

**Mathematics or Numeracy:
Education Indications from Steiner and the New Zealand Curriculum**

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ABSTRACT

Mathematics education has been of considerable concern in New Zealand for several decades. During the 1990s, following the *Education Act 1989* in which a quantifiable market value approach was imposed onto education, curricula changes and international mathematics and science surveys exposed low achievement levels among New Zealand students. A concentration upon number and algebra as 'numeracy' for years 1-10 in New Zealand schools was offered through the *Numeracy Development Projects* from 1999, and included programmes of pedagogical and mathematical content knowledge for teachers as well as resources and information for parents. By 2006 most New Zealand schools had taken up the Numeracy Projects, which are based on social constructivism and use a strategy-knowledge method with regular diagnosis of the student's level of understanding.

Social constructivism, as a philosophical approach to mathematics education, appears to be inconsistent with a market value ideology. As a contrast to both views, Rudolf Steiner's indications for mathematics education are centred on the development of the child; slowly maturing physiologically and cognitively towards the intellectual capacity for critical judgement. While the New Zealand Ministry of Education and Steiner provide indications only for mathematics education upon which the mainstream and Steiner Waldorf schools build their classroom curriculum, the 2009 legislation of *National Standards in Numeracy* unmasks the level of political control of mathematics education now in New Zealand.

This thesis uses a hermeneutic / bricolage method to review literature on the socio-cultural historical progression towards this 21st century position of mathematics or numeracy in New Zealand schools. It also introduces Rudolf Steiner and the foundational views of his educational approach. The Steiner Waldorf schools in New Zealand were initially privately funded, but eight of the present ten schools integrated into state support through *The Private Schools Conditional Integration Act 1975*, with varying degrees of financial aid coupled to a change of priorities and consequent amendments to classroom practices, especially in mathematics.

As mathematics is considered a language for thinking, this thesis presents some diagrammatic representations of mathematical thinking including a conjecture of the impact of the process towards algebraic thinking emphasised in the Numeracy Projects and National Standards. Future research in the classroom is needed to examine this conjecture.

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ATTESTATION of AUTHORSHIP

I hereby declare that this submission is my own work and that, to the best of my knowledge and belief, it contains no material previously published or written by another person (except where explicitly defined in the acknowledgements), nor material which to a substantial extent has been submitted for the award of any other degree or diploma of a university or other institution of higher learning.


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With thanks, I acknowledge Michael Hambrey and glaciers-online.net for his photograph of the Tasman River flowing out from the Tasman Glacier in the Southern Alps, and Adrian Meredith for his photograph of the Rakaia River flowing into the Pacific Ocean.

Mathematics or Numeracy: Education Indications from Steiner and the New Zealand Curriculum



Figure 1: Tasman River flowing out from the Southern Alps; Mt Cook to the right

M. Hambrey (2007). From <http://www.swisseduc.ch/glaciers>. Reprinted with permission.

Chapter 1. Introduction: Mapping the Journey

- 1.1. Aim
- 1.2. Background to thesis
- 1.3. Research Questions
- 1.4. Overview of Thesis

*Mathematics is a journeying and the journey's destination;
it is thinking and the process towards thinking;
it is a language and the languaging.*

I have begun with the piece of writing above as a reflection of the different ways in which mathematics can be both the product and the process within education. Before a child begins formal education language describing position and quantity is practised that is later used in mathematics lessons, and the beginning of causal and

sequential thinking develops. Once a student enters into schooling, the journey with and of mathematics as symbolic language is experienced. Bortoft (2012)¹ described the search for the essence of a phenomenon as shifting one's attention from the 'seen' to a point or place before the 'seen' appears. This thesis reports my journey through the New Zealand educational landscape from that place before the 'now' that has helped to build what we experience today as mathematics education in New Zealand in the 21st century.

The braided rivers common in the South Island of New Zealand epitomise for me this journey that I have made. The indications for mathematics teaching and learning provided in the New Zealand curricula (Ministry of Education, 1992, 2007) and those that Rudolf Steiner (1861-1925) gave for mathematics education (collated in Stockmeyer, 1982) weave together like the channels of the river. In New Zealand the source of the braided river is from the glacial and snow-capped mountain ranges, the Southern Alps. These mountains stand like pinnacles between the plains and the sky, just as mathematics has stood in the past. Braided rivers carry sediment scoured from the rock of the mountains and deposit it in small islands (braid bars) among the water channels along the way. The water flow can be but a trickle in the late summer and early autumn when there are few rains and little snow-melt, yet a roaring wide expanse in full flood when the rains and snow-melt combine in late winter and early spring. Eventually, the river waters reach the eastern coast emptying into the Pacific Ocean with perhaps sediment bars and lagoons or shallow lakes as the penultimate area for the deposit of sediment. From the grass-chequered, silt-formed plains the sharp peaks of the Southern Alps fill the horizon to the west, dominating the threshold between land and sky. To the east, the blue expanse of sea merges with the light of the sky; all appears as One.

Mathematics has a history of being infallible and stable, a pinnacle of knowledge that has been recognised as a discipline since the period of Classical Greece (5th century BCE). The word is derived from the Greek *máthēma*, meaning 'knowledge,

¹ See, especially, Bortoft (2012), pp. 19-27 in which he carefully describes this perspective.

study', and relates to mathesis,² 'learning', as was practised by Pythagoras (c.570 - c.495 BCE) and his School in relation to the study of harmony and music, number and proportion, geometry and astronomy. This Pythagorean study introduced mathematics as an intellectual discipline of a learned, knowledgeable, free person. Through Greek individualities from the 6th to 3rd centuries BCE, such as Plato (427 - 347 BCE), Aristotle (384 - 322 BCE) and Euclid (c. 325 - c. 265 BCE), mathematic statements became generalised and confirmed by proof. There was a focus on mathematic assumptions and mathematical issues were debated, publically, using mathematical language. *Máthēma/mathesis* became a way of thinking, to come of age during Renaissance times in the fifteenth and sixteenth centuries after Hindu-Arabic numerals had been introduced.³

During the twentieth century, the infallibility of mathematics was questioned from philosophic and scientific perspectives; Kuhn (1962/1996) and Lakatos (1978) brought the debate into the public arena during the 1970s. Yet, as 'mathematics as truth' was being questioned, mathematics had become an important tool in schools for the scientific age of space travel. By the early 2000s, a New Zealand governmental impetus towards a 'Knowledge Society' (Gilbert, 2005) brought numeracy back as an imperative, now channelled in New Zealand through the *National Standards in Numeracy* (Ministry of Education, 2009). Numeracy could be defined as 'fluency with number'; contracted down from the larger body of knowledge that was mathematics, and which now appears to dominate during compulsory formal schooling. Mathematics as a 'body of knowledge' has been particularised into smaller assessable bits, and what had provided a discipline in thinking and of itself now appears like a fractal, constantly devolving into parts.

1.1. Aim

In the beginning, my aim was to clarify the philosophical or ideological ground for the indications given for mathematics education in *The New Zealand Curriculum*

² See Kaplan & Kaplan (2003): "if mathematics is derived from Understanding (the real meaning of *Mathesis*, as science is from Knowledge, *Scientia*) we must clear the paths to and within it of rubble, and let the mind enjoy the pure play of form" (p. 126).

³ Fibonacci of Pisa (c. 1170 - c. 1250 CE) composed his book *Liber Abaci* in 1202 in which he presented the Hindu-Arabic symbols for numbers to fellow Europeans.

(2007) and for Steiner's indications (1907-1924)⁴ for mathematics education. A review of literature suggested that while both appear to arise out of a humanist view, there are differences in the processes used within the teaching/learning situations that may be explained through a description-analysis-interpretation of the fundamental philosophies of each.

During my 'study journey' my perspective of this destination changed. A certain dissonance has developed between the indications given in *The New Zealand Curriculum* (2007) and the delivery of mathematics education through the *Numeracy Development Projects* (NDP) since their launch in 2000/2001, and of nationalised assessment through the *National Standards in Numeracy* (2009). I have, therefore, given particular consideration to the impact of these two policy programmes on mathematics education in New Zealand.

Thinking is also an expectation of the different mathematics approaches, but what sort of thinking? Within the Mathematics and Statistics learning area of *The New Zealand Curriculum* (Ministry of Education, 2007) the thinking to be encouraged through mathematics education is listed as creative, critical, strategic and logical. Thinking *skills* are also defined: "to structure and to organise, be flexible and accurate when processing, to communicate and enjoy intellectual challenge, to predict, conjecture, justify and verify, to estimate and calculate" (Ministry of Education, 2007, p. 26). The *Numeracy Development Projects* use a strategy-knowledge model and focus on algebraic thinking (Ministry of Education, 2008a), while Steiner speaks of mathematics as encouraging moral thinking (Steiner, 1922/1947). What impact would these different emphases on thinking have?

My study thus broadened my initial aim of a reflective journey through literature concerning the philosophical or ideological foundations of the indications for mathematics education, to include theories of cognition relevant to mathematics education from both the mainstream and Steiner's writings. The question that now

⁴ Stockmeyer (1982), has collected and published in *Rudolf Steiner's curriculum for Waldorf schools*, the indications Steiner gave in his many lectures, discussions with teachers and writings on education.

provided the aim for this thesis concerned the impact of the Numeracy Projects and the *National Standards in Numeracy* on learning and on those indications for mathematics education by the New Zealand Ministry of Education and by Rudolf Steiner. Is it mathematics or numeracy that is the focus for mathematics education?

1.2. Background to Thesis

I first became interested in mathematics education through the Numeracy Projects and their impact. My grandson used to love numbers and mathematics but after exposure to the Numeracy Projects became very confused, and I wondered, “What is happening here?” In later university study my work with early research on the Numeracy Projects led me to further questions about the relationship of numeracy to mathematics and to thinking. It was apparent that in order to understand these sediment-clouded waters I needed to return to the source: mathematics, philosophy, *mathesis*, *máthēma*.

Jarman (1998, pp. xxvi-xxx) describes a progression from arithmetic to *mathesis*, which I portray in Figure 2, below.

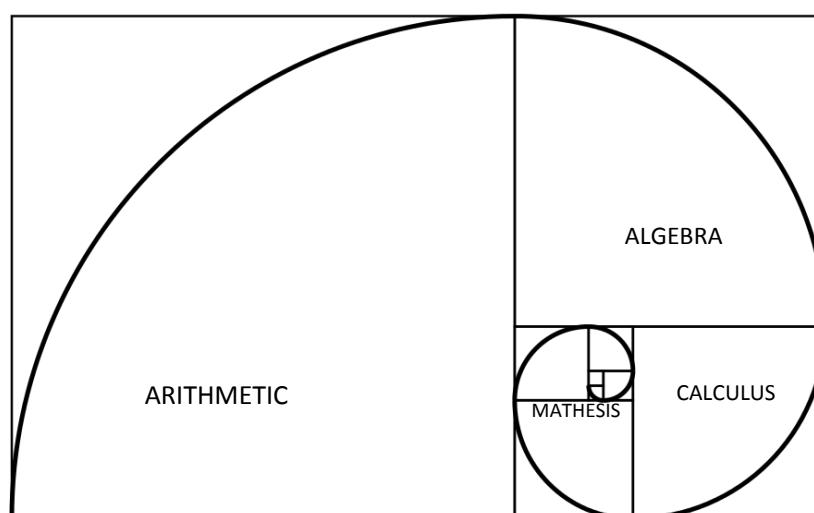


Figure 2: A progression from arithmetic to mathesis

The route takes us from the finite particular - the number in arithmetic - through the finite general (algebra), and the infinite general (calculus) to the philosophic centre - the infinite particular. But the journey could also move in the opposite

direction; from the philosophic centre out. Number had order and place, but what of the rest of mathematics? Where was Geometry, which had initially been used to represent numbers as triangular, square, pentagonal, and so on?⁵ It was time to climb the mountains.

Máthēma, during the sixth to fourth centuries BCE, had been the language of philosophy, (from Gk. *philo-* love, and *Sophia* wisdom) that ‘love of wisdom’. Mathematics is now a language of knowledge. Is knowledge wisdom? Does mathematical knowledge lead to wisdom? Beyond the knowledge, and as an integral part of knowledge lie values; the values or philosophic world-view of the one who shares such knowledge (either as teacher or pupil), and the values inherent in the knowledge itself. Thus, there are explicit and implicit values; the explicit and implicit philosophies within the mathematical knowledge shared, and within the sharer of that mathematical knowledge (Ernest, 1991).⁶ And with the particular philosophical approaches come particular ways of thinking, embedded and expected. Questions arose as to the forms of thinking that will be expressed by New Zealand adults in their twenties and thirties, who have experienced mathematics education in New Zealand schools since the 1990s when an outcomes-related approach was implemented (Sewell, 2007). Would these forms of thinking meet the expectations of the ‘Knowledge Society’ that New Zealand politicians hoped for in a 21st century New Zealand? Would this thinking be any different to the past when innovation and creativity was framed by the creative capacities of ‘the Number 8 wire’?⁷ The differences in emphasis on the sorts of thinking expected by the New Zealand curricula (1992⁸ & 2007⁹), by the *Numeracy Development Projects* (Ministry of Education, 2001-2012), and as suggested by Steiner’s indications, all hint at varying emphases to be expressed by mature adults. I felt there needed to be a theoretical-philosophical foundation built before moving on to attempt answers to such questions. Essentially, I needed to look upstream and

⁵ See Gregersen, 2011; Kline, 1990.

⁶ For further reflective discussion see François, K., & van Bendegem, J. P. (Eds.), (2007). *Philosophical dimensions in mathematics education*. Mathematics Education Library, Vol 42.

⁷ A gauge size of wire used in fencing farmland in New Zealand, and able to be utilised within many ‘make do’ situations. For fun, visit <http://www.fieldays.co.nz/no8wire>.

⁸ Mathematics in the New Zealand Curriculum (Ministry of Education, 1992).

⁹ The New Zealand Curriculum - Mathematics and Statistics (Ministry of Education, 2007).

frame my perspective over the history of mathematics education in New Zealand and its socio-cultural evolution, and to then describe the present with some conjecture regarding the future.

This research evolved out of my questions relating to the Numeracy Projects. My personal relationship to Steiner Waldorf education in New Zealand over the last thirty years, and that education's holistic approach, encouraged me to review the indications offered by Steiner for mathematics education, alongside a review of those indications and methods provided in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) and *The New Zealand Curriculum - Mathematics and Statistics* (Ministry of Education, 2007) and the *Numeracy Development Projects*. It is perhaps helpful to remember, at this point, that the New Zealand curricula (1992 & 2007) give only indications, also, and stress the need for schools to develop their own particular curriculum to suit the needs of their students and community (Education Review Office, 2013; Ministry of Education, 2007). Enrolment for formal schooling in New Zealand must have been made by the time the young child turns six years and last until the student turns sixteen years of age, unless there are specific exemptions such as home-schooling and youth apprenticeships (<http://www.minedu.govt.nz>). I have therefore limited this study of mathematics education to within the years 1 - 10 of schooling in New Zealand.

1.3. Research Questions

In the history of the present eleven Steiner Waldorf schools in New Zealand (the first acknowledged school was established in Hastings during the 1950s with others established from the 1970s on), there has been no review of the mathematics education according to the indications offered by Steiner. Since the implementation of the *Numeracy Development Projects*, many of the teachers in the Steiner Waldorf schools have used that approach in their mathematics lessons. A recent report made to the *Federation of Rudolf Steiner Waldorf Schools in New Zealand* in December 2013 (see Chapter 6, p. 72 below) showed a decrease in achievement in mathematics by students at the years 6-8 level since the use of the Numeracy

Projects in the schools. While irregular checks using Progressive Achievement Tests for mathematics (NZCER, 2006, 2009) had been made in some schools, there had been no full review of mathematics education, although curriculum review and adjustment had been carried out from time to time.

The implementation of the *Numeracy Development Projects* in many New Zealand schools had followed concern over the mathematics achievement of students, the pedagogical content knowledge of teachers, especially those of students in years 1 - 8, and the trialling of a New South Wales, Australia programme called *Count Me In Too* (New South Wales Department of Education and Training, 1999) in some New Zealand schools during 1999 and 2000. Although the *Count Me In Too* programme had been used for 6 - 9 year-olds in New South Wales, New Zealand developed the *Numeracy Development Projects* to cover years 1 - 10, that is, 5 - 15 year-olds. From year 10, unit and achievement standards tests that had been introduced during the 1990s, were taken by students towards their *National Certificate of Educational Achievement* (NCEA). In mainstream schools within New Zealand research has shown a 'plateauing' or levelling out of mathematical knowledge acquisition at the higher levels of the numeracy projects and slower acquisition among some ethnic and socio-economic groups (Young-Loveridge, 2006, 2009, 2011).

Since the 1990s when the *Education Act 1989* was implemented there had been considerable mistrust voiced over the change of orientation and emphasis in education, generally, in New Zealand (Codd, 1999; Gordon, 1992; Hazledine, 1998; Neyland, 1994, 1995; Sullivan, 1994, 1998). An economic model was imposed onto education, which was then considered to be a commodity, and economic values were placed onto measureable outcomes of students, teachers and their schools. Reports and reviews of the standard of education in New Zealand by businessmen (Kerr, 1991; Picot, 1988) had highlighted a negative view of education and teachers, bringing this view to political and public attention.

Towards the end of 2008, an amendment to the *Education Act 1989* legislated that *National Standards in Numeracy and Literacy* be implemented in years 1 - 8 in all

state-funded New Zealand schools from the beginning of 2010.¹⁰ Steiner Waldorf schools in New Zealand do not follow the exact progression of levels set in the *National Standards in Numeracy* and those receiving state funding¹¹ argued against the necessity to regularly test against those levels in their schools. A consequent legal campaign led to the introduction in 2012 of *Learning Steps in Numeracy (and Literacy)* for years 2 - 8 in Steiner Waldorf schools in New Zealand with the aim, through testing at year 8, of showing that their students reach or surpass the year 8 national level expected in the National Standards.

Given these developments in education in New Zealand, and in mathematics education in particular, I felt it was time to pursue research in this area as a foundation for possible further empirical research. Ernest (1991) formulates ideological models for mathematics education,¹² but neither the New Zealand curricula (Ministry of Education, 1992, 2007) nor the indications offered by Steiner (collated in Stockmeyer, 1982) fit neatly into any of these models. The *Numeracy Development Projects*, which use a social constructivist approach, had a programme support for teachers that now appears to be sidelined by the evaluation expectations of the *National Standards in Numeracy*. Thus, my journey has been guided by consideration of the following six sub-questions:

- What are the philosophical or ideological congruencies between *The New Zealand Curriculum - Mathematics and Statistics* (Ministry of Education, 2007) and the approach to mathematics education as indicated by Rudolf Steiner?
- What are the philosophical or ideological inconsistencies that are likely to affect the teaching and learning of mathematics in Steiner Waldorf schools in New Zealand?
- What ideological effect might the Numeracy Projects have on Steiner Waldorf schools and mainstream schools in New Zealand?

¹⁰ Section 60A(1)(ba): inserted, on 17 December 2008, by section 8 of the Education (National Standards) Amendment Act 2008 (2008 No 108).

¹¹ Of the eleven Steiner Waldorf schools in New Zealand eight are integrated, thus enabling state funding under the "Special Character" of *The Private Schools Conditional Integration Act 1975* (Munro, 1995).

¹² See Ernest (1991), pp. 138-139.

- What ideological effect might the *National Standards in Numeracy* (2009) have on the teaching/learning process in mainstream New Zealand schools?
- What impact might the *Learning Steps in Numeracy* (2013) have on the teaching/learning process in Steiner Waldorf schools in New Zealand?
- What effects might the philosophy/ideology of *The New Zealand Curriculum - Mathematics and Statistics* (Ministry of Education, 2007) and the approach to mathematics education as indicated by Rudolf Steiner, have on thinking?

Ideology is defined by Ernest (1991) as “an overall, value-rich philosophy or world-view” (p. 111), while François and Van Bendegem [(Eds.), 2007] highlight in their preface the implicit or unconscious values as well as the explicit.

1.4. Overview of Thesis

This thesis has eight chapters. Chapter 1, this introduction, provides some background and an overview for the reader, including the questions that have accompanied me during my study towards the aim of this thesis. Chapter 2 describes the methodology used in the analysis and interpretation of the review of literature that has been the basis of this study. Chapters 3 and 4 introduce the reader to the two main areas of study, mathematics education as indicated by the New Zealand curricula (Ministry of Education, 1992 & 2007) and by Rudolf Steiner (as collated in Stockmeyer, 1982), by providing some pre-21st century historical context for the further chapters. Chapters 5 and 6 are consistent with the two foci of study: the resultant philosophical or ideological impulse for mathematics education in New Zealand schools in this 21st century, and Steiner’s epistemology of thinking in relation to mathematics education. Chapter 7 presents a discussion of the results or findings of this study, including the influences of the different approaches to mathematics education as exemplified by the New Zealand Ministry of Education and by Rudolf Steiner. The final chapter, Chapter 8, offers reflections on the findings and suggestions for possible future research.

The reader of this thesis will observe footnotes, which are included to provide further clarification of certain terms used, as well as offering further reading or contextualising. A Glossary of terms, each initially provided in footnotes or text when first appearing, precedes the Appendix. I present this thesis as a foundation for dialogue on the teaching and learning of mathematics in New Zealand schools. I hope it stimulates questions and further dialogue and research.

Chapter 2. Methodology: Processing the Journey

2.1. Introduction

2.2. Hermeneutic/Bricolage Method

2.3. Methodology Background and Assumptions

2.4. Summary

2.1. Introduction

Here I relate the journey I have taken with this study and the methodology that has ensued. There is no empirical research; rather I have used freely available literature and publications (books, articles, transcriptions) on which to base my findings and consequent conclusions and recommendations. The review of literature that is central to this thesis is thematically undertaken throughout chapters 3 - 7, rather than in one chapter entitled 'Literature Review'. My personal learning experiences with psychology, education and anthroposophy have steered the methodology towards a qualitative search to interpreting meaning through the literature.

Figure 3 over the page is a diagram I have adapted from Denzin and Lincoln [(Eds.), 2011, pp. 12-13], of the research relationship between my personal history and views and those secondary sources of the 'world', with the recursive movement of observation and reflection that is so much a part of that relationship. Denzin and Lincoln (2011) would have the paradigm be the researcher's theory or philosophic orientation determining the ontology, the epistemology and the methodology of the research. I have found that the research process has impacted upon my original pre-research paradigm, and therefore perhaps comes closer to the "participatory / cooperative paradigm" as suggested by Heron and Reason (cited in Lincoln, Lynham & Guba, 2011, pp. 98, 100). There is necessarily an integration of my personal 'world view' with the 'public' writings through my approach and method of research.

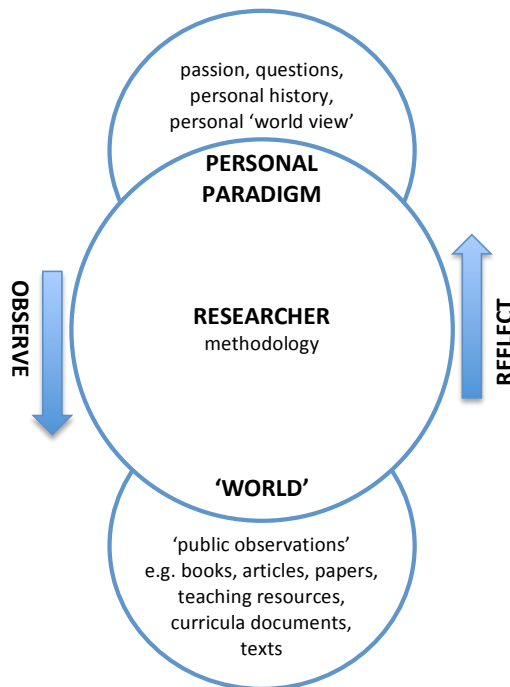


Figure 3: Research relationship between personal paradigm and 'world'

2.2. Hermeneutic / Bricolage Method

From both personal and socio-cultural perspectives, the development of a philosophical outlook accompanies a change in the conscious awareness of one's self in relation to one's social and learning environment. In the language of Henri Bortoft (2012), it is an activity that transforms *an appearing* of something into *the appearance*. To reflect on the method I have used means re-considering that cognitive space before the *appearing*. Kincheloe and Berry (2004) consider hermeneutics to be "a form of philosophical inquiry that focuses on the cultural, social, political and historical nature of research" (p. 82). My research has followed these routes within general education and mathematics education in New Zealand, and it could thus be argued that I am taking a hermeneutic approach: *to say, to explain, to translate*, as per Palmer (1969). Moore (2013) refers to Hans-Georg Gadamer and his hermeneutic view of 'tradition': one's particular understanding and judgement are intimately connected to that *appearing* of knowledge and as such, may be termed 'prejudices'; nevertheless, "according to Gadamer, these

prejudices not only condition our understanding but also provide the conditions without which understanding could not take place” (p. 30). My personal views therefore determine what I see or read and how I present the outcome of this study.

The choices I make in meaning-making within this thesis are inevitably ideological and ultimately political (Kincheloe & Berry, 2004). McCulloch (1992) reminds us, also, of the wider implications of the society or culture within which we live, and of the educational system and curriculum, on our understanding. Bernstein (1971) suggests that the determination of particular knowledge as public (for example, as a curriculum in education) “reflects both the distribution of power and the principles of social control” (p. 85). In working with hermeneutics, bricoleurs may explore the nature and effects of the social construction of knowledge and understanding (Kincheloe & Berry, 2004). This exploration is especially evident in my research on thinking in relation to mathematics education. My consequent interpretation exposes the relationship between power and the economic, cultural and social conditions inherent in meaning-making. There is therefore some justification in considering my method as that of a bricoleur; this thesis as bricolage presents something of the relationship of the ‘world’ to power (Kincheloe, McLaren & Steinberg, 2011). My study has subsequently led me through philosophical, sociological and political writings as well as educational and cognitive.

2.3. Methodology Background and Assumptions

My journey to this thesis began with concern regarding the Numeracy Projects. My personal relationship, for over thirty years, to anthroposophy and Steiner Waldorf education led me to a thesis topic: a two-school case-study of lessons in Fractions, one using the resources of the Numeracy Projects within a Steiner Waldorf school, the other only the indications as given by Rudolf Steiner. That thesis topic was originally to have been supervised by Jim Neyland. His premature death meant changes: in university, supervisors and topic. By the time I was ready to begin again, most of the Steiner Waldorf schools in New Zealand were using the Numeracy

Projects' approach in mathematics lessons, and I could no longer find a relevant 'control' group for empirical research. My present primary supervisor and I decided to work only with published or freely accessible literature and weave the review throughout the chapters.

Mathematics education in New Zealand is founded on a tradition of western education, which presented mathematics as having a philosophical origin arising from the concept of the 'certainty of Truth' (Bowen, 1972; Lucas, 1972). The transformation from Plato's sense of the infallible 'absolute truth' through to the influences of cognitivist approaches on learning styles and the present-day venture into fallibilistic constructivism were analytically reviewed by Dengate (1998). Constructivism is a philosophical response to what Gardner referred to as the "cognitive revolution" (cited in Schoenfeld, 2002, p. 442), which, in turn, was a consequence of the 20th century humanist approach to learning (by Dewey, Holt, A.S. Neill, and others) combined with the views on cognitive development from Piaget to Vygotsky (Begg, 1999; Elkind, 2004; von Glasersfeld, 1989, 1990). Questioning the perceived infallibility of mathematical knowledge, Kuhn (1962/1996) and Lakatos (1978) brought the debate into public and philosophical arenas during the 1970s. Ernest (1991) provided his readers with critiques of philosophies of mathematics from absolutist to social constructivism.

Constructivism has arisen out of the empirical scientific approach of developmental psychology. Focussed on the child and a biological evolution into adulthood, constructivism assumes a regular progression through discrete developmental stages from a pre-rational capacity to a rational meta-cognitive capacity in thinking (Dahlin, 2013; Piaget, 1969/1991). While Piaget posited that teaching could not accelerate what he felt to be a purely biological process, Bruner and Vygotsky both held to the view that the developmental progression could be influenced by teaching. In particular, Vygotsky was certain that mathematics could be learnt through building up one's knowledge and understanding in socially discursive interaction (as referred to in Bobis, Mulligan, & Lowrie, 2004). Ritchie and Carr (1997) described the impact of Vygotsky's studies of social interaction on learning,

especially within the mathematics classroom. Students were now no longer considered to be the passive recipients of teacher-led information and skills, but active participants in their own learning and cognitive development.

Although constructivism, as a philosophical approach, has been the foundation for mathematics education research and curricula changes since the latter part of the 20th century, there have also been critics among the post-modernists (Dahlin, 2013). Lerman (2001) reminds the researcher of “the dialectic between the theoretical and empirical; between mathematics education as a set of practices, and mathematics education as a field of knowledge” (p. 14). Making meaning within mathematics requires enculturation while mathematical understanding is dependent upon the teacher’s recognition of that understanding. According to Lerman (2001) and Bobis, Mulligan and Lowrie (2004) the classroom, as a learning community, is a ‘zone of proximal development’, and distinct from Vygotsky’s framing of that zone as being a ‘ripening’ area of learning within the child. Constructivism and socio-cultural perspectives in the teaching and learning of mathematics are argued by Cobb (1994) to be complementary theories, where constructivism focuses on what is learned and how, while socio-cultural perspectives focus on the conditions for learning.

The New Zealand Numeracy Projects presented a Number Framework (Ministry of Education, 2011) based on constructivism that included both strategy and knowledge derived from Pirie and Kieren’s recursive strategy model (1994) and Steffe’s counting types (Steffe, Glasersfeld, Richards, & Cobb, 1983). Hughes (2003) described the model in the *New Zealand Mathematics Magazine*, showing how Vygotsky’s *Zone of Proximal Development* (ZPD) had been adapted in mathematics education to provide a recursive strategy towards number knowledge.

Neyland (1995) referred to *Mathematics in the New Zealand Curriculum* (1992), in which strategy and knowledge are separated, highlighting the influence that behaviourism still has in teaching knowledge in discrete steps with specific outcomes (Neyland, 1995). The more recent *The New Zealand Curriculum* (2007) targets numeracy and literacy, and the prior socio-cultural knowledge that students

bring into the classroom community. The Number and Algebra learning strand, with a focus on algebraic thinking, is separated as a knowledge-strategy area, from the Geometry and Measurement strand, and Statistics strand. Algebraic thinking, regarded as one of the most important outcomes in the Numeracy Projects (Ministry of Education, 2008), is defined by Murray Britt (2008), as that “level of awareness” shown by students in explaining the “successful application of an operational strategy” (p. 18) within mathematics.¹³

While it appears that a fallibilistic perspective of mathematics may be the basis of *The New Zealand Curriculum - Mathematics and Statistics* (2007), it is not so clear what the fundamental philosophy is of the indications for Steiner Waldorf mathematics education. Conceptual models of phenomenological observation coupled with a Platonic search for an infallible archetype or *Ur*-ideal of Truth are used in some topics of the Steiner Waldorf curriculum (Steiner, 1886/1978; Stockmeyer, 1982), but how much is included in mathematics? A focus within Steiner Waldorf education is to enhance the inherent morality of children so that they may develop the values of social justice and reverence for their social and physical environment. The *Ur*-ideal of Truth plus social values do not match Ernest (1991) stating that an infallible mathematics cannot carry social responsibility. What is meant when Steiner (1924/1988) referred to every child as carrying a sense of number and geometry within their own body’s awareness that can be drawn out by the teacher to a level of consciousness and ultimately to mathematical understanding? When Jarman (1998) stated that mathematics involves “invisible, immaterial but exact and perfect entities” (p. 2), was he suggesting an absolutist philosophy is the basis of Steiner Waldorf mathematics? These are all questions that have guided my search through the literature towards this thesis.

