An Analytical Method for Centroid Computing and Its Application in Wireless Localization

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***Abstract*—This paper presents an analytical method for one to compute the centroid of the overlapping area of two intersecting circles with arbitrary orientation. We also extend the solution to estimate the centroid of the overlapping area of three intersecting circles. Furthermore, based on the analytical solution of centroid computing, we propose a cell-based localization technique, namely Centroid Based Location (CBL) for wireless sensor networks. CBL works by finding the centroid of intersection of any two circles. In particular, we study the effect of power level mismatch among anchors. Simulation results show that CBL can significantly improve the accuracy while reducing the transmission power of anchors.**

Keywords —Wireless sensor network, localization algorithm, centroid computing, power level mismatch.

I. INTRODUCTION

Coverage of a wireless transmitter is usually modeled as a circle. Despite of multipath fading, this model is fairly useful when the transmitter is equipped with an omnidirectional antenna [1]. In order to maximize network connectivity, omnidirectional antennae are usually adopted in wireless sensor networks (WSNs), which become increasingly popular due to their low cost and wide applications [2]. As we know, a WSN is basically a collection of small size, low power and limited computing capability nodes, which sense phenomenon such as light, temperature, movement or humidity. A node transmits its sensed data towards a sink node probably with multi-hopping as each node has limited radio transmission range. Then, the sink node will send the synthesized data, perhaps through satellite network, towards the end-user. Thanks to advances in modern radio frequency integrated circuits (RFIC), the cost of manufacturing a sensor node has been reduced significantly. Meanwhile, researchers and engineers start to embed more sophisticated computing capability onto a sensor node, and this further promotes the research activities in the area of wireless sensor networks.

Along with the information generated from sensor output, many WSN applications also require location information of sensor nodes to provide meaningful data. Interestingly, some WSN routing/clustering protocols could make use of location information to realize intelligent routing and clustering [3]. Therefore, localization, i.e., how to obtain the location information becomes a critical issue in WSNs. In the past few years, many localization techniques have been proposed and they can be classified as range-based and range-free localization techniques [4]. Range-based localization techniques utilize special hardware to measure distance or angle information among sensor nodes. For example, distance can be estimated by time of arrival (TOA) [5], time difference of arrival (TDOA) [6] and received signal strength indicator (RSSI) [7]. Angle can be measured by angle of arrival (AOA) [8]. On the contrary, range-free localization techniques that do not require special hardware to be equipped on sensor nodes and estimate location of a sensor based on the connectivity information among nodes [9]. Some range-free localization techniques first allow a sensor node to determine the region it resides in based on the connectivity to other sensor nodes, and then select a point in that region as its location estimation [10]. This type of localization techniques are usually referred to as Region-Based Localization (RBL). In particular, as pointed out in [11], it will minimize average location error using the centroid of a region as position estimation. Existing algorithms include DV-Hop [12], APIT [10], Monte-Carlo Localization (MCL) [13] and Monte-Carlo Box (MCB) [14]. In general, range-based localization techniques generally provide better accuracy than range-free localization techniques at the extra expense on special hardware, although the high cost may prevent them from being implemented for large-scale WSNs.

For infrastructure-based WSNs, we often need to deploy some sensor nodes with known positions, and these nodes are referred as *anchors* [4]. Viewing from this aspect, localization techniques can also be classified as anchor-based and anchor-free techniques. The former can map a sensor node to a global coordinate system while the latter can only provide relative node positions [15]. Generally, anchor-based localization techniques can provide better location accuracy but are less flexible in the sense that anchors are fixed at known positions. Interested readers are referred to [4] for more on classification of localization techniques. With infrastructure, Chu and Jan recently proposed an outdoor localization technique in [16] and the idea was based on cell-overlapping [17]. In brief, a sensor node determines its location by analyzing the beacon messages received from anchors. However, the localization algorithm works based on the assumption that all anchors have the same transmission range, which is unlikely in practice. In a later publication, they extended the location algorithm [16] to use multiple power-level approach [18], assuming that the coverage radii of anchors can be controlled by setting different transmission power levels. Nevertheless, this is extremely challenging to maintain the same transmission power levels, especially when anchors operate on batteries. Furthermore, they did not compute the centroid of the overlapping area of any two intersecting circles. The approximated method used in [18] will certainly lead to finite unknown error.

