / (1 + \gamma i) < i$. Both bonds have the same return as money. Type-2 households are indifferent between money and bonds. This case is represented by area 1 in Figure 2.

(8) When $\sigma_s > 0, \sigma_\ell > 0, \sigma_s + \gamma(1 + i) \sigma_\ell / [(1 - \gamma) i] > (1 - \omega) / \omega$ and $\sigma_s + \gamma(1 + i) \sigma_\ell / i < (1 - \omega) g(q^2) / [\omega g(q^1)]$ where (q^1, q^2) are solved from $\alpha_h(u) \lambda(q^1) = i, \alpha_h(u) \lambda(q^2) = 0$ and (22), both types of bonds exist and are valued by households. The equilibrium returns of the bonds satisfy $0 < i_s < i_\ell < i$. This case is represented by area 2 in Figure 2. We discuss the implications of changing (σ_s, σ_ℓ) in Proposition 4.

(9) When $\sigma_s > 0$, $\sigma_\ell > 0$ and $\sigma_s + \gamma(1+i)\sigma_\ell/i \geq (1-\omega)g(q^2)/[\omega g(q^1)]$ where (q^1, q^2) are solved from $\alpha_h(u)\lambda(q^1) = i$, $\alpha_h(u)\lambda(q^2) = 0$ and (22), both types of bonds exist and are valued by households. The relative high supply of bonds leads to a high return and $i_s = i_\ell = i$. Type-2 households strictly prefer to hold bonds, but are indifferent between the two types of bonds. Their consumption q^2 is q^* and consumption of type-1 households q^1 solves $\alpha_h(u)\lambda(q^1) = i$.

C Proof of Proposition 4

Suppose that $i_s > 0$. When (19) is satisfied, we know that both short-term bonds and long-term bonds are held by type-2 households. It is easy to verify that $i_\ell < i$ iff $i_s < i$. We label this type of equilibrium as scarce bonds equilibrium. When $i_\ell < [(1-\gamma)i + (\gamma+i)i_s] / [1 + \gamma i + (1-\gamma)i_s]$, long-term bonds are dominated by short-term bonds in terms of its return. Type-2 households hold only short-term bonds. This case is the same as the scarce bonds equilibrium in the basic model. When $i_\ell > [(1-\gamma)i + (\gamma+i)i_s] / [1 + \gamma i + (1-\gamma)i_s]$, short-term bonds are dominated by long-term bonds in terms of its return. Type-2 households hold only long-term bonds. This case is the same as the liquidity trap equilibrium discussed above. It follows that when $i_s > 0$, the only equilibrium where both short-term bonds and long-term bonds are valued features $0 < i_s < i_\ell < i$.

In the scarce bonds equilibrium, $(q^1, q^2, s_s, s_\ell, u)$ are determined by (10), (11), (18), (22), and (21). We derive (21) using the asset market clearing conditions. Recall that $B_s/M = \sigma_s$ and $B_\ell/M = \sigma_\ell$. The asset market clearing conditions imply that

$$\frac{(1-\omega)\hat{b}_s}{\omega\hat{m}} = \sigma_s \text{ and } \frac{(1-\omega)\hat{b}_\ell}{\omega\hat{m}} = \sigma_\ell.$$

Together with $g(q^1) = \phi_{m+}\hat{m}$ and $g(q^2) = \phi_{m+}\hat{b}_s + (\phi_{m+} + \phi_{\ell+})\gamma\hat{b}_\ell$, we can reach (21). Notice that s_s is determined by (11). One can find s_ℓ as a function of (q^1, q^2, u)

from (21) and substitute it into (18). Then we have three equations (10), (22) and

$$i - \frac{\omega\sigma_\ell\gamma(1+i)g(q^1)}{(1-\omega)g(q^2) - \omega\sigma_s g(q^1)} = \gamma\alpha_h(u)\lambda(q^2).$$

to solve for (q^1, q^2, u) .

Taking full derivation against these three equations, we have

$$\begin{aligned}\frac{\partial q^1}{\partial \sigma_s} &= \frac{\omega(1-\omega)\alpha'_h\lambda_1 g_1(g'_2 - c'_2)(\gamma\alpha_h\lambda_2 - i)}{D} \simeq D, \\ \frac{\partial q^2}{\partial \sigma_s} &= -\frac{\omega g_1(\gamma\alpha_h\lambda_2 - i)[\alpha_h\lambda'_1 H' + \omega\alpha'_h\lambda_1(g'_1 - c'_1)]}{D}, \\ \frac{\partial u}{\partial \sigma_s} &= -\frac{\omega(1-\omega)\alpha_h\lambda'_1 g_1(g'_2 - c'_2)(\gamma\alpha_h\lambda_2 - i)}{D} \simeq -D, \\ \frac{\partial q^1}{\partial \sigma_\ell} &= -\frac{\omega(1-\omega)\gamma(1+i)\alpha'_h\lambda_1 g_1(g'_2 - c'_2)}{D} \simeq D, \\ \frac{\partial q^2}{\partial \sigma_\ell} &= \frac{\omega(1+i)\gamma g_1[\alpha_h\lambda'_1 H' + \omega\alpha'_h\lambda_1(g'_1 - c'_1)]}{D}, \\ \frac{\partial u}{\partial \sigma_\ell} &= \frac{\omega(1-\omega)\gamma(1+i)\alpha_h\lambda'_1 g_1(g'_2 - c'_2)}{D} \simeq -D,\end{aligned}$$