The research and reading has been continual during the writing-up. I have focussed on the indications for mathematics education provided by both Rudolf Steiner (for the first *Waldorfschule* in Stuttgart), and by the Ministry of Education in its curricula

¹³ See also Britt & Irwin, 2011; Irwin & Britt, 2005, cited in Ministry of Education, 2008, p. 3.

publications.¹⁴ Steiner Waldorf schools throughout the world (over 1000), and New Zealand mainstream schools, build their schools' classroom curricula according to the needs of their school communities. While *The New Zealand Curriculum* (2007) is a "statement of official policy" (p. 6), Steve Maharey, the Minister of Education at the time, wrote in his accompanying letter that "(this curriculum) invites schools to embrace the challenge of designing relevant and meaningful learning programmes that will motivate and engage all students." Likewise, the *Waldorfschule*, which began in September 1919, developed a 'living curriculum' from indications given by Rudolf Steiner over the years 1907-1924 that would meet the needs of and challenge the students. E. A. Karl Stockmeyer gathered Steiner's indications from lectures, courses and books in his publication, *Rudolf Steiner's Curriculum for Waldorf Schools* (English edition, 1982), which is often used by Steiner Waldorf schools as a resource.

2.4. Summary

The combination of Hermeneutic / Bricolage as methodology in this thesis enabled an interpretation of the meaning of the literature and publications reviewed to expose the relationship between political and economic power and socio-cultural conditions on mathematics education in New Zealand. The descriptive chapters of this thesis (chapters 3 - 6) include much history; of education in New Zealand, and of the development of philosophical thought and consciousness. I found in my reading just how much the historical progression had impacted on the 'now', and sometimes, how little change there has apparently been! History provides a picture of the socio-cultural impact of and on education. A curriculum becomes a socio-political statement within the cultural context of the school, its pupils and teachers. A curriculum is also content plus methodology dependant upon the socio-cultural group within which it is used.

¹⁴ *Mathematics in the New Zealand Curriculum* (1992) and *The New Zealand Curriculum - Mathematics and Statistics* (2007).

Chapter 3. Background: Setting the Scene - New Zealand Mathematics Education

3.1. Introduction

3.2. Early New Zealand to End of Twentieth Century

i. Colonial and Practical: to 1930s

ii. Egalitarian and Liberating: 1930s to 1960s

iii. 'New' Mathematics: 1960s to 1980s

iv. Economics as 'Added Value' to Mathematics: 1980s to 1999

3.3. Summary

3.1. Introduction

This section describes some historical background of what I perceive to be the ideological basis of mathematics education in New Zealand's state-funded schools today. What is the ideology or socio-cultural framework that contextualises the curriculum document, *The New Zealand Curriculum - Mathematics and Statistics* (Ministry of Education, 2007) and consequent *National Standards in Numeracy* (Ministry of Education, 2009), and their implementation? In other words, what drives the New Zealand Ministry of Education's approach to the teaching, learning and assessment of mathematics, and how did this approach develop? McCulloch (1992) in his introduction in *The School Curriculum in New Zealand* put a strong case for reviewing the history, because of the relationship of an education curriculum to society, as a whole, and the impact of that society, its politics and culture on the curriculum. It is also important to contextualise the history in its time, as well as recognising the time in which we, the reviewers stand; both as influences on how the history is read. As Tarnas (1996) so succinctly states, one needs an "intellectual flexibility . . . to enrich, but not distort, the various ideas and world views we examine" (p. 1). Thus, curriculum approaches in New Zealand since the 1880s generally, and to mathematics education specifically, are considered in order to discover the extent to which any early attitudes still linger.

Bishop referred to mathematics as a “secret weapon of cultural imperialism” (1990, pp. 51-65; 1995, pp. 71-76), as its apparent neutrality is often distorted by the cultural loading placed on it through the teaching/learning process. In a keynote address at a MERGA (Mathematics Education Research Group of Australasia) conference, Barton (1993) spoke of “mathematics as process” (p. 2) that expresses the culture of those who use it. Not long after, Barton (1995) wrote of the way in which mathematics is a “prime gatekeeper” (pp. 165-174) of socio-political values. Mathematics is also considered a process of thinking, as in ‘mathematical thinking’, and in the more common term today, ‘algebraic thinking’. To bring some understanding to present developments in the teaching, learning and assessment of mathematics, I began by surveying the history of mathematics education in New Zealand since the early nineteenth century when European settlers first arrived.¹⁵

In terms of thinking, settled is an interesting word. It denotes a stopping, a setting down, an ordering and making comfortable. As Bishop reminded us, the mathematics knowledge brought by colonists, and the way in which it was taught, formed a particular process of enculturation in the ‘new’ country. Those colonies settled by English-speakers began to express the socio-political thinking of their homeland, Britain. Law, legislated for the maintenance of a civilised society, was expressed in policies that were to maintain the view of an English society at that time - white, male-dominated, elitist and hierarchical. The thinking preferred was rational, objective and ordered, with a heavy dose of religious morality (Cumming & Cumming, 1978). New Zealand, in the nineteenth century, was not immune.

3.2. Early New Zealand to End of Twentieth Century

i. Colonial and Practical: to 1930s

The New Zealand colony had developed from the 1830s, originally as part of New South Wales in Australia, then separately in its own right from 1841, to Dominion status from 1907. Education was initially piecemeal; initiated as required in the

¹⁵ In this thesis, I am not considering the Māori medium curriculum document - *Te Marautanga o Aotearoa* - which may or may not include reference to the use of mathematics in education prior to European settlement.

regions to “civilise the Aborigines”¹⁶ [sic] (Cumming & Cumming, 1978, p. 6) and to provide basic skills in numeracy and literacy. Once the colonial government had abolished self-managing provinces in 1876, nationally-available education became compulsory, secular and free for all children between the ages of seven and thirteen, irrespective of class, race or creed¹⁷ (Olssen & Morris Matthews, 1997). An earlier legislation had provided grants to schools managed by three churches only,¹⁸ and in 1877 religious-affiliated schools denied state assistance and retained their autonomy from the state educational system (Sweetman, 2002). According to the *Education Act 1877* the local state secular schools and their committees were answerable to regional boards, and these regional boards to the national Department of Education. A national curriculum was set by the Inspector-General of Education, and standards examined by a regionally-based Inspector-of-Schools. The arithmetic taught at primary level was elementary; sufficient to enable a leaving student to be an active numerate citizen in the developing democratic society (Olssen & Morris Matthews, 1997).

In 1936, the conservative Reform and Liberal parties made up largely of landowners and businessmen, and that had dominated politics since the late nineteenth century, formed a coalition, the National Party (<https://www.national.org.nz>). The Labour Party, which had formed in 1916, had arisen out of the workers’ trade unions (<http://www.teara.govt.nz/en/labour-party>).¹⁹ These major political parties reflected the differentiated school curricula, maintained since the early twentieth century through academic schools, and technical/vocational schools and courses. The academic schools followed a ‘classical’ curriculum, which focussed on mathematics as a discipline, a particular branch of knowledge; the technical/vocational schools focussed on ‘practical mathematics’. But the socially democratic ideals of a universal education were constrained by social prejudice;

¹⁶ Governors Hobson (1840 - 1842) and Grey (1845 - 1853); New Zealand was part of the New South Wales colony at the time.

¹⁷ *Education Act 1877* (see also Fraser, 1986, p. 249).

¹⁸ ‘*The Education Ordinance for Promoting the Education of Youth in the Colony of New Zealand, 1847*; first educational legislation in New Zealand, which provided annual grants to schools run by Anglican, Wesleyan and Roman Catholic churches only (Campbell, 1941, p. 27).

¹⁹ See also the relevant sections in Fraser, 1986.

Māori tended to be separated out and taught in schools especially provided,²⁰ homemaking was considered a priority for girls,²¹ and workers' children steered toward apprenticeships unless proficient enough academically to secure a 'free' place in the privately-funded secondary schools.²² New Zealand-wide social and economic changes brought about by the First World War (1914 - 1918) and the Great Depression years (1930s),²³ led to parents wanting more for their children - more education, more opportunity, more employment.

ii. Egalitarian and Liberating: 1930s to 1960s

In 1935 when Labour first became the governing body, the combined efforts of Peter Fraser, as Education Minister, and Clarence Beeby, as his Director of Education, advanced a curriculum that was child-centred, and an attitude that teachers were capable professionals in supporting children to reach their potential. Ernest (1991) would place this approach under the heading of "Progressive Educator"²⁴ (pp. 138-139). Parents and employers (and universities) wanted school-leavers to be suitably prepared for the employment or further education into which young New Zealanders would move.

Gazetted in 1945, a report by the Consultative Committee on the Post-Primary School Curriculum led by W. Thomas (*The Thomas Report*) recommended a compulsory secondary school 'core curriculum' that included elementary

²⁰ See the *Native Schools Act 1858*; and that of 1867, which allowed for 'day schools' in 'native areas' and *Amendment 1871*, when education of Māori was secularised and supported by central government (Cumming & Cumming, 1978, p. 67). In 1947, the schools were referred to as Māori Schools and did not come completely under Education Board control until 1969 (Jones, Marshall, Matthews, Smith & Smith, 1995, pp. 41-42).

²¹ In 1929, the *'Syllabus of Instruction for Public Schools'* (known as the 'Red Book') was gazetted, allowing (at the discretion of the head teacher) girls at Standards V and VI levels to be exempted from elementary mathematics if they took needlework (Cumming & Cumming, 1978, pp. 234-235).

²² *Secondary Schools Act 1903* in which secondary school governors were offered concessions and grants from the government for providing non-fee/'free' places in their schools according to the level reached by the student in their *Certificate of Competency* at Standard V (Cumming & Cumming, 1978, p. 150).

²³ See following link accessed August 14, 2014

<http://www.tepapa.govt.nz/whatson/exhibitions/sliceofheaven/exhibition/SocialWelfare/Pages/Greatdepression>

²⁴ Ernest (1991) labels the 'Society' a Welfare State with a 'soft hierarchy' and a liberal Political Ideology, and with a view of mathematics that is personalised and of 'process' rather than a structure to be constructed. For further reading and definition see pages 181-196.

mathematics (Committee, 1959; Cumming & Cumming, 1978; McLaren, 1980). The school leaving age was lifted from thirteen to fifteen years, and the compulsory core curriculum including elementary mathematics, came somewhere to meeting the appeals of parents and employers. A mathematics 'option' involving more intensive study, was taken by those who wished to sit the school-leaving examination (*School Certificate*) in mathematics after three years in secondary school (Committee, 1959).

During the 1950s, schools and teachers were overwhelmed by the number and variety of students. Immigrants fleeing from their war-torn countries,²⁵ Pasifika peoples imported from Pacific island groups to help maintain New Zealand's post-war growth, Māori moving from a rural to an urban lifestyle, and the post-war 'baby-boomers'; all exposed inadequate training and preparation in schools. Expected educational standards in many instances were not reached, giving credence to the sentiment that primary students were ill-prepared for secondary school. Students now tended to be 'socially promoted' with their age group, rather than by achievement of standards. The child-centred teaching was called 'play way'²⁶ and revision was called for, especially in arithmetic at primary school level. In response, Beeby is reported to have said that this 'modern' approach to education did not mean a student need not work; "some things like (times) tables still need hard work to be absorbed" (cited in Cumming & Cumming, 1978, p. 336). But a groundswell of negative public opinion in the 1950s²⁷ towards the changes and educational struggles post-war prevailed, and some recently qualified specialist mathematics teachers began to consider 'new maths' in the classrooms.

²⁵ After the Second World War (1939 - 1945) waged mostly throughout Europe and Asia.

²⁶ See Openshaw & Walshaw, 2010, p. 42, 132, referring to NZ Parliamentary Debates, Vol 309, 1956, 1731.

²⁷ Openshaw & Walshaw, 2010; the *Dominion* newspaper questioned the validity of the *Report* to the Minister of Education where "gains of liberalised curriculum had been secured without general losses in the basic skills, some of which in fact seemed stronger than ever" (p. 51). There were media and business sector allegations fuelling parents and politicians about "the problem" of standards from at least the 1940s (p. 168).

iii. 'New' Mathematics: 1960s to 1980s

In October 1957 *Sputnik 1* was launched into space by Russia, and a frantic race towards technological and scientific advancement began, stimulated by educational changes in the USA and Britain. High on the list of priorities was mathematics education. The 1960s and 1970s saw the disjointed and largely voluntary introduction of New Maths at every class level in schools in New Zealand.²⁸ Following the introduction of decimal currency the metric system terminology appeared, replacing the British terms. New Maths introduced the set theory and utilised a discovery mode of learning in primary schools through Manipulables, building on the social cognitive ideas of Jean Piaget (1896 - 1980), Lev Vygotsky (1896 - 1934) and Jerome Bruner (b. 1915). Arithmetic programmes in primary schools became Mathematics with the addition of set theory, algebra, geometry, probability and statistics, while secondary schools were updated to include recent topics in mathematics. Geometry moved from the Euclidian approach, standard for over 2000 years,²⁹ to coordinate geometry then transformational and vector, and calculus was introduced. Essentially, content previously contained within university and secondary school classes was pushed down into lower class levels.

It became clear through the introduction of set theory, algebra and statistics into lower-age classes that many primary school teachers lacked the mathematical content knowledge for a clear delivery of the concepts. These changes within mathematics had moved away from a more elementary content knowledge and practise of skills to an emphasis on the stages of ability and development of mathematical understanding (Cox, 1980; Openshaw, 1992). Not only should “children understand the arithmetic they are asked to learn”³⁰ (Lee, 1963, cited in

²⁸ See Openshaw, 1992, pp. 201-207: Some enthusiastic secondary mathematics teachers (the Christchurch Mathematics Group) with others around New Zealand, who wished to reform the subject during the 1950s, had begun innovative teaching methods with their students and the results were noticed! A full implementation of the ideas followed pilot classes (with some of the ‘better’ students) and a step-by-step introduction of different USA and British versions of ‘new’ mathematics, during the early 1960s. The same process was used to introduce CAS (computer algebra systems) into mathematics classes during the early 2000s.

²⁹ The Greek, Euclid (fl. 3rd century BCE) is celebrated as author of *Elements*, a book showing geometric proofs deduced from axioms and that was used as a standard mathematical text until the mid-20th century.

³⁰ *Suggestions for Teaching Arithmetic in Infant Classes* cited in Cox, J. (1980), *Mathematics in New Zealand Primary Schools 1965-1978*, p. 100.

Cox, 1980, p. 100) but teachers also needed to understand what they were to teach. Traditionally, women made up the majority of the primary teacher cohort and during their own education had often been exposed to arithmetic and elementary mathematics, rather than full mathematics. As generalists rather than mathematics specialists, the lack of confidence among primary teachers in their teaching of New Maths appeared to affect their classroom management. Although support for increased pedagogical content knowledge was provided through resources,³¹ this appeared to be insufficient. Test results showed that basic computation skills were reduced, a consequence thought to be due to lack of practice. An interview technique (*Individual Interview*) was used with students to evaluate their understanding of mathematical concepts. Moving from the simple to complex, and concrete to abstract, interview questions gave students the opportunity to describe their particular understanding of mathematical concepts (Cox, 1980). This interview style was to be re-introduced as a diagnostic tool (NumPA³²) for use by teachers in the *Numeracy Development Projects* from 2001 (Ministry of Education, 2008).

Thus, in spite of misgivings by many parents and primary teachers, who were struggling with the concepts of modern and pure mathematics being brought into schools, the New Maths curriculum prevailed. The alignment of this mathematics to science and technology and thus to “national prosperity and personal advancement” (Openshaw, 1992, p. 206) meant political and business interest in mathematics was high. In 1980, Cox wrote “the suitability of aspects of a mathematics syllabus . . . must be related to the aspirations of society and the aims it has for education and for its schools . . . they are socio-educational problems” (p. 109). Britain was moving closer to Europe and distancing itself from Commonwealth partners’ markets, and ‘economic diversification’ became the new catchword. New Zealand, during the late 1970s and early 1980s, began to see itself linked ever more tightly to changing economic values and to a change in the value of mathematics within education.

³¹ Textbooks and teaching manuals were provided, along with more specific activity cards and topic booklets. In 1975, an item bank of up to 500 multiple-choice questions was developed for each class level in order to help teachers assess a student’s learning. From this ‘bank’, a variety of tests for each class could be developed within a school (Cox, 1980, p. 106).

³² NumPA is the acronym of the Numeracy Project Assessment tool.

iv. Economics as 'Added Value' to Mathematics: 1980s to 1999

In 1984, the Labour political party won the election from the conservative National party, which had governed New Zealand since 1949 apart from two three-year terms of Labour in 1957-1960 and 1972-1975. Faced with a swiftly mounting external debt, Roger Douglas, Minister of Finance from 1984-1988, showed a strong interest in the New Right attitudes expressed in Treasury's 1984 briefing to the incoming government (*Economic Management*),³³ and introduced policies based on the American business economic model originating from Milton Friedman and the Chicago School (Smith, 2005). The 1984 Government adopted what Codd (1999) called "economic rationalism" (p. 45), sometimes referred to as neo-liberalism or 'market liberalism'. This approach is a reappearance of the economic view that enabled Victorian industrialisation (O'Neill, Clark, & Openshaw, 2004; Whitty, in Dahlin, 2010) and, as in the 19th century, encouraged that economic view onto socio-cultural expressions such as education.

A consequence of a review of education administration (published as *Administering for Excellence* and known as the 'Picot Report', 1988), followed by the Education Minister's brief *Tomorrow's Schools* in 1988, was the dissolution of the Department of Education and regional Education Boards. A Ministry of Education answerable to the Minister of Education, with the associated agencies New Zealand Qualifications Authority (NZQA) and Educational Review Office (ERO), was established. This new approach was a 'business model' (echoing the 'school committee' of the *Education Act 1877*) with local governance of schools now legally contracted to schools and their communities through an elected Board of Trustees. The philosophy and educational targets of the school were to be framed within the school's Charter,³⁴ which, according to Gordon (1992) was "an agreement" between the school and its community, as well as "a contractual undertaking" (pp. 187-203) between the school and the Government through the Ministry. The *Education Act 1989* (section

³³ <http://www.treasury.govt.nz/publications/briefings/1984i> - retrieved August 16, 2013.

³⁴ In 2011, school Charters were to be rewritten to include "specific school-wide targets for student achievement in relation to the National Standards" (Ministry of Education, National Standards pamphlet, 2009, p. 4).

60A)³⁵ introduced *National Education Guidelines*, included “all the national education goals, foundation curriculum policy statements, national curriculum statements, national standards, and national administration guidelines” (<http://www.legislation.govt.nz/act/public/1989/0080/latest/DLM179254.html>, retrieved 2013) set from time to time by the Minister and Ministry of Education. A change of government from Labour-led to the more conservative National in 1990 encouraged further New Right ideological implementation in education policy.

In line with the *Education Act 1989*, curriculum revision in all subject areas began with the first, in Mathematics, being published as *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) before the overarching *Curriculum Framework* (Ministry of Education, 1993). The New Maths that many primary teachers had finally begun to master was supplanted by a new curriculum. Curriculum development and revision had, since the time of Beeby in 1938, included input from many educational professionals. Curriculum change had been “initiated from *within* the sector, for *educational* reasons It was . . . extensive, lengthy, incremental, cumbersome and expensive” (O’Neill, 2004, p. 31; original italics). This process had engaged teachers and utilised the profession’s knowledge and skill base. But from the 1990s on, curriculum revision was to be led by the government through the Ministry of Education. The writers were required to orientate the curriculum information within the document into a hierarchy of achievable learning outcomes for each year/curriculum level. According to Neyland (2004) this led to a closed assessment-driven teaching/learning curriculum statement, leaving “little room for either open-ended learning, the pursuit of answers to questions posed by students, or the exploration of non-standard approaches to learning mathematical content” (p. 147). The concern of the writing team led to their integrating an important new strand - *mathematical processes*³⁶ (communication, problem-solving, logical reasoning), emphasising the ‘how to’ in mathematics - to the five ‘skills’ learning strands: number, algebra, geometry, measurement and statistics (Neyland, 2004).

³⁵ <http://www.legislation.govt.nz/act/public/1989/0080/latest/DLM179254.html> - retrieved August 16, 2013.

³⁶ See Howson (1994), where it is referred to as “most interesting and innovative”. . . “recognis(ing) that such skills can never be ticked off as having been attained” (p. 22).

In 1993 *The New Zealand Curriculum Framework* was published, to encompass and guide curriculum changes that occurred during the 1990s. Giving the appearance of the liberal-humanitarian but closer to Ernest's classification of "Public Educator" (pp. 138-139),³⁷ this *Framework* skewed education and the teaching of mathematics in favour of quantifiable positive outcomes. According to the Government's Achievement Initiative policy announced in 1990,³⁸ specific learning outcomes were set out in each of the subsequent subject 'curriculum statements', for each strand of learning within that subject knowledge area. Assessment procedures and examples, and guidelines for "appropriate teaching and learning approaches" (Ministry of Education, 1993, p. 23) also appeared in each curriculum statement. National monitoring was to occur with students in years 4 and 8, every three to four years (Ministry of Education, 1993). In fact, reports of the *Third International Mathematics and Science Study* of 1994-5 [TIMSS, Garden, (Ed.), 1997] and the trends seen in the *International Mathematics and Science* studies of 1998-9 and 2002-3 (TIMSS - *Mathematics and Science Achievement in New Zealand*, Caygill, Sturrock & Chamberlain, 2007) provided an international view of students in years 4/5 and 9 within the Mathematics and Science subject areas. Data collected during 1994 showed the performance of New Zealand's nine-year olds ranked near the bottom of the third quartile of the 24 - 26 countries participating.

Mathematics in the New Zealand Curriculum (1992), with its emphasis on strands within mathematics, had only just been introduced. These strands exposed middle and upper primary students and their teachers to secondary level mathematics concepts in a form different to the previous New Maths curriculum (Begg, 2006) maintaining a fractured view of mathematics as consisting of separated discrete areas of study and understanding. Concerns with the discrepancy between the level of numeracy as shown by the International Mathematics and Science Survey [TIMSS, Garden, (Ed.), 1997] results, and that preferred by a government wishing to enter a high knowledge-based society, led to the perception that students (and

³⁷ See Ernest, 1991, for further definition, pp. 197-216. The Theory of Society is one of an inequitable hierarchy needing reform, with the Political Ideology of a Democratic Socialist. The view of mathematics is that of social constructivism.

³⁸ Lockwood Smith in the Foreword to *The National Curriculum of New Zealand: A discussion document*, published by the Ministry of Education, 1991.

their teachers) were deficient in numeracy skills. Further consultative research of mathematics teaching and learning was undertaken in 1997-1998 with results suggesting that “limited teacher pedagogical content knowledge, poor teaching quality and confidence, lack of inspiring research-based professional development to meet student needs, and a lack of resources” (Ministry of Education, cited in Walls, 2004, p. 24-25) were all major areas of concern. Walls (2004) also reported “in 1998 New Zealand’s Literacy and Numeracy Strategy was launched with the overarching goal that by 2005, every child turning nine will be able to read, write and do maths for success” (p. 25).

3.3. Summary

For over 150 years mathematics education in New Zealand has carried the inherent values of the socio-political environment of the time. Initially expressing the outer pragmatism of a newly-colonised country but loaded with the implicit values of the 19th century Anglo-centric class system, mathematics teaching and learning was divided into two major approaches: basic elementary arithmetic or practical mathematics; and, a more extensive knowledge-based ‘classical’ or advanced mathematics. During the 1960s, after twenty-five years of a more openly egalitarian approach to education,³⁹ a mathematics curriculum for years 1 - 10 in New Zealand state-funded schools was introduced as New Maths. Higher-level mathematics was brought into lower classroom levels and knowledge-based mathematics began to supplement and replace the elementary arithmetic in primary classrooms, as New Zealand stepped from being a colonial hinterland into a global arena. But in the 1980s the increasing globalisation of New Zealand society and its products came up against the first major post-Second World War economic constraints. Education generally, and mathematics as a “paradigm case subject” (Neyland, 2002, p. 513-514), became mere commodities within this late 20th century socio-political

³⁹ Fraser was noted for the Labour government’s preference for egalitarian education (albeit stated in masculine terms!), originally written by Beeby and presented in the Department of Education’s 1939 Annual Report to Fraser: “every person, whatever his level of academic ability, whether he be rich or poor, whether he live in town or country, has a right as a citizen to a free education of the kind for which he is best fitted, and to the fullest extent of his powers.” (Appendix to the Journal of the House of Representatives, 1939, E-1 2-3, as cited in McLaren, p. 28. See also Fraser, 1986, p. 337).

portfolio. Kelly⁴⁰ (cited in Clark, 2004) is reported to have described the philosophical basis for *The Curriculum Framework 1993* as:

an approach of rigorous eclecticism with respect to the underpinning philosophies of its curriculum documents. No one single philosophic approach is adopted unquestioningly, but also no philosophy is dismissed without serious consideration being given to elements which can contribute to the overall quality of the work being undertaken. (p. 130).

Yet one could as easily argue that there is no foundational philosophy, merely ideological expediency within a universal educational system striving to meet new socio-political needs.⁴¹ The 19th century socio-political need for numerate workers has now moved into a 21st century socio-economic political need for participatory and contributing members of society “ultimately to succeed in the workforce” (Tolley, *National Standards pamphlet*, 2009, p. 1). New Zealand still needs ‘numerate workers’. It is just that the level of numeracy has moved from basic arithmetic that used formulae and repetitive memory recall to a new form of numeracy that involves stratagems and socially constructed knowledge towards understanding.

⁴⁰ Frances Kelly, Senior Manager in Learning and Policy at the Ministry of Education during the 1990s.

⁴¹ See Codd, 1999, p. 45 for further critical reflection on this argument.

Chapter 4. Background: Setting the Scene - Rudolf Steiner and his Educational Impulse in New Zealand

4.1. Introduction

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4.1. Introduction

There are five sections in this chapter: the first section provides a brief biography of Rudolf Steiner's life (1861-1925) in relation to the philosophical, social and educational changes at that time. The second section introduces Steiner's picture of the developing child and the terminology he uses, while the third presents the changing consciousness within different cultural epochs and the consequent changes in thinking and mathematics as expressed by some of the major personalities. The fourth section focusses on Steiner's particular attitude to social renewal⁴² and its relationship to the threefold perspective of the human being, as a further development of Schiller's concept of 'threefold', and how that influences his educational approach. The fifth and final section provides an introduction to the educative impulse as it was initially expressed through the development of 'Steiner Waldorf'⁴³ schools in New Zealand.

⁴² Steiner uses the term 'renewal' because of its continual futures orientation; 'reform', although a more common term, suggests a 'form' that lasts only until another 're-form' occurs.

⁴³ Such schools may be called 'Steiner', 'Rudolf Steiner' or 'Waldorf'; I refer to them in this thesis as 'Steiner Waldorf'.

4.2. Rudolf Steiner: a Man of Two Worlds

The biography of Steiner's early years coloured his relationship to education and mathematics. With his father working as a telegrapher and stationmaster for the new railway system, Steiner grew up in the 1860s with 'modern' technology while also enjoying the rural life of small south-eastern Austrian villages. From an early age, Steiner had supersensible, spiritual⁴⁴ experiences that he knew had nothing to do with the physical environment surrounding him at the time and he struggled to find a way to refer to them. When Steiner was about eight years old, he borrowed a book on geometry from his teacher and was, at last, able to see representations of shapes and things unseen by the physical senses and experienced by the mind alone. This marked the beginning by Steiner of a quest to understand thinking and the importance of mathematics in the realm of thinking. During his secondary school years at a *Realschule*,⁴⁵ Steiner was able to further his knowledge in sciences and mathematics, supplementing this with personal study in philosophy, Greek and Latin. From the age of fifteen, Steiner also tutored several school pupils to help with the costs of his education and, in so doing, began to develop an understanding of the art of education (Steiner, 1928).

Rudolf Steiner lived on the cusp of change - culturally, scientifically, philosophically and spiritually. Almost a century earlier, French revolutionists had shouted for Liberty, Equality and Fraternity, but the imperialism of the Habsburg era and Napoleonic and Victorian periods distorted these aims, respectively, into Liberalism, Socialism and Nationalism (Lucas, 1972). Following Rousseau (1712 - 1778) and Pestalozzi (1746 - 1827), there had been moves by Herbart (1776 - 1841) and Froebel (1782 - 1852) to adopt formal educational theories, both using an awakening awareness of the individual's sense of Self to extend the metaphysics of philosophy into psychological theories. Kant (1724 - 1804) had revolutionised philosophy with his three 'Critiques' that confronted the empirical limit of Hume

⁴⁴ In this thesis I mean spiritual to be that non-physical eternal enduring essence of everything that includes the 'archetype' (Goethe) and 'Idea' (Plato), and refers to those qualities of being which may not be perceived in this physical, material-sense-world. See Bortoft (2012), Gadamer (1986) and Zajonc (2009) for further clarification.

⁴⁵ During the 19th century, *Realschule* was a Germanic secondary schooling system that focussed on science and modern languages, in contrast to the *Gymnasium* that followed a more classical curriculum including philosophy, Greek and Latin.

and the rationalistic limit that had persisted since Plato, finally turning towards an aesthetic approach (Amrine, 2012). Fichte (1762 - 1814) had also spoken of the 'art of education' and its necessary relationship to philosophy (Cumming & Cumming, 1978; Lucas, 1972). Between 1788 and 1832, the Weimar Classicists, including Goethe (1749 - 1832) and Schiller (1759 - 1805), evolved the concept of *Naturphilosophie*⁴⁶ in Germany (Bowen, 1981), within the ethos of the Romanticism at that time, which encouraged intuition, imagination and feeling. Herbart's ideas influenced Freud (1856 - 1939), American educationists and the Austro-Hungarian education of Steiner's own schooling (Bowen, 1981; Lucas, 1972; Steiner, 1928), Froebel's pedagogy spawned a kindergarten movement and influenced Montessori (Bowen, 1981; Lucas, 1972), and Goethe's metamorphosis of plants encouraged Darwin (1809 - 1882) in his theory of evolution. Schiller's work *On the Aesthetic Education of Man in a Series of Letters* introduced the 'threefold man'⁴⁷ and the importance of play as creativity. However, a rationalistic attitude in scientific endeavour and method reigned supreme. The 19th century is often referred to as the Age of Progress, yet harsh social conditions in Britain, Ireland and Europe led to mass emigration. Steam power and industrialisation had added a mechanical approach to the developing atheistic, scientific experimentation. Burgeoning occult movements scanned the past and future to try and bring some understanding to the present. It was the time of the 'Individual', but he or she was for the most part trapped in revolutionary struggles, warfare or factories.