We noticed that it is trivial to compute the overlapping area of two intersecting circles and one can find several analytical solutions from textbooks on geometry. However, to date, we are not aware any analytical solution can be found on how to compute the centroid of that area. This paper makes two major contributions to localization techniques for WSNs. First, we managed to compute the centroid of the overlapping area of any two circles (regardless of their respective radii) and proposed a centroid based localization (CBL) algorithm similar to the one in [16]. Furthermore, we study the effect of power level mismatch in anchors. Simulation results show that CBL is able to improve the localization accuracy while reducing the transmission power level of anchors.

II. RELATED WORK

Cell-based localization technique [17] was originally proposed for localization in cellular networks, where base stations (BSs) are deployed in hexagonal layout or mesh layout. For example, Figure 1 shows a hexagonal layout where the location of BSs can be pre-determined. A cell-based location algorithm for WSNs was proposed in [16], where BSs were replaced by reference points (RPs), i.e., anchors. The area covered by a RP is referred to as a cell. With proper cell-overlapping, the area of interest can be divided into three types of region: region type A, region type B and region type C, as indicated in Figure 1, which are referred to as Type I, Type II and Type III regions, respectively. It is not difficult to notice that type A region, type B region and type C region is covered by one, two and three RPs' signal, respectively. Should the RPs have the same transmission range, the centroid ($\bar{x}$, $\bar{y}$) of each localization region can be found by

$$(\bar{x},\ \bar{y}) = (\frac{1}{n}\sum_{i=1}^{n} x_i,\ \frac{1}{n}\sum_{i=1}^{n} y_i) \tag{1}$$

where (x_i, y_i) is the i^{th} vertex of the region. It was assumed that each RP knows all centroids of its localization regions. Next, the beacon format contains the layout type, the centroids of different localization regions. Consequently, a sensor node can estimated its location by intersecting the centroid sets contained in the beacon frames from various RPs. The accuracy of cell-based localization can be further improved by using multiple power level approach [18]. In summary, each RP has a set of discrete value for transmission power level, which in turn leads to a set of discrete coverage radii. As a result, the area of interest can be divided into concentric rings of regions. By estimating centroids of different intersection regions of different rings, a sensor node could better estimate its location. Similar method was proposed in [19], in which grid scan algorithm was adopted to estimate the centroid of an intersection region.

We noticed that the aforementioned works did not provide a method to compute the exact centroid of intersection of any two circles [16]. Furthermore, they require strict control on the transmission power level of a RP, which is not easy to realize this in practice, especially when the RP is operating on battery instead of unlimited power supply. Enlighten by the cell-overlapping localization method proposed in [16] [17] [18], we figure out a method of finding the centroid of overlapping area of any two intersecting circles. Subsequently, we proposed a new localization algorithm, CBL, which computes the centroid of intersection of any two circles and relaxes the transmission power constraints of RPs. Hereafter, we use anchors instead of RPs for consistency.

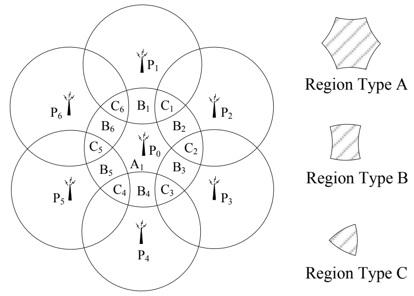

Figure 1. Layout of anchors.