where

$$\begin{aligned}D &= -\alpha_h\lambda'_1 H'(u) \{ \gamma\alpha_h\lambda'_2 [(1-\omega)g_2 - \omega\sigma_s g_1] + (1-\omega)\gamma\alpha_h\lambda_2 g'_2 - (1-\omega)ig'_2 \} \\ &\quad - (1-\omega)\gamma\alpha_h\alpha'_h\lambda'_1\lambda_2(g'_2 - c'_2) [(1-\omega)g_2 - \omega\sigma_s g_1] \\ &\quad + \omega(1-\omega)\alpha'_h\lambda_1(g'_2 - c'_2) [\sigma_s ig'_1 + \sigma_\ell\gamma(1+i)g'_1 - \sigma_s\gamma\alpha_h\lambda_2 g'_1] \\ &\quad - \omega\alpha'_h\lambda_1(g'_1 - c'_1) \{ \gamma\alpha_h\lambda'_2 [(1-\omega)g_2 - \omega\sigma_s g_1] + (1-\omega)\gamma\alpha_h\lambda_2 g'_2 - (1-\omega)ig'_2 \}.\end{aligned}$$

It remains to find the sign of D . We follow the same approach as we used above to reduce the equation system to two equations. In (21), q^2 is a function of (q^1, s_ℓ) . Recall that $s_\ell = \gamma i\lambda_2/\lambda_1$. We can transform (21) to

$$(32) \quad g(q^2) = \frac{\omega g_1}{1-\omega} \left[\sigma_s + \frac{\sigma_\ell\gamma(1+i)\lambda_1}{i(\lambda_1 - \gamma\lambda_2)} \right].$$

Now we have two equations (10) and (22) to solve for (q^1, u) . In (22), we view q^2 as a function of q^1 implicitly defined in (32), and

$$\frac{dq^2}{du} = \frac{\overbrace{\omega\sigma_s i g'_1 (\lambda_1 - \gamma\lambda_2) + \omega\sigma_s i \lambda'_1 g_1 + \omega\sigma_\ell \gamma (1+i) \lambda'_1 g_1 + \omega\sigma_\ell \gamma (1+i) \lambda_1 g'_1 - (1-\omega) i \lambda'_1 g_2}^{\Pi_1}}{\underbrace{(1-\omega) i g'_2 (\lambda_1 - \gamma\lambda_2) - (1-\omega) \gamma i \lambda'_2 g_2 + \omega\sigma_s \gamma i \lambda'_2 g_1}_{\Pi_2}} \frac{dq^1}{du}.$$

We can verify that $\Pi_1 > 0$ and $\Pi_2 > 0$. Substitute dq^2/du into (26),

$$\frac{dq^1}{du} = \frac{H'(u) \Pi_2}{\omega (g'_1 - c'_1) \Pi_2 + (1-\omega) (g'_2 - c'_2) \Pi_1} < 0.$$

It implies that in the (u, q^1) space, (22) is downward sloping. Moreover, when $u \rightarrow 0$, q^1 should approach infinity. As before, (10) is also downward sloping in the (u, q^1) space and q^1 is finite when $u \rightarrow 0$. The intersection of (10) and (22) gives equilibrium (q^1, u) . If monetary equilibrium exists and is unique (or we focus on the equilibrium with the smallest q^1), it must be the case that (22) is steeper than (10) at the equilibrium allocation. Mathematically,

$$(33) \quad \frac{H'(u) \Pi_2}{\omega (g'_1 - c'_1) \Pi_2 + (1-\omega) (g'_2 - c'_2) \Pi_1} < -\frac{\alpha'_h \lambda_1}{\alpha_h \lambda'_1}.$$

After some algebra, we can show that (33) exactly implies that $D > 0$. Again, $D > 0$ implies that the numerator in $\partial q^2 / \partial \sigma_\ell$ is positive. We conclude that

$$\begin{aligned} \frac{\partial q^1}{\partial \sigma_s} > 0, \quad \frac{\partial q^2}{\partial \sigma_s} > 0, \quad \text{and} \quad \frac{\partial u}{\partial \sigma_s} < 0; \\ \frac{\partial q^1}{\partial \sigma_\ell} > 0, \quad \frac{\partial q^2}{\partial \sigma_\ell} > 0, \quad \text{and} \quad \frac{\partial u}{\partial \sigma_\ell} < 0. \end{aligned}$$