The 19th century rolled into the 20th with a swelling undercurrent of social dissatisfaction as a reaction to the scientific and technological advances. The Industrial Age had helped produce a flourishing middle class, but the 19th century Marxian proletariat, filled with lower-paid workers from factories and mines, was also increasing in number. Adherents of the spreading socialism as well as *avant-garde* thinkers were questioning why the 'advances' had not benefitted the

⁴⁶ A German word used in English to refer to a particular philosophical approach by the German 'Idealists' Fichte, Schelling and Hegel, who were part of the German Romantic movement of the late 18th and early 19th centuries. The philosophers considered nature in its totality; *Naturphilosophie* was an attempt to be the philosophical foundation for natural science.

⁴⁷ A term Steiner also uses in reference to three qualities; in the body (sense-nerve, rhythmical, limb-metabolic), in the soul (thinking, feeling, will activity), in the realm of consciousness (waking, dreaming, sleeping), where there are two 'opposite poles' and a central dynamic mediating region.

‘masses’. A reconstruction of society was called for, and morality and values reconsidered.

Rudolf Steiner, who had grown up in a village working-class environment, was a modern, practical man, and a scientific researcher of the supersensible, spiritual world, which he strived to describe philosophically. Working with Goethe’s scientific writings in Weimar during the late 1880s and early 1890s, affirmed for Steiner the phenomenological approach that he took in his own philosophical writings. In 1886 he wrote *A Theory of Knowledge Implicit in Goethe’s World Conception* (1886/1978) for a collection of Goethe’s scientific writings, to provide an epistemology of Goethe’s thinking relationship to the spiritual in nature. This writing also provided the basis for Steiner’s own developing thesis of thinking as a spiritual activity, which he presented in his doctoral dissertation *Truth and Knowledge* (1892) as a prelude to his book *The Philosophy of Freedom* subtitled “the basis for a modern world conception” (1894/1979a). Amrine (2014) suggests that Steiner managed to work beyond the ‘letter’ of Kant to the ‘spirit’ of Kant, and surpass the empirical logical approach still common today to present a more transformational philosophy.

Steiner’s ‘spiritual science’ (also known as anthroposophy),⁴⁸ offers a philosophically based mathematical-scientific method, which an adult individual may use, to be awake ‘in the moment’ with an objective awareness of both the physical and non-physical spiritual worlds. Later, Steiner was to speak of using mathematical thinking in order to discipline such contemplative activity (1919/1994). Steiner posited that the individual human being is composed of a physical body, which derives from the past, a soul through which one engages in the present, and a spirit that links past, present and future (1904/1994). Steiner’s ideas that he expressed as author of over 28 books and essays, editor of magazines and lecturer of over 6000 lectures between the 1890s and 1925, led to new forms of drama and movement (Eurythmy),⁴⁹ new architectural forms (for example, the

⁴⁸ The name given by Steiner to his ‘science of the spirit’ where *anthropos* refers to ‘human’ and *soph(y)ia* to ‘wisdom’ (from the Greek); an approach that uses scientific method to examine the activity and effects of the sense-free (spiritual) domain in relation to the human being.

⁴⁹ Eurythmy consists of specific artistic movements expressing visibly the sounds of speech and musical tones (Steiner, 1920/1989). Rudolf Steiner and Marie von Sivers evolved this form of movement with the help of their initial students, from the end of the first decade of the twentieth century. Eurythmy is now used pedagogically and therapeutically, as well as being a performance art.

Goetheanum⁵⁰ in Dornach, Switzerland), and new approaches to education, agriculture, medicine and health.⁵¹ In an essay he wrote in 1907 (translated as *Educating Children Today*, 2008), we find the basic premises Steiner gave for an ethical renewal of education, but he was opposed to presenting a formal educational theory (Steiner, 1924/1971). During those years of the first decade of the twentieth century, Maria Montessori (1870 - 1952) had just begun her work with children in Italy, while John Dewey (1859 - 1952) in the USA had recently begun publishing his works on education stating that the school is itself a social institution through which social reform can and should take place (Cumming & Cumming, 1978).

Steiner's indications for mathematics education were given out of a background of social renewal, the development of the whole child, and the assimilation of prior cultures. Steiner saw education as the avenue towards a transformation of the post-war European society (Hahn, 1958; Steiner, 1920/1989). For him, education could help the proletariat raise themselves above the poverty of economics and soul in which many lived in the late nineteenth and early twentieth centuries. Guy Standing (2011) has written of a 21st century 'class', the *Precariat*,⁵² whom he suggests have replaced the proletariat and become marginalised as denizens, thus losing their status as citizens in our modern world. Without a 'place' in the modern economy-

⁵⁰ See <http://www.goetheanum.org> The building is a performance and conference centre, as well as the administrative centre, for the world-wide General Anthroposophical Society. The first Goetheanum was built by volunteer international labour during the First World War, and consisted of two interlocking wooden cupolas. It was destroyed by arson on New Year's Eve, December 31, 1922. The second, present, building is formed of concrete and was completed in the late 1920s. Both buildings were designed by Rudolf Steiner and are listed among the 1000 selected classics of world architecture by www.greatbuildings.com.

⁵¹ Steiner wrote four dramas ('Mystery Dramas' - in reference to ancient spiritual 'Mystery' centres); education initiatives are now referred to as Steiner or Waldorf schools; the agricultural approach is called 'Biodynamic'; the medical/nursing approach is termed 'anthroposophical' with Weleda and Wala medicaments especially prepared using some homeopathic principles.

⁵² The *Precariat* is defined by the following: 1. Precarious labour; income insecurity 2. No secure occupational identity; no personal work 'narrative' → existential insecurity 3. High ratio of work to remuneration → **denizens** with limited political / economic / cultural rights; no longer true citizens 4. Lack a 'social' memory → opportunistic 5. At 'war' with itself as includes three distinctive groups, which reject the centre right and centre left: (a) first generation from 'old labour' i.e. children of 'the miners, steelworkers', etc.; want job security back; preyed upon through fear; tendency to reactionaries and 'far right' (b) 'victims' made up of migrants and those with disabilities, etc.; detached from society (c) young educated with no career identity; frustrated but idealistic and utopian; tendency to non-conformists and 'far left' (Standing, 2011, pp. 9-16).

based society this class may well need a re-orientation of education to become socially integrated into 21st century society.

4.3. Steiner's Philosophical Premises for Education

i. The Nature of the Developing Child

In his 1907 essay Steiner wrote of the way in which our future development as individuals in society is inherent within each of us, an understanding of which provides us with the programmes for educational reform (1907/2008). Lejon (cited by Dahlin, 2010) referred to this approach as a “social philosophy of *Menschenbildung*”⁵³ (p. 50). When considering this development from the perspective of Steiner, I need to introduce the terms he used in relation to the developing human: the physical body, etheric, astral, and Ego-spirit. I suggest that Steiner, philosopher that he was, philosophically ‘deconstructed’ an individual human body into what he perceived to be four distinct qualities or areas of activity. He was then able to describe the development of the ‘child-becoming-adult’ through a picture of the progressive embodiment and maturation of these four areas of activity. Accordingly, Steiner posited the following. Each of us is made up of matter that is mineral and crystalline in nature and subject to the laws of physical existence; he named this the ‘physical body’. We also have a capacity to form and enliven this matter so that growth and reproduction, and the inner circulation of fluids, for example, are maintained; Steiner gave this capacity we share with plants and animals the following names: ‘etheric’, or ‘formative force’, or ‘life body’. There is also the capacity we share with animals for the inner sensation of pain and pleasure, of impulse, craving, passion and all other feelings, which Steiner termed ‘astral’ or ‘sentient body’. The Ego-spirit consciousness belongs uniquely to the human; it is our expression of our particular individuality, enabling us to call our self “I”, and is also our eternal aspect, linking our earthly phases of being to spiritual phases (Steiner, 1907/2008). Neurologist Vilayanur Ramachandran (with Blakeslee, 1998) referred to the epiphenomena ‘qualia’ (singular ‘quale’) and consciousness as qualities of mind-body that have a subjective nature in our perception and do not

⁵³ ‘*Menschenbildung*’ translates as ‘formation or development of the human being’.

appear to have a causal role in the activity of our physical brain. These definitions could help us to understand Steiner's description and reference to 'astral' and 'Ego-spirit', yet, Steiner's use of these latter terms suggest a greater depth and integration than Ramachandran's 'epiphenomena' appear to do.⁵⁴

Steiner went on to propose that an 'art' of education requires we know which particular area or capacity of the human constitution we should work with at a certain age. Although all four capacities are present in an individual growing child, each is at a different level of development or maturity. In the first years of a child's life, up to the 'changing of teeth', the physical is predominant, with the etheric supporting the rapid growth of the physical, the astral involved in the sense-dominated experiences, and the Ego consciousness slowly awakening. Once the milk teeth begin to be replaced by permanent teeth, the etheric becomes freed up to work in the soul transforming the child's tendencies of recall, character and temperament through formal education. The urge to imitate that had been the foundation of learning during pre-school years becomes an ability to assimilate under the guidance of authority, as the etheric helps the young student to increasingly develop memories, thoughts and ideas (Steiner, 1924/1968). The capacity for true intellectual thinking, capable of critical analysis, intellectual abstraction and discriminating judgement, is expected to only arise when the activity of the astral is freed from the physical at puberty to work in the soul, and the etheric to work more towards developing the adult-consciousness of the Ego-I spirit. Steiner suggested that formal education before the change of teeth imposes an intellectualising of the imitative process of learning, while pushing intellectual judgement into the years before puberty can 'freeze' thinking into rigid forms rather than maintaining flexibility and the potential for creative thinking as an adult (Steiner, 1907/2008; 1920/1989; 1922/1994).

Thus Steiner's general principles are: formal schooling should not take place before the 'change of teeth' in a child has begun, as the body's inherent energy would not have been freed up enough from the requirement of physical growth; and

⁵⁴ See Nagel, T. (2012), *Mind and Cosmos: Why the materialist Neo-Darwinian conception of nature is almost certainly false*; and Dahlin's review (2013, p. 147)

knowledge of human nature from the ‘change of teeth’ through puberty must underlie what is known as primary and secondary education (Steiner, 1919/1994). In lectures to the first teachers of the *Waldorfschule* in Stuttgart, Steiner stressed that the teacher must work through the soul of the student, thus maintaining freedom of the individual’s spirit-Ego (1919/1981). Table 1, below, presents the approach to learning that significant adults may orientate towards particular areas of the body and soul of the ‘child-schoolchild-youth-becoming adult’ during certain periods of growth and maturation:

Table 1: Presentation of developmental stages and related learning approaches

Period	Evolving Bodily Aspect	Soul Activity	Learning Approach
birth to change of teeth	physical	will - movement forces	Imitation
change of teeth to puberty	etheric	feelings - sensations	Authority
puberty to ‘adult’	astral	critical thinking	Individual Judgement

Following these principles, Steiner stated that:

- feeling is intimately bound up with the world of memory, and thus memory is enhanced through content that engages the feelings;
- the will may be cultivated by practical physical exercises (including knitting as preparation for logic) and artistic pursuits (for example, Eurythmy), adapted to the developing child-student;
- thinking must be fostered on whatever establishes each of us in the social realm (for example, through the instruction of languages other than the ‘mother tongue’, and including mathematics). Nothing abstract should dominate, with the approach being through the artistic realm - drawing, painting, music, handcrafts. (1919/1994).

According to Steiner, arithmetic acts as a bridge linking thinking and feeling; the feeling life is cultivated by all memory work in mathematics, especially through rhythm and story-picture. In his 1907 essay, Steiner stated that we need to make “intentional external efforts to cultivate memory” (1907/2008, p. 49) during the primary school years. He recognised that there was a prejudice against rote

learning; those against considered more important an intellectual understanding of the concepts before committing them to memory, not unlike the principle underlying the Numeracy Projects. But Steiner posited that it was far better pupils only later understood what they had memorised, as developing the memory helped develop the individuality (Ego-I) of the person (1920/1983).

Between the seventh year and puberty, two further stages of growth and change occur. The first is about the ninth year when the young student begins to differentiate an individual sense of self apart from the world, and is now capable of recognising processes and relationships between things in and of themselves, instead of purely to one's self. Once this threshold is crossed, the young student can begin to work with the concepts inherent in the work with fractions. The second threshold of growth and change begins about the twelfth year, when abstract concepts and cause-effect relationships may be introduced (Piaget, 1969/1991; Steiner, 1924/1971). Stockmeyer (1982) signalled the introduction of Euclidian proofs at this age.

ii. Stages of Human Civilisation and Changes in Mathematical Thinking

In the first nine years of the twelve-year curriculum within a Steiner Waldorf school, prior cultures and civilisations are presented through content and approach in order that assimilation of these cultures may occur. Our culture is not newly formed by each generation, but grows on what has come before. Working more consciously with cultural history helps to contextualise our present and future societal directions (Furinghetti, 2006). This can definitely be seen in the mathematics indications, and I summarise from both a history of mathematics (Gregersen, 2011; Kline, 1990) and Steiner's curriculum indications collated by Stockmeyer (1982):⁵⁵

- some of the processes within mathematics seen expressed in the ancient Sumerian/Chaldean/Egyptian periods are similar to those that Steiner suggested for Classes 1 to 3 (years 2 - 4);

⁵⁵ See Chapter 6, pp. 71-72 and the Appendix below where these indications are expanded in a table.

- some of those processes of mathematics used in the Graeco-Roman period up to Medieval times, are those Steiner suggested for Classes 4 to 6 (years 5 - 7); and
- those processes used from the Renaissance to about the sixteenth / early seventeenth century, are those Steiner suggested for Classes 7 to 9 (years 8 - 10).

Steiner wrote that conscious awareness among humans had changed during the development of civilisation. His book, *Occult Science* (1910/1979), outlined the results of his spiritual-scientific research into the evolution of the earth and the consequent development of consciousness in humans. Since the middle of the fifteenth century, humanity is now capable of moving through stages of soul-spirit development towards what Steiner termed “*consciousness-soul*”.⁵⁶ Steiner also provided pictures of the various earlier epochs of civilisation and the stage of development of consciousness, which that epoch expressed. He suggested that humanity from the Early Indian through the Chaldean/Sumerian/ Babylonian to Egyptian periods, expressed the *sentient soul* stage of consciousness. This earlier thinking had been largely a form of picture-based ‘story’ that seemed to ‘come upon’ a person (Steiner, 1914/2009). It related much more closely to the past experiences and present situation of a community or tribal group, while future intentions or future-oriented ideas were more likely to be the domain of the shaman, prophet or priest. While there is little evidence of this way of thinking, we can gather an understanding for it through the epics from those times, such as the *Mahabharata* and the *Vedas* of ancient India, the Gilgamesh and Inanna/Ishtar stories from the Chaldean culture, and the myths of ancient Egypt. Today, we can have a feeling for such ‘picture-forms’ in the re-telling of myths, fairy tales, fables and the way events in the early books of the Bible are described. Steiner indicated that much of the content for Classes 1 to 3 be delivered through such story-pictures. Kahneman (2011) writes of ‘faster’ thinking, and the capacity for intuition

⁵⁶ See Burkhard (1997) and Lievegoed (1985), both doctors, who further describe these stages of soul development; Burkhard from a biographical ‘whole life’ perspective, and Lievegoed from a medical-health perspective. Lievegoed, especially, points out the consequences to one’s physical and soul health when a particular phase develops at the expense of others.

and memory, that may be developed, added to and nourished by stories that engage the feelings.

Through their thinking there was a sense in these ancient peoples of being 'at one' with their environment, that we still witness now in very young children. There was no sense of individual self as we are aware today. What meagre historical evidence we have shows that the majority felt such a strong connection to the natural world and its rhythms, and to their family/tribal mores, gods and rituals, that there was little independent thinking, generally. There were, of course, particular individuals who showed a more independent mode of consciousness and thinking, but that was often attributed to their personal relationship to the gods.

In the realm of mathematics (an activity connected largely with the priesthood, while formulaic 'reckoning' was carried out by merchants and scribes), ancient documents suggest that a process of deduction was used before induction, and that there was counting and computation in arithmetic using formulae, with multiplication by repeated 'doubling' showing up in Egyptian texts. The geometry of volume and area (including an earlier form of what later became known as 'Pythagoras' theorem') and a basic practical application of what we now term algebra, were also practised by scribes (Kline, 1990). In mathematics lessons during the first three classes (years 2 - 4), Steiner indicated that such processes and approaches to mathematics from the practical, real world of experience be shared with the young students.

The early Greek period heralded the onset of the consciousness Steiner ascribed as *intellectual-mind soul*. The Homeric works, *Iliad* and *Odyssey*, provide a literary 'bridge' from the sentient soul to the intellectual-mind soul. There are Natural Laws, of which some mathematical truths form the basis, and there are mathematical laws or proofs that are derived through thinking. In a lecture Steiner gave in Amsterdam (1904), he reminded listeners that mathematical thinking is about something that the senses may perceive, but is, at the same time, not a thinking in terms of sense perception. Kline (1990) referred to Plato's ideas that "numbers and geometrical concepts have nothing material in them and are distinct from physical

things” (p. 43). The early philosophers, Plato⁵⁷ (427 - 347 BCE) and Aristotle (384 - 322 BCE), and Pythagoras (c. 570 - c. 495 BCE) before them, conceptualised the truth, as perceived, of mathematics and first formed the discipline, as such (Kline, 1990); laws given by the gods and discovered by the learner, so that an understanding of Natural Law could develop (Bowen, 1972). The Greeks⁵⁸ expressed a more objective although still dependent relationship to the world of spirit, with thought conceived as both an idea and a percept gifted by the gods. Through thinking, they strove to re-unite their soul with the divine in the ‘Form’ or ‘Idea’ as representative of the spiritual world⁵⁹. Kline (1990) reports from Plato’s *Dialogues Bk. VII*, section 525, that one must “learn arithmetic . . . for the sake of the soul herself; and because this will be the easiest way for her to pass from becoming to Truth and being . . .” (p. 43) This cognitive construction takes the thinker from the receiver of Truth to a co-creator, highlighting the proto-development of the ‘I am’, which is the signature of the ‘modern’ person since Descartes. Like the young child who begins to call herself “I” from between two and three years of age, and shows acute understanding from time to time that belies her chronological age, the philosophers of this early and Classical Grecian period foreshadowed what was to come. The Graeco-Roman period had ‘imported’ and mixed the mathematics of the earlier times with philosophical speculations. Deduction was considered a preparation for Logic (Kline, 1990); this latter practised for political and philosophical reasons. Mathematics was used to bridge the world of sense and the world of Form using systematic definitions and axioms, while Euclid’s “*Elements*” presented rigorous geometric proofs.

⁵⁷ P. Christopher Smith (1986) in his ‘Translator’s Introduction’ to Gadamer’s *The Idea of the Good in Platonic-Aristotelian Philosophy* notes that “Plato turned to mathematics, for he saw there a kind of reasoning that was self-evidently invulnerable to sophistic ‘tricks’” (p. xiii), such sophism at that time closely linked to “sensuous gratification and power” (p. xvi). See also the first book of *The Republic*.

⁵⁸ See such early philosophers and mathematicians as Thales (c. 624 - c. 546 BCE), Anaximander (c. 611 - c. 546 BCE), Anaximenes (c. 585 - c. 528 BCE), Xenophanes (c. 570 - c. 475), Heraclitus (c. 535 - c. 475 BCE), Parmenides (c. 515 - c. 450 BCE), Anaxagoras (c. 500 - c. 428 BCE), Empedocles (c. 490 - c. 430 BCE), Zeno (c. 490 - c. 430 BCE), Protagoras (c. 480 - c. 410 BCE), Socrates (470 - 399 BCE), Democritus (c. 460 - c. 370 BCE) and Leucippus (fl. 5th century BCE) and Euclides (c. 325 - c. 265 BCE) in Steiner, 1914/2009; Tarnas, 1991.

⁵⁹ See Plato, *The Republic*, Book VII: “The Philosopher Ruler”.

From the early centuries of the Christian era a religious impulse was also connected to thought (Augustine, 2001; Boethius, 1999; Steiner, 1914/2009), which did not regress to the earlier pictorial pre-Classical Greek consciousness, but was now more enumerative and hierarchical. The thinking person now perceived his thoughts according to the authority of the god-head (or 'divine messenger'). Instead of the 'authority' being within one's self as we perceive today, or through the Divine World of Form (Idea) as the Greeks perceived, it came through one's belief in a divine being.⁶⁰ Thought was considered connected to the thinking capacity of the soul, but the soul 'looked to the authority' in the spirit. In the late seventh-early eighth centuries as Moslems swept from Arabia to subdue and occupy northern Africa and Spain, they carried not only the Islamic teachings but also Hindu-Arabic learning that had been fructified by pre-Christian Greek thinking. Universities and centres of learning were built and flourished in Gundeshapur (now part of Iran), and in the Iberian Peninsula (Al-Andalus) now known as Spain and Portugal. Mathematics and philosophy, along with astrology and astronomy, medicine and surgery, science, architecture, arts and literature were among the areas of knowledge explored by scholars. For almost seven hundred years the Moors in Al-Andalus were dominant until overwhelmed and finally conquered by Christian armies in the fifteenth century. A mere handful of Europeans had earlier ventured into Al-Andalus for the learning offered there; the Islamists, Ibn Sīnā (Latinate: Avicenna, 980 - 1037 CE) in Persia and Ibn Rushd (Latinate: Averroës, 1126 - 1198 CE) in Al-Andalus had nurtured the writings of Aristotle and Plato in their Arabic 'nursery' and in uniting eastern philosophical thought with western, enabled the emergence of Platonic-Aristotelian philosophy into Europe.

The orientation towards mathematics and philosophy during this period from Classical Greece to the late Middle Ages was to 'play' and calculate with numbers - positive, negative, natural (whole) and rational (fractions) as well as with decimals and equations. As Kaplan (1999) so elegantly states, "our thinking's centre of gravity moved definitively . . . from co-ordinating the meaning of facts to subordinating facts to their significance" (p. 78). In keeping with this re-orientation from that

⁶⁰ Note the 'recitations' of Muhammad (c. 570 - 632 CE) he 'received' from the angel, Gabriel that make up the *Qur'an*, and the basis of Islam.

approach used in the early school years, Steiner suggested that almost two years (Classes 4 and 5) are to be spent on the practise of fractions and decimals, so that the students become fully comfortable with numbers and their factors before introducing formulaic algebra and Euclid's inductive method in the sixth class (ages 11-12).

Steiner gave the fifteenth century as the time when humanity as a whole began entering the period of *consciousness-soul* development. During the fifteenth century a new form of philosophical mathematical thinking that encouraged the awareness of a soul-Self separated from Thought and bringing objectivity to consciousness, was developing. Leonardo da Vinci (1452 - 1519) used mathematics and practical mechanics to determine laws of Nature, while Nicolaus Copernicus (1473 - 1543), Giordano Bruno (1548 - 1600) and Francis Bacon (1561 - 1626) used their still-religion-infused mathematical thinking to explain their growing awareness of the Ego-I requiring a 'new home' separated from the soul *and* from Nature. Copernicus developed a mathematical picture of a heliocentric universe, Bruno wrote of indestructible 'monads' of divine origin, that represented what lies beyond perception including the Ego-I, and Bacon introduced his inductive method (derived from Euclid's *Elements*) to explain all perceptions. Johannes Kepler (1571 - 1630) made the radical move to combine mathematics with physics (at the time, physics was considered part of the philosophy of nature, and therefore illusion) in his astronomical studies and writing, thus 'proving' that the sun, as the expression of the 'most high God', must be at the centre of our universe. Like Kepler, Galileo Galilei (1564 - 1642) worked with mathematics, physics, astronomy and philosophy, and although confined to house arrest late in life for heresy, gave conclusive mathematical arguments for the Copernican heliocentric view of the universe. He formulated a postulate that led to proving inertial motion, an important pre-cursor to Newton's theory of gravity. René Descartes (1596 - 1650) approached both perceived (sense-based) Nature and divinity (mathematical) in the human being with doubt and questions. His mathematizing logic (written initially in French; *Je pense, donc je suis*), "I think, therefore I am" separated the primary knowable from the sensory illusion. Using analytical thinking he bridged geometry and algebra and

is credited with the Cartesian coordinate system. Contra to Descartes' body-mind Dualism, Benedict (Baruch) Spinoza (1632 - 1677) modelled his philosophical approach on Greek mathematics; the 'universal' (*natura naturans* - nature naturing), and thought (*natura naturata* - nature natured) as a part of that universal substance, with human ethics as the highest expression of knowledge. (Bortoft, 2012; Steiner, 1914/2009) The need for laws to help humanity understand Nature and Self, that these philosophers and mathematicians wrestled with during the fifteenth to seventeenth centuries, is similar to the twelve year-old who requires 'rules' of moral behaviour to help her understand a changing awareness of her place in her world.

Steiner's indications for mathematics in classes seven to ten use the approaches towards mathematics expressed during these Medieval through Renaissance times. Platonic forms in geometry are introduced and the work with number becomes more algebraic and tends towards calculus.

From the seventeenth century onwards specific philosophies of mathematics develop, following one or another line of reasoning, and having adherents spanning the eighteenth to twentieth centuries. Ernest (1991) defined the philosophy of mathematics as "the branch of philosophy whose task is to reflect on, and account for the nature of mathematics" (p. 3). Gregersen (2011) considered that such a philosophy has two questions: one, that of whether non-material, abstract entities actually exist, and two, how one can define and explain the language used in mathematical 'sentences'. In lectures given in 1923, Steiner (1923/1985) stated that mathematical thinking was now combined with science, thus preventing the human being from experiencing the feeling in thinking. The physical was objectified to such an extent that the inner feeling associated with perception was no longer present. Man was merely a machine and thinking had become a tool of mathematics. During the twentieth century thinking was studied in the new science of psychology, and by late twentieth century mathematics had replaced Latin as the new language of knowledge with both knowledge and mathematical thinking economicised into commodities.

iii. Social Renewal

This section provides some background information on the way in which Steiner Waldorf schools could prepare their students to be active participatory members of society through an education that encourages in freedom rather than dictating or inculcating an expectation through the curriculum.⁶¹ Early in 1919, recognising that the natural scientific approach of the time could not answer the social needs brought about by the post-war chaos, Steiner published his book, known (in translation) as *The Threefold Social Order*. This built on ideas he had presented in *Riddles of the Soul* (Steiner, 1917/1999), where he suggested an integration of Ego-spiritual capacities into bodily functions through the soul faculties of thinking, feeling and will, further developing Schiller's picture of 'threefold man'. Steiner posited that the soul faculty of thinking is associated with the sense-nerve system throughout the body but localised in the head; the soul faculty of feeling is associated with the respiratory-circulatory systems localised in the chest; and the soul faculty of will with the limb-metabolic systems. This 'threefoldness' is used in teaching; the teacher's approach is through the soul of each student, activating the will through movement, the feelings through art and rhythm, and thinking through a sense-based phenomenological relationship to the physical environment of nature, and socially to people. The elements of the 'socialising' of this 'threefoldness' as presented in *The Threefold Social Order* can be summed up thus: that the spirit working in the soul faculty of thinking encourages the cultural realm within which each person may be free in the realm of thought and idea or belief; when the spirit works into the soul faculty of will, true economy may develop, which is associative and encourages fellowship and fraternity; the spirit working in the soul faculty of feeling offers a foundation for the rights/legal sphere within which all are equal. The whole forms the 'body social' (Steiner, 1920/1972), which I present diagrammatically in Figure 4 below:

⁶¹ See Schieren (2014, p. 19) who refers to a survey by Barz, Liebewein, & Randoll (2012) of Steiner Waldorf school graduates, and Gidley (2010), who all present findings of a greater focus on social responsibility by those Steiner Waldorf students in school and by school graduates in post-school work.

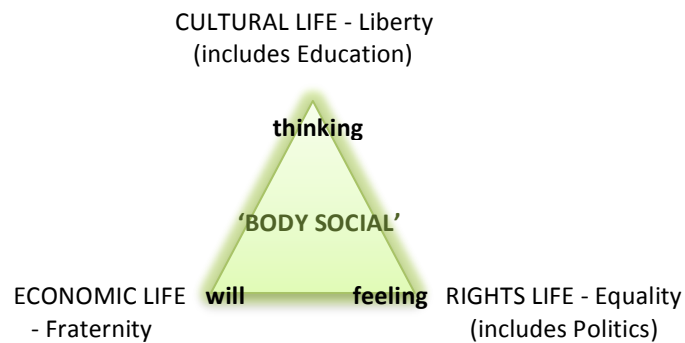


Figure 4: Diagram of the elements of the 'Threefold Social Order'

Steiner, in discussions at the time, stated that the poverty of soul experienced among the proletariat was due to two particular situations in the cultural life then. One was the demand of such abstract wide-awake thinking that the individual's egoistic impulse had grown at the expense of the awareness of the 'social body' (Hahn, 1958). The other was the lack of cultural development, as for many workers their schooling had been grossly inadequate (Hahn, 1958). In speaking of the ideals of a new social order, Steiner added that a new approach in education could provide an opportunity for renewal (see Uhrmacher, 1995). These ideas led Emil Molt of the Waldorf-Astoria factory in Stuttgart, Germany, to ask directly of Steiner for guidance to set up a school⁶² (initially for his workers) where a new art of education could be practised (Hahn, 1958). During the northern summer of 1919 Steiner gave three lectures, now published as *A Social Basis for Education* (1919/1994), where he referred to a different approach to education; as an organisation of the life of spirit (in its relation to thinking), education is to be independent of both the economic life and of the State (political or rights life). Published in 1850, Wilhelm von Humboldt had written in *The Limits of State Action* that the "economical sphere should *support* cultural life . . . and the state-run judicial system should *protect* it, but neither should *control* or *direct* it" (cited in Dahlin, 2010, p. 50, original italics). While Steiner emphasised that everyone should be able to receive an education that "contributes to the general culture of

⁶² The *Waldorfschule*, in Stuttgart, opened to 256 students over 8 classes in September, 1919; 150 were children of the workers at the Waldorf-Astoria factory. By 1928 there were over 1000 students in 28 classes covering the 12-year curriculum (Easton, 1980, p. 262).

humanity” (1919/1994, p. 16), and that that style of education develop such “social instincts” that people approach one other with genuine interest so that we never “break the thread of our study of the universally human” (Steiner, 1919/1994, p. 32), Humboldt emphasised an ideal of a social fellowship of individuals (Dahlin, 2010). These general premises are not so unusual among forward-thinking socially aware people; what made Steiner’s social reform through education different was his continual reference to the enduring Ego-spirit in the human being.

In addition, Steiner also suggested a totally different preparation for the teacher; there needed to be a complete change in the whole nature of the instruction and examination of teachers. Instead of teachers isolating themselves within their schools, they needed to be awake to what was happening in the whole world, to recognise what the spiritual life reflects as expressed in the environment and through the deeds of fellow humans, and therefore what changes have to be made in this present life (Steiner, 1919/1994). Inner guidelines needed to be sought for in the interweaving of the human with the world, as in the phenomenological approach used by Goethe. Personal experience as provided in the instruction to pupils only had value if prepared in a suitable way by the will of the teacher (that is, ‘digested’ first). These preferences given by Steiner were to be gathered within a background of a strong philosophical foundation in the teacher, and expressed in the delivery of the pedagogical content knowledge and through the ethos of the classroom community. As Kiersch also identified, the teacher must not only be a socially aware person engaged in the “social and political issues of the time”, but also have a “broad and deep knowledge about human culture and history (and) deep insights into human nature and child development” (cited in Dahlin, 2010, p. 49). Through a focus on *Menschenbildung* (development of the human being) Steiner Waldorf schools orientate towards the future and enable their students to become active members of society. Such an approach places Steiner among the socially progressive humanists.