III. PROPOSED CBL ALGORITHM

A. Centroid of Intersection of Any Two Circles

Let us consider a common scenario in WSNs, as shown in Figure 2, where a sensor node, e.g. S with unknown location is able to communicate with other two sensor nodes, e.g., A_1 and A_2 with known locations. Mathematically, we have

$$\begin{cases} (x-x_1)^2+(y-y_1)^2=r_1^2,\ (x-x_2)^2+(y-y_2)^2=r_2^2 \\ (x_S-x_1)^2+(y_S-y_1)^2\le r_1^2,\ (x_S-x_2)^2+(x_S-y_2)^2\le r_1^2 \\ |r_1-r_2|\le\sqrt{(x-x_1)^2+(y-y_1)^2}\le r_1+r_2 \end{cases} \tag{2}$$

In order to estimate the location of S, we need to determine the location of the centroid, e.g., point C of that particular region, as indicated as shaded area in Figure 2. As we know, the centroid of a region R with any shape can be found by

$$\left(\bar{x},\bar{y}\right)=\left(\frac{\iint_R xdxdy}{\iint_R dxdy},\ \frac{\iint_R ydxdy}{\iint_R dxdy}\right) \tag{3}$$

With a careful study, we notice that the centroid C will always be on the line connecting A_1 and A_2, because the region formed by the intersection of any two circles is symmetric respect to the line connecting the centers of the two circles. Therefore, we only need to find the value of $\bar{x}$.

Next, the equations for the left boundary and right boundary of the shaded region can be determined as follows.

$$\begin{cases} f(x_2)=\pm\sqrt{r_2^2-(x-x_2)^2},\ \text{for left boundary} \\ f(x_1)=\pm\sqrt{r_1^2-(x-x_1)^2},\ \text{for right boundary} \end{cases} \tag{4}$$

Consequently, we are able to find out the value of $\bar{x}$:

$$\bar{x}=\frac{\int_{x_2-r_2}^{0}2x\sqrt{r_2^2-(x-x_2)^2}\,dx+\int_{0}^{x_1+r_1}2x\sqrt{r_1^2-(x-x_1)^2}\,dx}{\int_{x_2-r_2}^{0}2\sqrt{r_2^2-(x-x_2)^2}\,dx+\int_{0}^{x_1+r_1}2\sqrt{r_1^2-(x-x_1)^2}\,dx} \quad (5)$$

Due to limited space, here we only provide the final result for $\bar{x}$, $\bar{x}=P/Q$, where P and Q are given by

$$\begin{aligned}P&=\frac{\pi}{4}(x_1r_1^2+x_2r_2^2)+\left(\frac{r_1^2}{3}+\frac{x_1^2}{6}\right)\sqrt{r_1^2-x_1^2}\\&-\left(\frac{r_2^2}{3}+\frac{x_2^2}{6}\right)\sqrt{r_2^2-x_2^2}+\frac{r_1^2x_1}{2}\sin^{-1}\left(\frac{x_1}{r_1}\right)-\frac{r_2^2x_2}{2}\sin^{-1}\left(\frac{x_2}{r_2}\right)\\Q&=\frac{\pi}{4}(r_1^2+r_2^2)+\frac{x_1}{2}\sqrt{r_1^2-x_1^2}-\frac{x_2}{2}\sqrt{r_2^2-x_2^2}\\&+\frac{r_1^2}{2}\sin^{-1}\left(\frac{x_1}{r_1}\right)-\frac{r_2^2}{2}\sin^{-1}\left(\frac{x_2}{r_2}\right)\end{aligned} \quad (6)$$

For the scenario shown in Figure 2, $\bar{y}$ is equal to y_1 (or y_2). Then, we will use ($\bar{x}$, $\bar{y}$) as the estimated position for node *S*.