References

- [1] Alvarez, F., A. Atkeson, and C. Edmond, "Sluggish Response of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand," *Quarterly Journal of Economics* 124 (2009), 911-67.
- [2] Alvarez, F., A. Atkeson and P. Kehoe, "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets," *Journal of Political Economy* 110 (2002), 73-112.
- [3] Alvarez, F., R. Lucas and W. Weber, "Interest Rates and Inflation," *AEA Papers and Proceedings* (2001), 219-226.
- [4] Badarinza, C., J. Campbell and T. Ramadorai, "International Comparative Household Finance," NBER working paper No. 22066, 2016.
- [5] Bai, Y., J. Rios-Rull and K. Storesletten, "Demand Shocks as Technology Shocks", mimeo, 2017
- [6] Berentsen, A., G. Menzies and R. Wright, "Inflation and Unemployment in the Long Run," *American Economic Review* 101 (2011), 371-98.
- [7] Bethune, Z., Rocheteau, G. and Rupert, P., "Unemployment and Household Unsecured Debt," *Review of Economic Dynamics* 18 (2015), 77-100.
- [8] D'Amico, S., W. English, D. Lopez-Salido and E. Nelson, "The Federal Reserve's Large-scale Asset Purchase Programs: Rational and Effects," *The Economic Journal* 122 (2012), 415-46.
- [9] Gu, C., and R. Wright, "Monetary Mechanisms", *Journal of Economic Theory* 163 (2016), 644-57.

- [10] Geromichalos, A., L. Herrenbrueck and K. Salyer, "A Search-Theoretic Model of the Term Premium," Working Papers 138, Department of Economics, University of California, Davis, 2013
- [11] Hall, R. E., "High Discounts and High Unemployment", *American Economic Review* 107(2017), 305–30
- [12] Herrenbrueck, L., "Quantitative Easing and the Liquidity Channel of Monetary Policy," working paper, Department of Economics, Simon Fraser University, 2014.
- [13] Khan, A., "The Role of Segmented Markets in Monetary Policy," *FRB Phila. Business Review*, Q4 (2016), 1-8.
- [14] Kaplan G. and G. Menzio, "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations," *Journal of Political Economy* 124(2016), 771-825.
- [15] Keys, B.J., T. Piskorski, A.Seru and V. Yao, "Mortgage Rates, Household Balance Sheets, and the Real Economy", NBER Working Paper No. 20561, 2014.
- [16] Lagos, R. and R. Wright, "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* 113 (2005), 463-84.
- [17] Lester, B, A. Postlewaite and R. Wright, "Liquidity, Information, Asset Prices and Monetary Policy", *Review of Economic Studies* 79 (2012), 1209-38.
- [18] Li, Y., R. Rocheteau and P. Weil, "Liquidity and the Threat of Fraudulent Assets," *Journal of Political Economy* 120 (2012), 815-846
- [19] Maggio, M.D., A. Kermani and R. Ramcharan, "Monetary Policy Pass-Through: Household Consumption and Voluntary Deleveraging", mimeo, 2015
- [20] Mahmoudi, B., "Liquidity Effects of Central Banks' Asset Purchase Programs," MPRA Paper No. 49424, 2013

- [21] Mian, A., K. Rao and A. Sufi, "Household Balance Sheets, Consumption, and the Economic Slump," *The Quarterly Journal of Economics* 128 (2013), 1687-1726.
- [22] Mian, A. and A. Sufi, "What Explains the 2007–2009 Drop in Employment?" *Econometrica* 82 (2014), 2197–223.
- [23] Mortensen, D. and C. Pissarides, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61(1994), 397-415.
- [24] Nosal, E. and G. Rocheteau, "Pairwise Trade, Asset Prices, and Monetary policy," *Journal of Economic Dynamics and Control* 37 (2013), 1-17.
- [25] Rocheteau, G. and A. Rodriguez-lopez, "Liquidity Provision, Interest Rates, and Unemployment," *Journal of Monetary Economics* 65(2014), 80-101.
- [26] Rocheteau, G., R. Wright, and S. X. Xiao, "Open Market Operations," mimeo, 2017.
- [27] Schuh, S. and J. Stavins, "The 2013 Survey of Consumer Payment Choice: Summary Results," Federal Reserve Bank of Boston, Research Data Reports, No. 15-4, 2015.
- [28] Silveira, R. and R. Wright, "Search and the Market for Ideas, " *Journal of Economic Theory* 145 (2010), 1550-73.
- [29] Wen, Y., "Evaluating Unconventional Monetary Policies—Why Aren't They More Effective," FRB of St. Louis Working Paper 2013-028B, 2013.
- [30] Wallace, N, "Optimal Currency-Creation in 'Pure Currency' economies – a Conjecture," *Quarterly Journal of Economics* 129(1981), 259-275.
- [31] Williamson, S., "Liquidity, Monetary Policy, and the Financial Crisis: a New Monetarist approach," *American Economic Review*, 102(2012), 2570-605.

- [32] Williamson, S., "Scarce Collateral, the Term Premium, and Quantitative Easing". Working Paper, 2013.