4.4. Steiner Waldorf Schools in New Zealand

Steiner's indications for a new impulse in education were first acted upon by members of the group 'Havelock Work', who developed a small preparatory school initially in what is now known as Duart House, Havelock North. From 1916 to 1929, this little-known school pre-dated the *Waldorfschule* in Stuttgart, Germany (Wright, 1996). Financial difficulties forced its closure, and another school working with Steiner's indications was not to appear in New Zealand until the 1950s.

In 1950 Ruth Nelson and Edna Burbury, who had established a centre for anthroposophy at their homestead *Taruna* in Havelock North and had visited the *Waldorfschule* in Stuttgart, purchased Queenswood School, founded 1921 in Hastings. Over the next decade Queenswood School was progressively transformed into a 'Rudolf Steiner' school as staffing changes allowed. Close connections with the Edinburgh Rudolf Steiner School and with friends in Europe, helped encourage the school's transformation, and it dropped the name 'Queenswood' from its title in 1975.⁶³ That same year, a school with kindergarten opened in Christchurch and in 1979, Michael Park School in Auckland and Raphael House School in Lower Hutt opened with classes and kindergartens. These 1970s initiatives had grown out of interest by parents and friends, who had prepared for the establishment of the schools through study, conferences and speakers visiting from overseas. During the 1980s, schools in Dunedin, Titirangi and Tauranga opened, and in the 1990s those on the Kapiti Coast, in Motueka and Hamilton followed, to meet the growing demands of parents. The schools in Hastings, Christchurch, Auckland (Michael Park and Titirangi) and Lower Hutt all follow a full school twelve-year curriculum supported by kindergartens. The other schools with their associated kindergartens provide a full primary (to year 8) curriculum (Munro, 1995, 1998).⁶⁴

The youthful enthusiasm of the founders of the schools opened during the 1970s to 1990s was complemented by the experience and Scottish-style pragmatism of the teachers in Hastings, which began offering training for these 'new' teachers during the 1970s. All the schools had begun as legally constituted private schools subject

⁶³ Information retrieved August 30, 2014 from <http://www.taikurasteiner.school.nz/index>

⁶⁴ See also <http://www.anthroposophy.org.nz/~anthropo/education.htm>

to the relevant articles of the Education Act and Private Schools Act, requiring particular standards in curriculum, staffing, accommodation, equipment, and so on. When the Misses Nelson and Burbury died in 1977 and 1978 respectively, *Taruna* was bequeathed by them to be established as a tertiary educational centre for those who wished to work in and lead initiatives arising out of Steiner's indications for education, agriculture, art and health therapies.⁶⁵ The first full-time course for teachers, at undergraduate Diploma-level, began there in 1982.

The impulse to follow through with an education to be available to all strata of society in keeping with Steiner's principles for an 'education in freedom' were sorely tested during the 1980s when the then Labour government announced its intention to abolish subsidies that had been available to private schools since the 1950s and 1960s. The *Private Schools Conditional Integration Act* had become law in 1975 and was initially designed to support the Catholic private school system. The Catholic Church had retained its educational autonomy outside of the *Education Act 1877* but was increasingly meeting financial difficulties. There was also the political and social history of a segregated schooling system that did not concur with the egalitarian approach fostered politically since 1939 (Sweetman, 2002). Initially, during the early 1980s the four oldest Steiner Waldorf schools approached the government to have Steiner Waldorf education recognised as a distinct and fully-funded alternative to the state education system. This was at a time when the Labour government was canvassing the country about preferences for education at primary, secondary and tertiary levels; the review being distilled into the report *Tomorrow's Schools* (1988). The result of discussions with the then Department of Education was for two of the Steiner Waldorf schools (Michael Park and Christchurch) to become integrated towards the end of 1989 (just before the department's dissolution), for Hastings-Taikura to follow in 1991, and Raphael House in 1992. Questions of autonomy versus state control of curriculum and educational standards, teachers and teachers' qualifications were as important as mediating the financial difficulties that had arisen in making available Steiner Waldorf education to as wide a group within society as possible (Munro, 1995).

⁶⁵ See the website <http://taruna.ac.nz/about/>; information retrieved August 30, 2014.

The 'Integration Act' offered private schools the opportunity for voluntary and conditional integration into the New Zealand state educational system, subject to approval from the Minister of Education. The Special Character clause of the Act allowed for any integrated school to maintain that educational approach, whether through a particular religion or philosophical outlook, that was the basis of its difference from the state education system. Among those (now 8 of the 11) Steiner Waldorf schools which have become integrated into the state system, that Special Character clause defines the school as "a Rudolf Steiner Waldorf school in which Rudolf Steiner's art of education is practised" (Munro, 1995, p. 6). Each school's Charter specifies the ways in which the school community of teachers, students and parents maintains this Special Character. As amendments to the *Education Act 1989* were added under legislation, the initial relationship between the schools and the state system changed. The recent legal argument (2011) won by the schools to not meet the requirements of the *National Standards for Literacy and Numeracy* by testing their students annually against the National Standards, reflects a tension between Special Character and the state.

4.5. Summary

Steiner's life and works introduce us to a different means of mathematics education that is unconventional, yet the approach has a social and moral validity that has survived almost 100 years. It asks for a reflective self-education in the teacher and an acceptance of the influence of history and culture, as well as the soul-physical development of the child-becoming adult, on the growing understanding within the student. The process of education becomes a practice of education as an art, rather than of a didactic method. In August, 1922, in England, Steiner (1922/1947) is reported to have said "(w)e must not learn at school for the sake of performance, but we must learn at school in order to be able to learn further from life" (p. 17). The indications Steiner offered for mathematics education encouraged a particular process of thinking towards an ethical morality that is at the same time social; towards a development of the Ego-I that is personal and autonomous with a thinking that reflects inner freedom for independent thought.

It is difficult to categorise Steiner, whose approach to the developing individual human is as a being intimately linked through thinking to the 'unseen' world of cosmic form, timeless relationships and sense-free dimension. A humanist, yes, but with a vision of the spiritual in the human; traditional in considering mathematics as a "body of structured knowledge" (Ernest, 1991, pp. 138-139), yet progressive in his child-centred educational indications and political in positing this style of education for social renewal.

Chapter 5. Results: Riding the Waters - New Zealand Mathematics: 21st Century

- 5.1. Introduction**
- 5.2. Numeracy Projects and Algebraic Thinking**
- 5.3 Curriculum 2007 and National Standards**
- 5.4 Summary**

This chapter headed ‘Riding the Waters’ follows on from Chapter 3, ‘Setting the Scene’ and continues the New Zealand mainstream orientation to mathematics through the introduction of the *Numeracy Development Projects* and *National Standards in Numeracy* within the socio-political environment of the time. This is offered to contextualise the ideological framework of present mathematics education and the consequent focus on algebraic thinking within New Zealand state-funded schools.

Because of the review focus of this thesis, this chapter (and the following Chapter 6) do not contain ‘results’ as an empirical study would; rather this elaborates the descriptions of mathematics education in New Zealand with recognition of the impact of the historical background referred to in Chapter 3. Mathematics education in New Zealand in the 21st century proceeds on from what has gone before. The river flows, braiding its channels across one another, and it is now a matter of ‘riding the waters’.

5.1. Introduction

Mathematics had been termed a “paradigm case subject” (Neyland, 2002, p. 513) in relation to it being the first subject to be restated as hierarchical levels of achievement outcomes after the *Education Act 1989* and *Education Amendment Act (No 4) 1991. Mathematics in the New Zealand Curriculum* (Ministry of Education) was published in 1992 before the 1993 *New Zealand Curriculum Framework* (Ministry of Education). This latter document presented eight hierarchic levels of learning (over school Years 1 - 13) and the way in which learning outcomes could be

assessed at secondary level, especially, through the newly-instated unit and achievement standards. Neyland (2002) continued by stating that the content of mathematics may appear suitable for hierarchic representation, but the schooling inherent within mathematics is distorted in the process.

Changes in education policy over the last fifteen to twenty years have led to numeracy being considered a priority by New Zealand's Ministry of Education.⁶⁶ In some Ministry publications the words 'mathematics' and 'numeracy' are interchanged.⁶⁷ The policy of National Standards' assessment in numeracy and literacy was actuated in January 2010 for all pupils in years one to eight of New Zealand state-funded schools. This policy followed the Numeracy Projects, which began to be implemented from early 2001, largely as a pedagogical content programme for teachers of years 1 - 10 with consequent assessment and national data collection of their pupils' achievements. The data then collected helped to frame the standards in numeracy.⁶⁸ By the time the *National Standards in Numeracy* came into effect in 2010, most schools in New Zealand had participated in the Numeracy Projects (Andrew Tagg, personal communication,⁶⁹ August 3, 2014). The Numeracy Projects' resources, used as a means towards meeting the National Standards in numeracy, have now become the 'default curriculum' in mathematics for many schools in New Zealand. In 1971 the sociologist, Basil Bernstein, had written, "Curriculum defines what counts as valid knowledge, pedagogy defines what counts as valid transmission of knowledge, and evaluation defines what counts as a valid realization of this knowledge on the part of the taught" (p. 85). It appears that the Numeracy Projects now define the 'valid

⁶⁶ See <http://www.minedu.govt.nz/theMinistry/PublicationsAndResources/StatementOfIntent.aspx> for 2009, '10 & '11 *Statements of Intent*, Operating Intentions, Priority 2. This priority was not set before the 2009 *Statement of Intent*, and was superseded by economic priorities from 2012.

⁶⁷ See, in particular, Learning Media publications: *Mathematics Standards for years 1-8* (2009a) and *Mathematics in years 4-8; developing a responsive curriculum* (ERO, Feb. 2013)

⁶⁸ Ministry of Education, May 2010, *Designing the Mathematics Standards for Years 1-8*. p. 4.

⁶⁹ Email text: "During the course of the rollout of the NDP almost every school in New Zealand participated in the Ministry funded professional development. That rollout ended about five years ago, and whether schools continue to 'use the Numeracy Projects' would be an almost impossible thing to quantify. I would imagine that teachers in almost every school continue to use ideas from the NDP in their teaching, but that very few would continue to use a structured system based fully on the NDP material. Sorry I can't be any more informative than that! Andrew"

knowledge’ and their teaching method the ‘valid transmission of knowledge’, while the *National Standards in Numeracy* define the ‘valid realization of this knowledge’.

5.2. Numeracy Projects and Algebraic Thinking

During 1999 and 2000 the New South Wales, Australian programme *Count Me In Too* (CMIT) for Class Years 1 - 3 (6 to 8 year olds) was trialled in some New Zealand primary level classes (Hunter, 2006; Ministry of Education, 2008a; Nicholas, 2006; Thomas & Ward, 2001). Since 2001 the New Zealand versions, *Early Numeracy Project* for years 1 - 3, *Advanced Numeracy Project* for years 4 - 6 and *Numeracy Exploratory or Intermediate Numeracy Project* for years 7 - 10, were all brought under the umbrella of *Numeracy Development Projects* (NDP), and largely replaced the teaching of mathematics in many New Zealand schools up to Year 10 (Nicholas, 2006; Young-Loveridge, 2009). The unease with curriculum content among primary school teachers, who are not mathematics specialists, had been partially met by the professional development insisted upon by the Numeracy Projects. This compulsory re-training and the provision of particular teaching tools and resources were intended to extend the pedagogical and mathematics content knowledge of the teachers, which was one of the three purposes of the Numeracy Projects. The other two purposes highlight: improved achievement by students of the Number and Algebra strand; and ensuring that schools and their communities recognise the significance of Numeracy⁷⁰, which is to “arise out of effective mathematics teaching” (Ministry of Education, 2008a, inside cover).

From the examples provided in the Numeracy projects’ Professional Development publications (Ministry of Education, ten booklets, revised 2007-2012), it appears that numeracy includes:

- number knowledge;
- addition, subtraction, place value;
- multiplication, division;

⁷⁰ See also Hogan, J. (2002). *Mathematics and numeracy - is there a difference?* Hogan offers a number of comparative definitions for Numeracy from a range of sources.

- fractions, decimals, percentages;
- number sense and algebraic thinking;
- number through measurement, geometry, algebra and statistics.

This list suggests that numeracy is a subset of mathematics, a point raised by Begg (2006) in his article *Does numeracy = mathematics?* Begg's main concern is that the focus on number in New Zealand primary (and lower secondary) classrooms is relegating mathematics teaching back to a form of arithmetic or 'core' mathematics. Walls (2004) notes, also, that numeracy, as practised through the Numeracy Projects, is almost exclusively "arithmetical skills" (p. 26). While the Numeracy Projects consider that mathematics is being taught, there is a question as to whether the *Number Framework*, incorporating a strategy model⁷¹ and number knowledge,⁷² and now a 'mathematics curriculum by default', is indeed delivering the expected outcome - a more numerate society.⁷³

These New Zealand numeracy programmes integrate a constructivist model of knowledge acquisition and cognition (Pirie & Kieren, 1994) with particular counting 'types' (Steffe, Glasersfeld, Richards, & Cobb, 1983) to form the *Numeracy Number Framework: Strategy and Knowledge* (Hughes, 2003; Hunter, 2006; Ministry of Education, rev. 2007-2012). The Numeracy Projects thus rely on a strategy model to focus the acquisition of number knowledge by students. Figure 5, on the following page, is a reproduction of the schema of the model (Ministry of Education, 2008a, p. 5).

The different stages of knowledge acquisition according to the *Numeracy Development Projects* incorporate a recursive learning 'loop' similar to Vygotsky's (1966/1991) Zone of Proximal Development (ZPD). It is these recursive learning

⁷¹ See Pirie, S. & Kieren, T. (1994) for a thorough description of the model that was adapted for use.

⁷² Hughes, P. (2003) writes of the 5 steps of Steffe's number knowledge being increased to 9 for use with the students from years 1-10 in New Zealand.

⁷³ See Young-Loveridge, in *Findings From the New Zealand Numeracy Development Projects 2009*

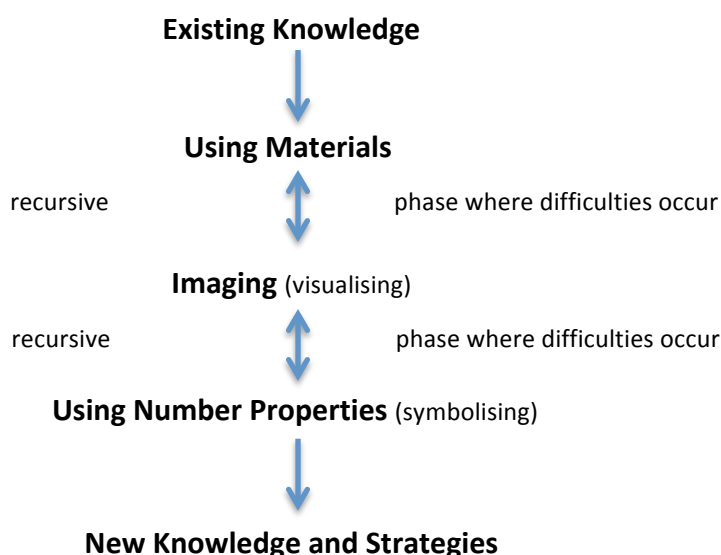


Figure 5: The Expected Knowledge Flow through the Numeracy Projects' Strategy Levels

areas, often stimulated by the teacher (Hughes, 2003) where opportunities for checking and validating in the self-construction of knowledge occur, but where it seems most difficulties in the integrative learning of mathematical knowledge and strategies arise.⁷⁴ Communication is central. It is expected that in asking students to explain their thinking, and using questions or explanations from other students to help progress them in their understanding, students will change from being passive users of rules to those who can communicate and reason mathematically (Hunter, 2006; Nicholas, 2006).

Constructivism is based upon an experimental, empirical, developmental psychology of the child, with the underlying assumptions of a linear progression from the immature child to the mature adult, and from a “pre-rational capacity” to a “rational capacity in thinking” (Dahlin, 2013, p. 18). It focuses on what students learn and the processes they use in learning (Cobb, 1994). Learning is considered to occur out of the gathering and building of knowledge by a child through interaction with her environment and with other people - children and adults - in order to make sense of that world or environment around her. In other words, new knowledge is built on previous learning (Hunter, 2006).

⁷⁴ In a conference abstract *Revisions and Extensions of a Pirie-Kieren-based Teaching Model*, Peter Hughes relates that teachers initially “mis-implemented the original 2001-2006 model which was later revised, especially in the area of imaging” (Hughes, 2007)

There are three main personalities whose work during the middle of the twentieth century (early 1930s to late 1960s) has led to what is now termed 'social constructivism', the main ideology behind the numeracy projects: the Frenchman, Jean Piaget; Jerome Bruner of USA; and the Russian, Lev Vygotsky (Harris & Butterworth, 2002; Light, Sheldon, & Woodhead, 1991).

- Piaget, a psychologist who began with an interest in biology and Binet's IQ tests, conceived of cognition as the capacity to think about things and to understand; that this cognition is adaptive towards and adapted by other influences; and with the 'child - becoming adult' to developmentally proceed through various cognitive stages to become an autonomous self-regulating individual.
- Bruner, a professor of psychology and cognitive studies, wrote of cognition being influenced by culture and growing also in a step-wise fashion. He introduced the concepts of 'scaffolding' and 'the spiral curriculum' into the field of education.
- Vygotsky, a social psychologist, posited a 'zone of proximal development' - the ZPD; a stage in learning which allows for the learner in the socially interactive learning process, to 'fall back' to prior knowledge if the new knowledge is too difficult to integrate. Through social engagement with their environment and with 'expert' others - children and adults - the child moves from the 'social' to the 'individual' by internalising their knowledge. To Vygotsky, communication is essential.

Thus, what has been brought from these psychologists into the teaching approach of the Numeracy Projects has been the understanding of cognition as not only the capacity to think about things and to understand, but also for that understanding to grow in a step-wise fashion adapted by other influences including culture (own social, the classroom) and adaptive towards such culture. Learning is encouraged through 'scaffolding', being socially interactive and communicative, and may be facilitated through a 'folding back' to prior knowledge in order to enable integration of new knowledge.

David Elkind (2004) suggested that in order to really work, constructivism needs teacher readiness (a thorough instruction in and understanding of child

development), curriculum readiness (matching the student's ability levels to the demands of the task), and societal readiness (an energised social consensus; not just political). This congruence was certainly stimulated in New Zealand in the early 2000s with much positive publicity about the Numeracy Projects following on from the heightened socio-political awareness of the need for targeted numeracy in mathematics in the late 1990s.

The Numeracy Projects combine teaching according to behaviourist methods, especially in the lower order skills, with learning according to a constructivist model (Hunter, 2006). *The New Zealand Curriculum 2007* combines a child-centred humanistic approach, carried over from the time of Clarence Beeby, with neo-behaviouristic step-wise, outcome-related achievement steps (Neyland, 1995). The tension of dissonance between these ideologies is not obvious until the higher stages of the Numeracy programme are attempted. In mainstream school research within New Zealand there is a recognised 'plateauing' of mathematical knowledge acquisition, believed to be attributable to increased knowledge at the higher levels, and slower acquisition among some ethnic and socio-economic groups (Young-Loveridge, 2006, 2009, 2011). Even so, there has been a concerted effort to link the curriculum level expectations in mathematics education to the language and stages of the Numeracy Projects and their diagnoses or assessments.⁷⁵

In 2005 Gilbert wrote that the document '*Growing an Innovative New Zealand*' from the Prime Minister's Department in 2002 stated:

Providing a quality education system . . . requires creating a culture of continuous improvement and a focus on quality teaching and learning. It involves focusing on better learning in our schools by emphasising literacy and numeracy, embedding ICT into learning processes, ensuring that schools reflect the curriculum's focus on giving students the skills needed for a modern economy and society, and outlining higher and clearer standards of achievement. (p. 45).

⁷⁵ See <http://nzmaths.co.nz>

It is interesting to note that those six countries ranked highest in the International Mathematics and Science surveys, stated that calculators ('embedded ICT'⁷⁶) were never or hardly ever used by nine-year olds, whereas New Zealand reported that over half of those classes sampled for the surveys stated almost all students had access to calculators during mathematics lessons, but *not* during tests [Garden in TIMSS, Garden, (Ed.), 1997].

Since the early 1990s, after the implementation of the *Education Act 1989*, mathematics-thinking skills have been specifically referred to in curriculum documents. In the 1993 *Curriculum Framework*, Numeracy skills were listed separately from Problem-solving skills (Ministry of Education, 1993). In *The New Zealand Curriculum* (Ministry of Education, 2007), thinking is set out as a Key Competency with the Mathematics and Statistics learning area defining *thinking skills* as the reason for studying mathematics and statistics. In particular:

By studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information and to enjoy intellectual challenge. (Ministry of Education, 2007, p. 26).

The Ministry considers that the long-term success of the Numeracy Projects will be best measured by “improvements in algebra performance” at secondary level (Ministry of Education, 2008a, p. 3). There is a specific ‘pathway’ proposed by Britt and Irwin (2011) that begins with young children exploring and describing in their own words, generalities arising in their activities of a proto-quantitative nature. Primary school pupils are then able to increase their range of strategies and knowledge, and intermediate and secondary students to use algebraic symbolisation to express generalisations derived from numeric and figural representations. The insistence on mental calculation and estimation in the teaching of number is meant to increase flexibility and facility of mathematical thinking, especially algebraic thinking. In their paper describing algebraic thinking, Irwin and Britt (2005) used the following definition: “developing an awareness of, and applying generality in any mathematical domain is in itself an indicator of

⁷⁶ ICT - Information and Communications Technology

algebraic thinking” (p. 172). MacGregor and Stacey, cited in Neill and Maguire (2007) define algebraic thinking as about generalising arithmetic operations, because in algebra symbols can be used to represent generalisations. They continue: “the language of arithmetic focuses on answers while the language of algebra focuses on relationships” (“Concept maps”, para. 2). But Whitehead, quoted by Howson in his *Review* (1994) of the 1992 mathematics curriculum “explained that the strength of algebra was that it allowed us to do things without thinking” (p. 27). Howson goes on to write that translating a problem into algebraic terms allows us to work through a process in the abstract without having to think about the original problem, thus ‘forgetting’ the original concrete contextual relationship.

5.3. 2007 Mathematics Curriculum and National Standards

The 2007 curriculum document also merged the Number and Algebra strands into one within the Mathematics and Statistics learning area, squeezing out the strand *mathematical processes* added in 1992, yet generalising these processes under *Key Competencies* (Ministry of Education, 2007). Geometry with Measurement make up a second strand while Statistics, as a single strand, completes the learning area. According to Book 3 of the Numeracy Projects, all strands are important in achieving numeracy, with different emphasis on particular strands at different year-levels (2008a). The Number - Algebra strand is considered most important during years 1 - 4, and is the predominant focus (by Venn diagram size⁷⁷) up to and including Level Five (years 9 - 11) of the curriculum. Only at Level Six (years 10 - 12) are the ‘ovals’ representing each of the three strands almost equal in size (Ministry of Education, 2007). While working with number is considered to support mathematical thinking (Ministry of Education, 2008a), algebraic thinking is considered more important as noted in the Numeracy Projects’ books and in the *Figure it Out*⁷⁸ series (Neill & Maguire, 2007).

⁷⁷ See *The New Zealand Curriculum 2007*, Levels 1-8 following p. 44.

⁷⁸ Resource booklets and sheets for students available through the nzmaths website.

Eight curriculum levels cover the thirteen years of schooling for all students five years and over in age. Based on each student having an equal opportunity to be engaged in learning, each curriculum level provides learning ‘milestones’ that need to be met through the application of the curriculum within specific knowledge areas (Ministry of Education, 2007). These ‘milestones’ have been further consolidated within the *National Standards in Numeracy and Literacy*. National Standards were first referred to in *Section 60A* of the *1989 Act*, under *National Education Guidelines*, but only formally developed and entered into legislation under *60A(ba)* on 17 December 2008, to be a compulsory evaluative tool from the beginning of the 2010 school year, for students in years 1 - 8 (equivalent to the full primary school): “National Standards aim to lift achievement in literacy and numeracy (reading, writing, and mathematics) by being clear about what students should achieve and by when” (<http://www.minedu.govt.nz/theMinistry/EducationInitiatives/NationalStandards.aspx> - retrieved August 21, 2013).

Data received during the years 2001-2009 from assessments made in the Numeracy Projects has helped develop the standard to be achieved at each year-level, and the Numeracy Projects’ definition of Numeracy: “ability and inclination to use mathematics effectively – at home, at work, and in the community” (Ministry of Education, back of all Numeracy Projects’ books) has provided the overall goal. While the standards link with the strands of the Mathematics and Statistics learning area within the Curriculum, “the expectations for Number are the most critical requirement for meeting the standard” (<http://nzcurriculum.tki.org.nz/National-Standards/Mathematics-standards/Layout-and-illustrations-of-standards> - retrieved 21 Aug 2013). An annual report by teachers to their school’s Board of Trustees (BoT) must include each student’s ranking within the National Standard for their year-level, at one of four prescribed levels: *above*, *at*, *below*, *well below*. The ministerial concern over the ‘tail’ of non-achievers within the statistical model, and a ‘deficiency belief’ place an emphasis on non-achievement in these reporting levels.

The ranking of students, as reported to the school’s Board of Trustees, must then be submitted to the Ministry. The following year, the numbers of students at each of the four levels of reporting is published, by school, on the Ministry’s website. This

process does not appear too different from the pass-rate percentage ranking of schools during the 1890s to mid-1930s! Today, the reporting focuses on literacy and numeracy; in the late 19th and early 20th centuries it was the *Certificate of Proficiency*, which covered a much wider range of subjects albeit of perhaps less, but particular, information. Is the narrower range of subjects a ‘distillation’ of the greater, into an ‘essence’ required for the 21st century? Teaching to the assessment (no longer is the word ‘examination’ used in this context) happens again, and teachers carry concerns, again, for the full education of their students. I am reminded of what Campbell (1941) wrote in the Centennial Survey series: “the individualist traditions brought from nineteenth-century England led to the acceptance by a large part of the community of almost any form of schooling provided only it opens the door to vocational success” (p. 107). Margaret Walshaw (2008), who was one of the authors of *Best Evidence Synthesis: Effective Pedagogy in Pāngarau / Mathematics*, wrote in the editorial of the *Mathematics Education Research Journal* that “student knowing, doing and thinking, taken together, are the cornerstones of the official policy document in mathematics, standing alongside a continuing belief in mathematics as a prime engine of economy, nation and identity” (p. 1). Compare this to the concepts of Liberty, Equality and Fraternity that became Liberalism, Socialism and Nationalism, respectively, during the nineteenth and early twentieth centuries (Lucas, 1972).

5.4. Summary

At the beginning of the 21st century, the Ministry of Education introduced the *Numeracy Development Projects*, ultimately to be effected in Years 1 to 10, the ‘compulsory’ years of schooling. By focussing on just one of the five mathematics ‘skills’ learning strands (Ministry of Education, 1992), and encouraging teachers through content, teaching strategies and mentorship, it was hoped to convert what was perceived as an anachronistic approach of teaching into a style of learning for the ‘Knowledge Society’. But the continued lower-than-expected achievement levels in mathematics, especially among Māori and Pasifika who also make up the greater number of school leavers with no official achievement accreditation, have

encouraged direct government intervention by the Ministers of Education. *National Standards in Numeracy* (and Literacy) must now be reported upon by teachers, twice-yearly (and by their school's Board of Trustees to the Ministry, annually) for all students in Years 1 - 8. Students are ranked according to their achievement (*above, at, below, well below*) of the Standard, as developed from a number of sources including the *Numeracy Development Projects* (Ministry of Education, 2010). Since 2011, schools' charters must include curriculum targets for their students' achievements in relation to the National Standards (Ministry of Education, 2009). Although the numeracy standards focus on the Number-Algebra strand, the Ministry of Education (2010) states that the "standards reflect the philosophy and content of the mathematics and statistics learning area of *The New Zealand Curriculum*. . . (They also) reflect the emphasis on key competencies, on modelling and problem solving in context across all strands and the progressions described through the levels" (p. 1). See Figure 6 over the page; a reproduction of an 1899 cartoon on the 'standards' in education at that time, retrieved from Campbell (1941). Have we come so very far?

There is a thread running through the history of mathematics education in New Zealand. With mathematics deemed an important part of the new science and industrial technology in the nineteenth century, it is now perceived to be equally important in the present 'Knowledge Society' of the 21st century. In both instances, mathematics education during the schooling years is seen as necessary to the society as a whole, in enabling its young members to become active and participatory citizens. This 'sedimentary warp' of socio-political aims in education is braided with the 'river channel weft' of mathematics education.

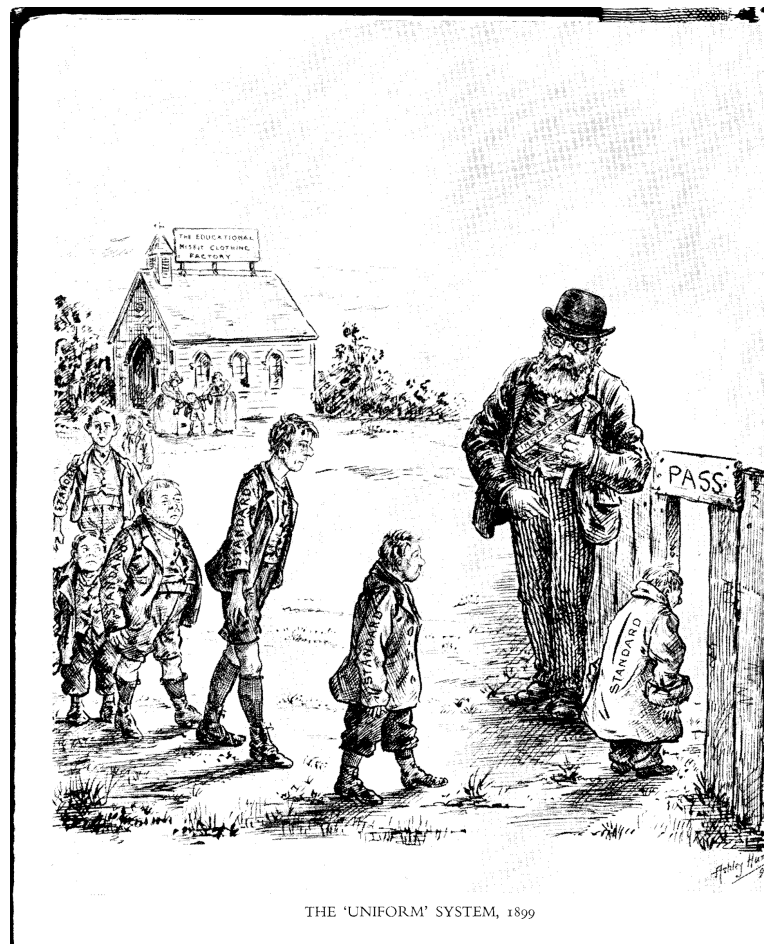


Figure 6: Standards and the 'Uniform' system

(reproduced from Campbell, 1941, Frontispiece)

Mathematics now equals Numeracy; the standards of which have been ordered by government legislation, and tied to the contractual undertaking between the school and the government. This level of control through authority is regarded as neo-liberalism, a 21st century version of the 'liberalism' practised by the Liberal government in New Zealand over 100 years ago. But this modern version is tied more strongly to an economic agenda; not in the way Foucault propounded (reported by Mitchell, 1999) as political economy, but as Gordon (cited in Mitchell, 1999) describes; *Homo œconomicus* has become 'manipulable man'. Whereas the individual between the ages of seven and thirteen in the nineteenth century, was to be educated at school to become a civilised member of society, it seems the individual is now subsumed through educational processes for the economic benefit of society.