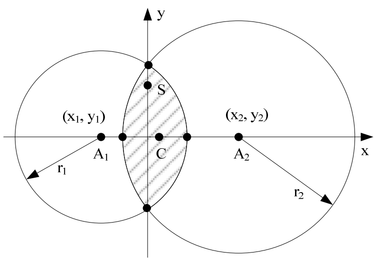

Figure 2. Intersection of two circles

Vigilant readers may notice that the scenario shown in Figure 2 is a special case as y_1 is equal to y_2 and the two intersection points are located on the *y* axis. Next, we consider a general scenario of the intersection of two circles in Figure 3. In order to use the formula we have developed, we first need to rotate both *x* and *y* axis anti-clockwise by a certain angle α,

$$\begin{cases}x'=x\cos\alpha-y\sin\alpha\\y'=y\cos\alpha+x\sin\alpha\end{cases}\quad \text{where } \alpha=-\tan^{-1}\left(\frac{y_2-y_1}{x_2-x_1}\right) \quad (7)$$

Then, after axis rotation, the line A_1A_2 will be parallel to x' axis. Next, we need to shift both x' and y' axis by certain amount as follows.

$$\begin{cases}x''=x'-a\\y''=y'-b\end{cases} \quad (8)$$

where *a* and *b* are given by

$$a=\frac{r_1^2-r_2^2}{2(x_2'-x_1')}+\frac{x_2'+x_1'}{2},b=y_1' \quad (9)$$

Eventually we can obtain the scenario shown in Fig. 2, which has two circle equations as follows.

$$\begin{cases}(x''-x_1'')^2+y''^2=r_1^2\\(x''-x_2'')^2+y''^2=r_2^2\end{cases} \quad (10)$$

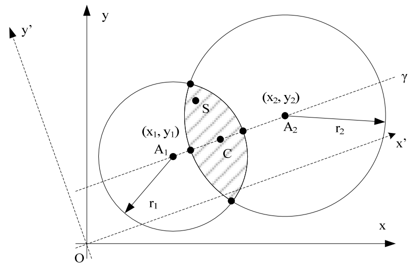

Figure 3. generalized scenario of intersection of two circles.

Finally, we can use (8) to solve for the ($\overline{x''}$, $\overline{y''}$) and then conduct the reverse procedures to obtain the respective ($\bar{x}$, $\bar{y}$) in the original coordinate system.

B. Centroid-Based Localization

Now we are able to find the centroid, C_{i-j}, of intersection of any two circles P_i and P_j. Mathematically,

$$C_{i-j}:(\bar{x},\ \bar{y})=F(x_i,\ y_i,\ x_j,\ y_j,\ r_i,\ r_j) \quad (11)$$

where *F* is the logic mapping function; (x_i, y_i) and r_i are the centre and the radius of circle *i*, respectively. The proposed localization technique is named as centroid-based localization (CBL) because it is based on finding the centroid of intersection of multiple circles. CBL works in four steps:

Step 1—Anchors Deployment: First, anchors are deployed with known locations to cover an area of interest. These anchors may not necessarily be deployed in a regular layout structure such as hexagonal or mesh structure used in [16]. Furthermore, these anchors are not required to know their localization regions and the corresponding centroid of any particular localization region, which is the distinctive feature that makes our CBL saliently different from the localization method proposed in [16].

Step 2—Beacon Frame Broadcast: Next, anchors periodically broadcast their beacon frames to notify all the sensor nodes residing in their coverage area. Similar to the beacon frame in [18], the CBL beacon frame from a particular anchor node A_i contains the following data:

$$I_i=\{(x_i,\ y_i),\ r_i\} \quad (12)$$

where (x_i, y_i) is the location coordinates of anchor node A_i and r_i is A_i's coverage radius. As mentioned in [18], the coverage radius may be obtained using the free space propagation model. In particular, if an anchor node is operated on battery, its coverage radius will change according to its transmission power level.

Step 3—Beacon Frame Synthesis: Subsequently, a sensor node with unknown location will listen to the beacon broadcast from anchors nearby. As shown in Figure 4, a sensor node *S* with unknown location receives beacon frames from anchor node

A_1, A_2 and A_3, thus S knows that it is residing in the intersection region of the three circles.

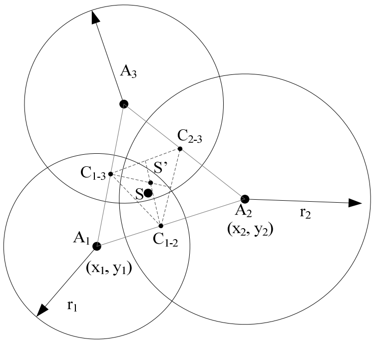

Figure 4. A general scenario of CBL

Step 4—Location Estimation: After listening to the broadcast of anchors for sufficient long time interval (about 10 beacon period), a sensor node may receive beacon frames from N anchors. The sensor node computes the centroid of intersection region of any two circles. Then, it takes the centroid of these centroids as its location estimations. Take Figure 4 as an example, sensor node S will estimate its location as follows.

(1) S calculates $C_{1\text{-}2}$ using the beacon frames from anchors A_1 and A_2 by applying (11).

(2) Then, S calculates $C_{1\text{-}3}$ and $C_{2\text{-}3}$ in a similar way.

(3) Finally, S estimates its location as

$$S':(x_{s'},y_{s'})=\left(\frac{x_{C_{1-2}}+x_{C_{1-3}}+x_{C_{2-3}}}{3},\ \frac{y_{C_{1-2}}+y_{C_{1-3}}+y_{C_{2-3}}}{3}\right)$$

Specially, if a sensor node, S, only receives beacon frame from a single anchor node, A, then S takes A's location as its location estimate.

C. Positioning Accuracy

Same as the metrics used in [16], we use absolute error distance, cumulative distribution function (CDF) of error distance and average value of error distance to measure the performance of our proposed CBL. For the sake of comparison, we revisit the definition of these parameters.

Error distance is mathematically defined as

$$D=\sqrt{(X-x_{est})^2+(Y-y_{est})^2} \tag{13}$$

where (X, Y) is the actual location coordinate of a sensor node, and (x_{est}, y_{est}) is the estimated location coordinate of the sensor node. The CDF of D is defined as

$$e(r)=P\{D<r\} \tag{14}$$

This is also referred to as the precision. If the sensor nodes were uniformly distributed over a region R, the probability density function $f(x, y)$ can be found as

$$f(x,y)=1/A_R \tag{15}$$

where A_R denotes the area of region R. Therefore, we can rewrite the precision as

$$e(r)=P\{D<r\}=\iint_{C_r} f(x,y)dxdy=A_{C_r}/A_R \tag{16}$$

where $C_r=\left\{(x,y)\middle|\sqrt{(x-x_{est})^2+(y-y_{est})^2}<r\right\}\cap R$. The smallest value of r that satisfies $e(r)$ = 1.0 is the critical radius and the worst-case accuracy is the maxima among the three critical radii of the three types of localization regions.

The average accuracy is calculated as

$$E[D]=\sum_{i=1}^{3} p_i E[D_i] \tag{17}$$

where p_i is the probability that a sensor node falls in the type i localization region, and $E[D_i]$ is calculated as

$$E[D_i]=\iint_{(x,y)\in R_i}\sqrt{(x-x_{c_i})^2+(y-y_{c_i})^2}\, f(x,y)dxdy \tag{18}$$

where R_i is the localization region type i and (x_{c_i}, y_{c_i}) is the centroid of R_i.

IV. Simulation Results

We first consider a hexagonal layout of anchors, where the distance between two neighboring anchors is one distance unit and the coverage radius of an anchor node is varied from $1/\sqrt{3}$ to $\sqrt{3}/2$ distance unit [17]. 10,000 sensor nodes are uniformly distributed over the area of interest and we study the performance of CBL and the technique presented in [16].