Chapter 6. Results: Riding the Waters - Steiner's Indications in the 21st Century

- 6.1. Introduction**
- 6.2. Steiner's Indications and 'Learning Steps'**
- 6.3. Mathematics and Thinking**
- 6.4. Summary**

6.1. Introduction

The spiral curriculum over the twelve years of a full Steiner Waldorf school presents and re-presents mathematics content according to the nature and the developmental period of the student, within the processes of 'forgetting' and recalling. The progressive increase in knowledge is integrated at ever-deepening levels with a sense of the relationship of the parts to the whole, enabling an overarching understanding. There is a sense that in mainstream education we get students to think by making them think (Oberski, 2006), whereas in traditional Steiner Waldorf schools, up to about puberty, thinking is trusted to develop partly through the organic growth and development of the student, and encouraged by teaching through the feelings and will in an aesthetic way (Steiner, 1919/1994). Steiner offered brief indications for mathematics education; leaving the classroom curriculum to the teacher to develop according to the needs of the students. Not an easy task if the teacher lacks confidence in mathematics education! Hence, some of the Steiner Waldorf schools in New Zealand have worked with the Numeracy Projects, and with *Learning Steps in Numeracy (and Literacy)* as their negotiated alternative to the *National Standards* in Numeracy.

6.2. Steiner's Indications and 'Learning Steps'

The indications Steiner proposed for mathematics education are based on a perspective of the young child bearing a potential for mathematical thinking, which may only slowly be drawn out of an unconscious embodiment (Donaldson, cited by

Davis, 1991; Steiner, 1922/1947; and 1924/1988).⁷⁹ As the very young child raises herself from a supine horizontal plane through crawling to a vertical upright stance, the capacity to geometrize in space develops. Then, as the growing child begins to speak and name, and to think, the capacity for counting develops (Harris & Butterworth, 2002). These capacities have nothing to do with the calculation of number, although it may appear as if the child is ready for such instruction, but are related to the development of the semi-circular canals of the ear.⁸⁰ While the child plays, a sense of space, magnitude and comparison are expressed in the gestures and language used. To Steiner (1907/2008), these are natural developments during the early years of socialisation through imitation, and are not consciously thought-through processes of knowledge acquisition.

Lakoff and Núñez (2000) also write of this natural development as ‘subitising’; that capacity for numerosity that young children express long before formal schooling, and which we humans share with some animal groups. According to these authors, as we become aware and differentiate between a significant person (‘mother’) and another person (not so significant), we are able to group and order. By the time we can call ourselves “I”, we are able to pair our fingers to objects in basic counting, and exercise a memory of names to objects. We also begin to have an awareness of when there is nothing more to count; a form of ‘exhaustion-detection’. All this happens before formal schooling. A study by Young-Loveridge (1989) of New Zealand five year-olds showed that a majority had begun formal schooling with a considerable informal knowledge of number concepts and skills.

Steiner likens the years from birth to the ‘changing of teeth’ as similar to a dreaming form of consciousness (1924/1988), when the faculties we consider appropriate for formal schooling are only slowly maturing. Arithmetic is not to be taught until formalised schooling has already introduced art (the drawing of straight lines and curves), and the writing and reading lessons that build onto the art

⁷⁹ See also Jarman (1998): “The genius is asleep to begin with, and rests within the beat of heart and lung - the true womb of arithmetic - and in the bones and muscles of the limbs - the true womb of geometry. . . . When the two parts or aspects ascend to the head, later to unite, the full genius can awaken and mathematics can become a conscious activity possessed by human thinking” (p. xiii).

⁸⁰ For further pedagogical information see Steiner, 1920/1989, p. 192.

(Steiner, 1919/1976; Stockmeyer, 1982). Reading follows writing, which follows drawing. And arithmetic notation begins by representing number using the Roman numerals. Piper (2012) reminds us that, historically, reading began with marks that represented quantity and time, and using the Roman numeral notation offers an initial 'grouping' of such tallies. Beginning with such a representation that allows the young student to 'find' the number within her own body - using fingers and toes - prevents the tendency to concentrate purely in the realm of head-thinking (Steiner, 1924/1988). The Steiner Waldorf teacher 'draws out' and awakens the young student's thinking by working with counting and numbers using real objects and rhythm to engage the memory, and does not move too quickly into abstract, symbolic terms. Rousselle and Noël (2007) showed that children with dyscalculia were only impaired when comparing magnitude using Hindu-Arabic digits (symbolic) but not when comparing collections using non-symbolic density patterns.

'Limb-learning' is emphasised by Steiner, not just head-learning (Schieren, 2012; Steiner, 1919/1981b). That is, educating the will through movement and bringing to a perceptual-conceptual consciousness the mathematics already embodied in the growing child. Lakoff and Núñez (2000) write of the stages of mathematics that physiologically are part of the developing human before enculturation through language occurs. Enculturation brings what Lakoff and Núñez call 'metaphor use' as part of the language preparation towards the abstraction and symbolisation that arises in formal schooling; for example, the metaphors below:

- 'object collection' (numbers - larger/smaller; take away/take out of/add),
- 'measuring stick' (can work with irrational and imaginary numbers - greater/less; a unit),
- 'motion along a path' (doesn't need an origin as 'collection' and 'stick' do; can include negative numbers), and
- 'object construction' (more specific than the 'collection' metaphor; put together/made up/ composed of).

Steiner also refers to the relationship of morality to arithmetic, and a long quote from his lectures at Oxford, England during August 1922, explains:

A child is able to take in the elements of Arithmetic at quite an early age. But in arithmetic we observe how very easily an intellectual element can be given the child too soon. Mathematics as such is alien to no man at any age. It arises in human nature; the operations of mathematics are not foreign to human faculty in the way letters are foreign in a succeeding civilisation. But it is exceedingly important that the child should be introduced to arithmetic and mathematics in the right way.

. . . It is not usual to hitch arithmetic on to moral principles because there seems no obvious logical connection between them. . . . The transition from one to two, and then to three, - this counting is quite an arbitrary activity for the human being. But it is possible to count in another way. And this we find when we go back a little in human history. . . . (W)hen I have an organic whole (a whole consisting of members) . . . I am starting with unity, and in the unity, viewed as a multiplicity, I seek the parts. . . . One did not think of numbers as arising by the addition of one and one and one, one conceived of the numbers as belonging to the whole, and proceeding organically from the whole. . . . Thus to get a living understanding of addition we start with the whole and proceed to the addenda, to the parts.

When a child has acquired the habit of adding things together we get a disposition which tends to be desirous and craving. In proceeding from the whole to the parts, and in treating multiplication similarly, the child has less tendency to acquisitiveness, rather it tends to develop what, in the Platonic sense, the noblest sense of the word, can be called *considerateness, moderation*" (1922/1947, pp. 72-74; original italics).

This quote identifies the reasoning behind the main principles that Steiner gave for mathematics education; working from the whole to the part, and using an aesthetic approach to engage the will and feelings.⁸¹ In lectures given in 1924 (1924/1988), he also encouraged a way of perceiving number out of the whole 'unit' by referring to the 'unit' line divided into equal sections, as in Figure 7, on the next page:⁸²

⁸¹ See also Steiner, 1924/1968, Lecture Four, especially pp. 66-68.

⁸² Lakoff & Núñez (2000) refer to this as the 'measuring stick' metaphor in arithmetic, such metaphorical representation necessary before moving into symbolic representation and abstract thinking (p. 68).

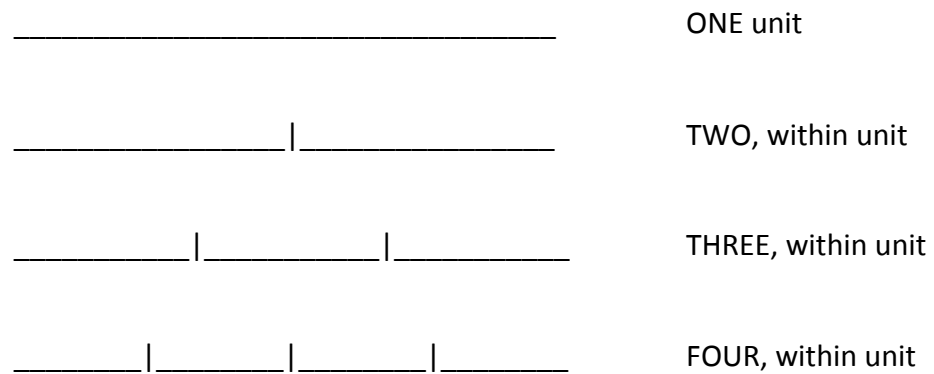


Figure 7: Representation of number as part of a whole Unit

In developing geometry from drawing, Steiner was insistent that the young student feel into the experience (Steiner, 1920/1989; Stockmeyer, 1982), this feeling experience aided by Eurythmy in Steiner Waldorf schools. Mathematics education, especially in the first classes, needed to focus on *feeling* the line as straight or curved; on *feeling* the number as a whole unit which nevertheless can have its parts ‘played with’ in counting, multiplying, and sharing out; *feeling* symmetry in completing a form; *feeling* the dissonance in some counting sequences. Through the realm of feeling, memory and recall are encouraged. The language of mathematics, in word, form and number, is practised until by the third class the magic code of mathematics begins to show and the student starts the exciting journey of decoding.

In a lecture given in Ilkley, England, August 16, 1923, Steiner was adamant that arithmetic and geometry be taught throughout the full twelve-year period of formal schooling “though naturally in a form suited to the changing characteristics of the different life periods” (1923/1981, p. 154). Table 2, on page 71 below, summarises the indications Steiner gave according to Class level (up to year 10), and developed from the collation by Stockmeyer (1982):

Table 2: Overview of indications for mathematics education by class/age level

<p>Class I - Year 2 (6-7yrs)</p> <ul style="list-style-type: none"> - all 4 rules from 'whole to part'; analysis to synthesis - counting to 100; use fingers at beginning - begin multiplication tables' learning, 'by heart' through rhythm
<p>Class II - Year 3 (7-8yrs)</p> <ul style="list-style-type: none"> - all 4 rules with greater range of numbers - solve simple problems (algorithms) mentally - reckon with 'concrete' numbers (using materials) and develop 'abstract' numbers (using imaging) in connection with objects
<p>Class III - Year 4 (8-9yrs)</p> <ul style="list-style-type: none"> - use more complicated numbers & apply to simple things in practical life ('house-building' main lesson introduces measurement)
<p>Class IV - Year 5 (9-10yrs)</p> <ul style="list-style-type: none"> - begin work with fractions, especially decimal - describe process out of artistic and practical exercises, using a 'feeling for' (Euclidian-style proofs not attempted until Class VI; induction follows earlier deduction)
<p>Class V - Year 6 (10-11yrs)</p> <ul style="list-style-type: none"> - continue with fractions and decimals, so that pupil 'moves freely' in reckoning numbers, whether whole, fractional or decimal
<p>Class VI - Year 7 (11-12yrs)</p> <ul style="list-style-type: none"> - calculate interest, percentage, discount and simple exchange using formulae and leading into algebra - study relations between calculation and the circulation of commodities and finance - introduce Euclidian proofs by deduction/induction
<p>Class VII - Year 8 (12-13yrs)</p> <ul style="list-style-type: none"> - raise numbers to powers; obtain roots - reckon with positive and negative numbers - theory of equations in connection to practical life
<p>Class VIII - Year 9 (13-14yrs)</p> <p>relate theory of equations to:</p> <ul style="list-style-type: none"> - calculations of length and areas of figures & surfaces - theory of loci
<p>Class IX - Year 10 (14-15yrs)</p> <ul style="list-style-type: none"> - advanced calculations as per systems of exchange - logarithms and plane trigonometry - descriptive geometry including π

While the above Table 2 shows the indications that Steiner offered, those New Zealand Steiner Waldorf schools that had integrated into the state system felt they needed to seriously consider the approaches offered by the Numeracy Projects. A reasonable number of Class teachers had the relevant state qualifications but insufficient experience or understanding of the philosophy and indications from Steiner to feel confident in their teaching of mathematics. Some teachers also embraced the social constructivism that lay behind the Numeracy Projects. The schools adopted the programme of support and mentoring provided for teachers, and many of the resources for use by students.

The integrated Steiner Waldorf schools in New Zealand recently had to mount a legal campaign to continue, as the then Minister of Education, Anne Tolley, threatened their closure (by de-integration) when their teachers refused to implement National Standards' testing before year 8.⁸³ The schools have now negotiated with the Ministry to work with their own formulated *Learning Steps in Numeracy (and Literacy)*, with the aim to have their students at or surpassing the national average by year 8, and to show this by a nationally moderated test. These *Learning Steps* are closely aligned to the National Standards and Numeracy Projects' terminology. The *Learning Steps in Numeracy* are under review this year following a recent (December 2013) report to the *Federation of Rudolf Steiner Waldorf Schools in New Zealand*, which showed a plateauing of achievement in mathematics from Class 6 (year 7) similar to that showing in the research within state schools.⁸⁴ While at least 7 of the 11 Steiner Waldorf schools in New Zealand use the Numeracy Projects' approach in most of their mathematics lessons to years 8/9, the Report to the Federation was built up from Progressive Achievement Tests (NZCER, 2006; tests 2,3,4, & 6A) in mathematics over a 3-4 year period.

⁸³ McKenzie-McLean, J. (2011). "Education Minister Anne Tolley wrote to the Rudolf Steiner Federation in May threatening the schools with privatisation if they did not comply and submit charters that set targets in relation to national standards. 'National standards are not optional ... If any board of a Steiner school cannot accept this, the proprietor has the option of requesting the cancellation of the school's integration agreement so that the school becomes a private school which does not have all the obligations of state schools. If proprietors wish the Steiner schools to remain integrated, I expect the boards to submit their charters as required by the ministry and that the charters will set targets in relation to national standards'." Posted on 'Stuff' July 27, 2011. Retrieved September 26, 2014, from <http://www.stuff.co.nz/the-press/news/5343976/School-forced-to-adopt-national-standards>.

⁸⁴ See the 'Findings' on <http://nzmaths.co.nz/numeracy-projects>

6.3. Mathematics and Thinking

Rudolf Steiner (1886/1978, 1919/1994, 1923/1985) considered mathematical thinking an important development towards an open investigative approach to the phenomena of the physical and spiritual worlds.⁸⁵ This is mathematical thinking from the perspective of a mature adult, a person who has integrated content (knowledge) and learning (processes) during the years of formal schooling. It is a thinking that includes critical judgement, logicity and rationality, argument and/or refutation as well as flexibility, imagination, inspiration and intuition. It is the ‘stuff’ of a scientific approach in one’s thinking, in the *spirit* of modern science, a spirit that is free from dogmatic attitudes (da Veiga, 2013). And, according to Steiner (1907/2008), this form of thinking has no place in the early years of childhood and formal elementary schooling.

In his early writings (1886/1978, 1892, 1894/1979) Steiner presented his epistemology of thinking. Using a mathematical approach to sense-free thinking he produced an explanation of thinking as an objective adaptation by an individual to their environment, the natural and social worlds. Because thinking / thought can be both the object and the subject of thinking, the process has implications in mathematics, such implications having been explored since early Greek times.

I initially presented Figures 8-12 along with an explanation (as is given on the following pages 74-80, up to the Summary of this chapter) at a Postgraduate Symposium held by AUT University on August 22, 2014. Figures 8 - 12 are developed from Steiner’s early books (1886/1978, 1894/1979), and articles from Schieren (2012) and Britt & Irwin (2011).

⁸⁵ See also the lecture Steiner gave in Christiania (Oslo) November 26, 1921: “The spiritual science of Anthroposophy is naturally not mathematics. But a significant example may be found in the way in which one penetrates into mathematical thought. It is not mathematics in itself which constitutes this example, but — if I may coin this expression, — “mathematizing,” the activity of mathematical thinking. If such a “mathematizing” culture shows us how to transcend any illusionary or suggestive element, we shall be particularly successful in concentrating upon concepts which can be surveyed and which are quite new (spiritual) to us” (Steiner, 1921, para. 13).

In thinking about thinking, we are doing something unique to humans, and it is a process of beginning by being aware of something other than our cognizing self, an object. Briefly, Steiner posited that an object (whether of physical matter or a thought) is perceived by the senses. Between this 'object' (Other) and our self, in body-mind, there is a threshold usually experienced through our neurosensory system by sensation-experiences, which lead to percepts.

We have many individual percepts, only some of which we attend to and which may involve a number of our sensory organs almost simultaneously. When we begin to attend to percepts, we begin to group them according to our experience. Steiner refers to this as the 'materialisation' towards a concept. There is a certain level of conscious awareness at this stage that I have labelled the 'realm of thought'.

Reason and experience (or common sense) combine the percept and concept to form a Representation of the Object, of Reality. Steiner also refers to this as 'mentalisation' towards a 'Mental Picture', as it is often a visual representation - but does not always have to be. It could be a smell, or a sound/tone, or a particular feeling of warmth or cold; all of which may or may not have some added experiential reference, for example: smell of coffee reminds us of that 'time-out' space we experience while drinking it in the past.

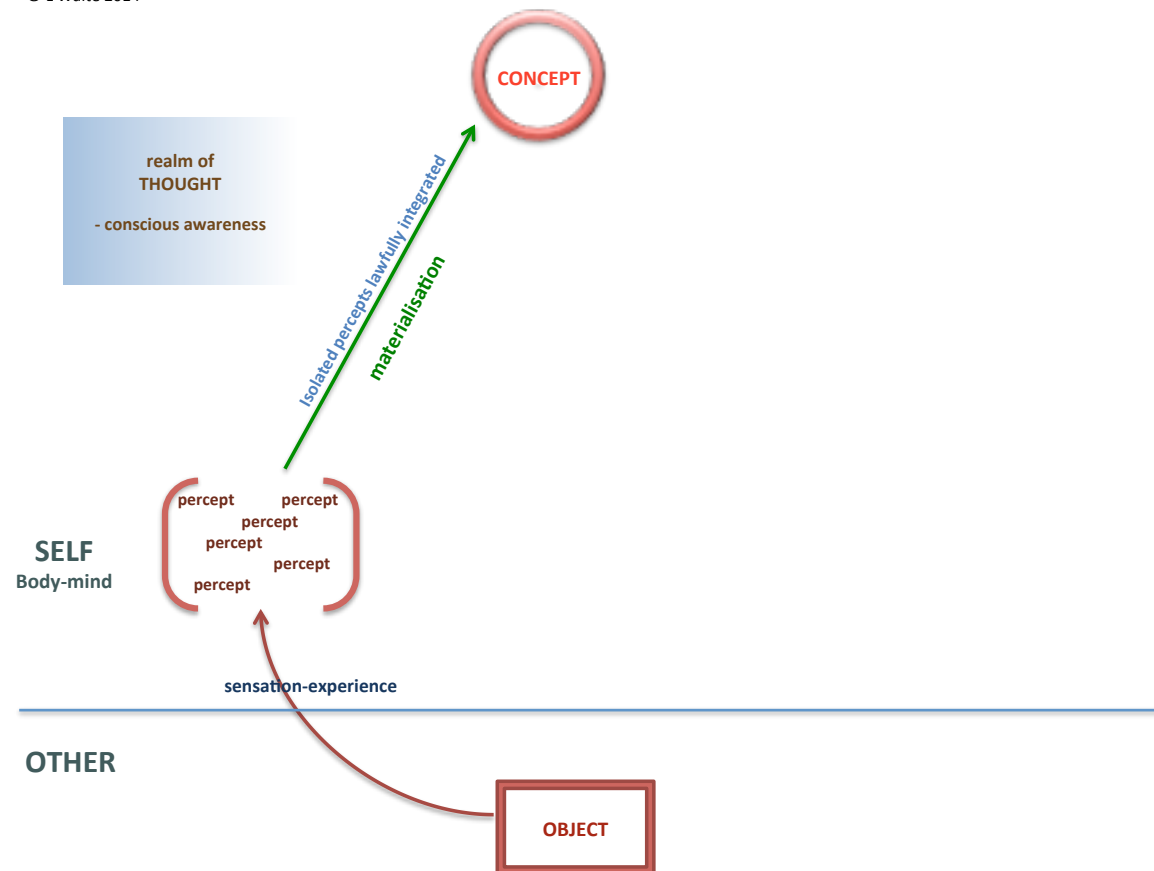


Figure 8: Diagrammatic interpretation of thinking 1.

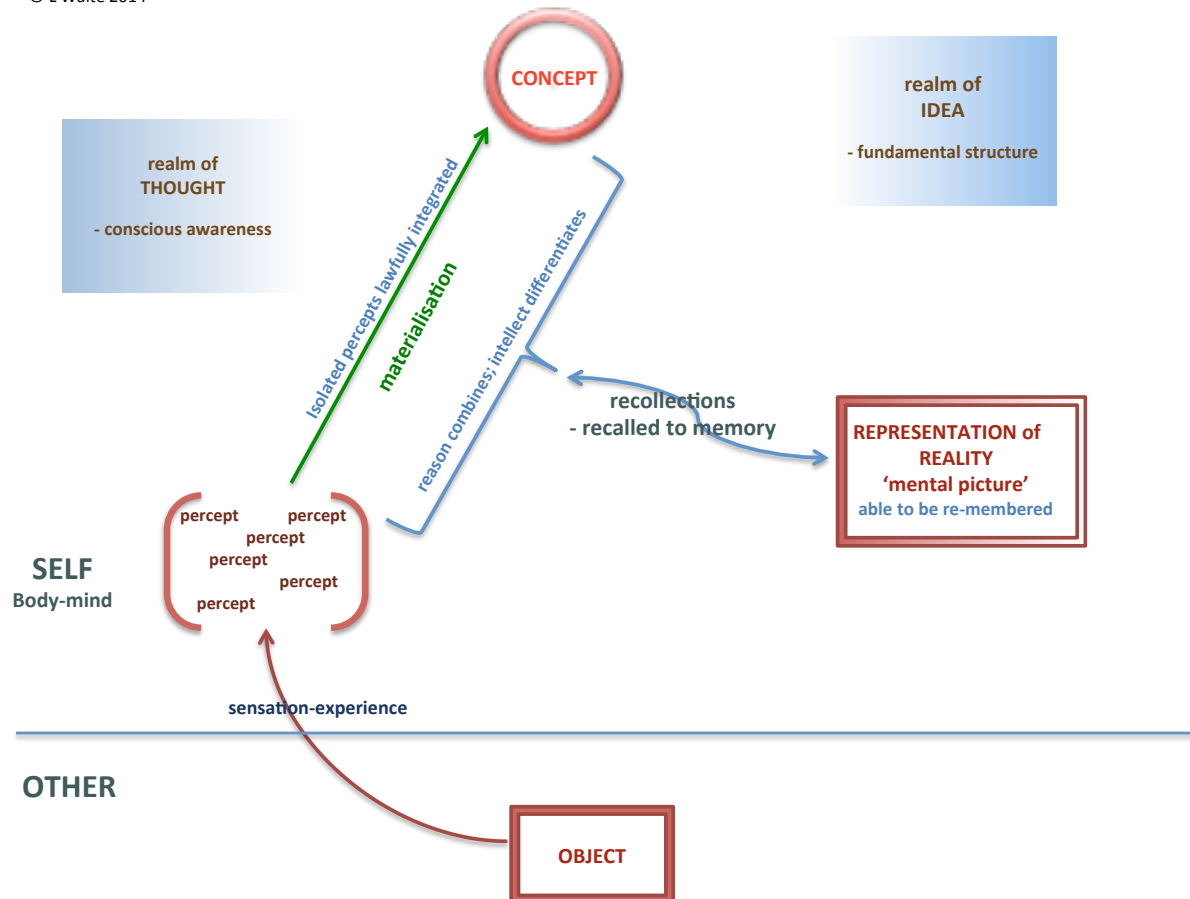


Figure 9: Diagrammatic interpretation of thinking 2.

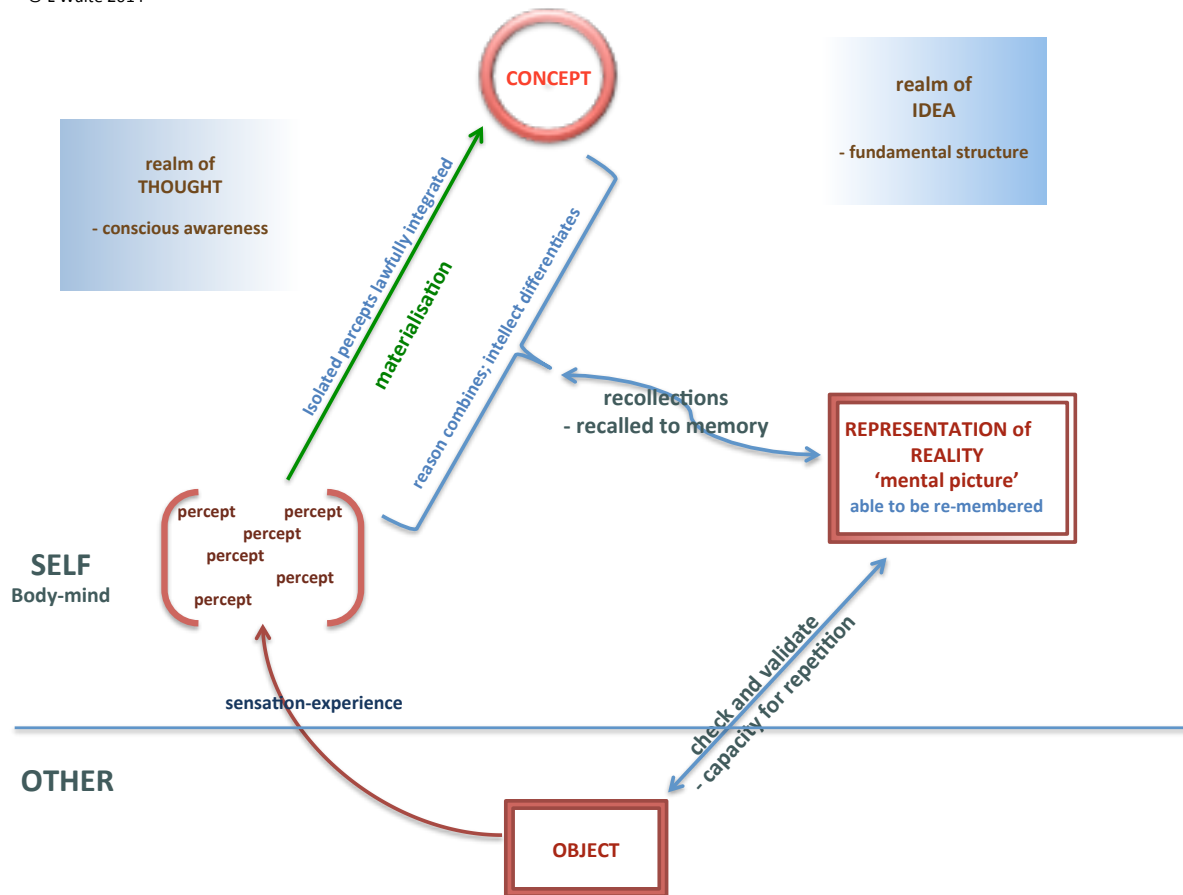


Figure 10: Diagrammatic interpretation of thinking 3.

It is this particular 'thought-representation of reality' that we recall and re-member. But, we re-build that memory anew each time; the 'representation' is always slightly different to the original; each recall differs slightly from former recalls. When, as children, we begin to form 'representations' and have 'names' for them, we are beginning to work in the 'realm of ideas'. And these 'ideas' have a fundamental structure. For example: the idea 'cat' has the fundamental structure of a smallish, four-legged, fur-covered animal with a tail at one end and two ears that stand erect over a face with side whiskers, and so on.

We mentally and through our senses, check this 'representation' we have formed, which as a mental image is essentially an 'illusion', against the original object of our perception, in order to validate this process so far. Sometimes in this process we individualise back to a particular percept, for example, a Manx cat has no tail. The more general concept-representation 'cat' is now perceived with no tail and thus needs a re-interpretation.

The process of validation is able to be repeated, and is involved when the student works with materials or Manipulables in the Numeracy Projects, and develops mental images of the activities. Jarman (1998) wrote "mathematical progress depends upon overcoming and freeing oneself from the pictorial and living in sense-free concepts" (p. 16). Steiner also refers to 'generalisation' where we become cognitively aware of the perceptual context of the concept formed. It is this capacity for 'generalisation', which I believe Britt & Irwin (2011) refer to as a pre-formal-schooling capacity towards algebraic thinking, when what they are most likely referring to is the capacity of generalising through abstraction and symbolisation, which follows on from forming the 'Representation of Reality'. This generalising capacity to symbolise is reflected in speaking, using language, and is transferable to mathematics and its 'language'.