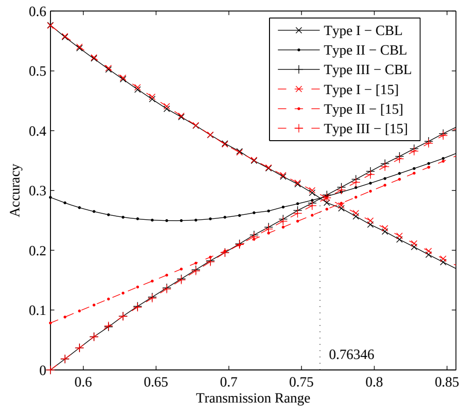

Figure 5. Worst-case Accuracy vs. Transmission Range

As shown in Figure 5, for type I and type III regions, our CBL algorithm has the same performance with the one proposed in [16]. However, for the type II region, our CBL requires higher error tolerance level. Interestingly, the minimum worst-case accuracy occurs at transmission range d=0.76346 at which the worst-case accuracy is 0.28286. This is better than the 0.2887 at d=0.7638 presented in [16].

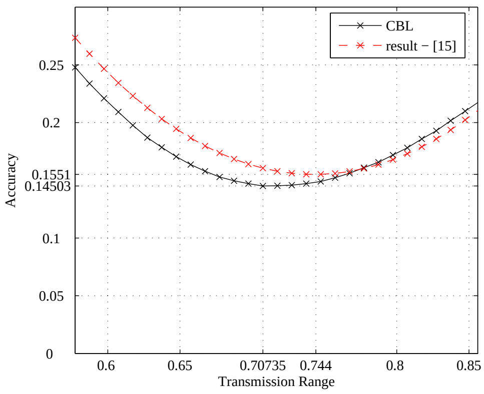

Figure 6. Average accuracy vs. Transmission Range

Next, we look at the average accuracy of CBL. As shown in Figure 6, the average accuracy $E(D)$ can be expressed as a convex function of transmission range d, i.e. $g(d)$. Furthermore, we found that the minimum of $g(d)$ has is 0.14503 where d=0.70735. This minima of $g(d)$ is lower than that found in [16], which is 0.1551 at d=0.744. Importantly, our CBL can achieve better average accuracy while using smaller transmission range that reduces transmission power level. This is desired as power is a scarce resource in WSNs.

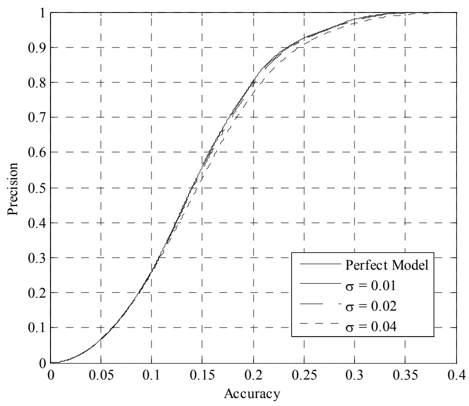

Figure 7. Effect of Power Level Mismatch on Average Accuracy

We have assumed that all the anchors have identical coverage radii in the previous simulations. However, in practice different anchors probably have different power levels and thus different coverage radii. Therefore, we shall take this effect into account by modeling their coverage radii as a Gaussian distributed random variable with mean value of d and standard deviation of σ. From above, we set the mean d to 0.70735 in order to achieve minimized average localization error. In this simulation, 100,000 sensor nodes are considered and anchors are deployed with hexagonal layout. The distance between any two anchors is one distance unit. During the localization process of each sensor node, the coverage radius of every anchor node is generated by Gaussian distribution with parameter (d, σ) and CBL is used to estimate the position of the sensor node. As shown in Figure 7, we found that mismatch in the power levels of anchors will affect the localization accuracy. The higher the mismatch, the higher the localization error will be caused.