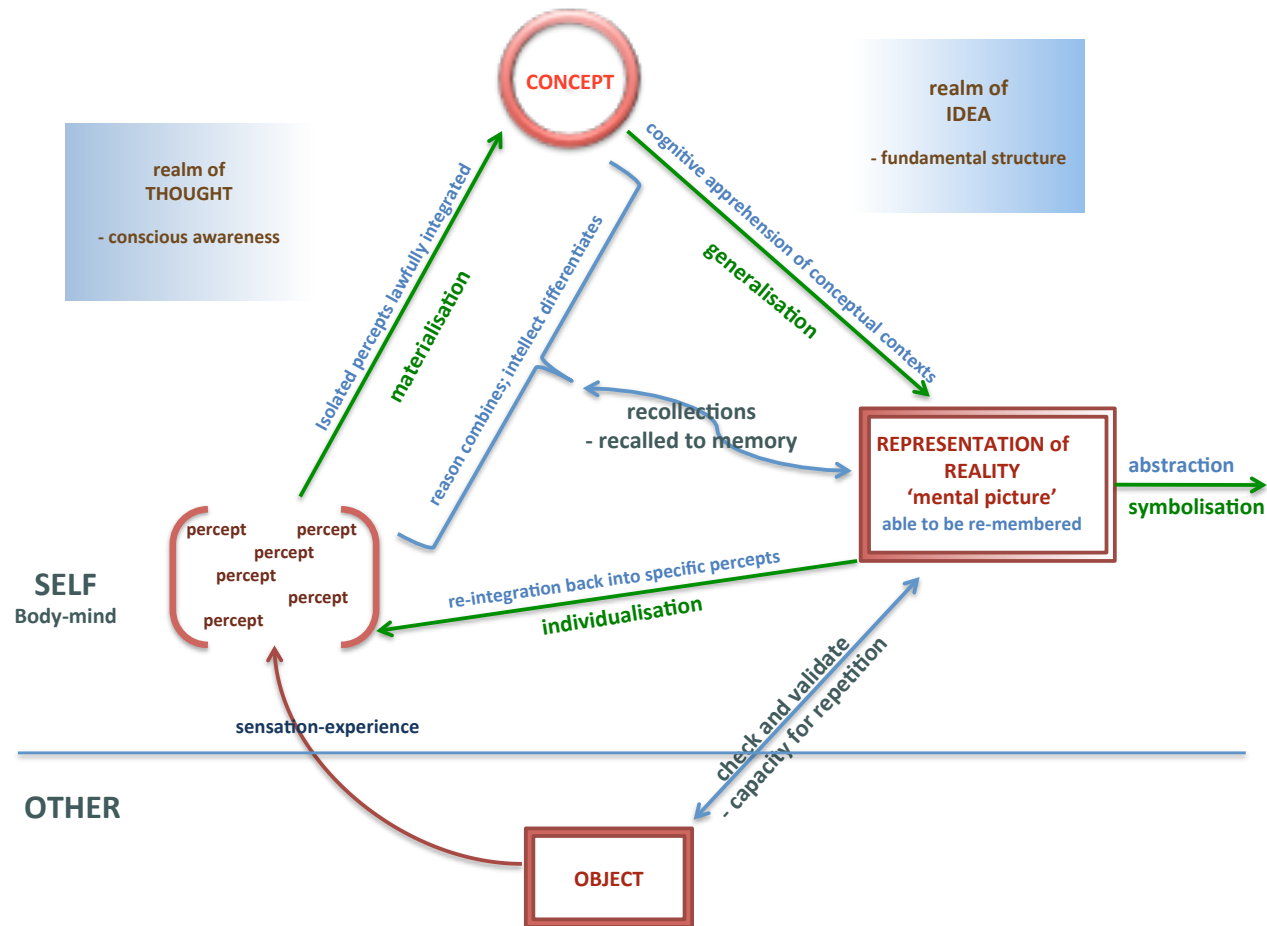


Figure 11: Diagrammatic interpretation of thinking 4

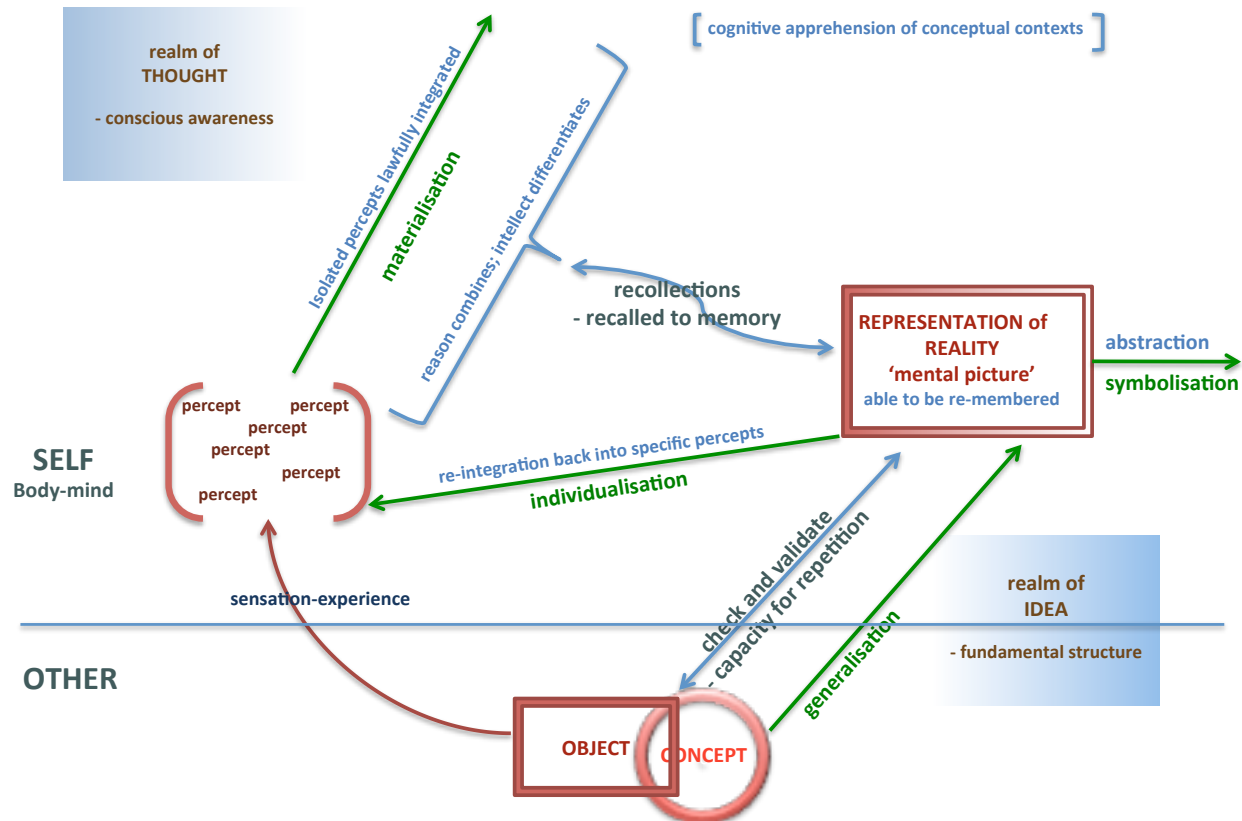


Figure 12: Diagrammatic interpretation of thinking 5

As the intention of a lesson is communicated to the students at the beginning of the lesson, the teacher provides concepts, and the work with materials and imaging moves the student through a process of generalising from that 'other concept' towards a new 'representation'. The student's capacity to become cognitively aware is not built up from a personal participatory process, but becomes 'managed' from without. The 'realm of idea' is initially the teacher's, not the student's. Here, I make a conjecture, which needs further study to 'check and validate': that without the inner participatory process from object through percept and concept to representation, driven by a student's own interest and will, a firm foundation for further 'refining' of the representations doesn't develop, with the eventual effect of not being able to extend one's mathematical knowledge at the higher stages of the *Number Framework*.

6.4. Summary

When the child enters school for formal instruction, she already exhibits a relationship to geometry and number, but this is not necessarily in a way to be 'thought about'. Steiner's indications for mathematics, in those first years at school, stress learning through movement; the senses rather than the 'nerves' ('nerves' = intellectualised head-thinking) of the sense-nerve system, and the limbs rather than the 'metabolism' ('metabolism' = behaviouristic learning) of the limb-metabolic system. Using the middle realm as mediator, the pulse of blood flow and the breathing of the lung system, rhythm is practised as a way of linking the 'representations' to the 'percept'. The pathway from sense experience through percept and concept to representation is practised using repetition and rhythm, allowing periods of forgetting to encourage recall. Validation may be encouraged, but understanding is allowed to develop in the student of its own accord.

One might argue that Steiner's picture of the formation of 'representations of reality' is constructivism by another name, yet it is significant that Jost Schieren (2012) stresses the "objective adaptation" (p. 65) of the process that begins with the

percept linked sensorily through perception to the object, rather than 'socially constructed' through interaction and communication with others. Maintaining this objectivity ensures a freedom in one's thinking; an aspect that Steiner was determined to foster through his indications for mathematics education. The individual can only be a fully participating member of society if she is able to pursue objectivity in her thinking, and mathematics education as indicated by Steiner will show her how to think in this way. As the student progresses through the curriculum, she develops the capacities for reflection and critical thinking, for rational thought and logic, for strategic and innovative thinking.

Chapter 7. Discussion: Tracking the Braids of the River

7.1. Introduction

7.2. Findings

7.3. Associated Influences

7.4. Concluding Observations

7.1. Introduction

My original intention was to describe - analyse - interpret in order to clarify the philosophical or ideological ground for the indications given for mathematics education in *The New Zealand Curriculum* (2007) and for Steiner's indications (1907-1924)⁸⁶ for mathematics education. Chapters 3 and 4: *Background: Setting the Scene*, and Chapters 5 and 6: *Results: Riding the Waters*, both provided description from literature; this chapter provides some analysis, while Chapter 8: *Conclusion: Journey's End* offers interpretation. Braided through this process has been the hermeneutic-bricolage methodology; reading, translating and transcribing within a philosophical approach that has continually met with a socio-political threshold in mathematics education in New Zealand.

I began my study knowing of confusion and concern surrounding the *Numeracy Development Projects*, which were introduced between the two hierarchic-level achievement-outcomes curriculum documents; *Mathematics in the New Zealand Curriculum* (1992) and *The New Zealand Curriculum - Mathematics and Statistics* (2007). These curriculum indications were the result of a commodification of mathematics education and knowledge arising out of the *Education Act 1989*, and aligned with the particularising of some levels of mathematics into unit and achievement standards during the 1990s. It appeared that numeracy had become the 'new' version of mathematics in schools for years 1 - 10 students, with the implementation of the *Numeracy Development Projects* (2001 to 2009) and the *National Standards in Numeracy* (2009).

⁸⁶ Stockmeyer (1982), has collected and published in *Rudolf Steiner's curriculum for Waldorf schools*, the indications Steiner gave in his many lectures, discussions with teachers and writings on education.

From personal teaching experience, I knew that the indications Rudolf Steiner offered for mathematics education are an ideal not easily realised in the classrooms of New Zealand Steiner Waldorf schools. Insufficient understanding of the underlying philosophy and lack of confidence in mathematical content knowledge often led the teacher toward formalising a 'recipe' into a didactic method. It appeared that the *Numeracy Development Projects* and Steiner's indications both required some inner ideological transformation of the teacher to align with the value system inherent in each approach and thus to enable the teaching/learning to occur as intended.

I had also gathered from life experience that mathematics is a language, that the learning of mathematics needs a learning community, and that mathematics is the one school subject that can show us how to think, whereas the other subjects also help us practise thinking. From my teaching and life experience I had become aware of a 'dumbing down' of the critical faculty in thinking towards a more dependent style.

I hoped that my study would either validate my concern about thinking, or show that I need not be concerned! I wondered if the political move to turn education into an economic commodity had a wider socio-political background and impact, and whether this economicising would appear in Steiner Waldorf schools in New Zealand. I formulated six sub-questions, which were presented above in the first chapter:
Introduction: Mapping the Journey.

I have learned that social constructivism, which is the philosophical basis of the *Numeracy Development Projects*, moves the child-individual through a social experience towards becoming an autonomous individual, whereas Steiner's indications would encourage the cosmic-social-child⁸⁷ towards a socially responsible individual. Both Steiner's indications and the commitment of the first half of *The New Zealand Curriculum* (2007) appear to have an egalitarian child-centred approach towards education; and both show a belief in the power of education to offer a futures-orientated renewal of society. I have found that a socio-political management

⁸⁷ My semantic way of expressing the manner in which a very young child feels 'at one' with the environment - physical, social and spiritual.

of education in New Zealand has had a cyclical difference in orientation towards mathematics education in the past, with that orientation difference now showing as a dissonance in the present curriculum. The ‘new’ language relating to the strategies and knowledge steps used in the *Numeracy Development Projects* is exclusive until specifically learned, and the language has been used to help frame the outcomes in the 2007 curriculum and in the mainstream *National Standards in Numeracy* (as well as in the Steiner Waldorf *Learning Steps in Numeracy*). I have also found that there is a ‘plateauing’ of learning and understanding in mathematics knowledge among those students exposed to Numeracy Projects’ approaches, and wonder if this ‘plateauing’ may affect thinking in the adult. Consequently, these findings fostered questions of the relationship of mathematical knowledge to power, especially when linked to the concept of a Knowledge Society. I am reminded of Marshall McLuhan’s statement: “the medium is the message” (1964/1987) and wonder at the message of the Numeracy Projects and *National Standards in Numeracy*.

7.2. Findings

I have set out the discussion of my findings according to the research questions that have guided me through this study. Each question acts as a ‘braid’ for the following analysis or translation towards understanding.

- What are the philosophical or ideological congruencies between *The New Zealand Curriculum - Mathematics and Statistics* (Ministry of Education, 2007) and the approach to mathematics education as indicated by Rudolf Steiner?

Both Steiner and the 2007 New Zealand curriculum appear to offer education as egalitarian and child-centred, and as an opportunity for social renewal. For Steiner, education could help all people, especially the proletariat, become fully participating citizens of the 20th century; educated to become life-long learners with an intimate connection to their whole environment (Hahn, 1958; Steiner, 1919/1994, 1920/1989). *The New Zealand Curriculum* (2007) records its “vision . . . for young

people (to) be confident, connected, actively involved, and lifelong learners” (p. 8). Whereas the egalitarian approach in the 2007 curriculum evolved from the Beeby-Fraser speech in 1939,⁸⁸ Steiner’s approach arose out of his own life experience and understanding (Steiner, 1928); and both approaches came from a progressive-socialist perspective. The educational indications of each are inclusive, offering a broad coherent curriculum and integration with the wider school community of student - teacher - parent - whanau/family (Ministry of Education, 2007; Munro, 1995; Steiner, 1919/1985). While these congruencies are not mathematics-specific, they act as an underlying foundation.

Arising from this foundation, the mathematics and statistics indications from the Ministry of Education and from Steiner are just that; indications only. The Ministry deems what it presents to be a “framework rather than a detailed plan” (Ministry of Education, 2007, p. 37). Steiner expected the curriculum to be developed according to an understanding of the needs of the child-becoming adult and stated, “the Waldorf School (in Stuttgart) will be a primary school in which the educational goals and curriculum are founded upon each teacher’s living insight into the nature of the *whole* human being” (1919/1985, p. 1, original italics). *The New Zealand Curriculum* (Ministry of Education, 2007) requires that a school community determine their school curriculum “in ways that best address the particular needs, interests, and circumstances of the school’s students and community” (p. 37). There is a proviso from the Ministry that such a school curriculum should demonstrate the principles, values and key competencies of the national curriculum “at all year levels” (2007, p. 37). Yet, a report on mathematics in schools from the Education Review Office in February 2013 expressed concern that schools did not always effectively develop their mathematics curriculum to meet the needs and strengths of their students. This report is further considered in relation to the effect of the *National Standards in Numeracy* below.

⁸⁸ “every person, whatever his level of academic ability, whether he be rich or poor, whether he live in town or country, has a right as a citizen to a free education of the kind for which he is best fitted, and to the fullest extent of his powers.” (Appendix to the Journal of the House of Representatives, 1939, E-1 2-3, as cited in McLaren, p. 28).

The work of the three psychologists, Piaget, Bruner and Vygotsky, which makes up the basis of social constructivism, shares similarities with Steiner's writings. Steiner and Piaget write of developmental phases of the child towards becoming an autonomous individual. In particular, both agree that the capacity for abstraction and critical thinking does not appear until the child is around 12 years of age. Piaget refers to this as the beginning of the '*formal operational stage*' when the capacities for abstract thinking, problem-solving, metacognition and hypothetico-deductive reasoning begin to show in the child (Piaget, 1969/1991). Steiner refers to a threshold through which the young person moves in their development around the age of 12, after which the capacities for abstract thinking and cause-effect deductive reasoning begin to be expressed more often (Steiner, 1924/1971). Bruner proposed a 'spiral curriculum', a teaching process of returning to the same subject, but with increasing content knowledge and skills development over a longer period of time, so that a deeper integration of understanding and knowledge acquisition occurs. This is the same practice that is part of the twelve-year curriculum indications from Steiner, for example, working with a 3-4-5 triangle in Class 3 out of the style of measurement used by the early Egyptians, then in Class 5 with the proof (Stockmeyer, 1982). While both processes share an evolutionary aspect, Bruner draws more from biological evolution (enactive with materials to imaging to language-based symbolisation), while Steiner draws more from cultural evolution (See above in this thesis: Chapter 4, pp. 39-45). Vygotsky's socio-cultural psychology posited that meaning is first socio-cultural then internalised through discursive practices. In mathematics, for example, inner speech is related to the processes of problem-solving, and young students may be heard to 'speak out' the process, considered a social activity by Vygotsky, before sub-vocalising and internalising the speech as they grow in autonomous individuality (Begg, 1999). Steiner posited that the young child exhibits an unconscious social 'at-one-ness' with the world, which progressively 'contracts' as the child-becoming adult awakens to increased consciousness and awareness, and matures into an autonomous individual, who is then able to independently direct their gaze socially out into the world (Mazzone, 2010). Although Steiner's writings and lectures bear similarities to certain underlying qualities of social constructivism, Steiner did not believe that knowledge could be built solely through social interaction. Steiner's

assertion of an enduring cosmic spirit-Ego within each human being is reflected in his portrayal of the spiritual dimension of thinking and of knowledge acquisition (Steiner, 1886/1978, 1894/1979).

In summary, the congruencies are a humanistic, child-centred approach to become a life-long learner, with a basis in progressive socialism and an orientation towards a renewal of society. Whereas some aspects of social constructivism are congruent, Steiner's particular emphasis on the enduring eternal spirit of the human being means that his picture of the conscious apprehension of knowledge includes spiritual input as well as social.

- What are the philosophical or ideological inconsistencies that are likely to affect the teaching and learning of mathematics in Steiner Waldorf schools in New Zealand?

Ernest (1991) defines ideology as “an overall, value-rich philosophy or world-view, a broad inter-locking system of ideas and beliefs” (p. 111) while François and Van Bendegem (2007) write in their ‘Prelude’ of an implicit as well as explicit philosophy or world-view. Those Steiner Waldorf schools, which became integrated through *The Private Schools Conditional Integration Act 1975*, found themselves employing teachers qualified to teach in New Zealand state schools, but not necessarily with the experience or understanding of Steiner's indications for mathematics education. Steiner expressed the following that teachers would strive to have: a thorough knowledge and understanding of human socio-cultural history; be capable of insights into human nature and the development of the child-becoming adult; be aware of and engaged in the socio-political issues of the time; and, cultivate an aesthetic relationship to the art of education (categorised by Kiersch and cited in Dahlin, 2010). And all this on top of the content knowledge required and the particular preferences for the teaching of mathematics! An ideal not easily realised that can also be unwittingly undermined by one's often unconscious, implicit or ‘hidden’ value system.

Consequently, there has arisen a confusion of goals within Steiner Waldorf schools. Each school's Board of Trustees and Charter must reflect the requirements of the *Education Act 1989* and its *Amendments*, and select teaching and assessment goals according to the priorities set in the National Education Guidelines, which now include specific *National Standards in Numeracy*.

There is also an inherent tension between a humanistic child-centred approach and an outcomes-based expectation for education to prepare an individual to be an economically worthwhile member of New Zealand society. As the integrated Steiner Waldorf schools have developed *Learning Steps in Numeracy* in place of the *National Standards in Numeracy*, and as all registered schools must acknowledge and follow the principles of *The New Zealand Curriculum* (Ministry of Education, 2007), the difference in approach between the underlying philosophies of each of these documents and that of Steiner's indications often lead to a conflict of intentions within the classroom. In December 2011, the New Zealand Qualifications Authority (NZQA) registered the *Steiner School Certificate* as a quality-assured national qualification with approved *Ad Eundem* status for level 3 by the Universities New Zealand Te Pokai Tara. It seems even more important that any inconsistencies between the two mathematical orientations are clearly defined, for both the teachers and for the 'teachers' of the teachers.

- What ideological effect might the Numeracy Projects have on Steiner Waldorf schools and mainstream schools in New Zealand?

The Numeracy Projects include a foundation in social constructivism and a metaphor of 'malaise' in the use of such terminology as 'Diagnostic Interview' and the programme of re-training for teachers to "help them become more effective teachers" (Ministry of Education, 2008a, pp. 2-4). The strategy-knowledge method of education is restrictive in its terms until learned by users (teachers, students,

parents), and thus presents as a 'Public Educator'⁸⁹ with a certain level of socio-educative control.

Knowledge becomes the content, the 'what'; strategy the process, the 'how'. The difference between the Numeracy Projects and the indications given by Steiner, apart from the emphasis on Number and apparent invisibility of Geometry in the Numeracy Projects, is not so much in **what** is taught, but in the **how**, the **when** and the **why**. Over two-thirds of the Steiner Waldorf schools in New Zealand had adopted the *Numeracy Development Projects*' programme, either in whole or in part. The Numeracy Projects encourage assessment with a 'Diagnostic Interview' at least twice-yearly, and have a bias towards number and strategy. These aspects are in conflict with Steiner's indications for reflection as an evaluative tool, for number and geometry (through 'form drawing') to be taught concurrently from the first class, and with less emphasis on strategy until a foundation of knowledge is developed. The recent Steiner Waldorf Federation Report on mathematics (December 2013) highlighted a 'plateauing' of knowledge acquisition during 2012-2013, in those students from year 6 onwards, similar to the 'plateauing' seen in mainstream schools, and there was a suggestion from a few teachers that the emphasis on strategy at the earlier class levels may have been a cause. Such a suggestion needs further research to clarify and validate. It is possible that this 'plateauing' of achievement arises from different perspectives on how children informally and in formal schooling, learn how to think. It is also possible that the PATmath tested mathematics achievement and not numeracy; the students could have achieved in numeracy at the expense of mathematics.

The emphasis on strategy in tandem with knowledge throughout the levels of the Numeracy Projects is related to the constructivist philosophy inherent in the programme. Not only is a new teaching method shared with teachers using the programme, but the style is also reflected in the learning process shared with the

⁸⁹ See Ernest, 1991, pp. 138-139 and for further definition, pp. 197-216. The Theory of Society is one of an inequitable hierarchy needing reform, with the Political Ideology of a Democratic Socialist. The view of mathematics is that of social constructivism.

students. Language and strategies are used to construct a knowledge of numeracy as mathematics. By comparison, Steiner's indications posit an awakening to consciousness and to the realm of thought, the student's own inner relationship to mathematical knowledge, building on that through repetition and memory before the independent choice of strategies. Scouller (2009) also noted the early emphasis on strategy and recommended a reconsideration of the balance of strategy and knowledge within the mathematics education as taught through Numeracy Projects' approaches.

The New Zealand Curriculum 2007 presents Mathematics and Statistics in three distinct strands: Number and Algebra; Geometry and Measurement; and Statistics, with the emphasis on the Number-Algebra strand up until Level 5 (years 8 - 11). The three years of pedagogical support by facilitators and mentors to teachers using the *Numeracy Development Projects* increased those teachers' confidence in their teaching of mathematics (Nicholas, 2006).

As the Numeracy Projects have been used in most New Zealand schools, they have become the 'curriculum by default' - the 'valid knowledge' - with the teaching method specified by the projects as the 'pedagogy' - valid transmission - and the *National Standards in Numeracy* since 2010 as the means of evaluation - the valid realisation.⁹⁰ The representation of mathematics as numeracy has effectively increased the status of number within mathematics and thus, the importance of generalised abstract calculation over mentally picturing towards a universal form. Both New Zealand mainstream schools and Steiner Waldorf schools show the effect of these influences of the Numeracy Projects. Additionally, Steiner Waldorf schools appear to be compromising the slower progression of intellectual awareness proposed by Steiner in his indications for mathematics education.

⁹⁰ See Bernstein (1971) quoted above in this thesis on page 54.

- What ideological effect might the *National Standards in Numeracy* (2009) have on the teaching/learning process in mainstream New Zealand schools?

The introduction of *National Standards in Numeracy* has moved the emphasis in mathematics education from the teaching to the evaluation. The Education Review Office (ERO) Report, *Mathematics in Years 4 to 8: Developing a Responsive Curriculum* (February, 2013), made reference to the 2007 New Zealand curriculum document stating, “ERO found variation in the extent to which schools effectively designed and reviewed their mathematics curriculum to respond to the strengths and needs of all students” (p. 6). *The New Zealand Curriculum* (2007) offers indications for mathematics education with a requirement for the Boards of Trustees “(w)hen designing and reviewing their curriculum, . . . (to) select achievement objectives . . . in response to the identified interests and learning needs of their students” (p. 44). The *National Standards in Numeracy* (2009) may be intended to provide criteria against which a student’s achievement can be measured, but the expectation within each year level, which may be in conflict with the needs of the student, produces a dissonance with the design aims of *The New Zealand Curriculum* (2007). The National Standards from 2009/2010 are presented as outcomes to be met or surpassed by 85% of students (Thrupp & White, 2013) and now presented as a regular and regulating evaluative tool, quickly becoming a ‘teaching tool’. The freedom to choose a school curriculum according to the needs and interests of the students appears to be subsumed by the exhortation to meet the standard. Evaluation now takes precedence over education.

- What impact might the *Learning Steps in Numeracy* (2013) have on the teaching/learning process in Steiner Waldorf schools in New Zealand?

Those Steiner Waldorf schools in New Zealand that are integrated for state funding through *The Private Schools Conditional Integration Act 1975* have, through recent negotiation with the Ministry of Education, developed *Learning Steps in Numeracy* (2013) to use in place of the National Standards in Numeracy.⁹¹ These *Learning Steps*

⁹¹ See page 72 of this thesis, and Footnote 85 on page 72, for background to this development.

are closely aligned to the National Standards, even using some of the Numeracy Projects' strategy and knowledge terminology. This brings into question the impact of the *Learning Steps* onto the Special Character implicit within the legal contract of integration that the schools have with the Ministry of Education. While those schools not integrated do not have to comply with the requirements to use the *Learning Steps* as an evaluative tool, the use by over half of the Steiner Waldorf schools in New Zealand, constrains their capacity to maintain their integrity with Steiner's indications, and their Special Character.

As the *National Standards in Numeracy* have impacted on the design of the school curriculum in New Zealand's mainstream schools, the *Learning Steps in Numeracy* have impacted on the design of the curriculum in Steiner Waldorf schools in New Zealand. In many instances, as gathered in conversations at a Steiner Waldorf conference in July 2014, teachers appear to be using the *Learning Steps* as a goal for their teaching rather than as an evaluative tool.

- What effects might the philosophy/ideology of *The New Zealand Curriculum - Mathematics and Statistics* (Ministry of Education, 2007) and the approach to mathematics education as indicated by Rudolf Steiner, have on thinking?

Mathematics has traditionally been linked to thinking and, as Numeracy, is now an important *language of knowledge* in years 1 - 8 in New Zealand schools. Knowledge is also now considered more of a 'verb' than a 'noun'; a process of relating, or a form of energy, rather than a fact from an expert (Gilbert, 2005). In that sense, knowledge has become a commodity in this 21st century. Steiner (1886/1972) recognised the activity within knowing-knowledge: "By means of our thinking, we lift ourselves from perceiving reality (as equivalent to 'knowledge') as product to perceiving it as that which produces" (p. 72). Bortoft (2012) describes the process as moving upstream in one's thinking to that place/moment before the conscious 'appearing', in order to become more awake to the creativity possible. The activity of thinking that Steiner (1886/1978, 1894/1979) described and as is also referred to in Figures 8-12 on pages 75-77 and 79-80 of this thesis, has become constrained towards algebraic thinking,

which is supposed to encourage a ‘mathematical thinking’ for New Zealand’s ‘Knowledge Society’.

Mathematics, as the subject that teaches us how to think - creatively, critically, strategically and logically (Ministry of Education, 2007) - has been captured by Numeracy for almost 10 years of schooling, with an encouragement towards an algebraic thinking that is abstract and generalised, rather than universal and creative. There has recently been reference to algebraic thinking as developing even before mathematical thinking, and definitely before formal arithmetic is taught. Britt and Irwin (2011) studied 4 - 7 year-olds in New Zealand and found they expressed the capacity to generalise. Vale and Radford (cited in Britt & Irwin, 2011) suggest there are three stages in algebraic thinking:

1. recursive or arithmetic generalising, where the young student sees and expresses repeated addition (e.g. $2 + 2 + 2 + 2 + 2$ or $5 + 5 = 5 \times 2 = 10$);
2. quasi-generalisation, where students replace the general rule with particular numbers (e.g. $3 + 5 = 5 + 3$ and $2 + 4 + 0 = 4 + 2$); and,
3. explicit or algebraic generalisation, where students express the general rule with language or symbols (e.g. $x [2 + 4] = 0$ and $5x = 10$).

But Hunter (2014), in a study of 9-11 year-old New Zealand students, noted that specifically designed tasks, using specific materials and representations, along with a constant urging for justification and generalisation, were all required to support students to link their numerical understandings to algebraic reasoning. While Britt and Irwin (2011) may consider algebraic thinking to be present early in the pre-school child, experiences with older children during formal schooling would suggest that making this thinking conscious is not easy. Recursive-generalisation may show in pre-school children, and a grouping or ordering similar to quasi-generalisation may also show in children before formal learning, yet Lakoff and Núñez (2000) would argue that to be an expression of the child’s capacity for numerosity.

Algebraic thinking is a term now commonly used for any capacity to generalise within mathematics, “as a way or practice of using and interpreting signs” (Britt & Irwin, 2011; Dörfler, 2008, p. 143). Mathematical thinking is also involved in generalisations,

where the activity of generalising in thinking is towards an 'archetype' or 'universal'. In comparison to the way in which the term 'algebraic thinking' is used, mathematical thinking actively brings to expression and to a cognitive awareness a lawful or universal form of the context of the concept (Schieren, 2012). Thus, mathematics, as an activity, exemplifies a lawfulness inherent within the foundational structure of the concept. 'Explicit algebraic thinking' works in the realm of generalisations of representations, maintaining a dualistic Self-Other quality and may not have the self-participated experience of the foundational structural progression of percept to concept to recollect.

Thinking begins with language and communication. Meaney (2006) writes of her mathematics classroom research, where she saw that similar skills are required as in learning a language. Very young children, beginning to speak their own 'mother tongue', need to relate a word-concept to an objective part (that is, outside of their self or another) of the sense-experienced world. We call it 'naming', and the children thus build a vocabulary of concept-representations according to their relationship with their environment. Mathematics also provides us with such concepts of the non-sense (sense-free) world. I use 'non-sense' advisedly here, because of the way in which we play with words in nonsense rhymes and riddles, thus building a feeling connection to language. Children love the tension in riddles; the knowing and not-knowing of the punch-line. We can do the same thing in mathematics; playing with 'magic squares' and other number games to build a feeling connection to mathematics. If numeracy is fluency with number and calculation then perhaps the mathematical understanding, the 'thinking about thinking' or metacognitive ability is not easily gained until the student is around 11 years old, a threshold-phase noted by both Steiner and Piaget.

7.3. Associated Influences

Although the Vision and preliminary pages of the 2007 curriculum document (Ministry of Education, 2007) suggest a humanistic, even holistic approach to education, the language used could be termed 'paternalistic'. Throughout the pages

referring to the Vision, Principles, Values, Key Competencies, Effective Pedagogy, and the School Curriculum (Design and Review), the word ‘students’ and its equivalent (‘young people’ and ‘child[ren]’) appear almost four times more often than the words ‘teacher(s)’ and ‘schools’. Not so surprising in a document about students and learning, but it is the way in which the words are used that belies the apparently inclusive holistic language. ‘Teachers’ is often used with ‘effective’ or a similar qualifying adjective; and ‘students learn best when’ occurs as often as ‘successful learners’, both phrases in relation to teachers or teaching. This language style plus a sparse reference to the community as ‘community’, ‘parent’, ‘family’, ‘whanau’ implies a paternalistic attitude, which was recognised by the Picot Taskforce as “a culture of dependence” (1988, p. 25). Roger Kerr (1991) rephrased this observation stating “the (Picot) report made the point that . . . the administration system remains at best paternalistic”. These comments were aimed at the previous Department of Education, but it appears that similar sentiments are seen in this Ministry of Education document. Nandy (cited in Dahlin, 2013) suggests that such paternalism is linked to colonisation by Britain during the 19th century.

Thales (c. 624 -c. 546 BCE) is reputed to have said, “I know that I know” (Kline, 1990). Vico (1688 - 1744) is reported to have stated, “the human mind can only know what the human mind has made”. Constructivism appears to develop this sentiment still further by suggesting that knowledge (or the understanding of facts and phenomena) is built through social interaction on previous knowledge. Constructivism seems concerned only with cognitive knowing; it does not explain that sense of ‘knowing that I know’ which is subjective rather than objective.⁹² The reliability and validity of prior knowledge will thus affect the integration of new knowledge, as does feeling and the role of emotions. Communication is central; asking students to explain their thinking, and using questions or explanations from other students to help progress students in their thinking (Hunter, 2006). In mathematics the focus is on number, which in itself is symbolic, a different language. This keeps the activity within the head-thinking realm instead of allowing an interplay between the thinking and the feeling. Kahneman (2012) wrote of the way the ‘faster’ intuitive type of thinking was

⁹² For further discussion and clarification, see Begg, 1999.

nourished and fostered by story, by contextualising, so that the feelings were also involved in the ‘quick remembering’.

7.4. Concluding Observations

The ‘Public Educator’ knows best. Such sentiment prevails in the *National Standards in Numeracy*, and in the programme and methodology of the Numeracy Projects. It derives from the initial impulse for a ‘free, secular and compulsory schooling’, which nevertheless was a political move in 1877 to rationalise the various styles of education within a newly-integrated colony. The initial broad curricula were in imitation of the more inclusive Scottish model⁹³ but knowledge teaching and learning was ordered into levels called ‘standards’; an expression of a political agenda towards ‘normalising’ the populace for a society requiring individuals who could help increase its wealth. The ideological emphasis in education arose out of the paternalistic economic liberalism of the 19th century and was succeeded by a period of progressive socialism during the 1930s to 1980s. From the late 1980s a quantifiable economic agenda was integrated with education, now an expression of neoliberalism, and included a form of educative and social control through audit. Mitchell (1999) describes neoliberalism as a tension between political control and pastoral protection.

Education now appears to be economicised socio-political management, rather than being supported by the economic realm and protected by the political realm, as Humboldt suggested (cited in Dahlin, 2010). Mathematics education, as expressed by its evaluation through the National Standards, fosters this form of management. Instead of encouraging freedom in thinking in a self-determined individual (autonomous and ethical), it would seem to encourage a dependence on the Other

⁹³ Otago, with its Presbyterian settler heritage, had imitated the Scottish style of education (schoolmasters with university training and a salary, and a classical curriculum), which although not free, was available through to tertiary level. Otago University was the first, opened 1869, in New Zealand. By 1876, Otago was the only province to have a fully-developed primary to tertiary level of schooling. The first two Directors-General of Education adopted the Scottish-style broad and classical curriculum (Campbell, 1941, pp. 33-35, 41-42).

with a consequent lack of development of one's own sense of judgement and discernment.

Theories of child development have moved from the developmental psychology of the late 19th and early 20th centuries to the cognitive psychology of the mid-late 20th century. The emphasis is now on the development of thinking and knowledge rather than on the whole child, yet there is still a perception that any such development will be linear, progressive and through discrete stages or phases.

Knowledge acquisition and thinking are now measured against the achievement of standards by level. In mathematics, the standards are biased towards number and calculation. Thinking out of mathematics has been diverted towards an abstract, generalised algebraic thinking with a consequential loss of an awakening to the innovative importance of archetypal, intuitive, spiritual thinking.

Chapter 8. Conclusion: Journey's End

8.1. Introduction

8.2. Implications

8.3. Reflections

8.4. Suggestions for Further Research

8.5. Final Remarks

8.1. Introduction

This concluding chapter offers some interpretation of what I have found in the landscape of educational literature in relation to the main question that has directed my review of these chosen channels of mathematics education in New Zealand: mathematics or numeracy? I began my research following concern about the inter-relationship of the *Numeracy Development Projects* and mathematics education. I now conclude with a clearer insight into the role of each within their ideological / philosophical frameworks in New Zealand schools considered 'mainstream', and in those known as Steiner Waldorf schools. Since 2010 *National Standards in Numeracy* have also contributed to mathematics education in New Zealand and have necessarily been considered in this research.

As the braided river flows from mountain to sea, its movement scours and carries the underlying substrate, eventually transforming the landscape. (See Figure 13 on page 100 below.) The underlying ideologies of the forms of mathematics and numeracy education revealed in this thesis require some inner transformation within those in the classroom, teacher and student; the learning community. We tend to call such a transformation in the pupil, learning, while such a transformation by the adult must be accompanied by a self-awareness of one's often-unconscious values and world-view. Elkind (2004) reminded us of the need for congruent readiness in teacher, curriculum and society in order that constructivism could be thoroughly developed; such congruence is also required whenever a paradigm change in personal values and world-view is to occur. Working with the Numeracy Projects or with Steiner's

indications depends upon an inner transformation; a redistribution of one's inner educational landscape to accommodate the paradigm changes asked for.



Figure 13: Braided river over Canterbury Plains to the Pacific Ocean

Adrian Meredith (n.d.). Retrieved from <http://www.waterways.ac.nz>. Reprinted with permission.

Dietz (2013) cites Bell (1996) who described the post-industrial society as a “society of knowledge”, not only through increased research and development, but also through the way in which “society . . . attaches more and more importance to the realm of knowledge” (p. 53). Gilbert (2005) also referred to the ‘Knowledge Society’ and the way in which the value of mathematics as knowledge has been modified. Mathematics as knowledge has now been captured by numeracy through the introduction of *National Standards in Numeracy* for years 1 - 8. A close look at these ‘standards’ shows a stronger affiliation to calculation than to thinking, while the language used links them to the strategy-knowledge method of the Numeracy Projects. An abundance of strategies might help us find our way through the ‘waters’ of number, yet will not necessarily reveal to us the bigger picture of the river of mathematics within the landscape of education. The Numeracy Projects and National Standards seem to have arisen out of a metaphor of malaise; teachers and students needed evaluation and diagnosis. The ‘cure’ has become not education in mathematics but a skills training in numeracy.

8.2. Implications

In 1989 New Zealand state education changed from a Department of Education with regional offices and opportunity for a teacher-led confluence of bottom-up with top-down, to a Ministry of Education answerable to a Minister of the Crown; top-down with 'advisors' and little opportunity for a 'bottom-up' sharing of ideas. The indications for mathematics education in the curriculum documents of 1992 and 2007 (Ministry of Education) gave a framework upon which schools could formulate their own curriculum according to the needs and nature of their students and school community. Yet the Education Review Office report of February 2013 disclosed the incapacity of a number of schools to manage this formulation. As the *Numeracy Development Projects* appeared, 'take-up' by schools was voluntary although there was much exhortation and positive 'spin' attached. Financial support to schools for their teachers' professional development and pedagogical support by facilitators and on-site mentors drew many into the programme. Resources became the new 'texts', which, in turn, prepared younger school students for the fragmentation of mathematics into unit and achievement standards at higher school levels. Legislation for National Standards introduced political coercion, and showed the progressive movement towards political control that mathematics education has undergone over the last two decades in New Zealand.

Steiner Waldorf schools in New Zealand have not been immune to this progressive control. While each registered school must allow itself to be guided by the principles of New Zealand Ministry of Education curricula documents, the integrated schools must also meet the educational priorities within the National Education Guidelines set by the Minister of Education. The relationship of political coercion to economic control was clearly displayed by the Minister in her threat to de-integrate the schools⁹⁴, when they appealed to their Special Character not to annually test their students through years 1 - 7. Such political and economic control is at direct odds to Steiner's representation of the 'body social' (See chapter 4, p. 47, in this thesis). In lectures and writing (1919/1994, 1920/1972) Steiner explained why the economic

⁹⁴ See Footnote 85 on page 72.

sphere and the political sphere should not overwhelm the cultural sphere within which education exists. It was during the 1850s that Wilhelm von Humboldt's educational writings were published, in which he stated that the cultural sphere should be supported by the economic realm and protected by the politico-judicial realm. Unfortunately, in this 21st century, 'protect' has veered towards 'controls' while 'support' has become 'determines'. See Figure 14 below.

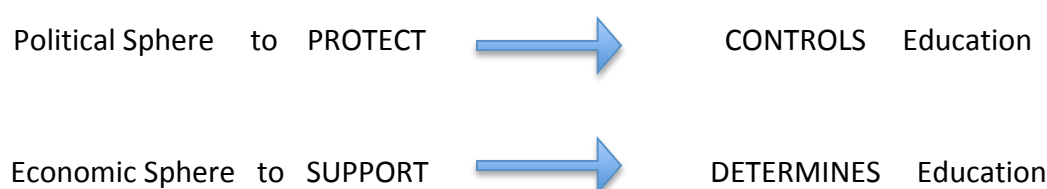


Figure 14: Progression of political and economic influence over education

The broader relationship of education to the economic welfare of the state has a cyclical history in New Zealand. The need for a numerate society within a fledgling colony led first to schools affiliated to particular churches (see Footnote 18, Chapter 3, p. 21 above in this thesis), then to a nationalised secular and compulsory system alongside church-sponsored and private schools. As the period of compulsory schooling has increased from the initial 6 years to 8 years (1940s) and now to 10 years, so have the expectations of the state government that financially provides this secular, compulsory schooling. Policies and audits now frame an accountability to the public purse, with politicians and business leaders having imposed Market Value onto education.

8.3. Reflections

Beeby and Fraser established an egalitarian and humanistic focus within education during the late 1930s. Ernest (1991) would frame this approach as 'Progressive Educator' with a hint of the 'Old Humanist' and a political ideology of 'conservative/liberal'. When, during the 1980s and 1990s, the Treasury and

Business Round Table in New Zealand endorsed Market Value onto education, the political ideology tended towards the 'democratic socialist' with 'meritocratic and conservative' leanings, combining the 'Public Educator' with the 'Technological Pragmatist' (Ernest, 1991). Mutch (2001) also noted the tension between the 'new right' and the 'liberal left' during the 1990s, with a consequent shift towards conservatism now expressed most clearly in the National Standards. There is no clear-cut ideological or philosophical framework for mathematics education in New Zealand, but rather a 'rigorous eclecticism' as Kelly stated (cited in Clark, 2004). Furthermore, it could be reasoned that the majority of the Steiner Waldorf schools in New Zealand have also taken up a 'rigorous eclecticism' by adopting some or all of the Numeracy Projects, and having to evaluate achievement in mathematics with their *Learning Steps in Numeracy*. Unfortunately, such an approach marginalises the indications and philosophical foundation that Steiner offered for mathematics education. This necessarily leads to a question of integrity if the *Steiner School Certificate* is awarded as a school-leaving certificate to students who have not had the opportunity for a full curriculum based on Steiner's indications.

So-called 'traditional' education, where the individual is socialised, is classified by Dietz (2013) as a conditioning towards understanding. He suggests that the individual must now set their own 'purpose' and "find their orientation within themselves" (2013, p. 52). In the process of becoming an autonomous individual, "uncertainty, disorientation and . . . self-alienation" (Dietz, 2013, p. 52) are faced. These are the existential consequences of a separation between one's self and the world, as represented diagrammatically in Figure 12, chapter 6, page 80 above in this thesis. The Cartesian statement "I think, therefore I am" defined as illusion that which the I cannot think. Later, Kant posited that the representations, which our inner senses perceive, are merely inner constructs that he referred to as 'the thing in itself' (Amrine, 2012); that our consciousness "exists in an autonomous mindscape, called reality" (Schieren, 2010, p. 6). These beliefs lead to the constructivist definition: the human mind can only know what the human mind has made (attributed to Vico [1668 - 1744]; see Begg, 1999).

Thinking is much more than cognitive development, and mathematics is much more than numeracy. See Figures 15 and 16, below, which encourage us to ‘think’ more than we can ‘see’!



Figure 15: Rabbit or duck?



Figure 16: Where is the dog?

We can see the non-sensory factor of cognitive perception in these wonderful figures where the ‘rabbit’ switches to ‘duck’, and the dark and light patches are ‘seen’ to eventually resemble a ‘dog’.⁹⁵ We apply meaning-making to a random sensory experience, because we want to make meaning of the world in which we live. This

⁹⁵ For further discussion on this topic see Code, 2011, p. 15.

meaning-making is a function of thinking and is adapted by and adaptive to our experiences. In 1986, Bortoft wrote *Goethe's Scientific Consciousness* in a pamphlet (#22) for the Institute for Cultural Research. According to Bortoft, re-presented in Figure 17 below, Goethe differentiated between the abstraction of the particular to the General, and the perception of the Universal reflected in the particular. We experience this differentiation between algebraic thinking which abstracts the particular to the General, and mathematical thinking which perceives the Universal reflected in the particular. This leads to the Universal being conceived as a Unity within the intuitive thinking realm, and the General being conceived as a Unity within the intellectual rational mind.

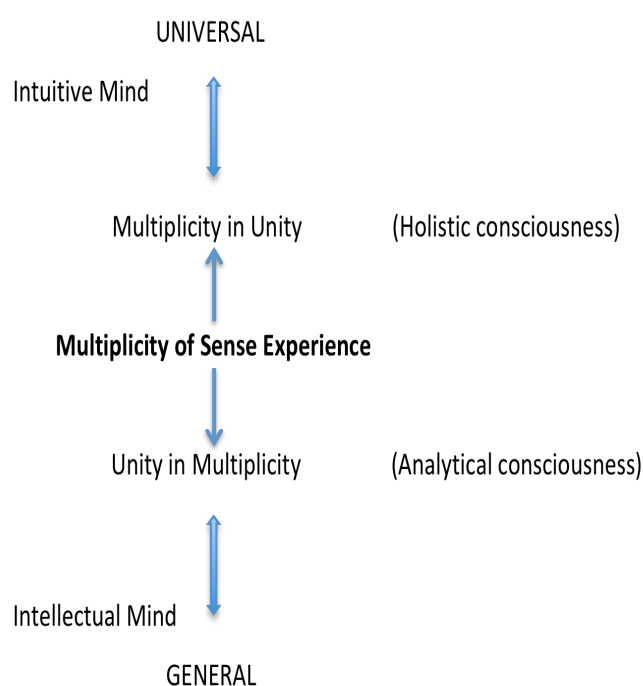


Figure 17: The 'Universal' and 'General' in relation to sense experience

8.4. Suggestions for Further Research

This literature-based thesis needs follow-up research with students in classrooms; preferably a longitudinal study of the relationship between a focus on numeracy in mathematics education, and the students' cognitive abilities. Shorter studies could consider which cognitive capacities are encouraged by a mathematics education

framed by the *National Standards in Numeracy* or the *Learning Steps in Numeracy*. As the *Learning Steps in Numeracy* are under review this year, a comparative study in Steiner Waldorf schools could be made of test results from 2012-2014, and from 2015-2017.

Future research could also examine such socio-cultural factors as the language used in the curricula and numeracy documents, in relation to the perceived paternalism, and consequent teaching styles and learning achievement, especially among different ethnic groups. A paternalistic language style, accompanied by the combination of prescriptive standards and a strategy-knowledge method could pre-define one's approach to mathematics education and learning, "forestalling something genuinely new from taking place" (Dietz, 2013, p. 53). What impact might this have on the hoped-for qualities of thinking outlined in *The New Zealand Curriculum* (2007); creative, critical, strategic and logical?

There is also a possible relationship between the early push towards algebraic thinking (involving the abstraction of the particular into the General) and a sense of isolation, as one's sense of Self (mind) is continually separated from one's sense of the World. Or one could research the sense of Self in those who have experienced the self-participatory exercising of their will in combining the concept to percepts in order to form their Representation of Reality (see Chapter 6 of this thesis, and Schieren, 2012).

8.5. Final Remarks

The *International Commission on Mathematical Instruction* reported on their 10th Study: the role of the history of mathematics in the teaching and learning of mathematics, in Bulletin #58 (2006). Of note is the recognition that the history of mathematics covers a number of discrete cultures leading to a more holistic and humanistic mathematics education (Furinghetti, 2006). Steiner's indications and his inclusion of the assimilation of cultures other than that of the 'modern western',

encourage this holistic and humanistic approach, which recognises the universal and cosmopolitan human being who has a sense of social responsibility to the world.

This manuscript provides an historical approach towards understanding the present relationship of mathematics to numeracy within New Zealand's 21st century mathematics education. The landscape of education supports mathematics and its strands - Number and Algebra, Geometry and Measurement, and Statistics - like the channels of a braided river. From their philosophical source of 5th - 3rd centuries BCE, when mathematics stood as a pinnacle of learning and knowledge, these strands have trickled and roared their ways around and over islands of doubt, heresy, and fear. Along the way, the sedimentary impacts of cultures and language, of ideologies and policies have impeded the flow. Eventually, the waters, freed of their burden, merge into a sea of interconnectedness, like the thinking that spans the physical and spiritual domains.

While we struggle with strategies to find a level of understanding we lose sight of the active principle in knowledge that links the thinker with the 'object' that initially encouraged the thinking; that arena of thought among the veils of consciousness and unconsciousness that links the capacity to perceive with what is perceived. The understanding can become a wall of answers that hides further questioning unless interest and curiosity prompt us to peep over and look beyond. More knowledge and strategies may only serve to confuse. In the past, a logical sequencing process materialised the path from perceiver to perceived. In this 21st century we now need to practise looking beyond both these points along the path; to consciously reconnect with that universal unity that is the precursor for true creativity, beyond the strategic, the critical and the logical.

REFERENCES

- Amrine, F. (2012, Autumn/Winter). The philosophical roots of Waldorf education, Part one: The revolution. *Research Bulletin*, XVII(2), 28-36.
- Amrine, F. (2013, Autumn/Winter). The philosophical roots of Waldorf education. Part three: From Schiller to Steiner. *Research Bulletin*, XVIII(2), 7-16.
- Amrine, F. (2014, Spring/Summer). The philosophical roots of Waldorf education. Part four: Rudolf Steiner as a philosopher. *Research Bulletin*, XIX(1), 7-18.
- Augustine, (2001). *The confessions* (P. Burton, Trans.). New York, NY: Alfred A. Knopf, Everyman's Library. (Original work composed 397CE).
- Barton, B. (1993). *Contemplating cultural constructs*. Retrieved from http://www.merga.net.au/documents/Keynote_Barton_1993.pdf
- Barton, B. (1995). The politics of mathematics education. In J. Neyland (Ed.), *Mathematics education: A handbook for teachers Vol. 2* (pp. 165-174). Wellington, New Zealand: Wellington College of Education.
- Begg, A. (1999). *Learning theories and mathematics: A, B, C, D, and E*. Paper presented at the NZAMT-6 Conference, 29 June - 2 July 1999, Dunedin, New Zealand.
- Begg, A. (2006). Does numeracy = mathematics? In *SET: Research Information for Teachers* (3), 21-22.
- Bernstein, B. (1971). On the classification and framing of educational knowledge. In M. Young (Ed.), *Knowledge and control: New directions for the sociology of education* (pp. 47-69). London, UK: Collier-Macmillan.
- Bishop, A. (1990). Western mathematics: the secret weapon of cultural imperialism. *Race Class*, 32: 51 DOI: 10.1177/030639689003200204
- Bishop, A. (1995). Western mathematics: The secret weapon of cultural imperialism. In B. Ashcroft, G. Griffiths & H. Tiffin (Eds.). *The post-colonial studies reader* (pp. 71-76). London, UK: Routledge.
- Boethius, (1999). *The consolation of philosophy* (P. G. Walsh, Trans.). New York, NY: Oxford University Press. (Original work written about 524CE).
- Bobis, J., Mulligan, J., & Lowrie, T. (Eds.), (2004). *Mathematics for children: Challenging children to think mathematically* (2nd ed.). Sydney, Australia: Pearson Education.
- Bortoft, H. (1986). Goethe's scientific consciousness. *Pamphlet #22*. Tunbridge Wells, UK: Institute for Cultural Research.
- Bortoft, H. (2012). *Taking appearance seriously: The dynamic way of seeing in Goethe and European thought*. Edinburgh, UK: Floris Books.
- Bowen, J. (1972). *A history of western education, Volume one: The ancient world*. London, UK: Methuen & Co.
- Bowen, J. (1981). *A history of western education, Volume three: The modern west*. London, UK: Methuen & Co.
- Britt, M. (2008, February). Getting maths right. *North and South*, p. 18. Retrieved March 16, 2008, from <http://www.pmca.co.nz>
- Britt, M., & Irwin, K. (2011). Algebraic thinking with and without algebraic representation: A pathway for learning. In J. Cai & C. Knuth (Eds.). *Early Algebraization: Advances in Mathematics Education*. Springer. DOI: 10.1007/978-3-642-17735-4_10

- Burkhard, G. (1997). *Taking charge: Your life patterns and their meaning*. (C. von Arnim, Trans.). Edinburgh, UK: Floris Books (Original work published "Das Leben in die Hand nehmen", 1992).
- Campbell, A. E. (1941), *Educating New Zealand*. Wellington, New Zealand: Department of Internal Affairs.
- Caygill, R., Sturrock, F., & Chamberlain, M. (2007). *Mathematics and science achievement in New Zealand: Tracking the changes of year 5 students in TIMSS 1994-2002*. Wellington, New Zealand: Ministry of Education.
- Clark, J. (2004). Rigorous eclecticism: The ministry of education's bizarre philosophy of the curriculum. In A-M. O'Neill, J. Clark, & R. Openshaw (Eds.), *Reshaping culture, knowledge and learning* (pp. 127-140). Palmerston North, New Zealand: Dunmore Press.
- Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23(7), 13-20.
- Codd, J. (1999). Educational reform, accountability and the culture of distrust. *New Zealand Journal of Educational Studies*, 34(1), 45-53.
- Code, J. (2011, July). The art of knowing: Epistemological implications for a schooling of the imagination. *RoSE - Research on Steiner Education*, 2(1), 12-23. Hosted at www.rosejournal.com.
- Committee appointed by the Minister of Education in November 1942 (W. Thomas, Chair). (1959). *The post-primary school curriculum (The Thomas Report)*. Wellington, NZ: New Zealand Department of Education.
- Cox, J. (1980). Mathematics in New Zealand primary schools. In D. K. Ramsay (Ed.). *Curriculum issues in New Zealand* (pp. 97-109). Wellington, New Zealand: New Zealand Educational Institute.
- Cumming, I., & Cumming, A. (1978). *History of state education in New Zealand 1840-1975*. Wellington, New Zealand: Pitman Publishing NZ Ltd.
- Dahlin, B. (2010, January). Steiner Waldorf education, social three-folding and civil society: Education as cultural power. *RoSE - Research on Steiner Education*, 1(1), 49-59. Hosted at www.rosejournal.com.
- Dahlin, B. (2013, July). Poetizing our unknown childhood: meeting the challenge of social constructivism. *RoSE - Research on Steiner Education*, 4(1), 16-47. Hosted at www.rosejournal.com.
- Dahlin, B. (2013, December). Rezensionen / Book reviews. *RoSE - Research on Steiner Education*, 4(2), 146-150. Hosted at www.rosejournal.com.
- da Veiga, M. (2013, March). Rudolf Steiner's anthroposophy - spiritual path or science? *RoSE - Research on Steiner Education*, 3(1), 126-131. Hosted at www.rosejournal.com.
- Davis, A. (1991). Piaget, teachers and education: Into the 1990s. In P. Light, S. Sheldon & M. Woodhead (Eds.), *Learning to Think*, pp. 16-31 [Child Development in Social Context 2]. London, UK: Routledge, in association with The Open University.
- Dengate, B. (1998). Constructivism and the mathematics classroom. In N. F. Ellerton (Ed.), *Issues in mathematics education: A contemporary perspective* (pp. 199-213). Perth, Australia: Edith Cowan University.

- Denzin, N., & Lincoln, Y. (2011). Introduction: The discipline and practice of qualitative research. In N. Denzin & Y. Lincoln (Eds.). *The Sage handbook of qualitative research* (4th ed.), (pp. 1-19). Thousand Oaks, CA: Sage Publications, Inc.
- Dietz, K.-M. (2013, December). Education as a way of life. *RoSE - Research on Steiner Education*, 4(2), 52-57. Hosted at www.rosejournal.com.
- Dörfler, W. (2008). En route from patterns to algebra: Comments and reflections. *ZDM Mathematics Education*, 40, 143-160. DOI: 10.1007/s11858-007-0071-y
- Easton, S. (1980). Rudolf Steiner: Herald of a new epoch. Hudson, NY: Anthroposophic Press.
- Education Act 1989 (with amendments). Wellington, New Zealand: Government Printer. Retrieved May 28, 2013, from <http://www.legislation.govt.nz/act/public/1989/0080/latest/DLM175959.html>
- Education Review Office, (2013). Mathematics in years 4 to 8: Developing a responsive curriculum. Retrieved from <http://ero.govt.nz/index.php/National-Reports/Mathematics-in-Years-4-to-8-Developing-a-Responsive-Curriculum-February-2013>
- Elkind, D. (2004). The problem with constructivism. *The Educational Forum*, Summer 2004. Retrieved from <http://search.proquest.com/docview/220663785?accountid=8440>
- Ernest, P. (1991). *The philosophy of mathematics education*. Oxford, UK: RoutledgeFalmer.
- Federation of Rudolf Steiner Waldorf Schools in New Zealand. (2013). *Learning Steps: Literacy and mathematics*.
- Federation of New Zealand Steiner Waldorf Schools (2013, December). *Report on Literacy and Mathematics*. Unpublished.
- François, K., & Van Bendegem, J. P. (Eds.). (2007). *Philosophical dimensions in mathematics education*. New York, NY: Springer.
- Fraser, B. (Ed.), (1986), *The New Zealand book of events*. Auckland, New Zealand: Reed Methuen Publishers Ltd.
- Furinghetti, F. (2006). The role of the history of mathematics in the teaching and learning of mathematics: A report on the 10th ICMI study. *ICMI Bulletin*, 58, 38-45. Retrieved from http://www.mathunion.org/fileadmin/ICMI/files/Publications/ICMI_bulletin/58.pdf
- Gadamer, H.-G. (1986). *The idea of the good in Platonic-Aristotelian philosophy* (P. C. Smith, Trans.). New Haven, CT: Yale University Press. (Original work published "Die Idee des Guten zwischen Plato und Aristoteles", 1978)
- Garden, R. A. (Ed.), (1997). *Mathematics and science performance in middle primary school: Results from New Zealand's participation in the Third International Mathematics and Science study*. Wellington, New Zealand: Research and International Section, Ministry of Education.
- Gidley, J. (2008). *Evolving education: A postformal-integral-planetary gaze at the evolution of consciousness and the educational imperatives* (Unpublished PhD dissertation), Southern Cross University, East Lismore, Australia.
- Gidley, J. (2010, December). Holistic education and visions of rehumanized futures. *RoSE - Research on Steiner Education*, 1(2), 139-147. Hosted at www.rosejournal.com.

- Gilbert, J. (2005). *Catching the knowledge wave? The knowledge society and the future of education*. Wellington, NZ: NZCER Press.
- Gilbert, J. (2005a). Catching the knowledge wave? 'Knowledge society' and the future of public education. In J. Codd & K. Sullivan (Eds.), *Education policy directions in Aotearoa New Zealand* (pp. 53-70). Victoria, Australia: Thomson/Dunmore Press.
- Gordon, L. (1992). The state, devolution and educational reform in New Zealand. *Journal of Educational Policy*, 7(2), 187-203.
- Gregersen, E. (Ed.). (2011). *The Britannica guide to the history of mathematics*. New York, NY. Retrieved from <http://www.aut.ebib.com.au>
- Hahn, H. (1958). Birth of the Waldorf school from the threefold social movement. In A. Freeman & C. Waterman (Eds.), *Rudolf Steiner: Recollections by some of his pupils*. (pp. 50-70). London, UK: The Golden Blade.
- Harris, M. & Butterworth, G. (2002). *Developmental psychology: A student's handbook*. Hove, UK: Taylor & Francis Group.
- Hazledine, T. (1998). *Taking New Zealand seriously: The economics of decency*. Auckland, New Zealand: Harper Collins.
- Hogan, J. (2002). Mathematics and numeracy - is there a difference? In the *Australian Mathematics Teacher*, 58 (4), 14-16. Retrieved 21 Aug 2013
<http://web.ebscohost.com.ezproxy.aut.ac.nz>
- Howson, G. (1994). *Mathematics in the New Zealand curriculum*. Auckland, NZ: Education Forum.
- Hughes, P. (2003). A model for teaching numeracy strategies. *The New Zealand Mathematics Magazine*, 40(2), 1-10.
- Hughes, P. (2007). Revisions and extensions of a Pirie-Kieren-based teaching model. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, p. 918). Adelaide, Australia: MERGA.
- Hunter, J. (2006, June). The Numeracy project: Foundations and development. In *ACEpapers 17*, Paper 6. Retrieved from
<http://www.education.auckland.ac.nz/en/about/research/research-at-faculty/research-publications/ace-papers/ace-issue17.html>.
- Hunter, J. (2014). Developing learning environments which support early algebraic reasoning: a case from a NZ primary classroom. *Mathematics Education Research Journal* (on-line Jan, 2014). DOI: 10.1007/s13394-013-0093-4
- Irwin, K., & Britt, M. (2005). Algebraic thinking in the numeracy project: Year one of a three-year study. In *Findings from the New Zealand numeracy development projects 2004* (pp. 47-55). Wellington, New Zealand: Ministry of Education.
- Jarman, R. (1998). *Teaching mathematics in Rudolf Steiner schools*. (Vol. 1). Stroud, UK: Hawthorn Press.
- Johnston, M., Ward, J., & Thomas, G. (2009). The development of students' ability in strategy and knowledge. *Findings from the New Zealand Numeracy Development Projects 2009*, pp. 49-57. Retrieved from
http://www.nzmaths.co.nz/sites/default/files/Numeracy/References/Comp09/comp09_johnston_ward_thomas.pdf

- Jones, A., Marshall, J., Matthews, K. M., Smith, G. H. & Smith, L. T. (1995). *Myths and realities: Schooling in New Zealand* (2nd ed.). Palmerston North, New Zealand: Dunmore Press.
- Kahneman, D. (2011). *Thinking, fast and slow*. London, UK: Penguin Books.
- Kaplan, R., & Kaplan, E. (2003). *The art of the infinite: The pleasures of mathematics*. New York, NY: Bloomsbury Press.
- Kerr, R. (1991, October 24). Joining the unreal world: Education politics in New Zealand. Retrieved April 12, 2007 from <http://www.nzbr.org.nz/documents/speeches/speeches-91-92/>
- Kincheloe, J., & Berry, K. (2004). *Rigour and complexity in educational research: Conceptualizing the bricolage*. Maidenhead, UK: Open University Press, McGraw-Hill Education.
- Kincheloe, J., McLaren, P., & Steinberg, S. (2011). Critical pedagogy, and qualitative research: Moving to the bricolage. In N. Denzin & Y. Lincoln (Eds.). *The Sage handbook of qualitative research* (4th ed.), (pp. 163-177). Thousand Oaks, CA: Sage Publications, Inc.
- Kline, M. (1990). *Mathematical thought from ancient to modern times* (Vol. 1). New York, NY: Oxford University Press.
- Kuhn, T. (1996). *The structure of scientific revolutions*, (3rd ed.). Chicago, IL: University of Chicago Press.
- Lakatos, I. (1978). In J. Worrall & G. Currie (Eds.), *Mathematics, science and epistemology: Philosophical papers volume 2*. Cambridge, England: Cambridge University Press.
- Lakoff, G. & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books, Perseus.
- Lee, J. (1963). *Suggestions for Teaching Arithmetic in Infant Classes*. Wellington, New Zealand: New Zealand Department of Education.
- Lerman, S. (2001). A cultural discursive psychology for mathematics teaching and learning. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Socio-cultural research on mathematics education* (pp. 3-17). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Lievegoed, B. (1985). *Man on the threshold* (J. M. Cornelius, Trans.). Stroud, UK: Hawthorn Press (Original work published "Mens op de Drempel", 1983).
- Light, P., Sheldon, S. & Woodhead, M. (Eds.). (1991). *Learning to think* [Child Development in Social Context 2]. London, UK: Routledge, in association with The Open University.
- Lincoln, Y., Lynham, S., & Guba, E. (2011). Paradigmatic controversies, contradictions, and emerging confluences, revisited. In N. Denzin & Y. Lincoln (Eds.). *The Sage handbook of qualitative research* (4th ed.), (pp. 97-128). Thousand Oaks, CA: Sage Publications, Inc.
- Lucas, C. J. (1972). *Our western educational heritage*. New York, NY: The Macmillan Company.
- McCulloch, G. (Ed.). *The school curriculum in New Zealand: History, theory, policy and practice*. Palmerston North, New Zealand: Dunmore Press.
- McKenzie-McLean, J. (2011, July 27). School forced to adopt national standards. *The Press*. Retrieved September 26, 2014, from <http://www.stuff.co.nz/the-press/news/5343976/School-forced-to-adopt-national-standards>.