V. Conclusion

We presented an analytical method of computing the centroid of the overlapping area of any two intersecting circles, and then developed a localization technique, namely centroid-based localization (CBL) for wireless sensor networks. Simulation results showed that CBL is able to improve localization accuracy while reducing the transmission power level of anchors. Furthermore, we studied the effect of power level mismatch of anchors on the localization error of CBL. The performance of CBL degrades gracefully when the power level mismatch in anchors increases.

References

[1] A. Goldsmith, *Wireless Communications*: Cambridge University Press, 2005.

[2] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Computer Networks,* vol. 52, pp. 2292-2330, 2008.

[3] K. Akkaya and M. Younis, "A survey on routing protocols for wireless sensor networks," *Ad Hoc Networks,* vol. 3, pp. 325-349, 2005.

[4] G. Mao, B. Fidan, and B. D. O. Anderson, "Wireless sensor network localization techniques," *Computer Networks,* vol. 51, pp. 2529-2553, 11 July 2007.

[5] B. H. Wellenhoff, H. Lichtenegger, and J. Collins, *Global Positioning System: Theory and Practice*, Fourth Edition ed.: Springer Verlag, 1997.

[6] B. P. Nissanka, C. Anit, and B. Hari, "The Cricket location-support system," in *ACM MobiCom'00*, Boston, MA, 2000, pp. 32-43.

[7] P. Bahl and V. N. Padmanabhan, "RADAR: an in-building RF-based user location and tracking system," in *IEEE INFOCOM'00*, 2000, pp. 775-784.

[8] D. Niculescu and N. Badri, "Ad hoc positioning system (APS) using AOA," in *IEEE INFOCOM'03*, San Francisco, CA, 2003, pp. 1734-1743.

[9] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-less low-cost outdoor localization for very small devices," *IEEE Personal Communications,* vol. 7, pp. 28-34, October 2000.

[10] T. He, C. Huang, B. M. Blum, J. A. Standkovic, and T. Abdelzaher, "Range-free localization schemes for large scale sensor networks," in *ACM MobiHoc'03*, San Diego, CL, 2003, pp. 81-95.

[11] S. Zhang, J. Cao, L. Chen, and D. Chen, "On accuracy of Region-Based Localization algorithms for Wireless Sensor Networks," in *IEEE MASS'09*, 2009, pp. 30-39.

[12] D. Niculescu and B. Nath, "Ad hoc positioning system (APS)," in *IEEE GLOBECOM '01*, San Antonio, TX, 2001, pp. 2926-2931.

[13] L. Hu and D. Evans, "Localization for mobile sensor networks," in *ACM MobiCom'04*, Philadelphia, PA, 2004, pp. 45-57.

[14] A. Baggio and K. Langendoen, "Monte-Carlo localization for mobile wireless sensor networks," *Ad Hoc Networks,* vol. 6, pp. 718-733, January 2008.

[15] G. J. Jordt, R. O. Baldwin, J. F. Raquet, and B. E. Mullins, "Energy cost and error performance of range-aware, anchor-free localization algorithms," *Ad Hoc Networks,* vol. 6, pp. 539-559, 2008.

[16] H.-C. Chu and R.-H. Jan, "A GPS-less, outdoor, self-positioning method for wireless sensor networks," *Ad Hoc Networks,* vol. 5, pp. 547-557, 2007.

[17] H.-C. Chu and R.-H. Jan, "A cell-based location-sensing method for wireless sensor networks," *Wireless Communication and Mobile Computing,* vol. 3, pp. 455-463, June 2003.

[18] J.-Y. Fang, H.-C. Chu, R.-H. Jan, and W. Yang, "A multiple power-level approach for wireless sensor network positioning," *Computer Networks,* vol. 52, pp. 3101-3118, November 2008.

[19] C. Liu, K. Wu, and T. He, "Sensor localization with ring overlapping based on comparison of received signal strength indicator," in *IEEE MASS'04*, 2004, pp. 516-518.