- McLaren, I. (1980). Curriculum making in New Zealand 1877-1962. In D. K. Ramsay (Ed.). *Curriculum issues in New Zealand* (pp. 19-31). Wellington, New Zealand: New Zealand Educational Institute.
- McLuhan, M. (1964/1994). *Understanding media: The extensions of man* (Ark paper ed). London, UK: Routledge.
- Maharey, S. (2007). *Letter from the office of Minister of Education*. (Inserted with *The New Zealand Curriculum*.) Wellington, New Zealand: Parliament Buildings.
- Mazzone, A. (2010). *A passionate schooling*. Australia: Aldino Mazzone & Susan Laing.
- Meaney, T. (2006). Acquiring the mathematics register in classrooms. set 3, pp. 39-43.
- Ministry of Education. (1991). *The national curriculum of New Zealand: A discussion document*. Wellington, New Zealand: Learning Media Ltd.
- Ministry of Education. (1992). *Mathematics in the New Zealand curriculum*. Wellington, New Zealand: Learning Media Ltd.
- Ministry of Education. (1993). *The New Zealand curriculum framework*. Wellington, New Zealand: Learning Media Ltd.
- Ministry of Education. (2007). *The New Zealand curriculum*. Wellington, New Zealand: Learning Media Ltd.
- Ministry of Education (2007a-2012) Numeracy Professional Development Projects:
Book 1: The number framework. (2011)
Book 2: The diagnostic interview. (2008)
Book 3: Getting started. (2008a)
Book 4: Teaching number knowledge. (2008b)
Book 5: Teaching addition, subtraction and place value. (2012)
Book 6: Teaching multiplication and division. (2008c)
Book 7: Teaching fractions, decimals and percentages. (2008d)
Book 8: Teaching number sense and algebraic thinking. (2008e)
Book 9: Teaching number through measurement, geometry, algebra and statistics. (2007a)
- Ministry of Education. (2009). *The New Zealand Curriculum: Mathematics standards for years 1 - 8*. Wellington, New Zealand: Learning Media Ltd.
- Ministry of Education. (2010). *Designing the mathematics standards for years 1-8*. Retrieved from <http://nzcurriculum.tki.org.nz/National-Standards/Key-information/Background-papers>
- Ministry of Education. (n.d.). *National Standards*. Retrieved August 21, 2013, from <http://www.minedu.govt.nz/theMinistry/EducationInitiatives/NationalStandards.aspx>
- Mitchell, D. (1999). *Governmentality, power and rule in modern society*. London, England: SAGE Publications.
- Moore, T. (2013). *Critical thinking and language: The challenge of generic skills and disciplinary discourse*. London, UK: Bloomsbury Academic.
- Munro, A. (1995). Rudolf Steiner schools in New Zealand and integration with the state system. *Anthroposophy at Work*, 2, 2-8.
- Munro, A. (1998). The work of the Federation of Rudolf Steiner Waldorf schools in New Zealand. *Anthroposophy at Work*, 5, 9-11.
- Mutch, C. (2001). Contesting forces: The political and economic context of curriculum development in New Zealand. *Asia Pacific Education Review*, 2(1) 74-84.

- Nagel, T. (2012). *Mind and cosmos: Why the materialist neo-Darwinian conception of nature is almost certainly false*. Oxford, UK: Oxford University Press.
- Neill, A. and Maguire, T. (2007). Algebraic thinking - our journey to a concept map. Presentation at NZAMT conference, Auckland, 25-28 September. Wellington, New Zealand: NZCER.
- New South Wales Department of Education and Training (1999). *Count Me In Too professional development package*. Ryde, Australia: NSW DET.
- New Zealand Council for Educational Research, (2006). Progressive Achievement Test (PAT): Mathematics (tests 2,3,4 & 6A). Wellington, New Zealand: NZCER Press
- New Zealand Council for Educational Research, (2009). Progressive Achievement Test (PAT): Mathematics Teacher Manual. Wellington, New Zealand: NZCER Press.
- Neyland, J. (Ed.). (1994). *Mathematics education: A handbook for teachers* (Vol. 1). Wellington, NZ: Wellington College of Education.
- Neyland, J. (1995). Neo-behaviourism and social constructivism in mathematics education. *SAMEpapers* 1995, 114-143. University of Waikato.
- Neyland, J. (2002). Rethinking curriculum: An ethical perspective. In B. Barton, K. C. Irwin, M. Pfannkuch & M.O. J. Thomas (Eds.), *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Group of Australasia, Auckland, pp. 512-519). Sydney, Australia: MERGA.
- Neyland, J. (2004). Towards a postmodern ethics of mathematics education. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 55-73). Connecticut: Information Age Publishing.
- Nicholas, P. (2006, June). Improving children's performance in numeracy: Two countries, two approaches. In *ACEpapers* 17, Paper 7. Retrieved from <http://www.education.auckland.ac.nz/en/about/research/research-at-faculty/research-publications/ace-papers/ace-issue17.html>
- Oberski, I. (2006). Learning to think in Steiner-Waldorf schools. *JCEP - Journal of Cognitive Education and Psychology*, 5(3), 336-349.
- Olssen, M., & Morris Matthews, K. (Eds.), (1997). *Education policy in New Zealand: The 1990s and beyond*. Palmerston North, NZ: Dunmore Press Ltd.
- O'Neill, A-M. (with J. Clark and R. Openshaw), (2004). Mapping the field. In *Reshaping culture, knowledge and learning: Policy and content in the New Zealand curriculum framework*. (pp. 25-46). Palmerston North, New Zealand: Dunmore Press.
- Openshaw, R. (1992). Subject construction and reconstruction: Social studies and the new mathematics. In G. McCulloch (Ed.), *The school curriculum in New Zealand: History, theory, policy and practice* (pp. 201-218). Palmerston North, New Zealand: Dunmore Press.
- Openshaw, R., & Walshaw, M. (2010). *Are our standards slipping? Debates over literacy and numeracy standards in New Zealand since 1945*. Wellington, NZ: NZCER Press.
- Palmer, R. (1969). *Hermeneutics: Interpretation theory in Schleiermacher, Dilthey, Heidegger, and Gadamer*. Evanston, IL: Northwestern University Press.
- Piaget, J. (1991). Science of education and the psychology of the child. In P. Light, S. Sheldon & M. Woodhead (Eds.), *Learning to think* [Child Development in Social Context 2], pp. 5-15. London, UK: Routledge, in association with The Open University. (Original work published in 1969, Harlow, UK: Longman).

- Piper, A. (2012). *Book was there*. Chicago, IL: The University of Chicago Press.
- Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics*, 26(2-3), 165-190.
- Plato (1974). *The republic* (G. M. A. Grube, Trans.). London, UK: Pan Classics.
- Radford, L. (2014). The progressive development of early embodied algebraic thinking. *Mathematics Education Research Journal*, 26(2), pp. 257-277. DOI: 10.1007/s13394-013-0087-2
- Ramachandran, V. & Blakeslee, S. (1998). *Phantoms in the brain: Probing the mysteries of the human mind*. New York, NY: William Morrow and Co.
- Ritchie, G., & Carr, K. (1997). Back with a future: Vygotsky, constructivism, mathematics education. In *SAMEpapers*, (pp. 115-143). Hamilton, New Zealand: University of Waikato.
- Rousselle, L., & Noël, M.-P. (2007). Basic numerical skills in children with mathematics learning disabilities: A comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, 102(3), 361-395.
- Schieren, J. (2012, July). The concept of learning in Waldorf education. *RoSE - Research on Steiner Education*, 3(1), 63-74. Hosted at www.rosejournal.com.
- Schieren, J. (2014, Spring/Summer). The spiritual dimension of Waldorf education. *Research Bulletin*, 19(1), 19-29.
- Schoenfeld, A. (2002). Research methods in (mathematics) education. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 435-487). Mahwah, NJ: Lawrence Erlbaum Associates.
- Scouller, D. (2009). Has strategy become the new algorithm? *The New Zealand Mathematics Magazine*, 46(3), 1-11.
- Sewell, K. (2007). Foreword. In *The New Zealand Curriculum*, p. 4. Wellington, New Zealand: Learning Media Ltd.
- Smith, P. C., (Trans.), (1986). In *The idea of the good in Platonic-Aristotelian philosophy*. pp. vii-xxxi. New Haven, CT: Yale University Press.
- Smith, P. M. (2005). *A concise history of New Zealand*. Melbourne, Australia: Cambridge University Press.
- Standing, G. (2011). *The precariat: The new dangerous class*. London, UK: Bloomsbury Academic.
- Steffe, L., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory and application*. New York, NY: Praeger.
- Steiner, R. (1886/1978). *A theory of knowledge implicit in Goethe's world conception* (O. D. Wannamaker, Trans., 2nd ed.). Spring Valley, NY: The Anthroposophic Press. (Original work published "Grundlinien einer Erkenntnistheorie der Goetheschen Weltanschauung", 1886 - GA2)⁹⁶.
- Steiner, R. (1892). Truth and knowledge. Introduction to the philosophy of freedom (GA3). Retrieved from http://wn.rsarchive.org/Books/GA003/English/RSPI1963/GA003_index.html
- Steiner, R. (1894/1979). *The philosophy of freedom* (M. Wilson, Trans.). London, UK: Rudolf Steiner Press, Pharos Books. (Original work published "Die Philosophie der Freiheit", 1894 - GA4).

⁹⁶ GA for Gesamtausgabe, the complete catalogue of Steiner's works in German.

- Steiner, R. (1904). *Mathematics and occultism* (Lecture given in Amsterdam, The Netherlands, June 21). Retrieved from <http://wn.rsarchive.org/Lectures/19040621p01.html>
- Steiner, R. (1904/1994). *Theosophy* (C. Creeger, Trans., 1922, 10th ed.). Hudson, NY: Anthroposophic Press. (Original work published "Theosophie, Einfuehrung in uebersinnliche Welterkenntnis und Menschenbestimmung" - GA35)
- Steiner, R. (1907/2008). *Educating children today* (M. Barton, Trans., Sophia Books ed.). Forest Row, UK: Rudolf Steiner Press. (Original work published as essay in journal "Luzifer-Gnosis", 1907 - GA34).
- Steiner, R. (1910/1979). *Occult Science: An outline* (G. & M. Adams, Trans.). London, UK: Pharos Books, Rudolf Steiner Press. (Original work published "Geheimwissenschaft im Umriss", 1910 - GA13).
- Steiner, R. (1914/2009). *The riddles of philosophy*. Great Barrington, MA: SteinerBooks. (Original work published "Die Raetsel der Philosophie", 1914 - GA18).
- Steiner, R. (1917/1999). *Riddles of the soul* (W. Lindeman, Trans.). Spring Valley, NY: Mercury Press. (Original work published "Von Seelenraetseln", 1917 - GA21).
- Steiner, R. (1919/1976). *Practical advice to teachers* (J. Collis, Trans.). London, UK: Rudolf Steiner Press. (Original work published "Erziehungskunst. Methodisch-Didaktisches" - Fourteen lectures given at the foundation of the Waldorf School, Stuttgart, Germany, from August 21 to September 5, 1919 - GA294).
- Steiner, R. (1919/1981). *Study of man* (A. C. Harwood, D. Harwood, & H. Fox, Trans.). London, UK: Rudolf Steiner Press. (Original work published "Allgemeine Menschenkunde als Grundlage de Paedagogik" - Fourteen lectures given in Stuttgart, Germany, August 21 to September 5, 1919 - GA293).
- Steiner, R. (1919/1985). *An introduction to Waldorf education* (E. Bowen-Wedgewood, Trans. Revised F. Amrine). (GA24). Retrieved from http://wn.rsarchive.org/Books/GA024/IntWal_index.html
- Steiner, R. (1919/1994). *A social basis for education*. Forest Row, UK: Steiner Schools Fellowship Publications. (Original work published "Drei Vortraege ueber Volkspaedagogik" - Three lectures given in Stuttgart, Germany, 11 & 18 May, 1 June, 1919 - GA192).
- Steiner, R. (1920/1972). *The threefold social order* (F. C. Heckel, Trans., 2nd ed.). Spring Valley, NY: Anthroposophic Press. (Original work published "Die Kernpunkte der sozialen Frage", 1920 - GA23).
- Steiner, R. (1920/1983). *The boundaries of natural science* (F. Amrine & K. Oberhuber, Trans.). Spring Valley, NY: Anthroposophic Press. (Original work published "Grenzen der Naturerkenntnis" - Eight lectures given in Dornach, Switzerland, September 27 to October 3, 1920 - GA322).
- Steiner, R. (1920/1989). *The renewal of education* (R. Everett, Trans.). Forest Row, UK: Steiner Schools Fellowship Publications. (Original work published "Die Erneuerung der Paedagogisch - Didaktischen Kunst durch Geisteswissenschaft" - Fourteen lectures given in Basle, Switzerland, April 20 to May 11, 1920 - GA301).
- Steiner, R. (1921). *Paths to knowledge of higher worlds - lecture given in Christiania (Oslo), Norway, November 26, 1921*. Retrieved from <http://wn.rsarchive.org/GA/GA0079/19211126p01.html> (Original work published with a series of lectures in "Die Wirklichkeit der Hoeheren Welten" - GA79).

- Steiner, R. (1922/1947). *Spiritual ground of education* (D. Harwood, Trans.). London, UK: Anthroposophical Publishing Company. (Original work published "Die Geistige-Seelische Grundkraefte der Erziehungskunst Spirituelle Werte in Erziehung und sozialem Leben" - Nine lectures given in Oxford, England, August 16 to 25, 1922 - GA305).
- Steiner, R. (1923/1981). *A modern art of education* (J. Darrell & G. Adams, Trans.). London, UK: Rudolf Steiner Press. (Original work published "Gegenwaertiges Geistesleben und Erziehung" - Fourteen lectures given in Ilkley, England, August 5 to 17, 1923 - GA307)
- Steiner, R. (1923/1985). *The origins of natural science* (M. S. Goar, Trans.). Spring Valley, NY: Anthroposophic Press. (Original work published "Der Entstehungsmoment der Naturwissenschaft in der Weltgeschichte und ihre seitherige Entwicklung" - Nine lectures given in Dornach, Switzerland, December 24, 1922 to January 6, 1923 - GA326).
- Steiner, R. (1924/1971). *Human values in education* (V. Compton-Burnett, Trans.). London, UK: Rudolf Steiner Press. (Original work published "Der paedagogische Wert der Menschenkenntnis und der Kulturwert der Paedagogik" - Ten lectures given in Arnheim, Holland, July 17-24, 1924 - GA310).
- Steiner, R. (1924/1968). *The roots of education* (H. Fox, Trans.). London, UK: Rudolf Steiner Press. (Original work published "Anthroposophische Paedagogik und ihre Voraussetzungen" - Five lectures given in Berne, Switzerland, April 13 to 17, 1924 - GA309).
- Steiner, R. (1924/1988). *Kingdom of childhood* (H. Fox, Trans., 2nd ed.). Hudson, NY: Anthroposophic Press. (Original work published "Die Kunst des Erziehens aus dem Erfassen der Menschenwesenheit" - Seven lectures given in Torquay, England, August 12 to 20, 1924 - GA311).
- Steiner, R. (1928). *The story of my life*. (GA28). Retrieved from http://wn.rsarchive.org/Books/GA028/English/APC1928/GA028_index.html
- Stockmeyer, E. A. K. (1982). *Rudolf Steiner's curriculum for Waldorf schools* (R. Everett-Zade, Trans.). Forest Row, UK: Steiner Schools Fellowship. (Reprinted from 1969).
- Sullivan, K. (1994). The impact of educational reform on teachers' professional ideologies. *New Zealand Journal of Educational Studies*, 29(1), 3-20.
- Sullivan, K. (1998). The great New Zealand education experiment and the issue of teachers as professionals. In K. Sullivan (Ed.), *Education and change in the Pacific Rim: Meeting the challenges* (pp. 137-159). Wallingford, England: Triangle Books.
- Sweetman, R. (2002). *'A fair and just solution?'*. Palmerston North, New Zealand: Dunmore Press, in association with the Ministry for Culture and Heritage and the Association of Proprietors of Integrated Schools.
- Taskforce to Review Educational Administration (1988). *Administering for excellence: Effective administration in education (The Picot Report)*. Wellington, New Zealand: Government Printer.
- Tarnas, R. (1991). *The passion of the western mind* (Pimlico, 1996 ed.). London, UK: Random House.

- The Treasury. (1984). *Economic management: Briefing to the incoming government 1984*. Retrieved from <http://www.treasury.govt.nz/publications/briefings/1984i>
- Thomas, G., & Ward, J. (2001). *An evaluation of the Count Me In Too pilot project*. Wellington, New Zealand: Ministry of Education.
- Thomas, G., Tagg, A. & Ward, J. (2002). Making a Difference: The early numeracy project. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland, pp. 49-57). Sydney, Australia: MERGA.
- Thomas, G., Ward, J., & Tagg, A. (2009). The performance of the longitudinal study students in year 8. *Findings from the New Zealand Numeracy Development Projects 2009*, pp. 3-14. Retrieved from http://www.nzmaths.co.nz/sites/default/files/Numeracy/References/Comp09/comp09_thomas_ward_tagg.pdf
- Thrupp, M. (2014). At the eye of the storm: Researching schools and their communities enacting National Standards. *New Zealand Journal of Educational Studies*, 49(1) 6-20.
- Thrupp, M., & White, M. (2013). *Research, analysis and insight into national standards (RAINS) project. Final report: National standards and the damage done*. Report commissioned by The New Zealand Educational Institute Te Riu Roa (NZEI). Retrieved from <http://www.waikato.ac.nz/wmier/research/projects/the-rains-research-on-national-standards>
- Tolley, A. (2009). *National standards information for schools*. Wellington, New Zealand: Ministry of Education.
- Uhrmacher, P. (1995). Uncommon schooling: A historical look at Rudolf Steiner, anthroposophy, and Waldorf education. *Curriculum Inquiry*, 25(4), 381-406. Retrieved from <http://www.jstor.org/stable/1180016>
- Vale, C. (2013). Primary teachers' algebraic thinking: Example from lesson study. In V. Steinle, L. Ball & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, Australia: MERGA.
- von Glasersfeld, E. (1989). Cognition, construction of knowledge and teaching. *Synthese* 80(1), 121-140. Retrieved June 12, 2014 from <http://www.jstor.org.ezproxy.aut.ac.nz/stable/20116670>
- von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C. A. Maher & N. Noddings (Eds.), *Monographs of the Journal for Research in Mathematics Education*, #4, 19-29. Reston, VA: National Council of Teachers in Mathematics. Retrieved September 21, 2008 from <http://www.univie.ac.at/constructivism/EvG/>
- Vygotsky, L. (1966/1991). Genesis of the higher mental functions. In P. Light, S. Sheldon & M. Woodhead (Eds.), *Learning to think* [Child Development in Social Context 2], pp. 32-41. London, UK: Routledge, in association with The Open University. (Original work published in 1966, Moscow, USSR: Progress Publishers.

- Walls, F. (2004). The New Zealand Numeracy projects: Redefining mathematics for the 21st century? *The New Zealand Mathematics Magazine*, 41(2), 21-43.
- Walshaw, M. (2008). Knowledge construction in mathematics education. Editorial for *Mathematics Education Research Journal*, 20(1), 1-2. DOI: 10.1007/BF03217466
- Ward, J., Thomas, G., & Tagg, A. (2006). Numeracy sustainability: Current initiatives and future professional development needs. *Findings from the New Zealand Numeracy Development Projects 2006*, pp. 87-98. Retrieved from http://www.nzmaths.co.nz/sites/default/files/Numeracy/References/Comp06/comp06_ward_thomas_tagg.pdf
- Wright, M. (1996). *Havelock North: The history of a village*. Hastings, New Zealand: Hastings District Council.
- Young-Loveridge, J. (1989). The development of children's number concepts: The first year of school. *New Zealand Journal of Educational Studies*, 24(1), 47-64.
- Young-Loveridge, J. (2006). Enhancing the mathematics achievement of Pasifika students: Performance and progress on the Numeracy Development Project. *Waikato Journal of Education*, 12, 101-116.
- Young-Loveridge, J. (2008). Analysis of 2007 data from the Numeracy Development Projects: What does the picture show? *Findings From the New Zealand Numeracy Development Projects 2007*, pp. 18-28. Retrieved from http://www.nzmaths.co.nz/sites/default/files/Numeracy/References/Comp07/comp07_young-loveridge.pdf
- Young-Loveridge, J. (2009). A decade of reform in mathematics education: Results for 2009 and earlier years. *Findings From the New Zealand Numeracy Development Projects 2009*, pp. 15-35. Retrieved from http://www.nzmaths.co.nz/sites/default/files/Numeracy/References/Comp09/comp09_young-loveridge.pdf
- Young-Loveridge, J. (2011). Assessing the mathematical thinking of young children in New Zealand: The initial school years. *Early Child Development and Care*, 181(2), 267-276. DOI: 10.1080/03004430.2011.536645
- Zajonc, A. (2009). *Meditation as contemplative enquiry: When knowing becomes love*. Great Barrington, MA: Lindisfarne Books.

GLOSSARY

anthroposophy

the name given by Steiner to his 'science of the spirit' where *anthropos* refers to 'human' and *soph(y)ia* to 'wisdom'; an approach that uses 'scientific method' to examine the activity and effects of the sense-free (spiritual) domain in relation to the human being.

astral

according to Steiner's use, this is an aspect or area of activity within the human being that enables the soul to experience consciousness, sensation and feelings.

body, soul, spirit

a threefold description of the human, where 'body' refers to the physical corporeality, the 'soul' refers to the faculties of thinking, sentient feeling and will-doing, and 'spirit' refers to an enduring, eternal and invisible essence of the individuality that is the particular human being.

Ego

according to Steiner's use, an aspect of the human being that refers to the unique individuality of that person; the capacity to name oneself 'I' as a separate entity to 'Other'.

etheric

a living, rhythmical capacity for a living body (plant, animal, human) to grow and reproduce, to excrete and secrete, to ingest and digest and to move.

Eurythmy

a form of artistic movement expressing visibly the sounds of speech and musical tones. Rudolf Steiner and Marie von Sivers evolved this movement form with the help of their initial students, from the end of the first decade of the twentieth century. Eurythmy is now used pedagogically and therapeutically, as well as being a performance art.

Goetheanum

the building situated in Dornach, Switzerland, that is a performance and conference centre, as well as the administrative centre, for the world-wide General Anthroposophical Society. The first Goetheanum was built by volunteer international labour during the First World War, and consisted of two interlocking wooden cupolas. It was destroyed by arson on New Year's Eve, December 31, 1922.

The second, present building is formed of concrete and was completed in the late 1920s. Both buildings were designed by Rudolf Steiner and are considered among the 1000 selected classics of world architecture by www.greatbuildings.com.

limb-learning

a form of learning described by Rudolf Steiner that involves activity in the limbs; today we might refer to such learning as 'kinaesthetic learning', but Steiner's use also includes a way of encouraging learning through the forces of the (unconscious) will activity.

limb-metabolic system

that system within the body that includes all limbs and the metabolic areas. Steiner suggested that this aggregated 'system' of the body expresses and is most easily activated by the unconscious will forces of the living body.

Naturphilosophie

a German word used in English to refer to a particular philosophical approach by the German 'Idealists' Fichte, Schelling and Hegel, who were part of the German Romantic movement of the late 18th and early 19th centuries. The philosophers considered nature in its totality; an attempt to be the philosophical foundation for natural science.

Numeracy Development Projects (NDP)

these projects were developed by the Ministry of Education in New Zealand, based on a New South Wales project called "Count Me In Too" (CMIT). CMIT was trialled in New Zealand schools in 1999/2000, and the NDP proper began in 2001. Using a social constructive strategy for knowledge acquisition and counting-types' knowledge in a recursive step-wise fashion, the projects cover years 1 to 10 of education in numeracy/ mathematics in New Zealand schools.

rhythmical system

a 'mediating' system within the body that includes the breathing and circulatory systems, centred but not exclusive to the area of the body encompassed by the rib cage. Steiner suggested that this 'system' is most closely associated with activity in the realm of feelings.

sense-nerve system

a reference to that physiological domain centred in the head but distributed throughout the body. Steiner suggests that this system is most involved in the activity of thinking and consciousness.

spiritual

that insubstantial eternal essence of everything that includes the 'archetype' (Goethe) and 'Idea' (Plato), and refers to those qualities of beingness which may not be perceived in this physical, material sense-world.

Spiritual Science

Steiner's reference to the research approach of using scientific method in exploring the activity and effect of the spiritual. An alternative name sometimes used by Steiner in place of 'anthroposophy'.

Steiner Waldorf schools

those schools (over 1000 world-wide) using Steiner's indications for education; the first begun in September 1919 in Stuttgart, Germany, and called *Waldorfschule*. In New Zealand, there are eleven schools using Steiner's indications and which may call themselves 'Steiner schools', 'Waldorf schools' or Steiner Waldorf schools. In this dissertation, I will always refer to the compound 'Steiner Waldorf' in referring to these schools.

threefold

a term Steiner uses in reference to three qualities; in the body (sense-nerve, rhythmical, limb-metabolic), in the soul (thinking, feeling, will), in the realm of consciousness (waking, dreaming, sleeping), where there are two 'opposite poles' and a central dynamic mediating region.

Waldorf-Astoria school

the first school to work with Steiner's indications for the formal education of the child; based in Stuttgart and enabled by Emil Molt the owner-manager of the Waldorf-Astoria factory. At a time when unemployment was reaching epidemic proportions after the First World War, Molt asked Steiner to initially begin some adult education with the factory employees, and consequently began a school for the children of these employees, as well as other local children. The school opened to 256 students over 8 classes in September, 1919; 150 were children of the workers at the Waldorf-Astoria factory. By 1928 there were over 1000 students in 28 classes covering the 12-year curriculum (Easton, 1980, p. 262).

APPENDIX

Overview of Steiner's Indications for Mathematics Education as compared to Steiner's picture of the development of consciousness

Sumerian / Chaldean / Egyptian sentient soul consciousness
<ul style="list-style-type: none"> - ancient documents suggest deduction prior to induction - counting & computations in arithmetic and algebra - geometry of area & volume; use of 'Pythagorean' theorem - multiplication by repeated doubling (Egyptian) - practical application of algebra & geometry
Class I - Year 2 <ul style="list-style-type: none"> - all 4 rules from 'whole to part'; analysis to synthesis - counting to 100; use fingers at beginning - begin multiplication tables' learning, 'by heart' Class II - Year 3 <ul style="list-style-type: none"> - all 4 rules with greater range of numbers - solve simple problems (algorithms) mentally - reckon with 'concrete' numbers and develop 'abstract' numbers in connection with objects Class III - Year 4 <ul style="list-style-type: none"> - use more complicated numbers & apply to simple things in practical life - introduce measurement through 'house building' main lesson and time
Graeco-Roman including Mediæval intellectual-mind soul consciousness
<ul style="list-style-type: none"> - prior epoch imported and 'mixed' with philosophical speculations - mathematics for philosophical and political reasons (Plato) - mathematics bridge between world of sense and world of forms - (Euclid) systematic definitions and axioms; general theorems and proofs
Class IV - Year 5 <ul style="list-style-type: none"> - begin work with fractions, especially decimal - describe process out of artistic and practical exercises, using 'feeling for' - Euclidian-style proofs not attempted until Class VI Class V - Year 6 <ul style="list-style-type: none"> - continue with fractions and decimals, so that pupil 'moves freely' in reckoning numbers, whether whole, fractional or decimal Class VI - Year 7 <ul style="list-style-type: none"> - calculate interest, percentage, discount and simple exchange leading into algebra - study relations between calculation and the circulation of commodities and finance - introduce Euclidian proofs by induction/deduction

Aristotle practised a two-step process in his writings on scientific proof. Induction was the first step and followed that which was known, 'facts'; either an 'observation' by the senses, or an 'intuition' through thought. Aristotle processed from the 'known' particular to a generalised or universal antecedent. Deduction from this more universal *Idea (Form)* back to the perceived 'facts' demonstrated the explanation of them and completed the second step. While induction was more readily used in science, deduction was more common in mathematics.

Euclid (c.325-c.265BCE) gathered together empirical geometric constructions and number theories with deductive proofs in "Elements", a text that shaped the study of geometry and mathematics for over nineteen hundred years. This deductive logic was extended beyond mathematics to all forms of reasoning in later centuries (Bortoft, 2012), sitting uncomfortably on the shoulders of the social and organic worlds, as well as the inorganic for which it was best suited

Renaissance to 16th Century consciousness soul
<ul style="list-style-type: none"> - Indo-Arabic impulse imported into Europe; more numeric and algebraic - calculus 'anticipated' from Indian mathematics - Chinese introduced higher equations & advanced geometry - Islamic algebra, Number theory and geometry
Class VII - Year 8 <ul style="list-style-type: none"> - raise numbers to powers; obtain roots - reckon with positive and negative numbers - theory of equations in connection to practical life Class VIII - Year 9 relate theory of equations to: <ul style="list-style-type: none"> - calculations of length and areas of figures & surfaces - theory of loci Class IX - Year 10 <ul style="list-style-type: none"> - advanced calculations as per systems of exchange - logarithms and plane trigonometry - descriptive geometry incl. π

These Tables for Classes I to VI and VII to IX are derived from consideration of the following:
Stockmeyer, E. A. K. (1982). *Rudolf Steiner's Curriculum for Waldorf Schools*. (Trans. R. Everett-Zade), Forest Row, UK: Steiner Schools Fellowship;
Kline, M. (1990). *Mathematical Thought from Ancient to Modern Times* (Vol. 1). New York, NY: Oxford University Press;
Gрегersen, E. (Ed.). (2011). *The Britannica guide to the history of mathematics*. New York, NY. Retrieved from <http://www.aut.ebilib.com.au>